

**Electronics**  
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**Module No. # 03**  
**Small Signal BJT Amplifiers**  
**Lecture No. # 01**  
**h and r Parameters and their use in Small Signal Amplifiers**

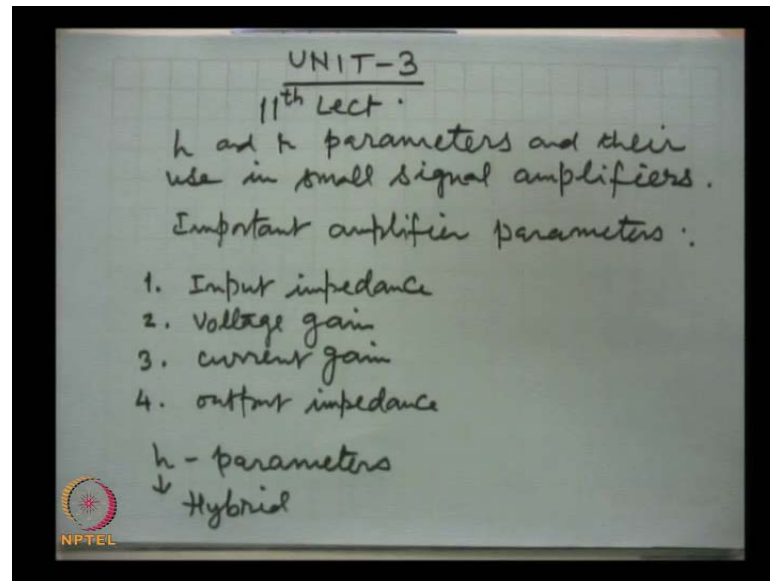
We are going to start the next unit, which is on h and r parameters and their use in Small Signal Amplifier analysis. What are small signal amplifiers? Small signal amplifiers are in which the signal strength is low as compared to the DC operating point parameters. You remember, we said that the real objective of amplifiers is to amplify AC signals.

And for proper application and proper operation of the amplifiers, we have to bias the transistor in the circuit properly; and the biasing which we used are that emitter junction forward biased, collector junction reverse bias. Then the operating point has to be chosen properly, and there should be a proper design for the biasing circuit, this was discussed in previous lectures.

On these DC currents and voltages, which we have set on the operating point, the AC signal which we want to amplify is superimposed. So, when we say a small signal amplifier, these currents and voltages, AC part which is superimposed on the DC, they are small. AC part, AC current and voltages are small in comparison to the DC current and voltages, which we have set at the operating point.

So, these are a small signal amplifiers and the parameters and the analysis, which we are now conducting we are going to be study, this is applicable only for a small signal amplifiers. Large signal amplifiers which are actually called power amplifiers, they are analyzed differently and that we will see later. Now, for an amplifier, there are four most important parameters.

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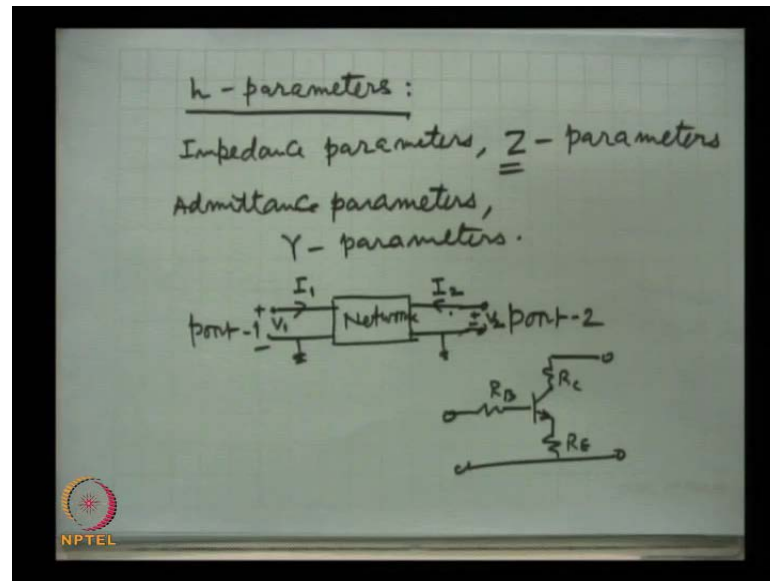


These parameters are (No audio from 03:03 to 03:15) important amplifier parameters, one is the input impedance **input impedance** you know that, resistance and impedance **they are** they may differ at high frequencies; when the frequencies are small, then resistance and impedance they **they** are very close to each other. And so, we may call input resistance or input impedance, but it is better to call input impedance, because frequency we are not yet defining.

The other parameter is the voltage gain, voltage gain of the amplifier. The third parameter is current gain, current gain of the amplifier; and the fourth parameter is output impedance, output impedance of the amplifier. So, remember these are the four important parameters, and they characterize a small signal amplifier; the input impedance, the voltage gain, the current gain and output impedance, this we are going to estimate by using expressions.

And expressions **they are** they will be based on two models which we are going to study, one is h parameter h stands for hybrid parameter h parameters **h parameters**, and this h stands for hybrid **hybrid** parameters, but popularly they are known as h parameters. And then after we finish this analysis, we will take another analysis which is through r parameter analysis, r stands for resistance parameters.

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So, first h parameters hybrid parameters why, this hybrid these are called hybrid parameters? Because, before the invention of the transistor, there were in electronics there were tubes, all circuits were designed in tubes. And tubes could be analyzed comfortably with two sets of parameters any one of them in fact, the impedance parameters **impedance parameters** these are also called the Z parameters. All these four quantities for the circuit amplifying circuit could be obtained in terms of Z parameters.

And then there were admittance parameters **admittance parameters** and popularly, these are called Y parameters. So, there were four Z parameters, there were four Y parameters, and they were able to define to give the estimation of all the four quantities of interest; the two impedances input and output, and two gains the voltage gain and current gain.

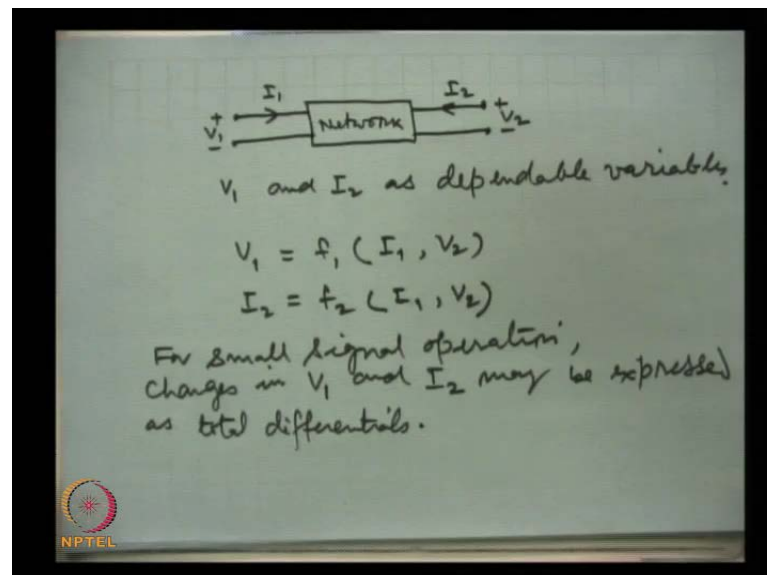
But, when the transistor was discovered was **was** invented then, there was problem in defining in measuring these parameters, either Z parameters or Y parameters for the transistor. So, then another set of parameters was developed and that is known as h parameters. Now, h parameters are applicable **are applicable** for any two port network, this is two port network, this is port 1 (Refer Slide Time: 08:37), this is port 2 **port 2**; and here, we are not concern what is inside this box, we are concerned with the terminal currents and voltages.

So, let this current be  $I_1$  and this voltage here be  $V_1$  similarly, the current here is let it be  $I_2$  and the voltage here is  $V_2$ . Now, these currents and these voltages they are total

current and voltages, because as I have said earlier that, in a amplifier the AC signals are superimposed over the DC **over the D C**. So, this is the instantaneous total values, total currents and total voltages.

Now, our transistor fits in that is also a two port device like this, here is the transistor circuit, this is simplified circuit, actual circuits we will be drawing (No audio from 10:01 to 10:13) this is also a two port network, this is port 1 here and this is port 2 as it is here. So, now we will define first the h parameters that we will take this **(( ))** network, let me redraw the network.

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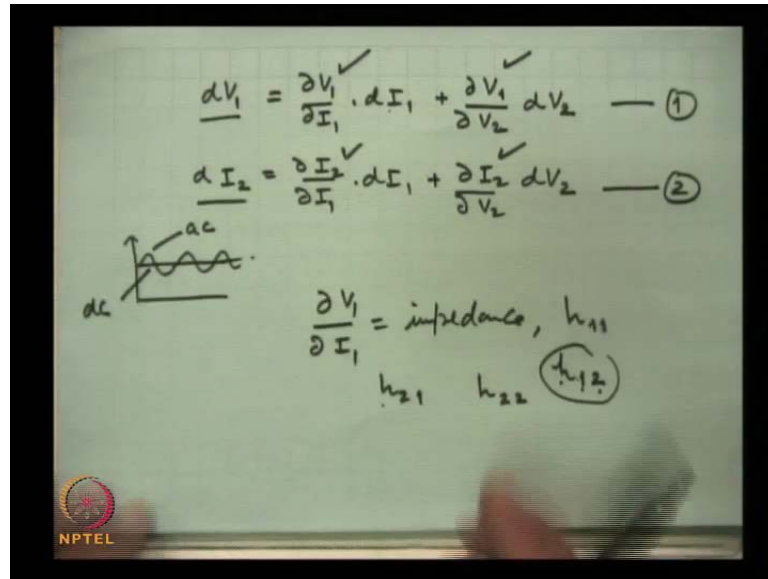


The network is or this is just the transistor and these are this is I 1, this is V 1 and this is I 2 and plus minus V 2. The parameters can be defined by taking out of these four quantities, the two currents and two voltages, two are taken as independent and two are taken dependent. So, we can write the fundamental relation for dependent variables, we take V 1 and I 2 as dependable variables.

Then, the functional relationship with other two parameters, **I 2** I 1 and V 2 can be written can be expressed like V 1 is equal to f 1 (I 1, V 2). And I 2 is equal to f 2 the function, I 1 and V 2. What these equations represent? They are representing that, I 1 and V 2 are independent variables the input current for example, in a transistor we can decide and we can choose whatever we want.

And  $V_1$  will be dependent on  $I_1$  and of course, and  $I_2$  as we will see. Similarly, this equation tells the relationship the dependence of  $I_2$  as a function of the variations in  $I_1$  and  $V_2$ . For a small signal operation, **for a small signal operation changes in** changes in the voltage  $V_1$  and current  $I_2$  may be expressed **may be expressed** as total differentials **as total differentials**.

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$$\frac{dV_1}{dI_1} = \frac{\partial V_1}{\partial I_1} dI_1 + \frac{\partial V_1}{\partial V_2} dV_2 \quad \text{--- (1)}$$

$$\frac{dI_2}{dI_1} = \frac{\partial I_2}{\partial I_1} dI_1 + \frac{\partial I_2}{\partial V_2} dV_2 \quad \text{--- (2)}$$

ac

dc

$\frac{\partial V_1}{\partial I_1} = \text{impedance, } h_{11}$

$h_{21} \quad h_{22} \quad (h_{12})$

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And this is  $dV_1$  the total change in  $V_1$ , this depends on (No audio from 14:21 to 14:42) this term the first term, this is the rate of change of  $V_1$  on  $I_1$ ; and if  $I_1$  changes by  $dI_1$  then this gives the total change in  $V_1$  as a result of change  $dI_1$  change in the current. And this term is the change in  $V_1$ , because of  $V_2$ , **when** so this is the rate and this is the total change in  $V_2$  and hence, this is the total dependence of  $V_1$  on  $I_1$  and  $V_2$ .

Similarly, we can write the another equation (No audio from 15:33 to 15:54) let us call this equation 1 and this as equation 2. And remember, we were talking about  $V_1$  as the total current, like this here this was the DC current and voltage. And so, this is let us see in this case, this is that voltage over which the AC signal a small signal is superimposed, so this is AC, this is DC.

So, when we talk of variation then this accounts for actually just the AC component similarly, here the AC component. Now, let us look at these derivatives, what do they represent this  $\frac{dV_1}{dI_1}$  you will recall that, this has the dimensions of impedance **impedance** and this we will see; and so, this we replace for the time being with  $h$

parameter  $h_{11}$ , hybrid parameter  $h_{11}$ . Similarly, this quantity is a voltage ratio, so this is represented by the quantity, another h parameter  $h_{12}$ ; and this one as  $h_{21}$  and this one as  $h_{22}$ . Now, this is not random see,  $h_{11}$  both quantities involved in  $h_{11}$  or from port 1  $V_1 I_1$ , so  $h_{11}$ . Here, it is  $h_{12}$ , so this is  $h_{12}$ ; here the two the ratios are of current and two ports are involved, the output port and input port. So,  $h_{12}$  here the two the ratios are of current and two ports are involved the output port and input port. So,  $h_{21}$  and similarly here  $h_{22}$ .

Now, these are the variables the varying quantity and what will vary, DC is constant, AC part will vary. So, taking this voltage, AC quantities we write with a small letters. So, we write this as  $v_1$  and this we write as  $i_1$  as small  $i_2$ .

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$$v_1 = h_{11} i_1 + h_{12} v_2 \quad \text{--- (3)}$$

$$i_2 = h_{21} i_1 + h_{22} v_2 \quad \text{--- (4)}$$

CB, CE, CC

$$h_{11} = h_i, \quad h_{21} = \frac{i_2}{i_1} = h_f$$

$$h_{12} = h_{re}, \quad h_{22} = h_o$$

$$h_{11} = h_i = \left. \frac{v_1}{i_1} \right|_{v_2=0}$$

Input impedance measured with output short circuited for ac

And then, we can write these equations 1 and 2 like this  $v_1$  equal to  $h_{11} i_1$  plus  $h_{12} v_2$ ; and  $i_2$  is equal to  $h_{21} i_1$  plus  $h_{22} v_2$ , this we call equation 3 and this we call equation 4. Now, these equations are applicable to all the configurations of the amplifiers, whether it is common base, common emitter or common collector, these equations are valid. And in fact, the whole analysis which we are trying to study, they are applicable to all these circuits.

And now and these are the h parameters four h parameters and the hybrid nature of these parameters is again that will soon be clear as I said that,  $h_{11}$  has the dimensions of impedance. So, this is actually  $h_{11}$  is often written as  $h_i$   $i$  stands for input impedance  $i$

for input. And so, similarly, this  $h_{12}$  this is a reverse voltage ratio. So, this is  $h_r$ ,  $r$  for reverse; and similarly,  $h_{21}$  this is  $i_2$  by  $i_1$  under certain conditions, but this is written as  $h_f$ ,  $f$  for forward. And the fourth parameter  $h_{22}$  has dimensions of admittance. So, this is the and both properties, both parameters involved in this are from for the output port two; and hence this is as we will see the output admittance. So, this is written as  $h_o$ .

So, four parameters and the hybrid nature of these parameters become clear right now. For example, earlier in the beginning we talked about admittance parameters or impedance parameters, all the four parameters were either impedance parameters or admittance parameters. Here, the hybrid nature becomes clear that, this is impedance  $h_i$  is the impedance, and  $h_o$  is the admittance.

So, impedance admittance and this is the forward current gain, actually the ratio of the two currents  $i_2$  by  $i_1$  we remember the fundamental definition of gain for any quantity is output quantity in the same quantity at the input that gives the gain. So,  $i_2$  by  $i_1$  is the gain the forward current gain. And this is a feedback term reverse voltage ratio  $h_r$ . And so, these are the two, this is current ratio which is forward current gain, this is voltage ratio and these are the impedance. So, that is why these parameters are called hybrid parameters.

Now, let us define that **what** under what terminal conditions these are measured and defined. The manufacturers almost for all transistors give the  $h$  parameters  **$h$  parameters**. And so, this let us define that  $h_{11}$  which we have written as  $h_i$ , this if you look at this equation (Refer Slide Time: 23:46), **this equation** if  $v_2$  is 0 then, we can define  $h_{11}$  from here as the ratio of  $v_1$  by  $i_1$ . So, this is defined as  $v_1$  by  $i_1$  under the terminal condition that  $v_2$  has to be 0,  $v_2$  is 0. So, I write in full, this is input impedance **input impedance** measured with **measured with** output shorted or short circuited for A C;  $v_2$  will be 0, only when this output is short circuited for A C.

So, this is remember, these are very important, if this is not 0 then we cannot define this input impedance by this ratio, this ratio will give that also implies that if we change the terminal conditions if we change this, then this will also change. So, that is not the motto, motto is to find the value of  $h_{11}$  and this is defined under the condition  $v_2$  is 0.

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$$h_{12} = h_r = \frac{v_1}{v_2} \Big|_{i_1=0}$$

Reverse voltage ratio  
measured with input  
open circuited for a.c.

$$h_{21} = h_f = \frac{i_2}{i_1} \Big|_{v_2=0}$$

forward current  
gain with output  
shorted for a.c.

$$h_{22} = \frac{i_2}{v_2} \Big|_{i_1=0} = h_o$$

output admittance  
with input open circuited  
for a.c.

$h_{ie}$  — CE amplifier  
 $h_{ic}$  — CC amplifier  
 $h_{ib}$  — CB amplifier.

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Similarly, we define other parameters  $h_{12}$ , which is the reverse and this is the ratio of, here again from equation 3 (Refer Slide Time: 25:47), if  $i_1$  is 0 then, this ratio  $h_{12}$  comes out to be  $v_1$  by  $v_2$ . So,  $v_1$  by  $v_2$  under the condition that  $i_1$  is 0,  $i_1$  is 0 means this is input current; and so, this is reverse voltage ratio **reverse voltage ratio** measured with **measured with** input open circuited for A C, when this is open circuited in a open circuit, the current is 0. So, the input has to be open to measure this and this is written as  $h_r$ .

And similarly,  $h_{22}$  which is the, which is written as a  $h_f$   **$h_f$**  and that is the ratio of  $i_2$  by  $i_1$  with  $v_2$  0. So, this is forward current gain **forward current gain** with output shorted for A C. And finally,  $h_{22}$  this is equal to  $i_2$  by  $v_2$  and under the condition  $i_1$  equal to 0; and this we have said that, this is equal to  $h_o$ . So, this is the output admittance **output admittance** with input open circuited for A C.

So, these are the four parameters we should understand, their definition and here the terminal conditions are very important. If terminal conditions are changed then, these parameters will also change and our analysis will give inaccurate results. So, as we have defined, we have to obtain these conditions to **to** measure these parameters.

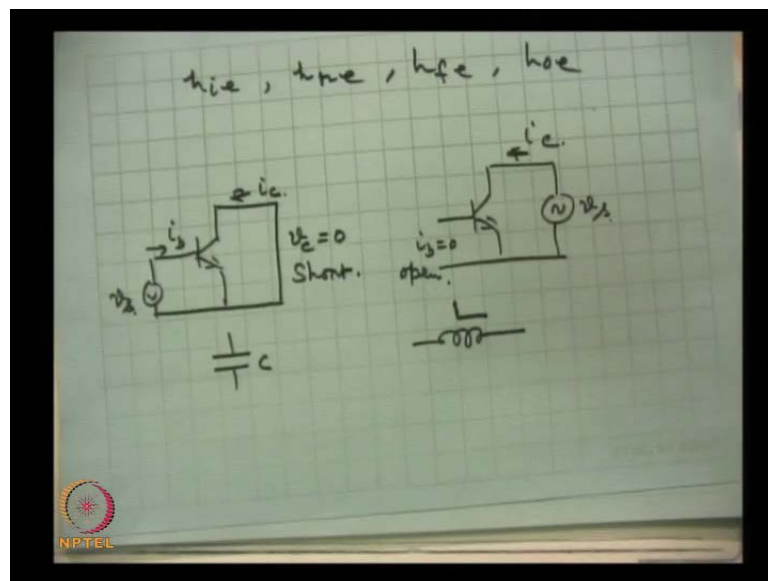
Now, we move further. So, now these parameters as I said, they are applicable for every circuit. Now, the input impedance  $h_i$ , this is again very important, in nomenclature how we express for different circuits, e here small e  $h_{ie}$ , e means that this quantity is for C E



amplifier e. And **if it is** if we write  $h_{ic}$ , this is for common collector amplifier; and similarly,  $h_{ib}$  this is for common base amplifier.

So, this distinction is to be maintained and we have to specify that, for which circuit we are defining and we are measuring in **in** which configuration we are measuring, because  $h_{ie}$  will not be equal for example, to  $h_{ib}$  because, this will be in C B configuration, this will be in C E configuration.

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So, they are not the same, because C E circuit is most widely used, so C E amplifier. So, for C E amplifier the four h parameters will be  $h_{ie}$ ,  $h_{re}$ ,  $h_{fe}$  and  $h_{oe}$ . These are the four h parameters, hybrid parameters for common emitter circuit, and how we can get the boundary conditions? Let us, first express for example, these two fundamental equations 3 and 4, this equation 3 and 4, we replace this h parameters. So, **for a common emitter** for common emitter like here, this circuit (No audio from 31:49 to 31:59) this is a signal we will apply, we will measure  $i_b$  and this will give you  $i_c$ , this is simplified circuit; just a transistor connected in common emitter and then, these are the parameters. So, this is  $v_c$  equal to 0, this is the short, because for defining for example, these two parameters the input impedance  $v_2$  has to be 0.

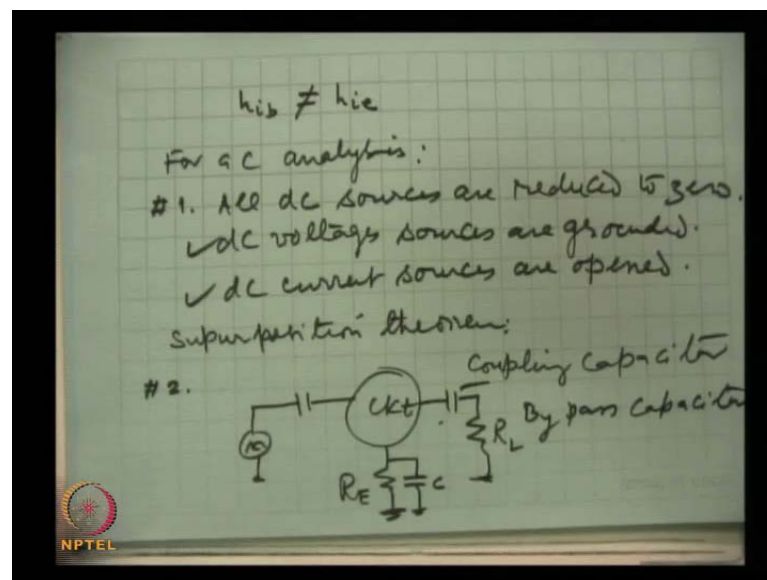
Similarly, for the forward current gain  $v_2$  has to be 0, so that is to be measured under these conditions. And similarly, the two parameters, other two parameters require  $i_b$  equal to 0, so this is open, input is open, we apply a test signal here which will establish

a current  $i_c$ . So, this is the open circuit condition, this is the short circuit condition. Now, practically because, we **we** how we achieve this in practice, not just by shorting it otherwise, how we will apply the signals. And so, this shorting is achieved by using a **a** appropriate capacitor in shunt. So, AC signal will pass through it without disturbing other thing. So, this condition of output shorted is obtained by use of a capacitor.

And this open condition is obtained, because we have to bias this properly. So, we cannot just left it open. So, this open condition can be obtained by using an inductance of appropriate value in series with this. So, use of a capacitor at the output, we can establish short conditions; and by using an inductance of appropriate value in series at the input, we can establish this open conditions.

This is being said repeatedly that, different equations are invariant whatever expressions which we will be deriving for example, for voltage gain, current gain, input impedance and output impedance; these equations we would be deriving in general and then, we will see that.

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That  $h_{ib}$ , this represents the input impedance in the case of the common base and this is certainly not equal to  $h_{ie}$  that means the hybrid parameters, the same parameter in different configurations common base, common emitter and common collector, they are different. And we will see that, how we can get the value of the parameter in other configuration like common base or common collector, because the manufacturers specify

the h parameters only for common emitter; because common emitter circuit is the one common emitter amplifier which is most widely used, this we have been talking repeatedly we have been saying repeatedly.

Now, before we proceed for the analysis, there are certain thumb rules of the analysis. So, these at least two points let me make it clear that. For AC analysis two things we should keep in mind that number 1, all DC sources **all DC sources** have to be reduced to 0, are reduced to 0. So, voltage sources are ground DC voltage sources are grounded, and DC current sources are opened. This is the rule of the analysis.

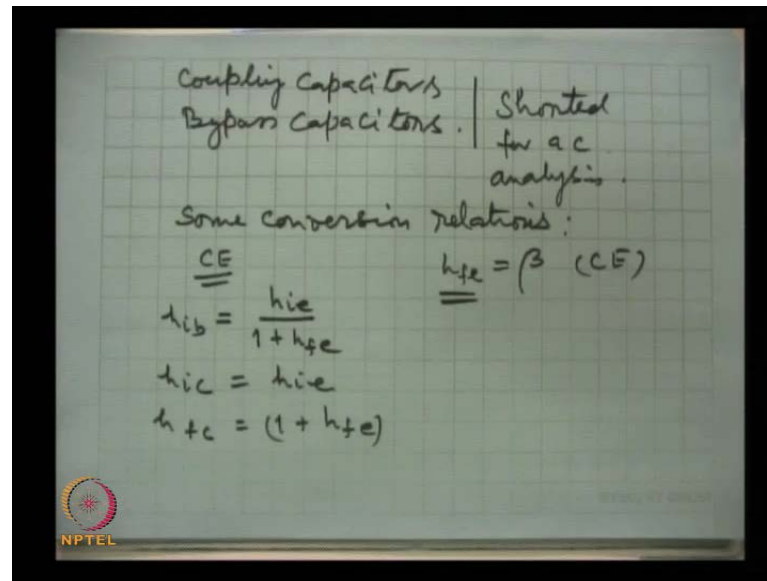
And when of course, when we want what is the effect of this D C, we can use super position theorem **super position theorem**. It is very widely used super position theorem, is very widely used in the analysis of electronic circuits. And this we will see when we come to this point. So, this is first point.

Another important point is for AC analysis, capacitors **you know** that when electronic circuits, capacitors are used in two ways. One is as a coupling capacitor, this is the circuit and the signal is applied, this is AC signal through a capacitor. So, this is known as coupling capacitor.

Similarly, at the output of the circuit when we connect the load this is normally this is the load  $R_L$ , where we take the amplified signal; this may be for example, the **the** resistance the impedance of the loud speaker, if this is audio signal amplifier, then the output we will take and that will be this. So, this is connected through a capacitor, which is again a coupling capacitor.

The other use of capacitor is bypass capacitors for example, the emitter resistance is often bypassed; so, that AC signal goes through this capacitor and we save that drop AC drop across this resistance  $R_E$ , because here the drop of AC signal will be a net loss. And hence, the gain will fall, this we will talk in details later. But, this way when the capacitor is used, this is called bypass capacitor. So, remember the two ways capacitors are **are** used.

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These are coupling capacitors and bypass capacitors. So, AC analysis needs and this is the requirement as the first requirement was that, the DC sources have to be reduced to 0. Similarly, these capacitors are all taken as shorted capacitors these are shorted for AC analysis. Now, I give you first some conversion relations **some conversion relations** that is I said that, common emitter is most widely used amplifier.

And h parameters by the manufacturer are provided only for the C E circuit Common Emitter circuit, but there are the expressions have been derived that, we can get without further measurement; we can use those common emitter parameters to get parameters for other circuits; like  $h_{ib}$  that means input impedance,  $h_i$  input impedance in common base is related to  $h_{ie}$  by  $1 + h_{fe}$ . This is same, it gives a very important information that,  $h_{fe}$  as we will recall  $h_{fe}$  this is the current gain measured under the appropriate boundary conditions; and this is the same beta which we have been talking beta, beta is the current gain in common emitter circuit. So, this represents the beta **beta**.

So, the input impedance in common base is heavily reduced as compared to the input impedance of common emitter circuit. If this is for example, 100 then this input impedance in common base is reduced by 2 orders of magnitude to that, what we will observe in common emitter. So, similarly,  $h_{ic}$  is equal to  $h_{ie}$ ; and  $h_{fc}$  is  $1 + h_{fe}$ , this way actually all quantities have been calculated, and we can get h parameters for common base circuit and common collector circuit by using h parameters of the **common**

**collector** common emitter. Now I give you the typical values of the h parameters in three configurations.

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Typical h-parameter values :

Parameter	CE	CC	CB
Input impedance, $h_i$	$2.5\text{ k}\Omega$	$2.5\text{ k}\Omega$	$20\Omega$
Current gain, $h_f$	100	101	0.99
Reverse voltage gain, $h_r$	$2 \times 10^{-4}$	$\sim 1$	$2.2 \times 10^{-4}$
Output impedance, $\frac{1}{h_o}$	$30\text{ k}\Omega$	$30\text{ k}\Omega$	$2\text{ M}\Omega$

So, typical h parameter values, so here is the parameter, and here for C E configuration, C C configuration and C B configuration. If this parameter is input impedance  $h_i$ , so here this is having this is order mind it, this may vary from transistor to transistor, but to give you an overall idea about these quantities. So, I am giving this typical values 2.5 kilo ohms; while in common collector this is as I said here (Refer Slide Time: 44:52), input impedance in common collector is same as input impedance in the common emitter. So, this is also having 2.5 kilo ohms; and this is highly reduced this is 20 ohms only.

And the current gain **current gain**  $h_f$ , this is around 100, this is 101 same as for common emitter, but this current gain; you remember alpha in the common base, which is close to 1, but less than 1, so this is 0.99. And reverse voltage gain, which is  $h_r$  this is very small quantity  $10^{-4}$ , here it is around 1 and here also  $2.2 \times 10^{-4}$ .

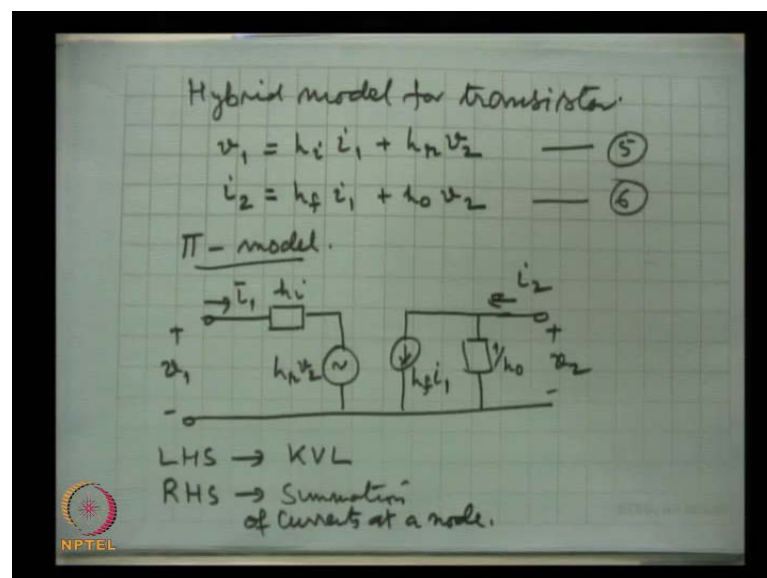
And then, output impedance **output impedance** is inverse of admittance. So, output impedance, which is  $1/h_o$  this is around 30 kilo ohms, same here 30 kilo ohms, but here it is much higher 2 mega ohms. So, these are the h parameters typical values of h

parameters and three circuits. And briefly, we can see in when we cascade when we use multi stage amplifiers, which is very commonly done in systems.

Then these input and output impedances here, they are drastically different that means two common base circuits cannot be connected together, because of the large difference in the input and the output impedance; you must have done a power transfer theorem, which indicates that from one circuit to the next circuit, power transfer is most efficient when the impedances are matched **when impedances are matched**; but here this is complete mismatch, here they are not identical, but order is the same it is 2.5 kilo ohms, it is 30 ohms, order is the same.

So, several common emitter amplifiers can be cascaded without involving a buffer stage, which is kind a transformer stage, which will be required for matching; this much is good enough, they are having similar magnitude, it is not drastically different from matching angle. So, this is additional advantage plus in common base, the current gain is very small. Current gain as we will see will enter even in voltage gain, so here because the current gain is a small. So, this is another drawback with the common base. Common base circuit is having only few applications in very high power circuits.

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Now, the hybrid model hybrid h model, hybrid model for transistor for example, the two equations are,  $v_1$  this is  $h_i i_1$  plus  $h_r v_2$ ; and similarly,  $i_2$  is equal to  $h_f i_1$  plus  $h_o v_2$ . These equations we will be now using and let us call these equations, and **and** from

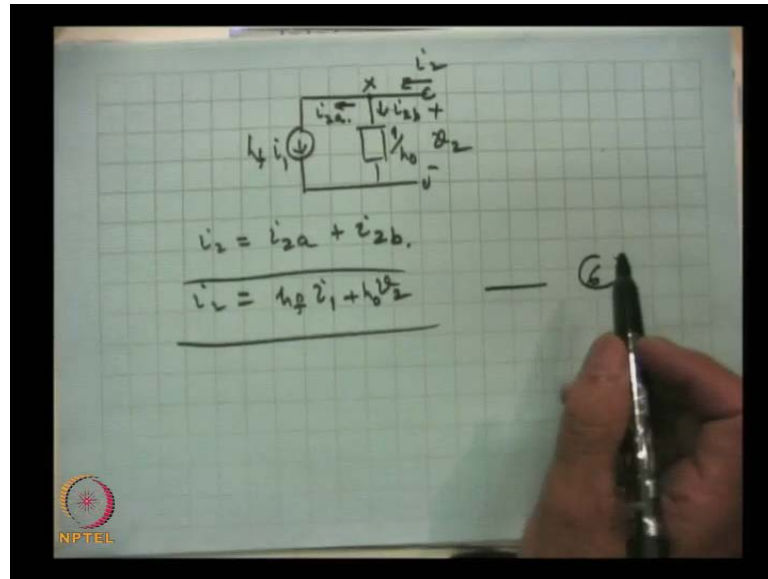
here, this is equation 5 and this is equation 6; and from here, we can develop the hybrid model, this is actually pi model as we call pi model.

And let us see, this model comes out to be like this (Refer Slide Time: 50:26), (No audio from 50:27 to 50:44) this is inverse of the admittance, so this represents the impedance; here is the voltage  $v_2$ , the current  $i_2$ , here this is  $v_1$ ,  $h_i$  and current  $i_1$ , and this is a voltage source  $h_r v_2$ , and this is the current source  $h_f e$   $h_f$  and  $i_1$ . Now, how (( )) arrived at this model, the first equation this we get from the left hand side of this model pi model; if we use this left hands side then, this is exactly what this equation is.

This is left hand side that, Kirchhoff's summation Kirchhoff's voltage law for any loop, the summation of voltages have to be equal to 0; or they applied voltage has to be taken as drop at at all places. So, this will give  $v_1$  equal to the input impedance is  $h_i$ . So, the voltage drop here will be represented by the first term. And here this is as I said, voltage source which is there, because of the in in transistor, the two junctions do not function in complete isolation. It is very important. What you are doing at the output side also reflects at is reflected the input side. So, this is that reflection a feedback term.

So, anyway this is the first term in equation 5 and this is the second term. So, left hand side gives you that the first the equation 5. And this equation, the right hand side this is summation of currents at node summation of currents at a node for example, let us elaborate this point that how we get this equation; and this is the right hand side actually represents this equation 6.

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And this is let us consider just the right hand side and a node that this is the node **this is the node** at a point x and this is  $v_2$ , and this is  $1/h_o$ , the impedance. Now, this current  $i_2$  will be divided into two branches here. So, this is **this is**  $i_{2b}$  and this is current  $i_{2a}$ , then simple summation gives that  $i_2$  is equal to  $i_{2a}$  plus  $i_{2b}$ . And  $i_2$  is equal to this is given by this,  $h_f i_1$ ; and if this is impedance, then what will be the current here you know that, if the voltage  $v_2$  is there, then  $v_2$  divided by  **$v_2$  divided by** this impedance and that will be  $1/h_o$ , so  $h_o$  will come here (Refer Slide Time: 55:19). And this is exactly what the equation 6 is here.

So, we have arrived at the model, and this model is to be used for the analysis of small signal amplifiers that we will continue, I hope you have followed that h parameters their definitions, and then this is the model which we get on the basis of these two equations, and which can be used for the small signal amplifier analysis that we will continue in the next lecture.