Group Theory Methods in Physics
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Lecture - 55

1) Quadrupole moment tensor
(Wigner - Eckart theorem)
2) Decimet baryon wavefunction

So now, let me get to the problem of Quadrupole moment tensor ok.

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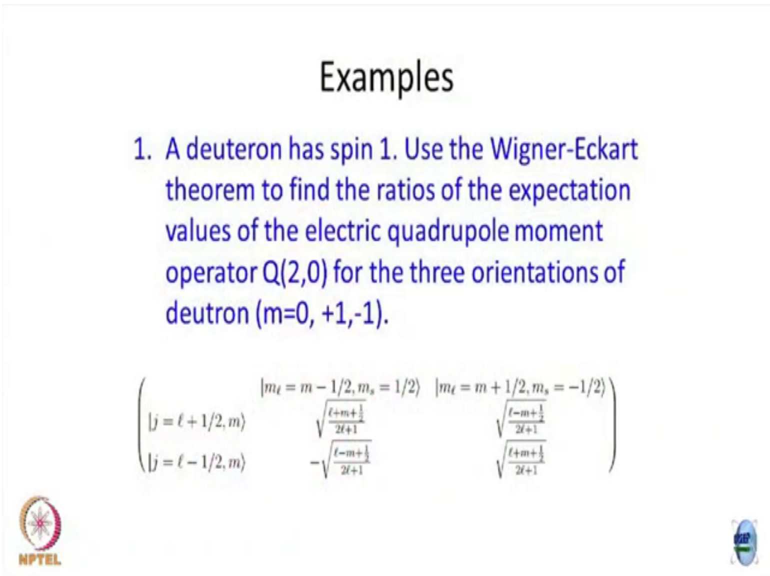

So, the deuteron has spin 1, use a Wigner Eckart theorem to find the ratios of the expectation values of electric quadrupole moment operator. Specifically I have taken the component; it is a rank two tensor. I have taken its Q value rank two tensor with Q equal to 0 for the three orientations of deuteron ok. So, that is the question. So, for this c g coefficient is not really applicable here because this one was a linear combination of one was l equal to 1 and the other one was l equal to half. So, let us not use this ok. But what will this b is the question using Wignar-Eckart theorem.

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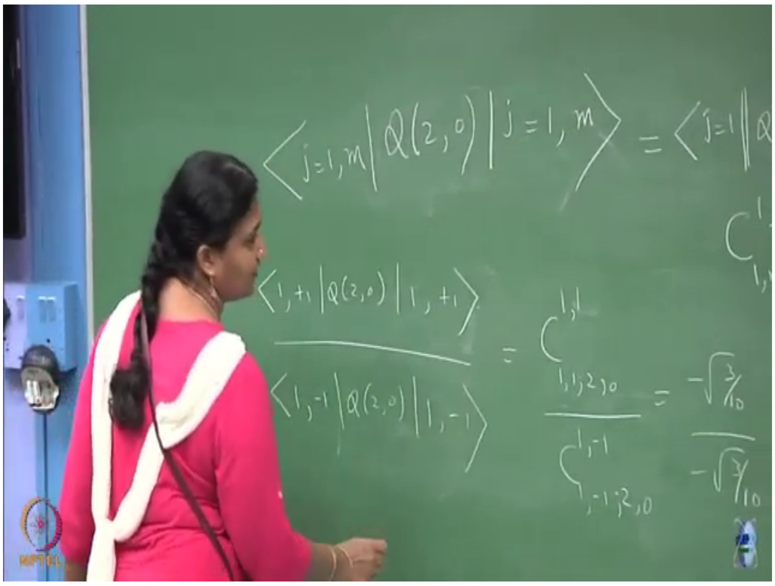

So, how do you go about this? So, the deuteron is given to you as j initial equal to 1 and m and you have an operator which is the quadrupole moment tensor operator and you are asked to find the expectation value; expectation value means it is the same state ok. So, I can forget about this i, j equal to 1 m I want to find what is j equal to 1 m ok. So, to find this using Wigner-Eckart theorem; using Wigner-Eckart theorem you can work out this expectation value to be some constant as I said which depends on the total j 2 and 1 which we call it as a reduced matrix element

So, j equal to 1 Q operator which depends on 2 j equal to 1 which is some constant, I do not know what it is, but I have a C g coefficient. The C g coefficient is going to combine this angular momentum with this angular momentum. So, it is 1 comma m semi colon 2 comma 0 right and the final stage should also be 1 comma m, clear. So, this is the expectation value. This coefficient is always same irrespective of what value of m is there; things which are going to change as only the C g coefficients for different depths.

So, if I ask you the ratio what is the ratio for m equal to 0 plus 1 and minus 1. So, that ratio will be what? So, you can write it out expectation value between 1 plus 1 Q 2 0 1 plus 1. What will this p? This is just going to be the ratio of your C g coefficients. This is 1 1 for these two orientation. Similarly, you can find it for the other one and you can write it as a all the possible ratios. The C g coefficients just like the table I gave you, I explained to two of the tables for you. This table will be given to you and you have to only figure out what is that coefficient from the table and substitute it ok.

So, the coefficients actually turns out to be. So, let me just give you the answer, it turns out to be this ratio turns out to be 1.

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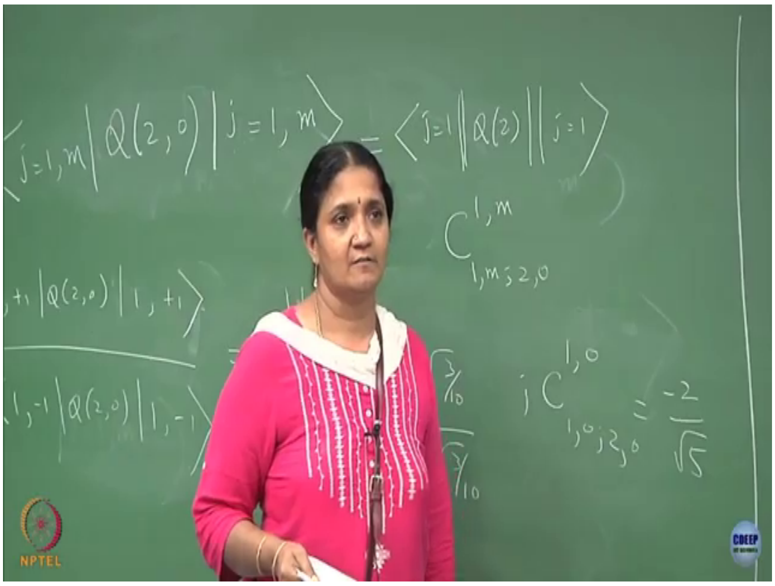

So, essentially it is minus root 3 by 10 and this is also minus root 3 by 10 ok. But if you want to find 0; if you put in 0, then you get that coefficient C g coefficient is, this c g coefficient is minus 2 by root 5 ok. So, the table will be given and you can look at the table and find out what the coefficients are. So, if you want to find the ratios of all the three orientation, it is like 1 is to 1 is to this fact ok, this is clear

So, experimentalists; if they give you one if they give you this result to you, then you do not even need the experimentalist to determine the coefficients for the other orientation Wigner-Eckart theorem will fix, it is that clear? If we gives Q to 0 expectation value for a deuteron with m equal to plus 1, then the rest of the 2 orientations, you do not need the experimental data. In fact, you can make sure that you can validate the experimental result by showing as Wigner-Eckart theorem has actually given that ratio correctly ok.

So, that is the power of the SU 2 group symmetry of most of the spherical systems. Any spherical systems if you are looking at, they are given by a potential which is like a 1 over r potential right. All these systems have rotational symmetry. If it has rotational symmetry, the generators of rotational symmetry in the position space would be just orbital angular momentum. But if you include internal spin space, then it is the total angular momentum. Once you put in total angular momentum, it is called the SU 2 symmetry because j can be half odd integers or integers, is that clear.

So, most of the systems with rotational symmetry, Wigner-Wckart theorem is really powerful to give you the matrix elements or selection rules of transition from an initial state to a final state due to some operator like electric dipole moment or quadrupole dipole moment also, yeah. So, now let us see this interesting thing which I have already given to you in the next problem on the slide. Use the Clebsch Gordan coefficient ok. So, this notation is equivalent to this notation which I have already explained.

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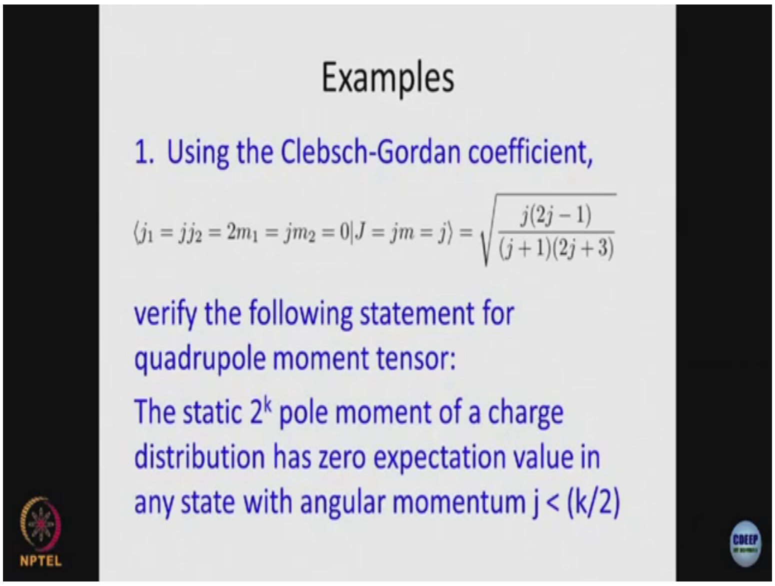

But, let me put that rotation also for you. Sometimes people write this also as a inner product ok.

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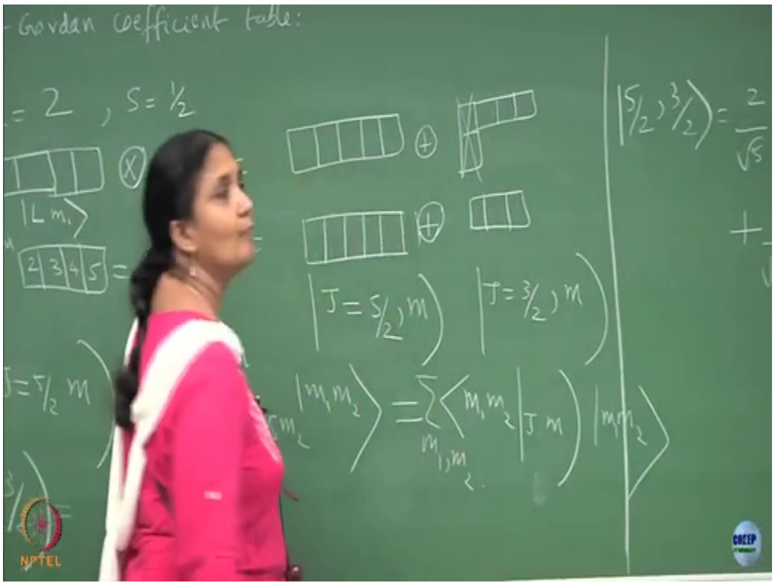

People do write this way; both are equivalent.

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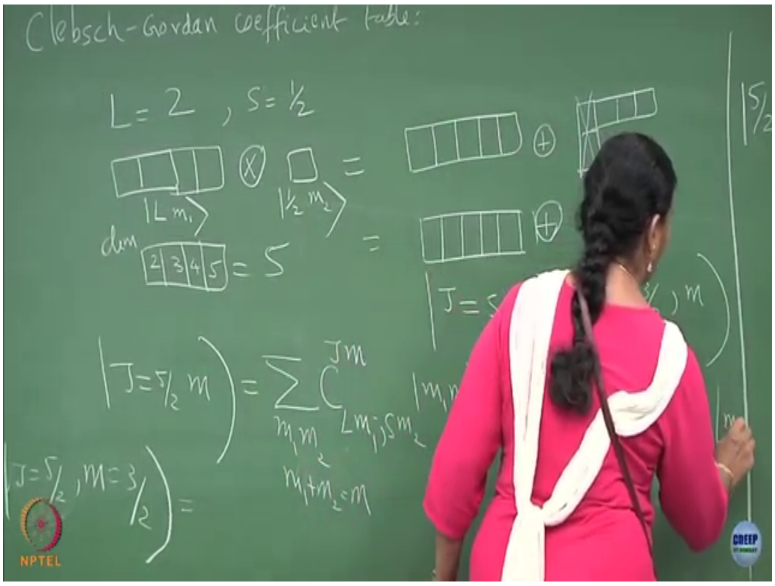

The C g coefficient which I have written sometimes, they write it as an inner product of 2 states ok. So, that is a coefficient which I have given in your problem. Use the Clebsch Gordon coefficient which is given to you and you are asked to verify dipole moment, quadrupole moment and so on ok. So, you have to verify that the static 2 k pole moment of a charge distribution has 0 expectation value in any state with angular momentum j less than k by 2. How many of you have already verify this? So, what is to be done here? You have to find the expectation value. So, let us do the same quadrupole moment tensor which we did here ok

So, if you take the quadrupole moment, it is 2 to the power of k equal to what k is . So, k is 1if I put that in. So, if you take j to be less than 1, you have to get it to be 0, clear. You have to get it to be 0. So, let us check it if you put this to be half, you use this coefficient. The coefficient which is given to you has to be used. If j is half, then you can see that the coefficient itself vanishes, clear; the C g coefficient vanishes.

So, to do an expectation value of Q to 0 between instead of deuteron if you take spin half particle that expectation value will be 0 ok. So, this is what you can check for one or 2 cases from using this coefficient and make the statement that a powerful statement that to find the 2 k pole moment expectation value, it is going to be 0 if the particle has j less than k by 2, this is clear.

Now, I also want you to appreciate not just seeing it as a problem. It is just that you know without even doing computation, you can tell the experimentalist these things you do not even need to measure, you will never see it in the lab; it will be 0 that kind of a thing you can give as an input to the experimentalist. That is why the Wigner Eckart theorem is really powerful, it is not just a passing theorem just stated. It is just working out the C g coefficient and looking at the ratios. So, this is the statement saying that 2 k pole moment expectation value for j less than k by 2 will vanish.

So, you do not even look for all the j's which is not going to show any signature in the lab. You will say if you try to do that expectation value or any transition elements, you will find it to be 0 that is all we are trying to do ok. It is just giving some flavor of I am sure, you have done Wigner-Eckart theorem in quantum mechanics code and Wigner-Eckart theorem does not stop there, it is going to be useful as I said that when I do a decay process or a scattering process to go from initial state a to a final state which is mediated by a tensor, typically the scattering matrix is a tensor of rank 0.

It is like a scalar operator ok; if you have a scalar operator those expectation values if you want to see, you can still invoke the Wigner Eckart theorem ok

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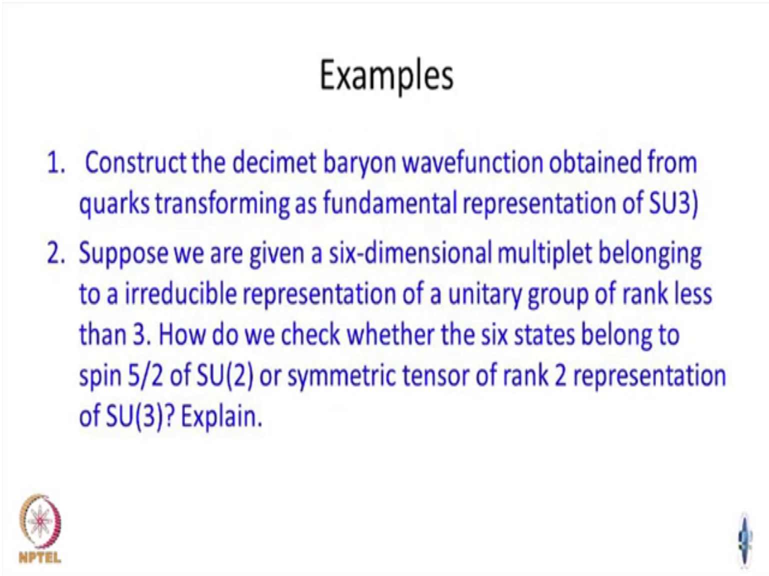

So, the next problem decimet baryon wave functions. So, what was the decimet? How did we find the decimet baryon? So, baryons are made of three fundamental quarks which come in three flavors; up, down and strange right.

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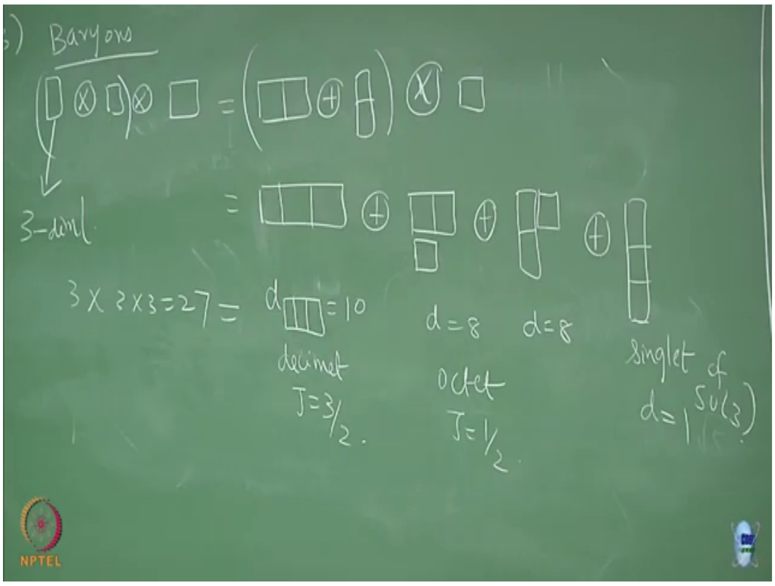

And then we denoted the baryons to belong to a tensor product of three fundamental representations and if you try to compose it in two ways. The first you compose these two and then this one. You get up sorry this is only three boxes. I am going to put this separately just to remember that the first two objects are symmetric; first two objects are anti-symmetric and then you have one more which is the trivial representations ok.

So, you have to write the wave function for this tensor product which gave you this one height. What is the dimension of this? This we worked it out, this was 10 and they represent the decimet baryons; baryons in the decimet. They all have spin to be 3 by 2 and then you have a singlet; singlet or trivial representation of SU 3. See all these things which I am doing, it is SU 3 baryons.

What is the meaning of SU 3 baryons? This has to be three dimensional correct, this is three dimensional and we work out the dimensions of them and as I said this one is d equal to 8, this one also d equal to 8 and this one is d equal to 1 and you have to get 3 into 3 into 3 which is 27 is to be 27 is a reducible representation that is decomposed into irreducible components 10 plus 8 plus 8 plus 1 which adds up to 27, clear.

And this some combinations of this will represent your octet, octet baryons the eight pole (Refer Time: 15:48) ok. So, this is what is the particle content which is seen. In fact, this corresponds to j equal to half. This includes the baryons and the sorry the protons and the neutrons this one has the other baryons fact. As I said one of the baryon which was omega was detected after Gell Mann's Quark model ok.

So, now what is the question? I have asked you to find the just like we found the j j m states which is in the binary basis. Now you have to find the states for them which is tertiary basis right. You compose fundamental objects, three of them and you should get the tertiary basis wave functions for. The same procedure or finding C g coefficients, but this c g coefficients will be the SU 3 C g coefficients. Instead of j and m, you will have highest weight vector lambda and the corresponding weight vector mu; you have to play around with that ok.

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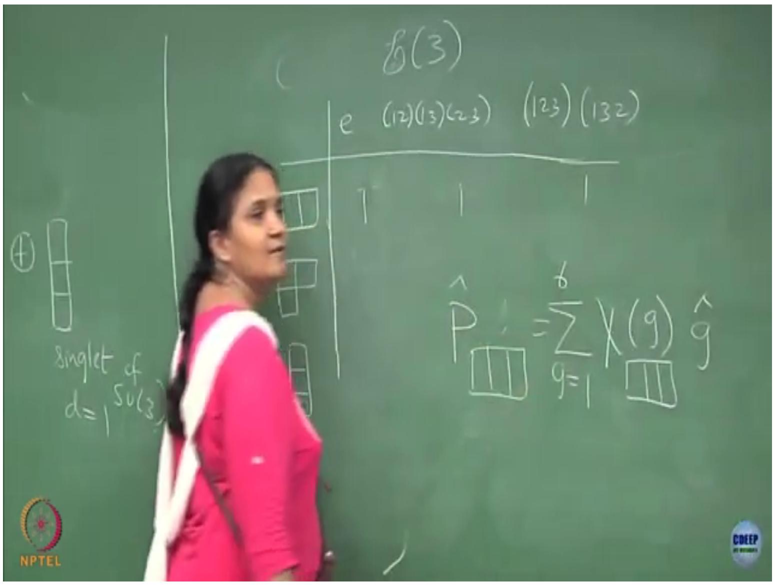

As a nice trick where we connect the symmetric group of degree 3 so, you remember the symmetric group of degree 3, what is the character table? Sure now by even in the middle of the sleep, you should know this right e, then you have 1 2, 1 3 and 2 3 cycle and then you have 1 2 3 and 1 3 2. These two belong to the same class, am I right? And then we had irreducible diagrams right.

So, when I want to write the projection operator for this one, one. I can write it as summation over characters for that representation every element g times g operator fair enough. The g is running between 1 to 6. This is a symmetric group of degree 3 permutation of three distinct objects. Order of the group is 3 factorial which is 6. What I am going to make a modification here is that this projector forces that these three boxes are for distinct objects.

But now when I go to SU3, I am going to say that these three objects could be repeating that is the modification. If it was distinct, you will get only one dimensional representation because it is repeating, you get 10 dimension right.

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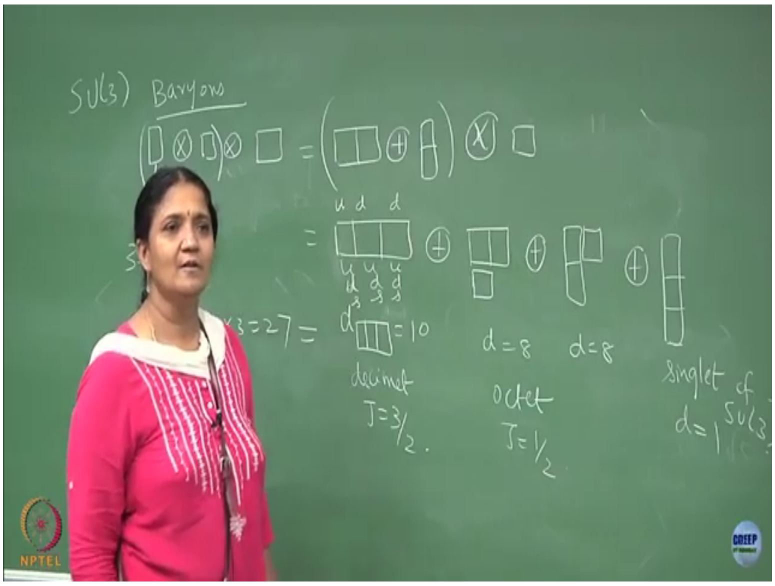

What are the things possible here in this 10 dimensional state? You can put u u u d d d s s s then you can put two of them u d u d d and so on. So, list it out, you will find that there are only 10 possibilities where you have one distinct object u d s, but the remaining 9 will have something repeating ok. So, when I do this projection, I am still going to use the character table. Character table was 1 1 and 1 for this representation and I can find out what is the state corresponding to the diagram ok.

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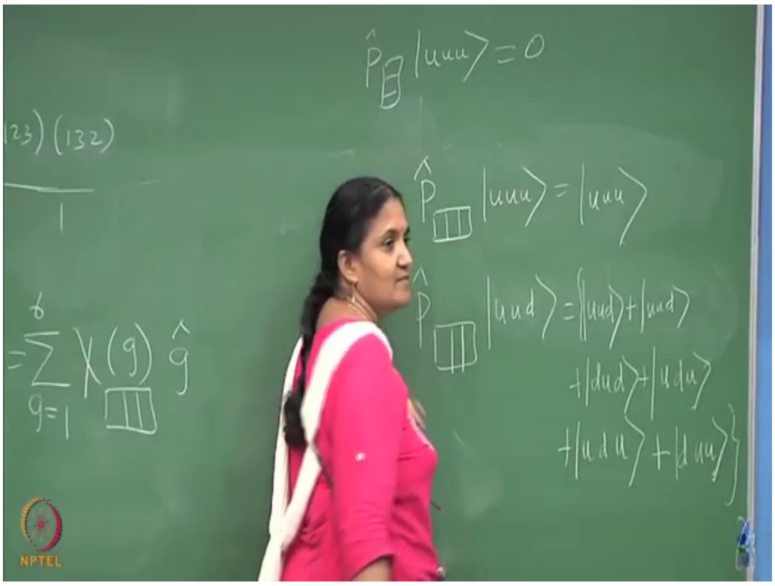

So, let us write it out. Suppose I start with a state u u u and if I do a projector of this, what will I get? This g operator will interchange the first position and second position when g is 1 2 first and third position and so on. All the three positions are identical particles, what will happen? It will give me back the same thing ok.

But if you do it on if you do that 2 3, 2 3 will do what? Will interchange? Second particle with third particle, third particle with second particle; if you do 1 2 nothing happens, 1 2 3 become 1 will become 2 2 will become 3, 3 will become 2, 3 will become 1. So, write it out and see what you get, can you write it out. So, the first element is identity element, it does nothing. So, you will have u u d 1 2 also does not do anything. It will be again u u d 1 3 will be d u d, 2 3 will be 2 3 on this u d u and then 1 2 3 will be u d u and then 1 3 2 will be d and then you put in the normalization.

So, what it tells me is that you have to symmetrize completely. If you interchange any two, you should get back the same state totally symmetric. So, the 10 decimet states which I have here is a totally symmetric state where you could use this projection operator to get your results for the symmetric states ok. That also tells you how you do for the anti symmetric case, what will happen?

Let me the 1 2 we will change it with a negative sign right. So, essentially you will find if you add up everything, you get it to be 0 clear. So, this is one way of determining, this could you could have done it for even SU 2 right. What I did by 1 over root 2, 1 over root 2 can also be done by using the symmetric group of degree 2 character table. But this is going to give me all the symmetric wave functions which corresponds to the decimet baryons.

I am not done everything, but you can do for all the 10 of them and each one will be having a different quark content, their states are all orthogonal. All the ten states have different quark content this, is clear? So, that was the question, construct the decimet baryon wave function obtained from quarks transforming as fundamental representation of SU 3. So, the u u u is one state, then you can show that it is a linear combination of u u d which will be the other state and so on; all the 10 states you can write it out.