

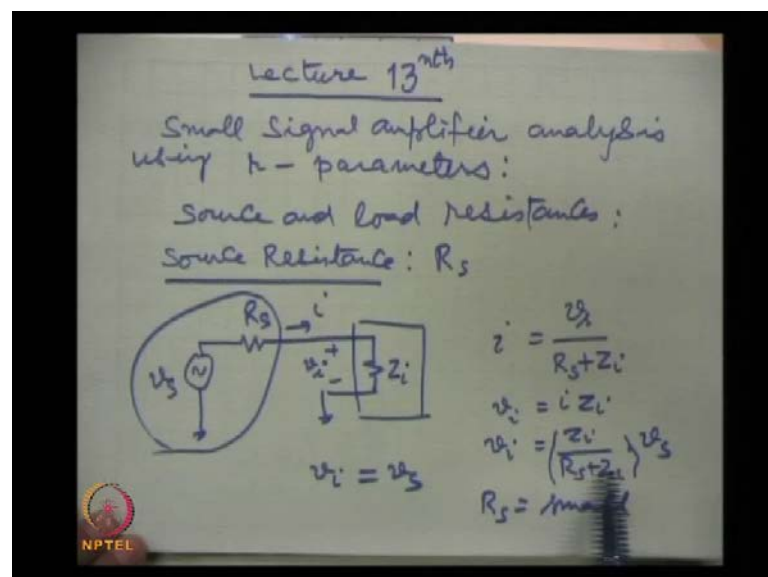
Electronics
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Module No. # 03
Small Signal BJT Amplifiers
Lecture No. # 13
Small Signal Amplifier Analysis using r-parameters

We were discussing Analysis of Small Signal Amplifier and as mentioned earlier, this can be used by using hybrid parameter or r-parameters. Hybrid parameter analysis, we discussed, now we will be taking r-parameter analysis. There are various reasons, which of course, I will specify because of these reasons, r-parameter analysis has become quite popular in recent years; and in fact in many times, it has been preferred over h-parameter analysis.

Now, before taking the actual analysis, let us talk about important thing that is source and load impedances often; because is often half set that when the frequency is low, impedances resistance does not differ. And hence, we loosely talk at low frequencies at about resistances or impedances.

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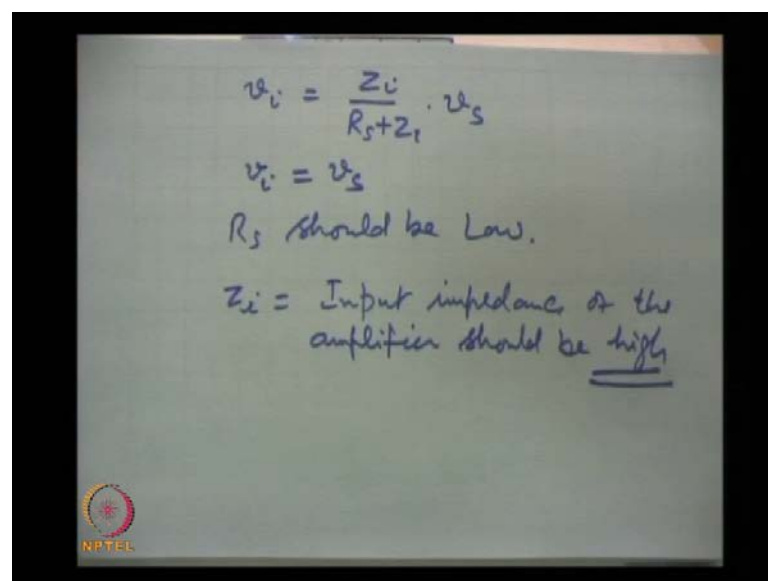
So, source resistance first we talk, this is often written as R_s . Every source, which we connect at the input of the amplifier; this may be micro phone, this may be some other oscillator or the previous amplifier stage to which **is** this stage is connected, they have all impedance.

And a circuit can always be thevenised like this (No audio from 02:16 to 02:37). This is the source impedance v_s (Refer Slide Time: 02:38) and this can be thevenised with the voltage source and resistance. Now, this current i as, we simply we can write, this will be v_s the signal voltage and the two resistances are coming in series, R_s and Z_i .

Now, the voltage which is available for amplification actually is v_i . So, v_i is equal to i into Z_i . So, when I substitute the value of i here, I get v_i is equal to $Z_i R_s$ plus Z_i into v_s . Now, from this expression you can see that, v_i will be equal to v_s ; that means, there is no signal loss as much as we are applying the same is applicable for amplification at the amplifier.

This is amplifier, Z_i is the input impedance of the amplifier, R_s is the source resistance source impedance that is, if it is oscillator then the oscillator what impedance it has. Now, this is obvious from here, that v_i will be equal to v_s , if a R_s is small and it is small enough as compare to Z_i . So, if the input impedance is the high, I write the expression.

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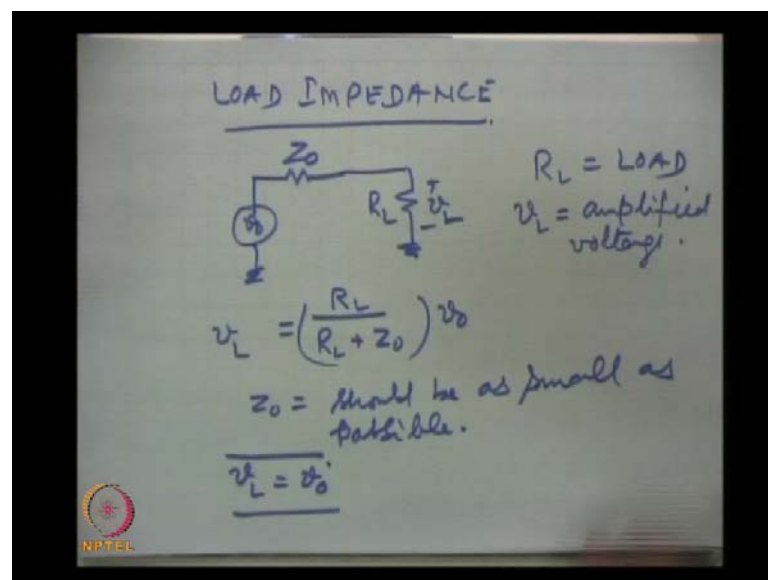
$$v_i = \frac{Z_i}{R_s + Z_i} \cdot v_s$$
$$v_i = v_s$$
$$R_s \text{ should be Low.}$$
$$Z_i = \text{Input impedance of the amplifier should be high}$$

The image shows a handwritten slide with mathematical derivations and conditions for maximum signal transfer. It includes the voltage divider formula, the condition for no signal loss, and the requirement for high input impedance. An NPTEL logo is visible in the bottom left corner.

v_i is $Z_i R_s$ plus Z_i into v_s and the condition that v_s v_i v equal to v_s , R_s should be low, so we should use a source which has low resistance, R_s should be low. And this condition can also be satisfied that, Z_i that is the input impedance of the amplifier, **input impedance of the amplifier** this should be high should be as high as possible **should be high**.

So, remember a voltage amplifier should have high input impedance. This point we have been emphasizing so far and this become very obvious here, that for a voltage amplifier, input impedance should be high and output impedance should be low; and that point we shall take now and that is the load impedance (No audio from 06:14 to 06:23) **load impedance**.

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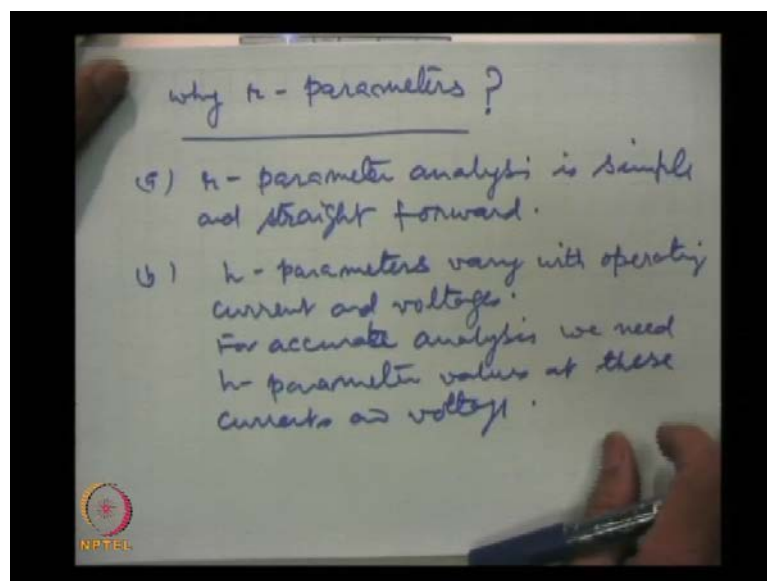


The amplifier the output of the amplifier will give the output v_o and every amplifier has the output impedance, which we are writing at Z_o . Now, to this we connect the load, so this is R_L ; and across this is R_L is load and across R_L , we measure what we call the amplified **amplified** voltage at the load v_L , v_L is amplified **voltage** ac voltage.

Now, from this figure we can make out that, **v_L** v_L is equal to $R_L / (R_L + Z_o)$ into v_o , this is the voltage which we will be observing at the load resistance. Now, **there is** there should not be loss of voltage, this will require that, Z_o should be as a small as possible only then, v_L will be equal to v_o .

So, this is the justification for the statement, which we have been making that the output impedance of the voltage amplifier should be low. When this is low then, the two impedances are equal and there is no loss of impedance. So remember that, source which we used with the amplifier; its input impedance should be low, R_s should be low. And the output impedance of the amplifier should also be low, if we proper voltage at the load. Now, we come to r-parameter analysis, r is tends for resistance. And soon, we will realize we will look at the good features of this analysis and that is why, this has become quiet popular, first why r-parameters analysis?

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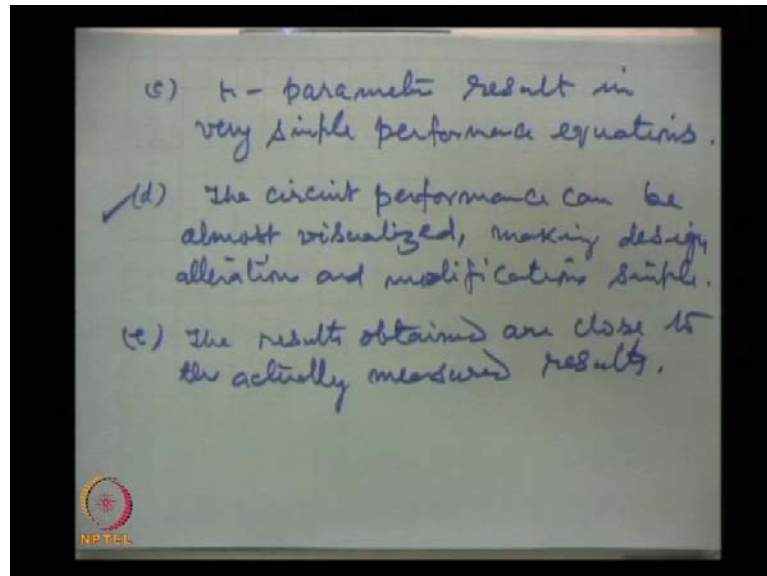


Why r-parameter? First reason is that, the r-parameter analysis is simple and is straight forward. Second point is, we have seen in the h-parameter analysis and these h-parameters vary with operating current and voltages. For accurate analysis for accurate analysis we need h values h-parameters values at these currents and voltages, which is again it is not very straight forward.

And the r-parameter analysis on the other hand requires only one parameter r-parameter r-parameter analysis requires only one parameter and this is actually calculated for the case for our circuit; and hence, it is very accurate that makes analysis very simple and the calculation of the one parameter as we will see is very straight forward and very simple. The performance, the third point which goes in favor of the r-parameter analysis that, the

equation performance equation, which we will soon be getting they are very simple not as complex as in the case of s-parameter analysis.

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So, r-parameters result in very simple performance equations. You will see it soon that for example; the voltage amplification with h-parameter that was the equation was quite complex. Here, it will be extremely simple, so this is that point and then very important two points, one is that the circuit performance is almost that can be visualized the circuit performance can be almost visualized, making design making design alterations and modifications simple, this is very important (Refer Slide Time: 14:01).

And lastly, the results obtained by r-parameter analysis are very close to the experimentally observed results; that is, these results obtained are close to the actually measured results. So, this is the reason, these are the reasons why r-parameter analysis is becoming popular.

Now, there is one limitation of this r-parameter analysis also and that is, this analysis actually does not take into account the feedback effect you know, what is feedback effect? That, the two junctions in the transistors, the emitter junction and the collector junction, do not work, do not operate totally independently, there is some little dependency; and this dependency is for example, the input current (()) becomes a function of the collector junction voltage, this is because of feedback effect, this is one of the feedback effects.

Now, r-parameter analysis normally ignores does not take into account this feedback effects; but, feedback effects are very small and they are influence on the performance can often be neglected; the magnitudes which are affected, those magnitudes are really negligibly small. So, this is not something very serious. So now, we will come to the r-parameters, what are the r-parameters?

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The r-parameters

(a) the value of current gain β
 $\beta = \text{CE amplifier} = \frac{I_E}{I_B}$
 $\beta = h_{FE} = h_{fe}$

(b) value of emitter dynamic resistance
 $r_e' = r_e = \frac{26 \text{ mV}}{I_E (\text{mA})} (\Omega)$
 $\approx \frac{25 \text{ mV}}{I_E (\text{mA})} (\Omega)$

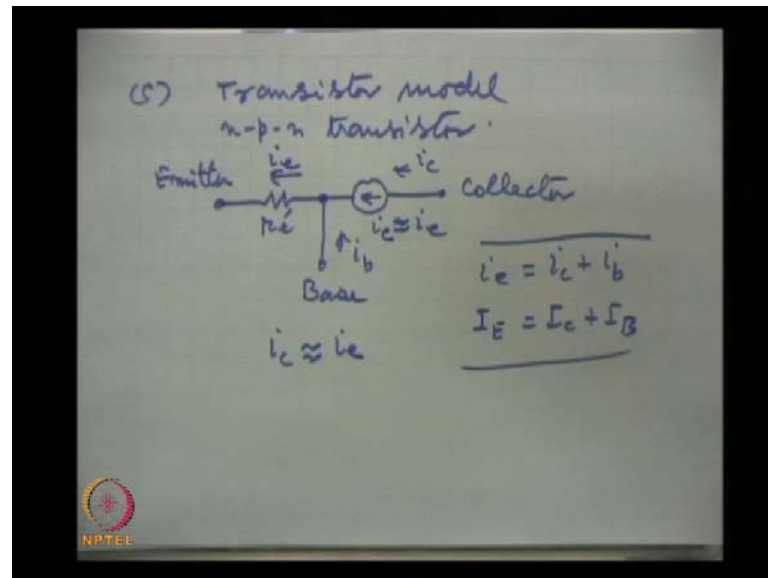
The r-parameters, there are only two parameters required for this analysis, one is the value of current gain beta you remember beta, we define for C E amplifier and this was the ratio of the output current to the input current. And this beta is seen as in h-parameters the forward current gain h_{FE} forward current gain or sometimes this is for dc and as I said that, dc and ac parameters current gain do not differ significantly with hardly difference of 1 or 2 percent, so they are the same, so this is beta which we need.

The other parameter is which is required for this r-parameter analysis and **this is** that is the value of emitter dynamic resistance, **emitter dynamic resistance** **which is** which we have talked earlier also, when we developed with the small signal model of the transistors and this is r_e prime, which is sometimes written just simply r_e and this is 26 millivolts by I_E in milliamperes; then it is the value in ohms.

And this 26 millivolts is also sometimes taking just 25 for the shake of the convenience of calculation 26 millivolts, one in the same thing it will not incur much in accuracy. And this is I_E in milliamperes then we get the values in ohms. So, these are the two

parameters we required. Now, you remember that we developed a small signal model for moderate frequency application, when we talked about when we are talking about the transistors BJT.

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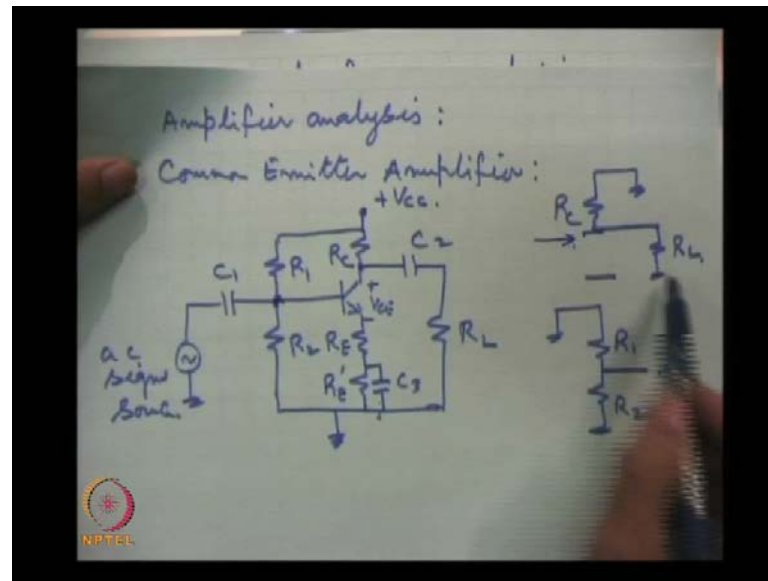


And you will recall that, this is what we arrived is simple model transistor model, that we will be using in this analysis, transistor model this model was for n-p-n transistors. This model was which is of course, equally applicable for p-n-p transistors only difference will be in the direction of the current, that is it. So here, this is collector, this is base and this is emitter and this is the current source, which is i_c and this is equal to very close i_e and this resistance is the $r_{e'}$ (Refer Slide Time: 20:07).

The emitter dynamic resistance which we were just talking and this will be the direction of current, this is i_c , this is i_b and this is i_e . Few things are very fundamentals for the analysis of transistors. One thing is that, i_e is equal to i_c plus i_b this is for ac currents. And this is the same similarly, for dc currents I_E is equal to I_C plus I_B , this is the very fundamental.

And i_c is I have written here is very close to i_e because, the base current which is i_b current is often two orders of magnitude are higher three orders of magnitude this smaller than emitter current or collector current. Now, we take the amplifier which we are going to analyze. Let us, take CE amplifier.

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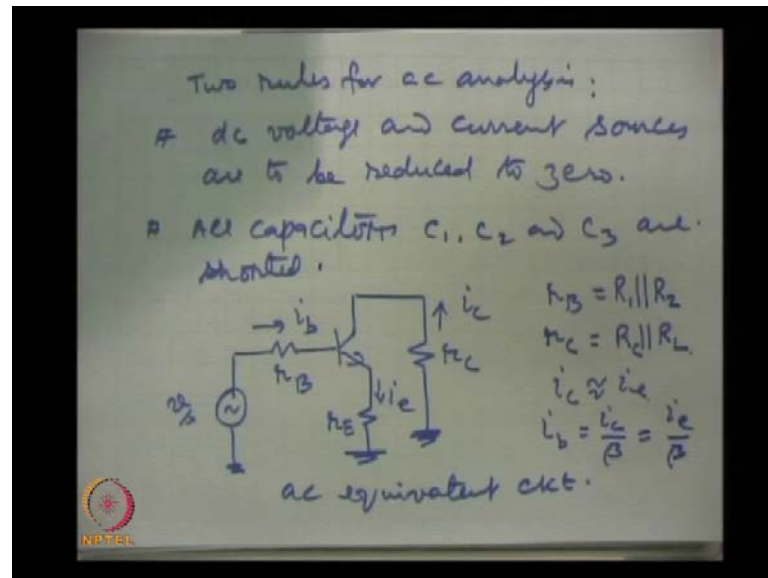
So, we start from here the amplifier **the amplifier** analysis **the amplifier analysis**. And we first take C E amplifier, Common Emitter amplifier. We take first the common emitter amplifier in the complete amplifier is this (No audio from 22:43 to 23:12). This is the circuit must be looking familiar to you, because we have ground it several times. (No audio from 23:22 to 23:42) This is the circuit, these are the resistances which are used to provide **in** a proper bias.

And always remember that, the emitter junction has to be forward biased, collector junction has to be reverse biased. So that, we are in the active region and then, we have to choose proper values of the dc currents, the i_c and this voltage drop here V_{CEQ} , this we have to derive properly; So, that the circuit function.

And this we have talked that, choosing appropriate values of the voltage as in current dc currents in voltages in the circuit that decides the operating point. And operating point should be nearly some where in the middle of the active region of the characteristics. So, any way this is the C E amplifier circuit, this is the source ac signals source and amplified output is taking at the load.

Now, **we applied** we apply the rules of the analysis which of course, we applied even in the case of hybrid parameters analysis and I summarize just in two lines, these rules again.

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So, the two rules for ac analysis, this is true for **ac** any ac analysis actually that these rules **this** have to be followed. The one is that, dc voltage and current sources are to be reduced to 0; that means the dc voltage source when we are conducting ac analysis then, dc voltage source is to be grounded.

And current source to reduce it to 0, this is to be open, open circuited and this is to be grounded, voltage source. And all capacitor C_1 look here in this circuit C_1 C_2 and C_3 all these capacitors we have chosen of the right value and taking in to the account, what is the frequency region in which **(())** suppose to work. So, **one** they will offer hence the negligibly small impedance. And so we short them C_3 are shorted.

Now, if we follow this rule of the analysis then, the C amplifier which we have shown here this gets reduced to its ac equivalent circuit. So, the ac equivalent circuit will be (No audio from 27:29 to 28:15) this is the **ac signal** ac circuit, ac equivalent circuit **ac equivalent circuit**. The actual circuit is this (Refer Slide Time: 28:33). And when we short this, we short this, we short this and dc voltage source is have to be grounded, so this point is grounded.

Then we will see that, r_B this r_B this is actually **when we** when we ground this terminal, they actually become like this, this is grounded and this is R_1 and this is R_2 then, as seen between this two points this two resistances will be in parallel and there that is why the R is R_1 in parallel with R_2 (Refer Slide Time: 29:06).

Similarly, on the output side when this is grounded, this terminal is grounded, this is R_c and this capacitor grounded and this is R_L . So, R_c R_L as seen between these two points here and here, these two resistances will be in parallel. This is the parallel combination is the impedance, which is seen by the collector (Refer Slide Time: 29:59). So, this is the effective value of base resistance, this is the effective value r_c is the combination of the collector resistance with the load resistance.

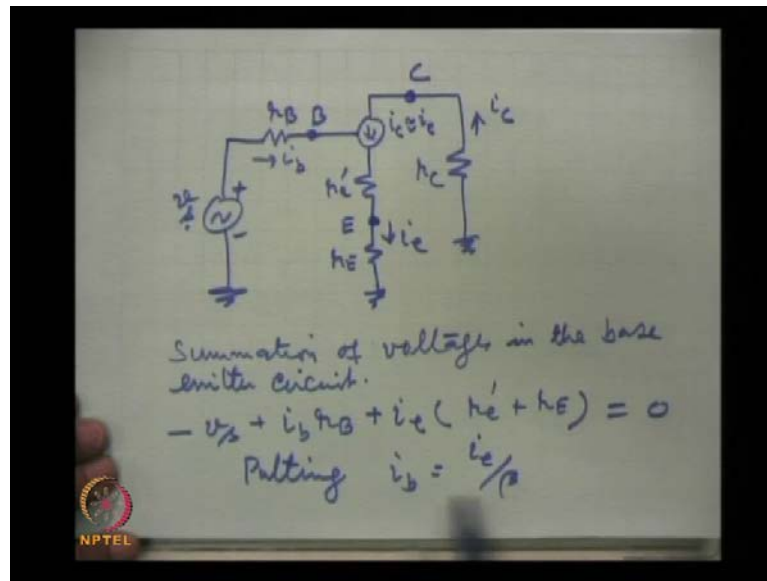
And also all we remember that, collector current is very close to the emitter current. And i_b you know is i_c by beta and this can also be written as i_e by beta. So, I hope things are clear. Let me, recollect things, this is the actual common emitter circuit (Refer Slide Time: 30:53). And we are going to apply r-parameter analysis for its analysis. Now, the rules are that dc current and voltage sources are reduced to 0.

This is the dc voltage source, so we ground for ac analysis (Refer Slide Time: 31:10). And this capacitors are shorted we take them of appropriate value, so that they offer negligible impedance as compared to the other impedances in the circuit. And hence, they are shorted. And when we do that then as I have shown here that, if this is grounded (()) complete the circuit at the input then the between base and ground; the impedance is r_B this r_B which is parallel (()) of R_1 and R_2 .

Similarly, on the collector side when this is battery is grounded. Then, R_c and R_L (()) seen by the collector, they are in parallel. And hence, R_c is this and these of course, have been talking. And i_b as you know i_c by beta, which is same as i_e by beta. Now, we replace in this model these models which have developed ac equivalent circuit. We replace this transistor by model, which we developed.

And you will remember that, the model was this, this is the emitter, this is the collector very simple one resistance is there, which is emitter dynamic resistance and this one is the current source (Refer Slide Time: 32:30). And this we a write at when we discuss the transistor. So, now we replace this transistor in the circuit, in this ac equivalent circuit we replace this we will keep of $i_r B$, we will keep r_E , we will keep r_C and this transistor we replaced by that model.

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Then, what we get is this, (No audio from 33:08 to 33:30), this is emitter lead, this is collector lead and this is base lead, this is r_B , this is r_C and this is the dynamic resistance r_e' . And here, this is our ac voltage source v_s . So, this is actually the equivalent circuit of this is the transistor, which is being replaced by this and this source is i_c or equal to i_e .

Here is the current i_e , this is the current i_c and this is the current i_b , these are all ac currents (Refer Slide Time: 34:25). Now, we so I am sure (()) clear that, how we arrive at this the basic circuit of the amplifier. First we came to the ac equivalent circuit and in the ac equivalent circuit in the ac equivalent circuit we replaced the transistors by its small signal model and we get this.

Now, we are ready to go for the analysis. So, summation of voltages summation of voltages in the base emitter circuit in the base emitter circuit, this results in for example, at any instant we take the polarity like this we start from here. So, we write minus v_s plus i_b into r_B plus i_e is flowing here, so i_e into r_e' plus r_E . These are the two resistances in series, this is r_E and this is r_e' that dynamic resistance (Refer Slide Time: 36:02). So, we get this.

Now, in this equation this is equal to 0. Now, putting i_b equal to i_e by beta actually the base current equal to the collector current by beta, but collector current and emitter

current are taken as equal, so this is that. Then, this equations becomes I_e by βr_B plus $i_e r_E$ equal to v_s .

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Handwritten derivation on a whiteboard:

$$\frac{i_e r_B}{\beta} + i_e (r_E' + r_E) = v_s$$

$$i_e \left(\frac{r_B}{\beta} + r_E' + r_E \right) = v_s$$

$$\text{or } i_e = \frac{v_s}{\left(r_E' + r_E + \frac{r_B}{\beta} \right)}$$

(ac emitter current).

ac voltages at 3-terminals of the transistor are:

Which can be written as, i_e we take out and r_B by β plus r_E prime plus r_E is equal to v_s . Or the emitter current ac emitter current this is I will write this is v_s r_E prime r_E plus r_B by β , this is ac emitter current. Now, we can write ac voltage is at the three terminals, the basic current; if we known i_e , we know all the current. We know i_c , which is equal to this. We know i_b , which is equal to i_e by β ; β as I said in the **I** said in the beginning (()) the value of β for the amplifier.

So, all the currents are known and once we know all the currents we can find out all the ac voltages. So, ac voltages at the three terminal of the transistor are these are, so these equations are v_b , which is i_e into r_E prime plus r_E , let us call this equation 1.

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$$V_b = i_e (r_c + r_e) \quad \dots (1)$$

$$V_e = i_e r_e \quad \dots (2)$$

$$V_c = i_e r_c \quad \dots (3)$$

the voltage gain for a CE amplifier:
same as voltage amplification factor.
By definition

$$A_v \equiv \frac{V_c}{V_b}$$

From Eqs (1) and (3),

$$A_v = \frac{i_e r_c}{i_e (r_c + r_e)}$$

How, what is this V_b , the base terminal and ground what is this voltage here, this is V_b between base and ground. Now, these two resistances are in series and the current flowing from here is i_e . So, obviously the voltage which we will get will be what I have written, i_e into the two resistances in series. And V_e is equal to $i_e r_e$, this we call this we will be using further, so that is why you should note down these equation numbers also.

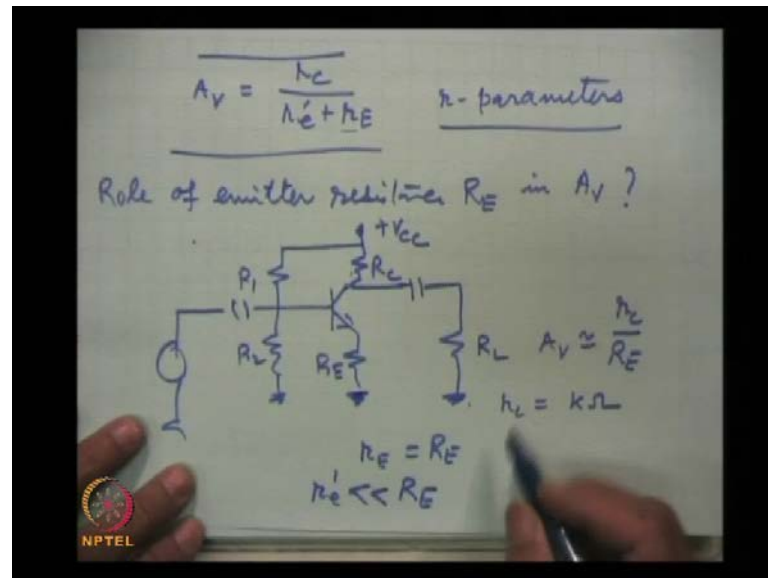
What is V_e , V_e is this voltage here this is plus minus V_e the voltage across this is emitter lead and this is the r_e resistance and this is current is flowing. So, between this end ground will be this voltage actually I am sorry this is not, this is (Refer Slide Time: 40:55), so that is V_e . And then, V_c the collector current is i_c which we can write as i_e and r_c this i_c which is same as i_e is flowing through this lead.

Then, what is the voltage which we will observe across this, this is i_e into r_c . So, these are the three voltages and once we know them, then we find out the value of performance parameters of the amplifier. So, first we write an expression from these equations is very simple to get the voltage gain. The voltage gain for a Common Emitter amplifier C E amplifier, this voltage gain is sometimes called **amplification factor** voltage amplification factor.

So, this is same as voltage amplification factors same and by the definition **by definition** this is written of course, as A_v . And this is output voltage to the input voltage and you

will recall that, output voltage is v_c by input voltage is v_b . This output voltage and this is v_c and input voltage is v_b , so this is that (Refer Slide Time: 43:24). Now, value of v_c is here, the value of v_b is here (Refer Slide Time: 43:35). So, this from equations 1 and 3, this becomes A_v equal to v_c we substitute as $i_e r_C$ divided by r_i divided by v_b , v_b is this r_e prime plus r_E prime and this get cancelled.

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So, this simple expression for the voltage gain r_C r_e prime plus r_E , this is very important, this is the expression for the voltage gain. Now, look at the simplicity of this expression plus these are all resistances, the expression is simple it contains the resistances only and that why it is called r-parameter analysis.

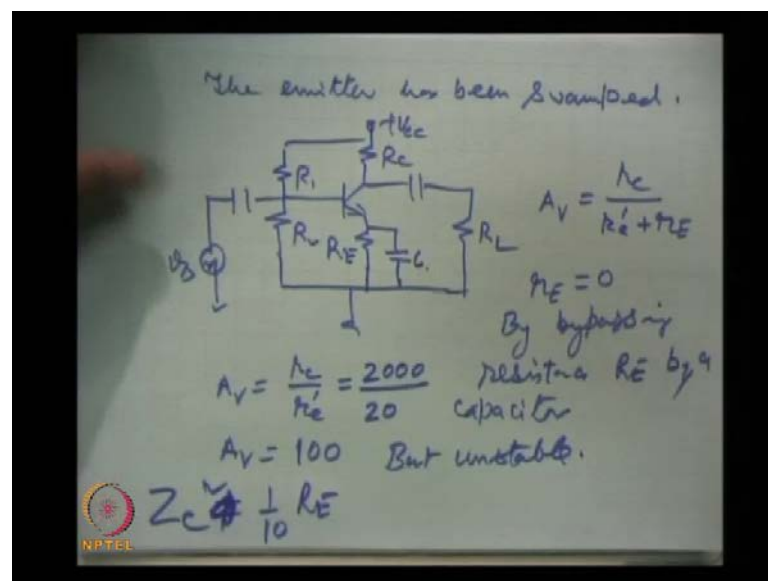
Results are obtained in terms of the resistances and (()) one more parameter which may be there, that is beta which is we had talked. Now, you know what are these r_C r_C is this parallel (()) of the load resistance with the collector resistance. And r_E r_E is the effective resistance seen by the emitter and this is the emitter dynamic resistance.

So, if we know the values of the three resistances we can immediately and very easily calculate value of the voltage gain. Now, let us consider the some discuss the role of the emitter resistance. The role of emitter resistance R_E in the voltage gain often, you look at the original circuit here, suppose we use only one resistance here like this. If I use one resistance here, so that this circuit is becomes this, this is R_E R_1 R_2 R_C and from here this capacitor another capacitor (Refer Slide Time: 46:57).

Suppose, we use this resistance and we do not bypass it then, what is the influence? In that case, this r_E will be equal to R_E . And this will be this dynamic resistance is few Ω only just 10, 15, 20 ohm's. And this will be R_E we will be using same 500 ohms 800 ohms 1 kilo, so we can neglect. In this case, $r_{e'}$ is very small in comparison to the resistance which we are using.

Then, the voltage gain will be close to r_C by R_E the collector resistance will be also in kilo-ohms 1 or 2 kilo-ohms 3 kilo-ohms and this is also in kilo-ohms. So, the gain will be extremely low, because r_C is in kilo-ohms and this will also be almost the same order. So, the gain A_v will be low 3 4 5 it will be drastically reduced. But, why we use R_E ? If R_E is made 0 then of course, the gain will increase, because this resistance will be 0 and I come to that, one important thing is that here we have made the voltage gain independent of R_E totally, this is known as swamped of the emitter. Emitter has been swamped.

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And this process is called swamping. What is swamping? Making the voltage gain independent of $r_{e'}$, because $r_{e'}$ is temperature sensitive as you will recall it was said earlier. So, making their independent of the changes in the value of $r_{e'}$, this is known as swamping. We have swamped the emitter, but what praised we have paid that **we have** the gain is reduced quiet a bit. We will take some example to illustrate this point.

So, what is the other way, other way is that we can by pass the capacitor r_E and that case if we by pass it, this circuit become this we by pass it (No audio from 50:50 to 51:14) we have by passed it. And then, original relation was r_C by $r_{e'}$ plus if we by pass then this resistance simply 0, r_E has been made equal to 0 by bypassing resistance R_E by a capacitor.

We use this capacitor then **then** we get for voltage gain r_C by $r_{e'}$ but this way actually the gain has become very large, because this is in kilo-ohms. If r_C suppose is 2 kilohms; that means, 2000 ohms and this is 20 ohms, then the voltage gain is 100 it is very high. But this is temperature sensitive; this is will be high, but unstable.

So, if we do not use this by pass capacitor let me talk little more about this by pass capacitor; how we choose the value of bypass capacitor **bypass capacitor**. What is the purpose? Purpose is ac will find a path of least resistance through this capacitance; this is higher impedance as compared to this.

Now, how high it should be or in other words how low this capacitance should be, this is the thumb rule is that the impedance at the lowest frequency signal frequency should be one-tenth. The impedance Z_c **should be one-tenth** approximately should be one-tenth of R_E , this is the thumb rule. So, we choose this like that. And but then, it becomes unstable.

So, there is intermediate position, which is often taken the designer will make the circuit, which from where we started that one resistance is maintained in the circuit and one part is bypassed. So, this will be best compromise, this will be smaller resistance than this and this will be this best part. So, **((C))** will not be as low as in the case of not bypassed, and it will not be unstable, because of the effect of dynamic resistance. So, as a compromise we break that resistance R_E into two resistances; a small resistance, a little higher resistance and this higher one is blocked. So, we will continue this analysis further.