Group Theory Methods in Physics
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Lecture – 54
Clebsch – Gordan coefficients

So, today I thought I should tell you how the Clebsch Gordan coefficients are written down, before we attempt this quadrupole moment using Wigner Eckart theorem ok. How is Clebsch Gordan coefficients done? So, I already explained in the Young diagram language yes, in the last lecture on Saturday. By the way today, I will stop the lecture at 4:45, because I think they have some recordings. So, we need to leave by 4:45 ok.

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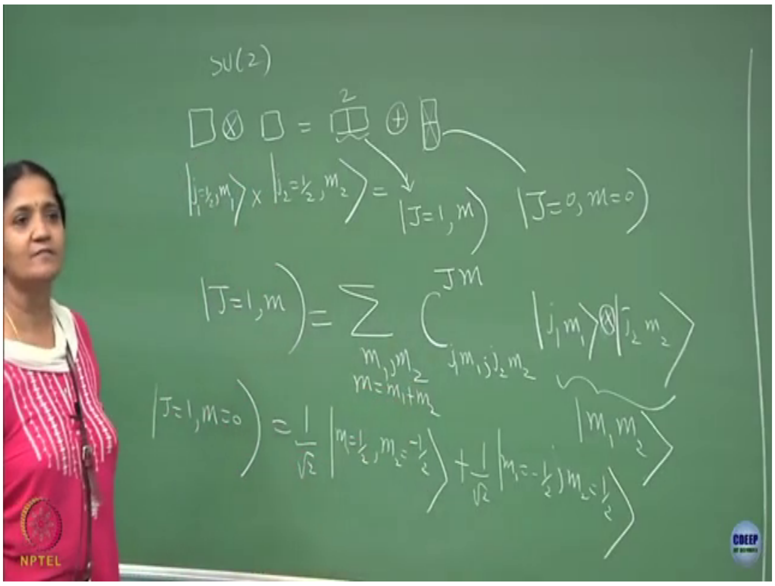

So, this cross this. So, let me do only S U 2; S U 2 group. Now, you understand what this means is, the way to see it is that the state for this can be written as J 1 equal to half m 1 can be anything that is the state which belongs to this two dimensional representation m 1 could be plus or minus j j 1 ok.

And then you take a product with j 2, which is half and m 2. So, you are taking now a fundamental basis or a primary basis you are taking a product of, tensor product of two primary basis and you want to find out this one, the corresponding thing will be J equal to what? This one corresponds to 2 J boxes someone?

So, this one is this has 2 boxes J has to be 1 correct and m will be any arbitrary one. Let me put a curved line to remember that it is for a binary basis obtained from taking tensor product of primary basis and then this one is trivial as I said, this is like a unit representation or a trivial representation this you can treat it like as if it is not there right. So, this corresponds to J equal to 0 m equal to 0, is that right.

So, essentially if I want to write J equal to 1 m state, I should be able to rewrite in terms of this basis, is that right. So, I should be able to rewrite in terms of that basis. So, that is what we write, all possible values of m 1 and m 2 correct, j 1 and j 2 are fixed, because the primary basis has fixed this j 1 and j 2. So, no j 1 j 2 summations only the corresponding states m 1 and m 2 can vary between minus j 2 plus j ok.

So, m 1 m 2, then the notation which we are going to follow is C J m J 1 m 1 semicolon; j 2 m 2 and then you have j 1 m 1 product width j 2 m 2. Technically, a tensor product, but many times they do not put that tensor product they write by the side of it. Many books do not write it like this since, j 1 and j 2 are fixed they do not even write j 1 and j 2, they start with, let us look at the system with j 1 and j 2 ok.

So, and then this one they formally, write it as m 1 m 2, clear? This is what they quantum mechanics textbooks well look at it and there is a procedure of determining these matrices these elements and many of them in the textbook, if you any tables if you call it as Clebsch Gordan tables ok.

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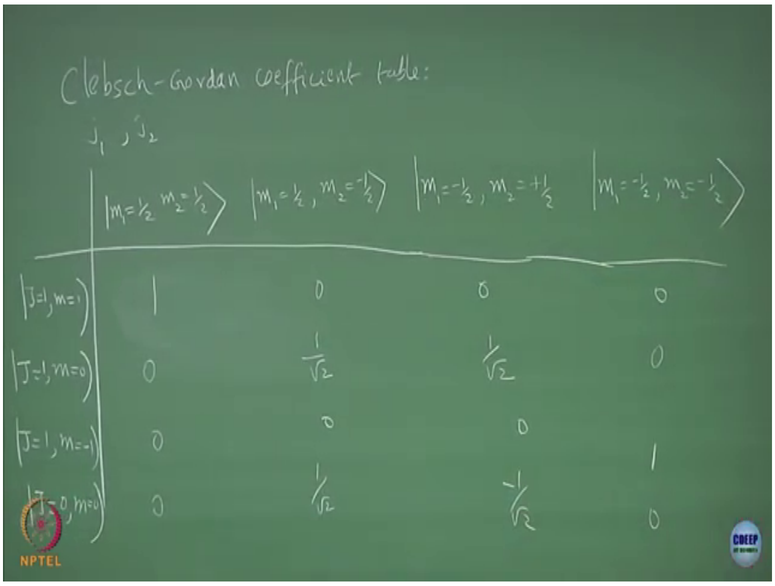

So, if you look at the Clebsch Gordan coefficient table, which you can go and look it up there is a way of determining it, somebody has worked out every. It is like multiplying some digit of numbers, many times you use a calculator or a log book log table. So, similarly, you can fall back on this Clebsch Gordan table, at least for this course, because the course is not for deriving the Clebsch Gordan, but you should learn, if you have not learnt it in quantum mechanics words please, learn it how to do it.

At least for simple cases like spin half composition and spin one composition ok, but for this course you just need to go on look at the table as I said in even in the exam, I will give the table to you ok. So, but you should know how to read the table. So, that typically the table will be like so, they will say j 1 and j 2 and then there will be values here.

In this particular case it is m 1 half m 2 half right, you can have m 1 half m 2 minus half m 1 minus half m 2 plus half and then m 1 minus half m 2 minus half that is the tensor product rule. When you wrote the four bases you wrote it as up up up down down up down down that is what I have written here, up up up down down up down down that is a tensor product.

Here, you will have J equal to 1 m equal to 1 J equal to 1 m equal to 0 J equal to 1 m equal to minus 1 and finally, you have J equal to 0 m equal to 0. These are these states ok. So, we need to find these elements. So, this element I would write it as J equal to 1 m equal to 1 as I said j 1 and j 2 are fixed sorry, the m and j I am putting it as a superscript. Let me just follow the same notation, half semicolon j 2 equal to half m 2 equal to half and so on. So, every coefficient here is the corresponding projector coefficient which helped due to determine the binary basis, clear? Why binary, because it is a tensor product of primary base.

So, this coefficient table will be given to you. So, suppose I give you this coefficient as 1, I think this one is 1, then you will have ok. So, if I give this coefficient to you, you can basically plug it in and write out the states right. Suppose, I want to write J equal to 1 m equal to 0. So, that from looking at this J equal to 1 m equal to 0 is a linear combination of this piece and this piece with these coefficients right.

So, it becomes 1 over root 2. I am going to write a shorthand notation m 1 m 2 here. So, it is going to be m 1 is half m 2 is minus half plus 1 over root 2 m 1 is minus half m 2 is plus half. So, this is the way one could read out from the Clebsch Gordan matrix table what those coefficients are and you can write the binary basis state, corresponding to a three dimensional irreducible representation of S U 2, in terms of fundamental objects which are spin half particles, composite is spin 1 with these coefficients.

This is the superposition of the spin states of the elementary objects, which constitutes the composite object. Is this clear?. So, now we will let us get to the problem, which we want to do.

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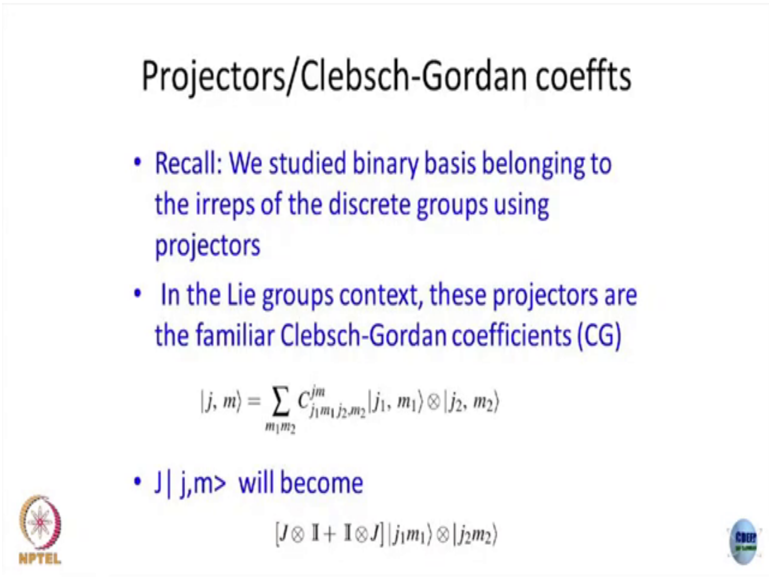

So, for completeness on the slide, I have again put this thing ok. So, if you see the slide, I have written this formally and then I, there is a procedure of determining the Clebsch Gordan which is not pertinent to this course, but if you do it then you can appreciate this table otherwise, you can use the table ok.

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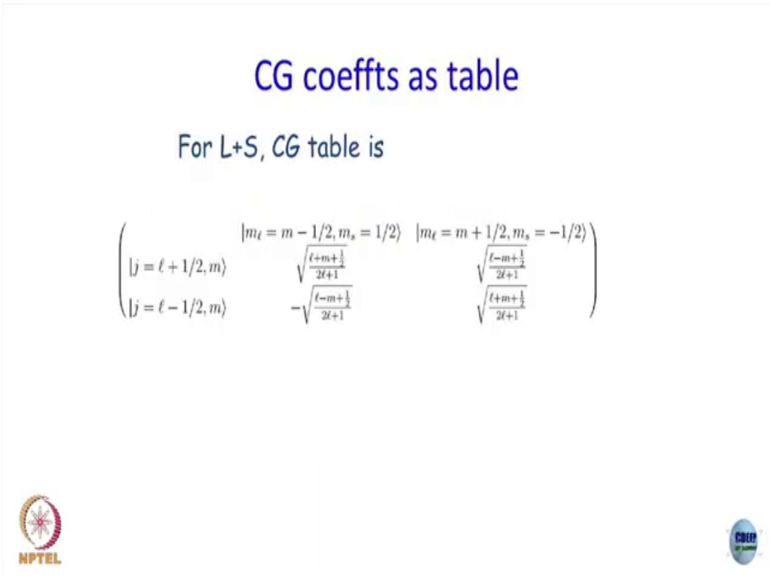

So, suppose I give you spin orbit coupling ok. Suppose, I want to look at a system where you have an orbital angular momentum L and spin angular momentum S S is anyway spin is half you can combine those two L plus right, if we can do that. What will that be?

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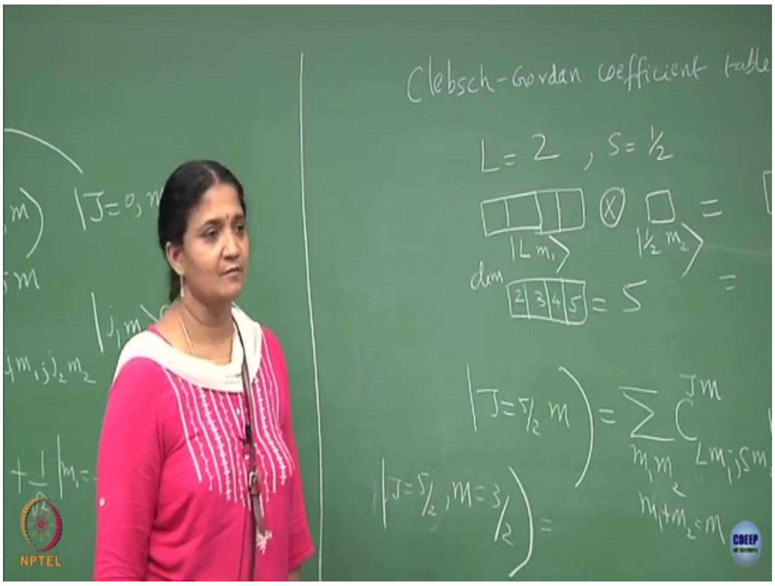

So, I can look at a situation where, let us take L to be 2 and S to be half. What is the Young diagram for L equal to 2 that is this, is that right? This is far as L equal to 1 twice; this number should be the number of boxes. So, how many should be there? 4 boxes, L is the orbital angular momentum, 2 L should be the number of boxes representing that, because dimension of this will be, what is the dimension of this?.

I am doing only S U 2 2 3 4 5. What is the answer? So, L equal to 2 will be 5 dimensional. So, it is 4 boxes correct and then you take tensor product, but spin half; spin half is always a single box right and then what are the possibilities you can get here?

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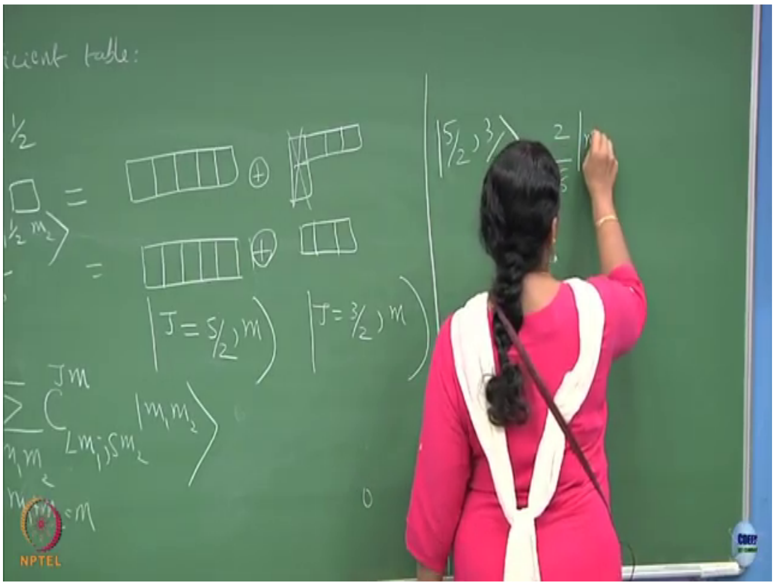

You will get 5 boxes together and then 1 box can go below, 3 boxes that side, this can be ignored, because it is like a unit representation. So, essentially it is 5 box 5 box and then 3 box. So, what will the value J be for this 5 by 2 half of it and J to be 3 by 2. So, these are the states for this.

These are the states for this, this m will take values from minus 5 by d 2 plus 5 by 2, this m will take values from minus 3 by 2 to plus 3 by 2 clear. Now, suppose I ask you to find what is J equal to 5 by 2 and m, what will you do? Again, the same procedure m 1 m 2. So, for this case it is L m 1 and this case is half m 2, m 1 will take values from minus L 2 plus L L is 2.

So, minus 2 2 plus 2 and the m 2 is minus half and plus half. So, what will this p there will be a C g coefficient of J m, then L m 1 S m 2 and then I am putting it as m 1 m 2 clear. So, this is what I would write it as a state for some specific only thing you have to remember is that this magnetic quantum number that is true with everything here also you can you have to put in the condition that m is m 1 plus m 2 always.

The C g coefficients are going to satisfy at least, if you see the table m 1 plus m 2 should be this m. So, this was 0 half and minus half adds up to 0 ok, same thing here ok. So, if I ask you to write this state and if I give you a table like this, what I show on the screen so, if you see the screen I have given you column, states and the row states with these coefficients. So, from there you can read out what is going to be your.

So, L S 2 2 plus half is 5 by 2 and then m and then a specific m I could take it to be, let us say if I take m to be 3 by 2 can somebody work it out and tell me what it will be by looking at this slide. J equal to 5 by 2 m equal to 3 by 2, what is that going to be? J equal to 5 by 2 m equal to 3 by 2 will be linear combinations of two states. What are the coefficients when you substitute the coefficients? You want to see the screen again? Just put the screen back.

So, you have to see which one you have to take do we have to take this column or this column, the first column and then if you look at here it will be a linear combination of m is 3 by 2 3 by 2 minus half is 1. So, 1 and half with the coefficient L is always 2. So, 2 plus 3 by 2 plus half, it is 2 divided by 5 root 5 ok. So, that is one, what about the next one? So, you get. So, first one you are all set can you stay the other one? 1 by root 5 is everybody with 5 by 2, 3 by 2 is 2 by root 5 with what is the m 1 and m 2?

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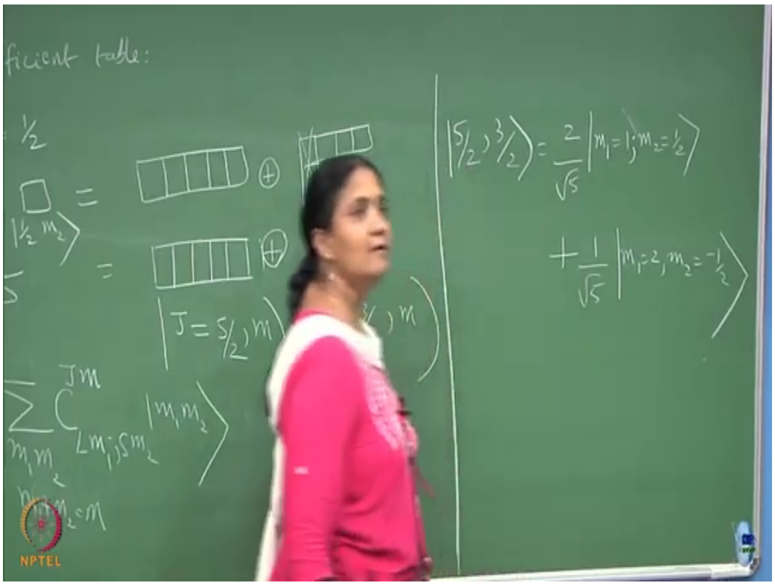

M 1 is 1 m 1 is 1 and m 2 is half that is 1 plus 1 by root 5 m 1 is 2 m 2 is minus half, is that right? Are you all with me? Yes, this is a way to read the table and write out the states.