Group Theory Methods in Physics
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Lecture – 57
Symmetry breaking in continuous groups

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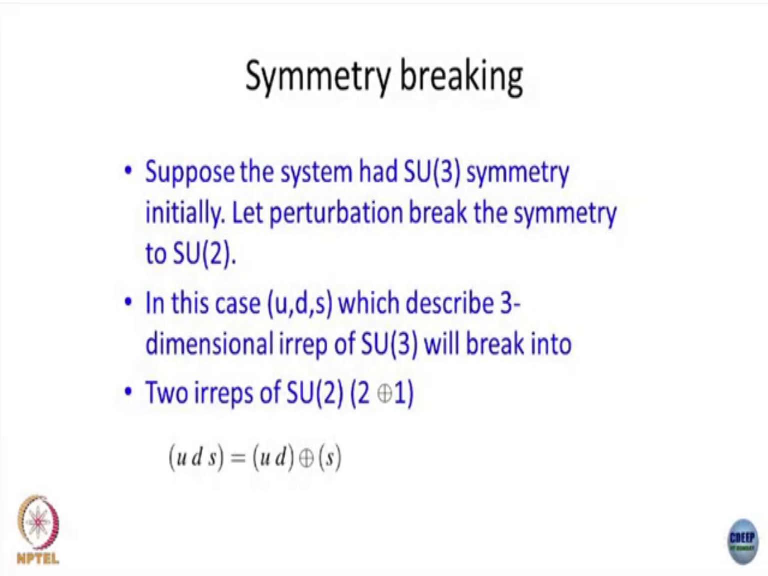

So, I kind of explained tensor product of defining representations tensor product of irreducible representations like decimate baryons right or octet baryons. One of the problems I asked you to continue and finish it off how many of you did it. Nobody not a single person has done it. So, I will give it in the exam. That is a good way to make you people work ok. So, I am not going to do it for you, but it will be there in the exam.

So, a couple of things just like we started with in a discrete group you have a bigger symmetry like a tetrahedral symmetry. And then I said there is a defect or a perturbation which breaks the symmetry to a lower symmetry like C 3v and then we were looking at with respect to C 3v the irreps of t d r all reducible or irreducible? With respect to C 3 v they are reducible right. Then we broke them broke the irreducible representation of t d into irreps of C 3v.

So, that is what is the symmetry breaking which we studied in the context of discrete groups. You all remember. So, now, the same thing you can do in the context of continuous group zones. So, you start with a system which has a SU 3 symmetry right. What do I mean by saying a system has SU 3 symmetry? The Hamiltonian if I write a Hamiltonian.

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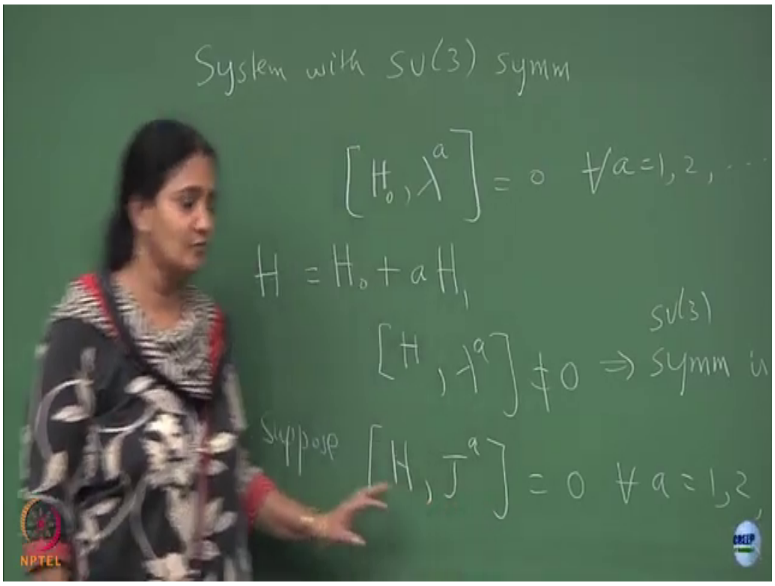

So, system with SU 3 symmetry means Hamiltonian for such a system should commute with all the generators of SU 3. Let me call those generators as lambda a just to remember that they are. There will be Gell-Mann matrices for defining representations. Then other representation there will be higher dimensional matrices, but satisfy the SU 3 algebra right.

So, all these things should be 0 for all a 1, 2 up to 8 of them ok. There will be 8 generators. So, let me put this to be the starting Hamiltonian which I call it as H naught. Let me perturb this H naught by some constant small value. Let me call it as some a which is a small perturbative parameter times H 1.

This is the perturbation right. We add a perturbation to the Hamiltonian and claim that the system is no longer having SU 3 symmetry. It is like adding a perturbation and claim is that this let me call it as total Hamiltonian is not equal to 0. At least for one a if you find this not equal to 0 you can say that at least for even one generator of SU 3. If the commutator is not 0 then you say that, this implies symmetry is broken.

SU 3 symmetry is broken right. Even for some generators if this happens then you say SU 3 symmetry is broken suppose you find that the H the total H where you have done the perturbation is such that the SU 2 generators are 0. Suppose you are all with me. SU 2 generators are your J x, J y, J z. You can call J 1, J 2, J 3. Suppose this is happening then what do we say. After you added a perturbation the symmetry of the system is SU 2 ok.

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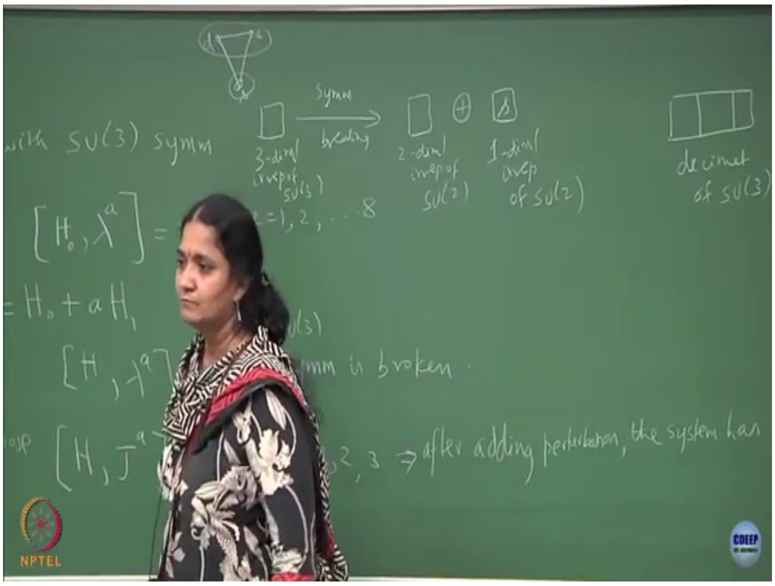

So, this implies after adding perturbation the system has SU 2 symmetry. So, what have I shown you started with a system with SU 3 symmetry you added a perturbation and the perturbation broke the SU 3 symmetry which has rank 2 to a lower symmetry group which has rank 1 which is SU 2 clear. Now if I take an irrep belonging to SU 3. So, let us take an irrep which is the defining representation to start with, this irrep under symmetry breaking.

So, this is 3 dimensional of 3 dimensional irrep of SU 3 SU 3. What happens to it. It breaks down to a 2 dimensional irrep SU 2 plus there is a box with s. This has nothing to do with up down quarks. This will behave like a singlet or a 1 dimensional irrep of SU 2.

Now, your group is SU 2. Whatever irreps which you are studying will be reducible representations as far as SU 2 is concerned clear. As far as SU 2 is concerned every irrep of SU 3 is reducible and the breaking is such that whenever you find an s squawk that is going to be treated like a trivial representation of SU 2 ok. So, this is the breaking pattern the 3 dimensional representation irrep. Once you add a perturbation which breaks the SU 2 symmetry breaks a SU 3 to SU 2 then you get this to be 2 plus 4 clear.

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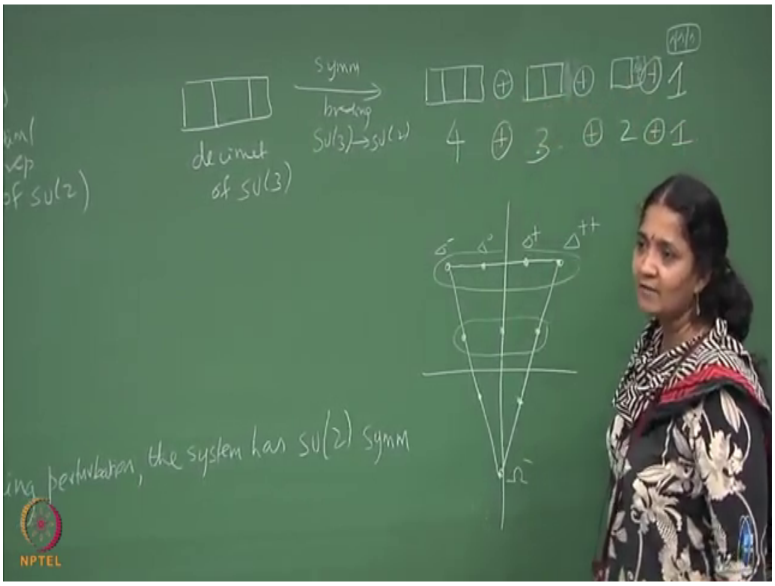

So, now tell me what happens to the decimet. This is decimet of SU 3 ok. When you do a symmetry breaking of SU 3 getting broken to S U2. That is what I mean here in this particular examples what happens. How does it break? You will still get this diagram, but the entries in the diagrams are only u and d. There will be one more diagram where one of the box has s squawk I remove it. That is like a singlet. Two of the box can have a s squawk clear. So, that is like a singlet a single box with s squawk is like as if it is nothing and what else. All the 3 boxes could have s squawk which is like a trivial representation ok.

Let me call that as 1 dimension clear. So, how much is this dimension. Spin it is like spin 3 by 2 which is 4 dimensional. This one is 2 no this is 3 right spin 1. Are you all with me and then this is 1. No this is 2 and this is 1. So, the total dimension should add up 2 plus 1 is 3. The 10 this is an irrep of SU 3 adding a perturbation will break it up into irreps of SU 2 ok. Another way of seeing it in the diagram is that you remember the diagram. Some kind of a diagram like this with 4 points are there 1, 2, 3 4.

So, this line is like a SU 2 sub algebra. These 4 are irreps of the residual s 2 SU 2. Then you have the other 3 particles which are here which is again another irrep of SU 2 then you have 2 particles here. I think this one is your delta plus plus delta plus delta 0 delta minus and so on. There is sigma and then cascade and then this particle is what I explained is omega minus.

So, it is actually breaking up into various pieces in the horizontal line as 4 plus 3 plus 2 plus 1. So, SU 3 irrep under perturbation will break up into 4 plus 3 plus 2 plus 1. That is what happens even in the fundamental representation right. In the fundamental I can draw like this. You had an up squawk a down squawk and a strange squawk. So, it breaks up into 2 plus 1 ok. So, this is a way to see the symmetry breaking in the context of continuous (Refer time: 12:28).

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How do you break up? No I said know strange squawk is always like a singlet. In the SU 2 context any an amount of strange squawks you add it is not going to show up in the representation. So, in the 3 boxes which you put here these 3 if it does not have a strange squawk then you can find the dimension of this to be 4. If you had one more box with strange squawk it is like nothing. Similarly you can add 2 boxes here with strange squawk that is like adding nothing as far as SU 2 is concerned ok.

This one is all the 3 boxes as s which is like nothing. After you have done it then you use the same Young diagram language for SU 2 to find the dimensions, but this is the breaking pattern. Irreducible SU 3 representation becomes reducible for SU 2 and how in the reducible representation how much it breaks into a 4 dimensional representation yes. Once it breaks it can break it into a 2 3 dimensional representation yes, 2 dimensional representation yes under 1 dimensional representation.

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So, in discrete case there it was only a character table. Here I am looking at not a character table, but all possible dimensions for the SU 3. So, once I take that then when I break it up if there are 2 possibilities then you can get that dimensional representation twice. Otherwise you will always have it to be once right. Do it for the octet. Just do it for the octet and see what happens ok. How does the 8 break. From the diagram you know how it breaks. It should break into a 2 then a 3 and another 2 and at the center you should have 1 right. You know that for the octet which is our joint representation. So, you see whether this argument helps you to see that (Refer time: 14:59). It is that clear.

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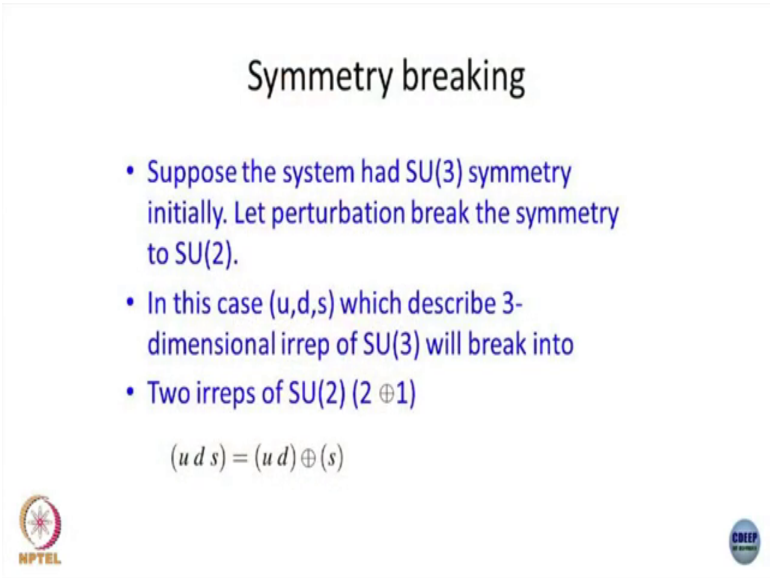

Suppose the system has SU 3 symmetry initially let perturbation break the symmetry to SU 2 then in this case u d s which describes a 3 dimensional irrep of SU 3 will break into 2 irreps of SU 2 one is 2 dimension another one is 1 dimension. I showed you in the Young diagram language you can also write it compactly like this that the u d s vector space breaks into a u d vector space and another 1 dimension vector space.