Group Theory Methods in Physics
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Lecture – 56
Higher dimensional multiplets in the quark model

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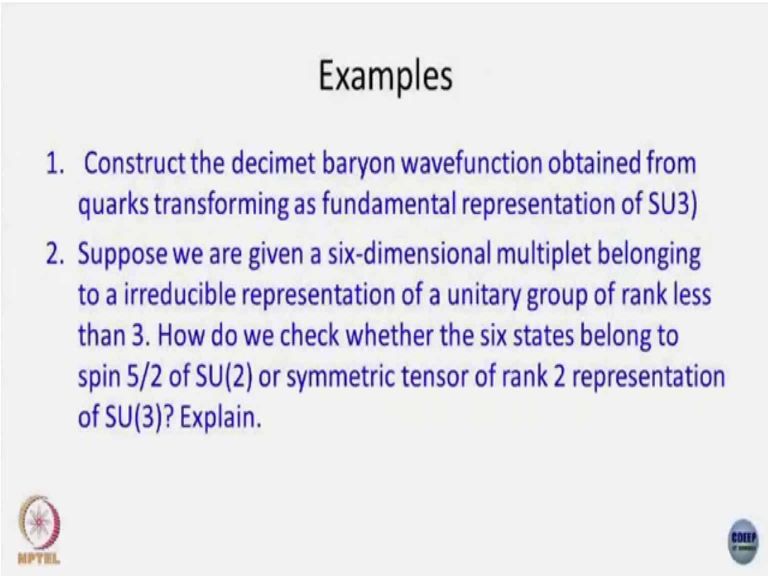

So, this second question. Suppose I give you a 6-dimensional multiplet. Yeah, I do not tell you whether I am looking at SU 2 group or an SU 3 group. I say the dimension of the multiplet is 6-dimension and then I also say that the rank is less than 3, the group it is a 6-dimensional representation of a group with rank less than 3 which means I could be in the unitary context it could be either SU 2 or SU 3. I do not know whether it is SU 2 or SU 3. If it is SU 2 then what is the diagram for 6-dimensional representation? Tell me.

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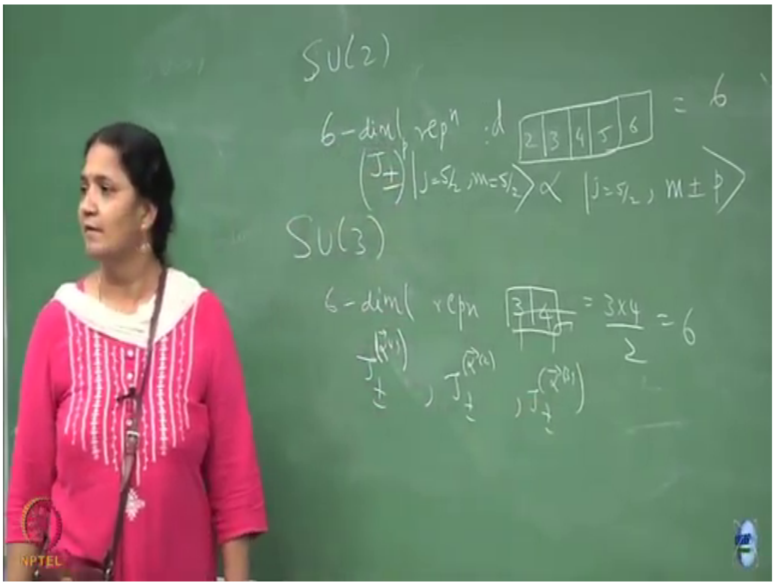

So, let us take SU 2. 6-dimensional representation as a young diagram I denote it by 5 boxes. Is that right? The dimension of this will be 2, 3, 4, 5, 6 with the hook number. So, the dimension of this is 6. So, this is 6-dimensional representation. SU 3 6-dimensional representation is. Is that right? This is 3, 4, hook number is 2 I will give you. Are you all with me? If you have not done this then yeah, so this gives you 6, both have 6-dimensions

So, suppose I give you in your hand a 6-dimensional representation I ask you does it belong to SU 2 or SU 3, what will you do? How do you say that the 6-dimensional representation is a spin pi by 2 particle or it is a symmetric rank 2 tensor which is like a die quark system of SU 3? How do you, how do you decide this? Tell me what is the distinction?

Suppose, you use the ladder operators here, suppose the J plus and J minus acting on the highest weight state; highest weight state will be j equal to 5 by 2 m equal to 5 by 2 if it generates for you, by doing this N number of times. How many times? Up to a maximum p should be 5, right. Am I right?

So, by doing this if you can generate the states then you know that, right, proportional to this then you know that it belongs to the SU 2 multiplet, clear. The generators should take you through a linear you should take you to all the other states in the 6-dimensional vector space. If it can do the generator of SU 2 are only J plus and J minus, if you can span all the 6 states using only J plus and J minus you can blindly say that you are in SU 2, given a 6-dimensional vector space the states can be spanned using the generators of SU 2 then you know it is a 6-dimensional vector space of irreducible representation of SU 2.

But if you do SU 3, if you need the other generators you are not able to get all the 6 states, you get only 3 of them let us say by J plus J minus, but you are not able to generate some of them using only J plus J minus, then you are forced to look at; what are the things. You will have J plus J minus with alpha 1, right that is like J plus J minus then J plus J minus with alpha 2 and J plus J minus alpha 3.

If you need these to span the 6-dimensional vector space, you cannot just work with only one of them, to generate 6 of them, then you know that the group requires a generators of SU 3, clear. So, that is the way one should distinguish whether a vector space belongs to SU 2 or whether the vector space belongs to SU 3, the dimension does not decide to see whether you can span all the states in the vector space using the generators of that group, clear.

So, that is the question it is asked here in the question and I have explained it on the board for you, ok. So, suppose you are given a 6-dimensional multiplied belong to an irreducible representation of a unitary group of rank less than 3, how do you check whether 6 states belongs to spin 5 by 2 of SU 2 or symmetric tensor of rank 2 of SU 3 depending on whether you can achieve all the states using only J plus J minus or you need the other 4 generators, ok, yeah.

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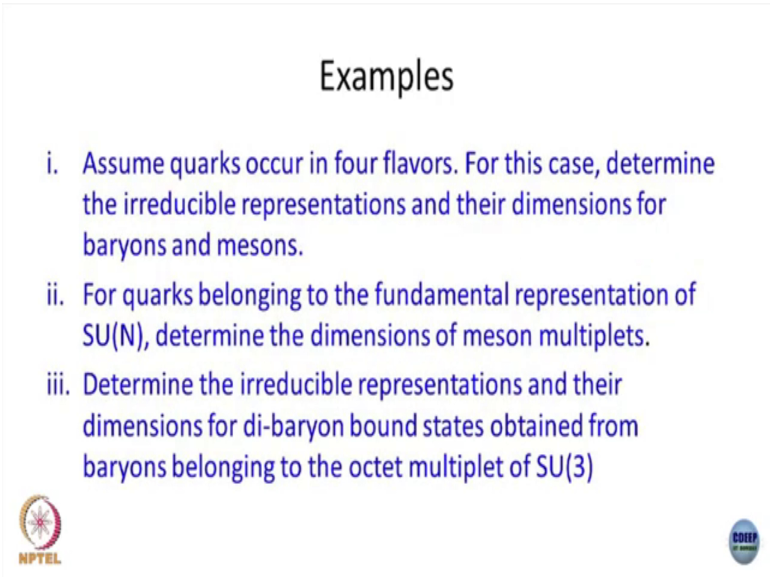

So, now the next question. We were going now going to do a hypothetical thing. Like the way we did rotations in 3-dimension then I went on to abstract notation of saying let us look at rotations and arbitrary in dimensional abstract space, we will start looking at that let the quarks does not need to be only in 3 flavors, let it have 4 flavors, up, down, strange and charm. This is extrapolating from SU 2, SU 3, to SU 4. The fundamental objects which you can put on a single box will be either up, down, strange or charm, ok. Is that clear?

So, assume quarks occur in 4 flavors for this case determine the irreducible representations and their dimensions of baryons and mesons, ok. So, baryons you know what to do. Baryons are made up of 3 quarks, right. So, suppose technically you know baryons should be always made of the flavor is N it should be like N minus 1, but let us just take a composite object made of 3 quarks. Let us see how the product should be. We have already done this.

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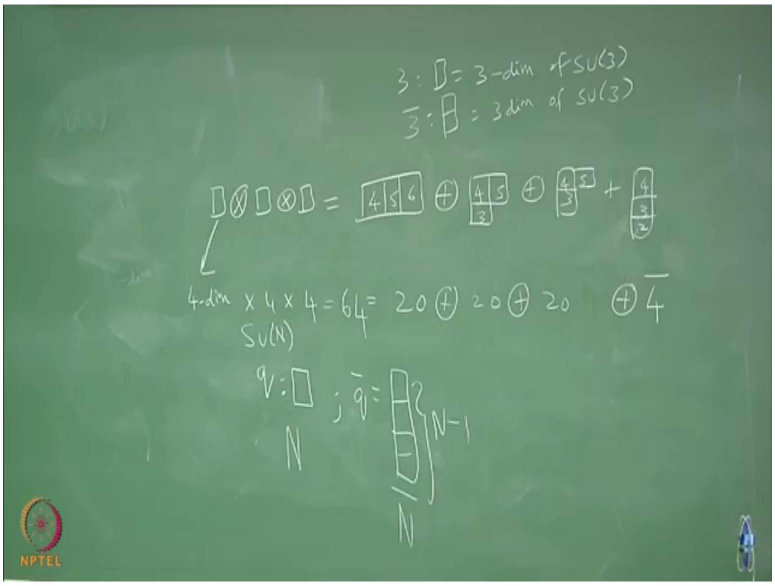

Suppose I take this, ok. What will be the dimensions now? If I say that this is 4-dimensional, 4 into 4 into 4 will be 64. So, one can work it out 4, 5, 6 that is 20, hook number is 6. What about here? Hook number is 3; 4, 5 and 3; 4, 5 and 3; 4, 3 and 2. So, this is 4-dimensional and this one is 3 s by the hook is gone, so it is 20 plus 20. Is that right? So, this is your 64 dimensional representation.

Incidentally, this is 4-dimensions. You remember SU 3 quarks and anti-quarks. This was 3-dimension of SU 3, this one is SU 3 which we call it as 3 bar. So, any vertical box of N minus 1 boxes if you are going to call it as 4 bar, ok. So, that is the anti-quark representation.

So, given a quark representation, if this quark is represented by this q bar is represented by N minus 1 boxes, both will have the same dimensions. If this dimension is N, right for SU N this dimension will be N and we denoted by N bar just to remember that it is a anti-quark or antiparticle of quarks, ok.

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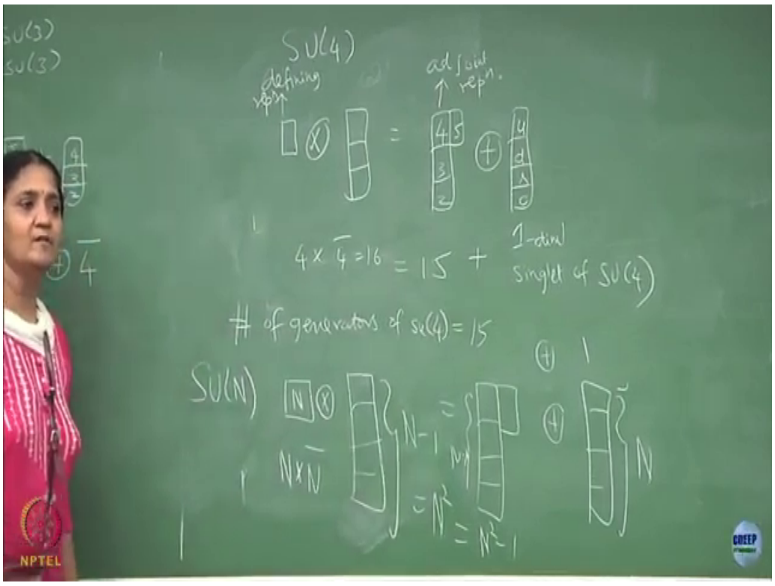

So, now mesons will be what? Mesons are made of quark with anti-quark, but now I am doing it for SU 4. SU 4 this is the anti-quark and this is the quark. What are the diagrams possible? What is this diagram? That is the trivial representation because if you put up, down, strange and charm. So, this is one-dimensional singlet of SU 4, correct.

What is the dimension of this? Someone; is that right? What does 15 dimensional mean? 15. How many generators are there for SU 4? SU 4 is N squared minus 1, unitary matrices with determinant plus 1 corresponding to the generators which are traceless and Hermitian turns out to be N squared minus 1, right. So, that is 16 minus 1 which is 15, that is what I call it as the number of states 15 dimensional states. There is a one to one correspondence to the number of generators what is that representation called? Adjoint representation that is what I called, right.

Like a regular remember regular representations in your discrete groups where the group elements decides the dimensions of the matrix representations, the number of elements here denotes the number of generators, so this is what we call. So, this we call it as a defining representation and this we call it as adjoint representation. What is the beauty of this? The weight vectors here are exactly the root vectors, clear. So, that is the other problem which I had asked you to look into.

For quarks belonging to the fundamental representation of SU N, determine the dimensions of meson multiplets. Quark belonging to SU N whatever I did here you can do it for SU N. What will be the anti-quark? N minus 1, ok. And then what do you get? With a single box plus all of them put together, always. What is this? This is SU N, this is the trivial representation or a singlet representation. So, this will be 1 and this adds up to N squared, right, N into N bar, this is N bar, that is N square and if I want to write the dimension of this it has to be this is 1, has to be N squared minus 1.

So, it is always the adjoint representation plus the singlet. There will be only two multiplet us for the mesons which is made of composed of two fundamental objects one is a quark another one is an antiquark and what is the dimension of the quark representation depends on how many flavors you allow, fundamental flavors. If you have allow 3 it is SU 3, if you allow 4 it is SU 4 and then for a general N flavor system you can work out what are the irreducible representations of the mesons which are bound states of quarks an anti- quarks, is this clear, ok.

So, as a last problem. These are all there in the assignment problem, ok. I am not doing anything new and that let me take it like a tutorial. So, that you are clear about things. What is the third problem? Third problem is determine the irreducible representations and their dimensions for dibaryon bound states. Now, I am looking at a dibaryon bound states obtained from baryons belonging to the octet multiplet of SU 3 what is the fundamental object baryon is a fundamental object and I am going to look at a bound state which is a dibaryon bound state. Is the question clear?

So, what is the procedure? Here I looked at bound state of 3 fundamental quarks. Now, I want to look at bound state of two baryons which belongs to the octet. Octet diagram is this, right. So, let us do that for the SU 3.

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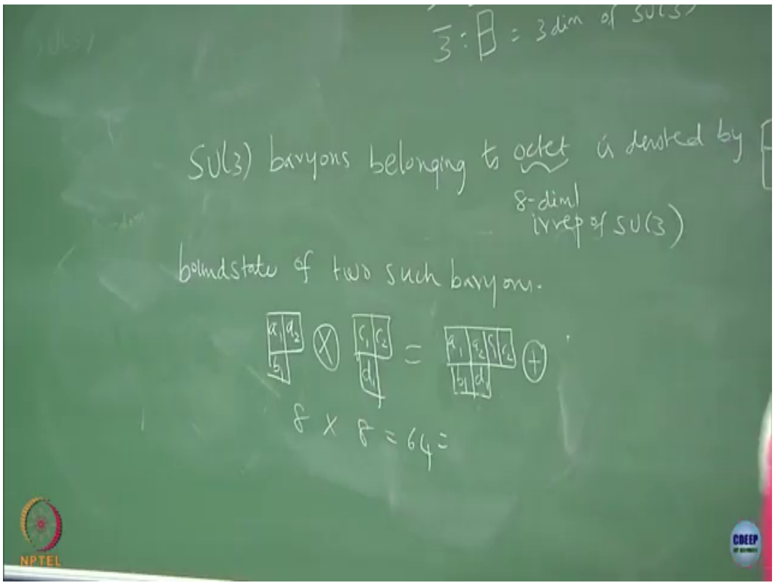

So, baryons, SU 3 baryons belonging to octet as denoted by this diagram, ok. So, octet is a irreducible representation, ok. This is 8-dimensional irrep of SU 3, using all kinds of notation, so that you do not forget the group theory jargon.

Now, we want to look at bound state of two such baryons, ok. This belongs to a 8-dimension another baryon will belong to a 8-dimensional irreducible representation I have to take a tensor product of these two, ok. Now, you are to struggle a bit to draw the diagrams without violating the symmetries. Earlier it was simple because you know these 3 objects are anti-symmetric, this is has no symmetry to it, you could just attach this to it or put it below for simple. Now, you have to play around making sure that if you call it as a 1, a 2, b 1, c 1, c 2, d 1, try and start mixing things here and draw the diagrams, ok.

So, let me give you some simple diagrams. This is one possibility. Why is it possible? d 1 has no symmetry to c 1, it is ok. No symmetry means it can be either symmetric or anti-symmetric. So, this is a allow diagram, ok. So, now, you have to list out all the diagrams I leave it you to do the listing. One way of checking that this listing is exhausted without violating the symmetries.

Remember that a 1 and a 2 symmetric, a 1 and b 2 is anti-symmetric. You can make it no symmetry, but make sure that you do not make a symmetric one to look you cannot put a 1 and below a 2 you understand, right. So, take care of the symmetry properties of the fundamental objects combine them and write out all the irreps. Each irrep will be an irreducible representation of a dibaryon whose fundamental objects are octet baryons, ok. And this should be such that what happens to the dimension? 8 into 8 is 64. After you list it out write down the dimensions, for check whether you get 64 into total. Yeah.

Student: (Refer Time: 22:27).

No, no, no. Here when I put d 1, a 2 is anti-symmetric to d 1, a 2 has no relation to a 1 a 2 has no relation to this, but c 1 is anti-symmetric to d 1. If I put the d 1 here, it is not below this. Once I put the d 1 here, c 1 has no symmetry relation to d 1. No symmetry relation can be either symmetric or anti-symmetric, it will always have a non-trivial projection for anti-symmetric. So, this is allowed. But you should not put c 1 and on the side of it at d 1. So, this is not allowed. You understand what I am saying?

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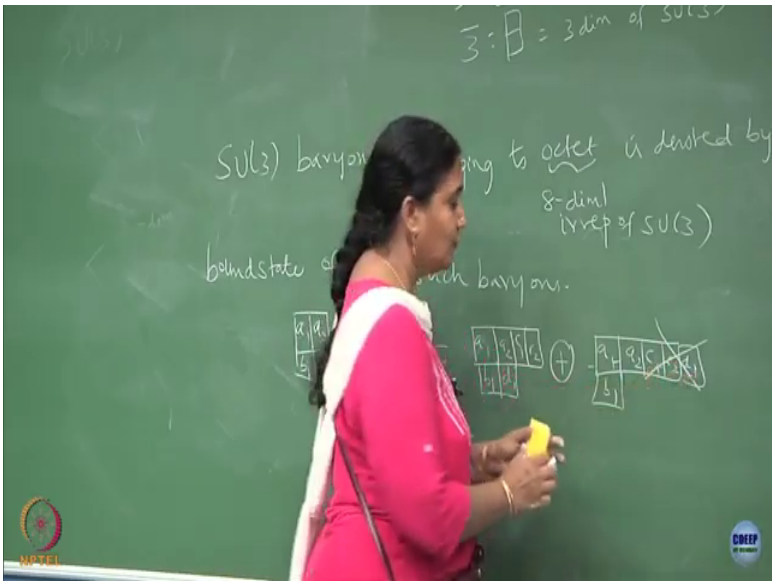

So, suppose I want to put something like this, I cannot put this, this is not allowed. That is distinctly making it symmetric which is not allowed. So, this diagram is not allowed. No symmetry is allowed, ok. So, take care of it and write it out and see whether the total adds up to give you 64.