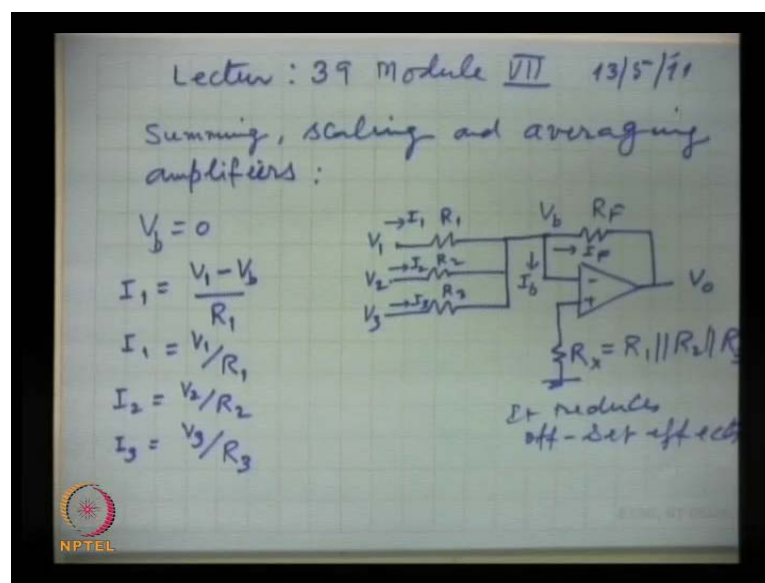


**Electronics**  
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**Module No. 07**  
**Differential and Operational Amplifiers**  
**Lecture No. 39**  
**Summing, Scaling and Averaging Amplifiers**

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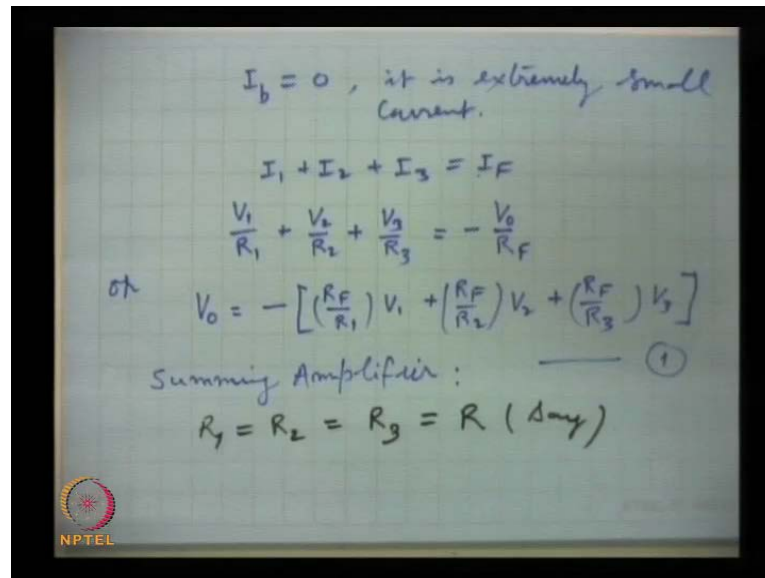
We were discussing the method, mathematical operations through amplifiers, and op amps are used in this process, and in fact that is the reason, that is why these are called operation amplifiers. So, the mathematical operation, which was currently under discussion, is summing amplifier is scaling amplifier and an averaging amplifier. These are three different amplifiers, but because they all the three, can be drawn from the basic amplifier. So, that is why I am taking them all under single heading. Now, these amplifiers can be realized by using a inverting amplifier, inverting op amp or by non inverting amplifier equally well. We are taking just one case that is of inverting amplifier, so how to realize a summing amplifier with inverting configuration. The summing amplifier is the one, in which the output is the sum of all the inputs. I repeat, that summing amplifier in is the one which gives a output which is the sum of all the inputs. This is summing amplifier. Now, the inputs can be more than one, so may be two, three, four, five, six or it can be number eight or ten, but we just to make the diagram

simpler. We take three inputs and when we use an inverting configuration; then this circuit is this.  $v_1$ ,  $v_2$ ,  $v_3$  are the three voltages, which are to be summed up and output is here, and this non-inverting input.

Normally, so far we have been putting it at a ground potential, by directly grounding it, but for a better arrangement is, when this resistance  $R_x$  equivalent to the parallel combination of these resistances is attached. It is still at ground potential, but not directly, but through a resistance and this reduces the offset effects; we have not yet talked about the offset effects. We will talk a little later about these things, but they arise, because of the symmetry in the design of the circuit. And even when the inputs are absent some currents still flow. They are of course, very mild, very small currents, but still, so those offset effects can be reduced drastically, if instead of directly connecting the non-inverting input to ground. It is connected through a resistance equal to the parallel combination of all the input resistances. Now here this is at a potential  $v_b$ , and the three currents  $i_1$ ,  $i_2$  and  $i_3$  they flow here, and from here will flow  $i_f$ .

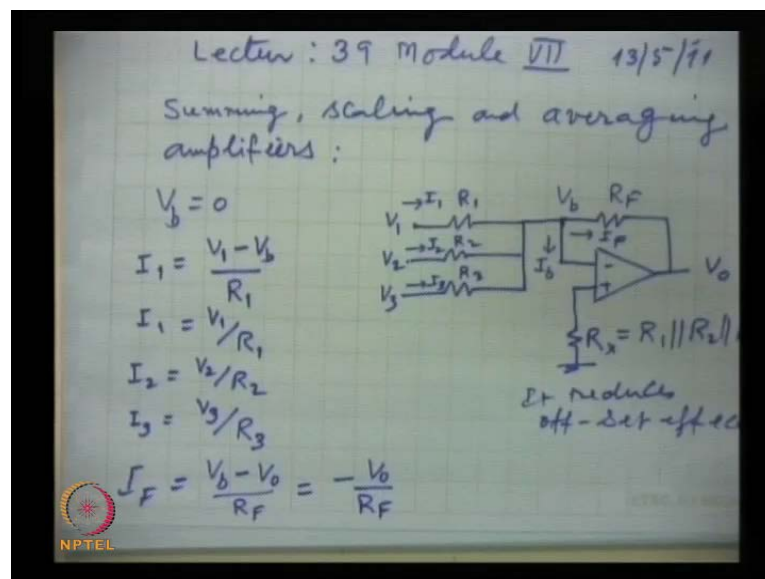
Now, remembering that  $v_b$  is at zero potential, remember we have said that the inverting input is virtual ground. So, this is at ground potential. Hence  $v_b$  is zero, if this is so, this voltage is zero. Here it is  $v_1$ , then we can simply write the expression for the current  $i_1$ , this is actually speaking  $v_1 - v_b$  by  $R_1$ , but because  $v_b$  is zero. So,  $i_1$  is simply  $v_1$  by  $R_1$ . Similarly, we can write for  $i_2$  and  $i_3$ . So,  $i_2$  is  $v_2$  by  $R_2$ ,  $i_3$  is  $v_3$  by  $R_3$ . These are the currents and now we are seeing that this is simplified. And the whole expression gets simplified by this concept of virtual ground, and that is why emphasis was given to this point, that the inverted input is a virtual ground. Now, these three currents are flowing, there will be a current here  $I_B$ , but  $I_B$  as we are seeing that, because the input impedance of the amplifier is extremely high. So, this  $I_B$  can be taken as zero.

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$I_B$  is equal to zero, we neglect this current. It is extremely small current it is extremely is small current, if the neglect this current  $I_B$  is equal to zero, then; obviously, the summation of these currents is equal to  $I_F$ . Therefore,  $I_1$  plus  $I_2$  plus  $I_3$  this is equal to  $I_F$ . And we can write for  $I_F$ , all the three currents we have written. We can also write for  $I_F$  the current we can write here.

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This three currents we write here. Now, what will be  $I_F$ . This is  $v_B$  minus this terminal is at potential  $v_b$ , which is zero, we will write, finally zero minus  $v_0$  by  $R_F$  and this is

equal to minus  $v_0$  by  $R_F$ . So, substituting the values of all the currents in this expression, we get  $v_1$  by  $R_1$ , which is equal to  $I_1$  plus  $v_2$  by  $R_2$  plus  $v_3$  by  $R_3$  and this is equal to minus  $v_0 R_F$ , or the output we can write from here, or  $v_0$  is equal to minus  $R_F R_1$  by  $v_1$  plus  $R_F R_2$   $v_2$  plus  $R_F R_3$   $v_3$ . This is the relation which is we may, say the basic relation for this circuit, and then by choosing. You recall that  $R_F$  by  $R_1$  is the magnitude of the gain by which  $v_1$  will be amplified. Similarly,  $R_F$  by  $R_2$  is the gain by which  $v_2$  will be amplified, and  $f$  appears at the output and so on. So for now, from this basic equation which we call as first equation, we can now go for summing amplifier. Summing amplifier in this equation one, if we choose  $R_2$  equal to  $R_3$  equal to  $R_1$ , say this is  $R$ .

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$$V_o = -\left(\frac{R_F}{R}\right)(V_1 + V_2 + V_3)$$

Further,  $R_F = R,$   
 Magnitude Gain = 1.

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$$V_o = -(V_1 + V_2 + V_3)$$


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Then what we have is this,  $v_0$  is equal to minus  $R_F$  by  $R$  and  $v_1$  plus  $v_2$  plus  $v_3$ . If further we choose  $R_F$  this gain as one. So,  $R_F$  further, if  $R_F$  is taken equal to  $R$ , then magnitude of gain is equal to one, and in that case this term will be one, and  $v_0$  is simply minus  $v_1$  plus  $v_2$  plus  $v_3$ ; that means, the output voltage for this summing amplifier under these conditions of choosing  $R_1$  equal to  $R_2$  equal to  $R_3$ , and let us say that is  $R$ . And  $R_F$  also equal to have that is alright this resistance is in this circuit are taken as single value resistance. So, all these resistance is  $R_F$   $r_1$   $r_2$   $r_3$  are taken as a single value resistors say 10 k, 20 k and so on. Then the output will be, the sum of all the input voltages with a reverse sign, with a negative sign; that is sign is not really material, if you want positive we can use the sign changer circuit, after that which we have already

discussed, and because this minus sign is coming, because we have taken an inverting amplifier. If we take a non-inverting amplifier, this sign automatically will disappear, but as I said it is not a significant point. So, this is the summing circuit.

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Scaling of weighted Amplifier: -

$$V_o = 4V_1 + 0.5V_2 + 1.5V_3$$

Let  $R_F = 12\text{ k}\Omega$

$$R_1 = 3\text{ k}\Omega,$$
$$R_2 = 24\text{ k}\Omega$$
$$R_3 = 8\text{ k}\Omega$$
$$V_o = - [4V_1 + 0.5V_2 + 1.5V_3]$$

Then we can go for scaling, scaling or weighted amplifier, in the simple summing amplifier, we have seen that all the inputs are amplified by the same amount, choosing all the resistance as a single value as in  $R_F$  also of  $R$ . Then simple summing amplifier results in scaling. A weighted amplifier is the one, in which the inputs, each input voltage is amplified differently. I repeat that in a scaling, a scaling amplifier is also called a weighted amplifier, in which the amplification factor for individual input voltage is weighted, is not the same, but they are different, they are weighted. And so then the amplifier is called a scaling or weighted amplifier. Such amplifiers are needed. For example, if we want that the output should be when  $v_1$ ,  $v_2$ ,  $v_3$  are the input voltages, then  $v_o$  the output should meet this requirement; that means, this equation should be satisfied, this is the equation.

The output should be according to this equation, where  $v_1$ ,  $v_2$ ,  $v_3$  are the inputs. Now here same circuit and same fundamental equation, that circuit is the same, but here the values of  $R_1$ ,  $R_2$ ,  $R_3$  and  $R_F$  we have chosen differently, and we will take help of this basic equation, which we have written for that amplifier. Now, in this case if we choose a particular value of  $R_F$ . For example, let  $R_F$  the feedback resistor is of the value of

twelve kilo ohms. We can choose 24, but just as an example I am taking 12. Then we should choose  $R_1$  is 3 k, 3 kilo ohms. So, that this will be amplified by  $R_F$  by  $R_1$ , look here  $R_F$  by  $R_1$  into  $v_1$ . Here we want affecter of four, according to the requirement. And if we choose  $R_F$  at 12 kilo ohms, then  $R_1$  necessarily we will have to take at 3, so that if factor of 4 comes.

And  $R_2$ , we are suppose to take as 24 kilo ohms, because  $R_F$  by  $R_2$  here, we need is point five. So, this will  $v$  point five, when 12 kilo ohms divided by 24 kilo ohms. So that means, 12 by 24, which is point five. So, we will get again of point five. And similarly  $R_3$  we take as at 8 kilo ohms. These resistance are whole different  $R_1 R_2 R_3$ , they are not same they are all different, and they are different, then  $R_F$  as well. So, this scaling can be done according to this equation, by choosing this resistances, and then keeping that we will get, which will be  $4 v_1$  point  $4 v_2$  plus  $1$  point  $5 v_3$  with a reverse sign, with a minus sign, if this minus sign we already talked, it is not really much important, it can be taken care of without much problem. So, this is the scaling are weighted amplifiers.

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Averaging Amplifier:

✓ If  $R_1 = R_2 = R_3 = R$  (say)

✓  $\frac{R_F}{R} = \frac{1}{N}$ , where  $N = \text{no. of inputs}$ .

In the present case under discussion,

$$\frac{R_F}{R} = \frac{1}{3}$$

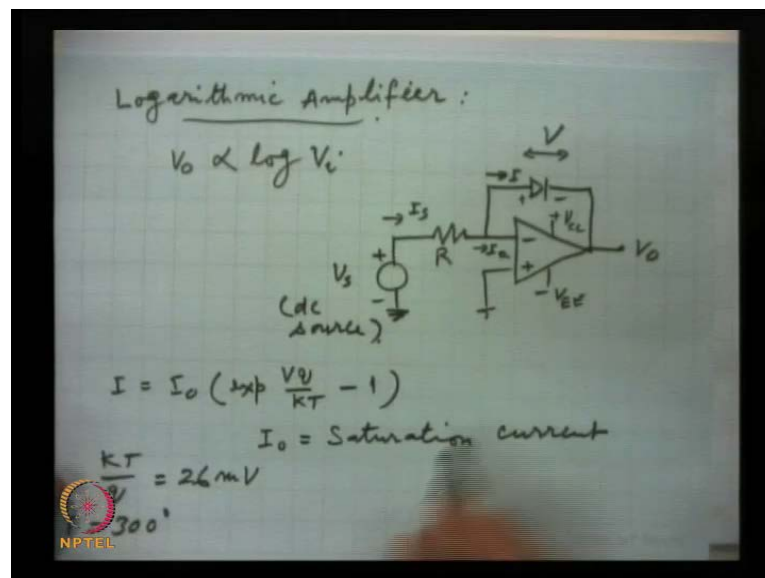
$$V_o = -\left(\frac{R_F}{R}\right)(V_1 + V_2 + V_3)$$

$$\left\{ V_o = -\frac{(V_1 + V_2 + V_3)}{3} \right\}$$

Then the third one was the averaging amplifier. Averaging amplifier, what say averaging amplifier. Averaging amplifier is the one in off the inputs. Average of all the inputs put together, SO that is averaging amplifier. Now averaging amplifier again by the same fundamental equation we can get. If  $R_1$  is equal to  $R_2$  is equal to  $R_3$ , say  $R$ . This is the

condition and  $R_F$  by  $R$  is taken as  $1$  by  $n$ , this is additional point, first point is this. This is the second point in  $R_F$  by  $R$  should be taken as  $1$  by  $n$ , where  $n$  is the number of inputs, in the present case which is under discussion. There are three inputs. So, this factor  $R_F$  by  $R$  should be equal to, in the present case under discussion, we should have  $R_F$  by  $R$  equal to  $1$  by  $3$ . Now, if we substitute  $R_F$  by when we take  $R_1$  equal to  $R_2$  equal to  $R_3$  equal to  $R$ . So, here this is common term which we can take out,  $R_F$  by  $R$ , and  $R_F$  by  $R$  in this case, is taken as  $3$ . So, obviously,  $v_0$  is minus  $R_F$  by  $R$  into  $v_1$ , plus  $v_2$ , plus  $v_3$ . And since this is taken as one, so  $v_0$  will be minus  $v_1$  plus  $v_2$  plus  $v_3$  by  $3$ , because  $R_F$  by  $R$  is  $1$  by  $3$ . So, this is the result for averaging amplifier. The output is the average of all the inputs with this sign reversed. So, this is the averaging amplifier, we continue with these amplifiers, we will take other circuits also and we continue, let us talk of a logarithmic amplifier.

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Logarithmic amplifier is the one, in which the output is the log of the input. I repeat what is logarithmic amplifier. Logarithmic amplifier is the one in which output is proportional, to the log of input, let us say, that is  $v_i$ , this is the logarithmic amplifier. Now if in the simple inverting amplifier, if we replace the resistance  $R_F$  by a diode, what will the circuit. This will be the circuit,  $v_o$  and this is a DC voltage  $v_s$ , dc source. This is the circuit of a logarithmic amplifier. Here the I B characteristic of diode, we remember that  $I$  equal to  $I_0$ , exponential  $v_Q$  by  $K T$  minus  $1$ . This is the equation of diode, which describes the current voltage relationship. This is applied voltage in this the resulting



current, and here  $I_0$  is the saturation current; the saturation current. This we are going to use and here this  $K T$  by  $q$ , this is Boltzmann's constant, this is temperature in Kelvin and this is  $q$  we charge electronic charge when. So, the thermal equivalent of that room temperature, the voltage equivalent this is that, the voltage equivalent of thermal energy at room temperature. This comes how to be roughly 26 milli volts at temperature  $T$  equal to 300 degrees Kelvin. So, as long as this  $v$  applied field, is say of the order of 500 milli volts or one volt. This one will be neglected, because this is exponential term which will rise very fast.

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$$I = I_0 \exp\left(\frac{Vq}{KT}\right)$$

$$\text{or } \exp\left(\frac{Vq}{KT}\right) = \frac{I}{I_0}$$

$$\text{or } V = \frac{KT}{q} (\ln I - \ln I_0)$$

contribution of this term is negligibly small

$$I = I_s = \frac{V_s}{R}$$

$$V_o = -\frac{KT}{q} \ln \frac{V_s}{R}$$

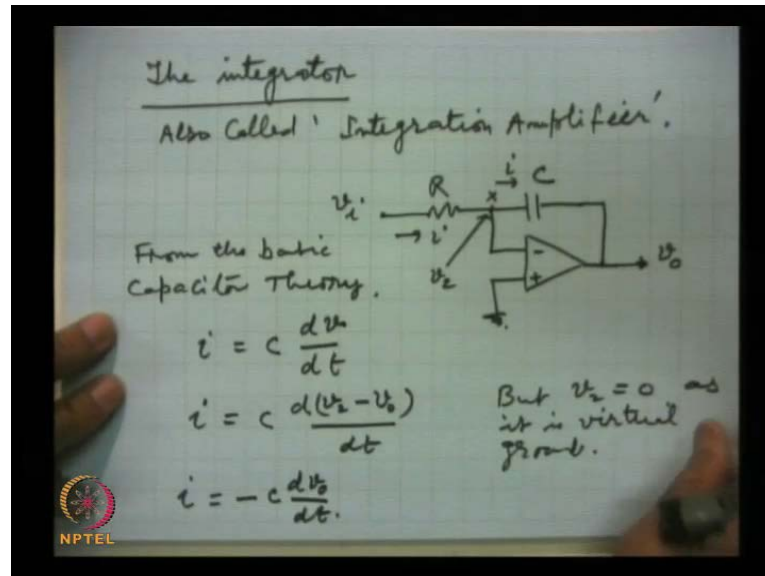
output voltage is proportional to the natural log of the input voltage.

And so neglecting that one, we can use the equation  $I$  equal to  $I_0$  exponential  $v q$  by  $K T$  from here, or exponential  $v q$  by  $K T$  is equal to  $I$  by  $I_0$ , or we take log of both sides;  $v$  equal to  $K T$  by  $q$  into  $\log I$  minus  $\log I_0$ , and this current  $I$  here in this circuit. This is  $I_s$  from the source, this current and this current is  $I$  and this voltage across this which will be applying that is  $v$ . And this current the current here, this is actually zero, because of very high impedance of the operational amplifier. The current going in the inverting input is zero. So, taking  $I$  as  $I_s$  equal to  $v_s$  by  $R$ . And further neglecting small contribution, this contribution of this term is negligibly small. So, we neglect that, in that case this  $v$  will be equal to minus. It will  $v_0$ , because see here. Here this is the voltage developed across this, and because it is inverting. So, with the minus sign it will be  $K T$  by  $q$  into  $\log v_s$  by  $R$ . Thus this is a constant. So, the output voltage is proportional to the natural log of the input  $v_s$ . Output voltage is proportional, to the natural logarithmic



of the input voltage. This is the logarithmic amplifier. Now we will go for other mathematical operations, using these op amp circuits, we go for the integrator.

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The integrator, this is also called integration amplifier. Here the output voltage is integral of the input voltage, output voltage is the integral of the input voltage then the amplifier is called, the circuit is called the integrator. Integrator can be easily realized, by using the op amp in the inverting mode, and by replacing the feedback resistance R F by a capacitor. So, the circuit of the integrator, the basic circuit is this. This is R, this is that cap stance, this is the point x. And here we attach the input signal, and output is taken here v out. This is the circuit, R F has been replaced by the capacitor c. And here this is the current i which would be flowing, and here also, because this current we taking as zero. So, the same current will flow here, in the capacitor.

Now, from the simple basic capacitor theory; from the basic capacitor theory or the theory of the capacitor, the voltage developed across the capacitor and the charging current have a relationship; that i is equal to c d v by d t. If this is the relationship between the charging current and the voltage, developed across the capacitor, this is simple relationship. The charging current gives a rate of change of voltage, and the capacitor is charged. Now, a applying this equation in the present case, then it will be. Let us say which of course, this voltage is v 2, which is zero of course, but for the sake of the argument, this is c into d v 2 minus v 0. The two sides of the capacitor, one is at a

voltage of  $v_2$ , and the other end is at  $v_0$  and this is  $d t$ , but  $v_2$  is 0, as it is virtual ground.

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$$i = \frac{v_1 - v_2}{R} = \frac{v_1}{R}$$

therefore

$$\frac{v_1}{R} = -C \frac{d v_0}{d t}$$

or,

$$\frac{d v_0}{d t} = -\frac{1}{R C} \cdot v_1$$

Integrating both sides,

$$v_0 = -\frac{1}{R C} \int_0^t v_1 dt + C_1$$

$C_1$  is a integration constant —  
initial voltage at capacitor at  $t=0$

So,  $i$  is equal to  $\frac{v_1 - v_2}{R}$ , and  $i$  is the same as  $\frac{v_1}{R}$ . So,  $i$  is; obviously, here  $v_1 - v_2$  by  $R$ ,  $i$  is equal to  $\frac{v_1 - v_2}{R}$ ,  $v_2$  to  $v_0$ , is equal to  $\frac{v_1}{R}$ . This is  $i$ . So, this  $i$  we substitute in this expression, therefore we get  $\frac{v_1}{R}$ , the current is equal to  $-\frac{1}{R C} \frac{d v_0}{d t}$ . or from here we can write  $\frac{d v_0}{d t}$  is equal to  $-\frac{1}{R C} \frac{v_1}{R}$ . Integrating both sides, we get  $v_0$ . So, this will not be there, when we take this, this is simply this will be in the case. So,  $v_0$  is equal to  $-\frac{1}{R C} \int_0^t v_1 dt + C_1$ , where  $C_1$  is a integration constant,  $C_1$  is a integration constant and represents initial capacitor voltage at  $t=0$ . And it represents initial voltage at capacitor at  $t=0$ .

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$c_1$  can be set to zero.

$$v_o = -\frac{1}{RC} \int v_i dt$$

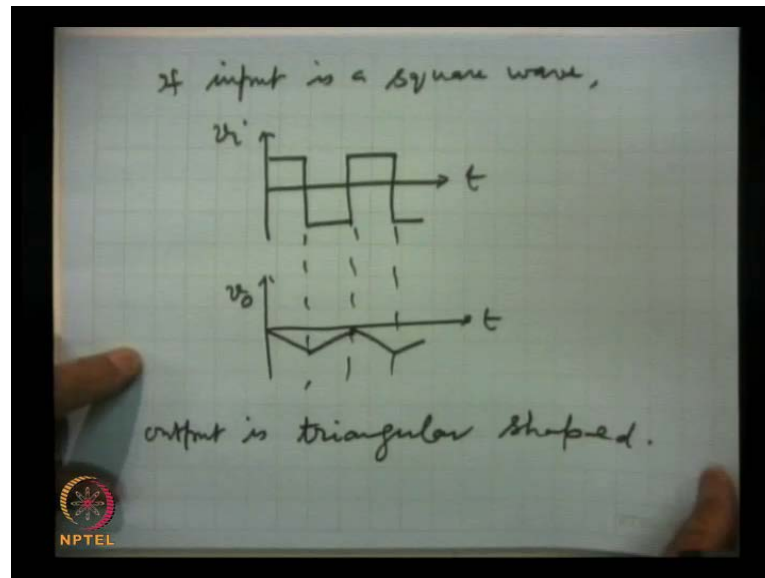
if the input voltage  $v_i = V$

$$v_o = -\frac{Vt}{RC}$$
$$v_i = -V$$

The image contains two graphs of output voltage  $v_o$  versus time  $t$ . The top graph shows a downward-sloping ramp starting from a positive value on the  $v_o$  axis, representing a positive input voltage  $V$ . The bottom graph shows an upward-sloping ramp starting from a negative value on the  $v_o$  axis, representing a negative input voltage  $-V$ .

And this we can set  $c_1$  can be set to zero. In that case  $v_0$  the output voltage will be  $\frac{1}{RC} \int v_i dt$ . This is the expression for the output voltage, and the amplifier, therefore provides an output voltage, which is proportional to the integral of input voltage. Output is proportional this is a constant  $RC$ , time constant of the circuit. So,  $v_0$ , the output voltage is proportional to the integral of the input voltage. This is the integration and the integrator or an integration amplifier. Now let us discuss further more, and what are the problems in this basic integrator, and how we take care of those things. Now here if the input voltage is constant, that is  $v_i$  is a constant, say let us write with capital  $V$ , then by substituting that in this equation, we will get  $v_0$  will be  $\frac{Vt}{RC}$ . So, from time to zero to time  $t$ , this will be keeping on changing. So, it will be a ramp, the output will be a ramp, because of this negative sign, it will be going this way. This is time and this is the voltage  $v_0$ . So, a constant input voltage if it is positive, the ramp will be like this. If  $v_i$  is a negative voltage then the ramp will go in the other direction, this is  $t$ . So, this will be the ramp. This is  $v_0$  versus  $t$ , and for a square wave input, we will get a triangular shape output.

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If the input is a square wave, the output will be triangular; that means these once. This is input in square view, this is time, then the output, this is constant, so it will be a ramp, and then this is also constant, it will be ramp and that these two ramps will be going in a opposite direction. This is  $t$ , so this is the output, a triangular. So, if the input is a square wave, output is triangular. In square wave input will give a triangular wave.

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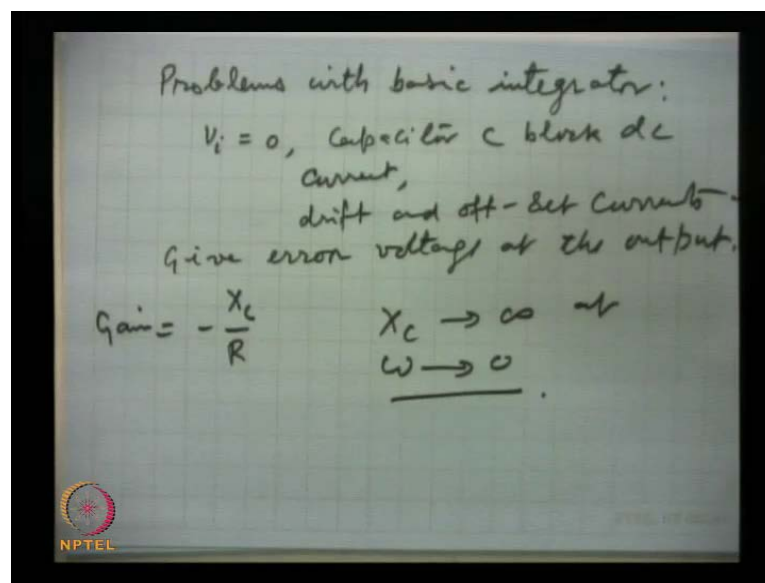
let input voltage be sinusoidal  
 $v_i = a_0 \sin \omega t$   
 $v_o = -\frac{1}{RC} \int a_0 \sin \omega t dt$   
 $v_o = \frac{a_0}{\omega RC} \cos \omega t$   
Frequency dependent maximum  
output voltage.

The slide contains handwritten mathematical derivations. It starts with the statement 'let input voltage be sinusoidal' followed by the equation  $v_i = a_0 \sin \omega t$ . Then, it shows the integration of this equation to find the output voltage  $v_o$ , resulting in  $v_o = -\frac{1}{RC} \int a_0 \sin \omega t dt$  and  $v_o = \frac{a_0}{\omega RC} \cos \omega t$ . The final line states 'Frequency dependent maximum output voltage.' An NPTEL logo is visible in the bottom left corner.

And if the input is a sinusoidal signal, let input voltage be sinusoidal, say  $v_i$  is a zero  $\sin \omega t$ . We put this in the fundamental equation, in this equation. Then  $v_o$  will be

minus 1 by  $R_c$  into a  $0 \sin \omega t$ , and this  $v_0$  comes out to be a  $0, \omega R_c \cos \omega t$ . Here this is important that the maximum amplitude of the output, will be is frequency dependent. This is frequency dependent. Therefore, whatever is the maximum output for a signal of a frequency, say one kilo hertz at two kilo hertz, it will be half. Maximum output will be half, so it will be frequency dependent. Frequency dependent maximum output voltage. Now let us talk, we continue with the integrated amplifier, that what are the problems with the basic circuit, which we have drawn simply by replacing  $R_f$  by a capacitor  $c$ .

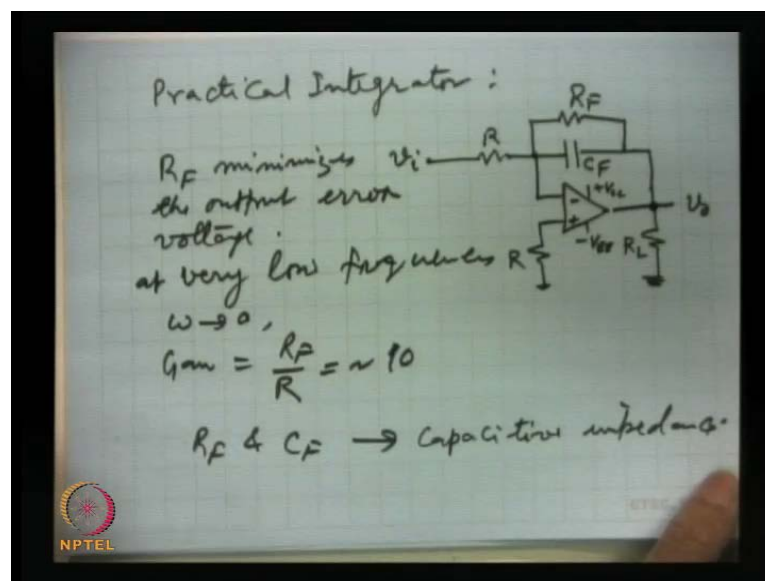
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Now, there are two problems; problems with basic integrator, problems are that, for  $v_i$  when it is zero, the integrator  $x$  as in open circuit for d c. Look at this circuit, here when  $v_i$  is zero, then in this circuit, the integrator  $x$  is a op amp circuit, because this capacitor will act as a blocking capacitor, and it will not permit any d c currents to be fed back to the input. I repeat that when  $v_i$  is zero capacitor  $c$  blocks d c current, and from where this current will come. These are drift currents, drift and off set current. I said about offset currents, we will be talking later, but at least for the time being, we can say offset currents are there, because of a symmetric design of this circuits, which are almost impossible to get them in perfect mast conditions. So, this small current, there will be blocked by the capacitor, and these currents will not be fed back to the inverter, but they will continuously charge the capacitor.

And hence the voltage gets developed, and then it is amplified and the output voltage appears, while the input is zero. So, this presents of this output voltage, it is a error voltage and it should not b when input voltage is zero, the output voltage should be zero, but it will be finite, because of the charging of the capacitor, and then that voltage is amplified by the circuit and there will be a error voltage, gives error voltage at the output. Another problem; that in this circuit at very low frequencies close to, for example zero. The gain of this inverting amplifier, is impedance of the capacitor divided by R F by R. R F is now replaced by X c, that is the capacitive reactants of the capacitor. So, the gain is this, and this is close to, this is this is very high reactance, because the capacitor 1 by omega c is the reactance. So, omega approach is zero, this reactance X c approach is infinity, as omega approach is zero. So that means, the gain will be very high for is small frequencies. These problems can be taken care, by attaching a register R F with the capacitor, and that is the practical circuit.

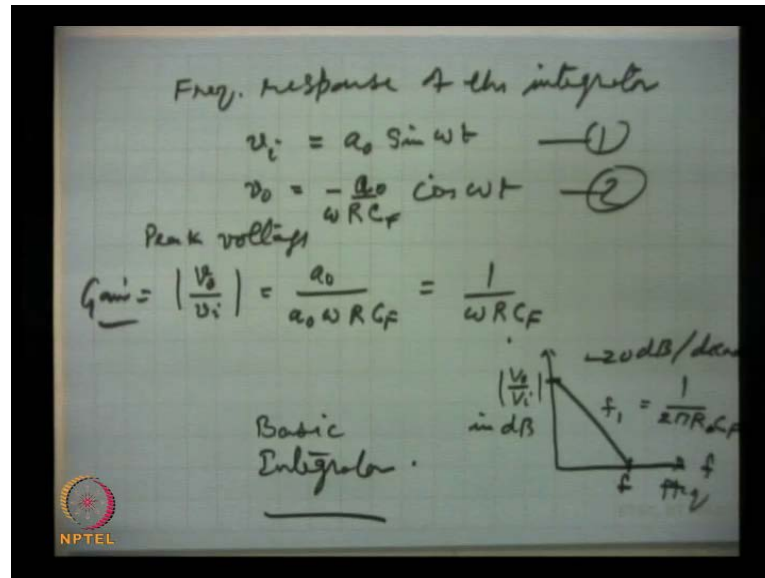
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Practical integrator; the circuit is this. C F is that capacitor, and v zero we take here. Now, this R F we will take care of the problems, which we have discussed, that it minimizes. R F minimize is the output error voltage, and it will also take care at very low frequencies, the gain will not be infinity, which will be at very low frequencies; that means omega going to zero. Now, this is infinity we can neglect, because R F is in parallel, and gain will be R F by R which is finite. And we will see that normally this is kept around ten by design, we keep it around ten. So, this takes care, but it puts a

limitation. True integration will be there, when the combination R F and C F produce a capacitive impedance. So, there is a restriction that at very low frequency, this circuit will not be a true integrator, and what is that condition we will just see, but this is the practical circuit and inclusion of R F that gives the modification and the performance.

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Now, we discussed the gain magnitude and frequency response of this circuit. Frequency response of the integrator; if we take  $v_i$ , we have done it above as  $\sin \omega t$  and  $v_o$  is  $\frac{1}{\omega R C F}$  times  $\cos \omega t$ . This is input. This is output after integration. And if we take the ratio of the peak voltages,  $v_o$  peak and  $v_i$  peak, peak voltages, ratio of peak voltages, then this is  $\frac{1}{\omega R C F}$ . So the gain, this is gain, gain falls. This is  $v_o$  by  $v_i$  by taking the peaks. This is gain and this is in dB. this is frequency  $f$ , and this is  $f_1$ , and we will see what it is. This will come out from here.  $f_1$  comes out to be  $\frac{1}{2\pi R C F}$ . Let us have taking only  $R C F$ . And this is a slope minus 20 dB fall per decade of change of frequency. So, this is the frequency response, and this is for the basic amplifier, basic integrator and how this is modified for the case of the practical integrator, we will take further and discuss other circuits also.