Group Theory Methods in Physics
Prof. P. Ramadevi
Department of Physics
Indian Institute of Technology, Bombay

Lecture – 53
Tensor Product of Irreducible Representation – II: Decimal and Octet Diagrams in the Quark Model

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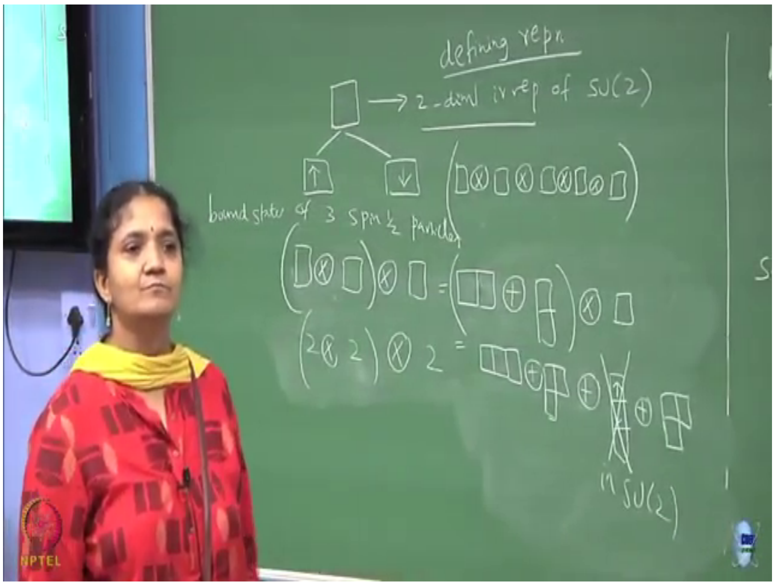

So, now I am doing S U 2. So, I take this combination, this any way we did. It is anyway associative. It does not really matter. Let us take the bound state of 3 spin half particles ok. So, in dimensions, it is 2. This is two-dimensional; this is two-dimensional and then, another two-dimensional.

So, this part we have already done there, the first bracket. The first bracket turns out to be this and then, you take it with a box because I am going to look at a composite made of three fundamental objects with hand side that it should not give me anything other than spin 3 by 2 or spin half, that is the experimental requirement ok.

So, now, you can re-do this. So, we can combined with this box, a single box along with this. Then, this box can go below not violating any symmetry. So, let me write that ok. That is from the first box multiplying this one.

Similarly, I should do it with this. The box can go below and it can also be, can just attach itself without violating the symmetries. You do not violate any of the symmetries by putting this below and do it. Because you are doing S U 2 spin half particles, the group is SU 2 the fundamental objects. If you put an up spin here, you have to put a down spin; then, you cannot repeat anything because this is totally antisymmetric.

So, that is why is it always a vertical diagram with more than 2 boxes is not allowed in SU 2. So, this is not allowed in SU 2 ok, such a state does not exist in SU 2. So, what are we got? I have got this, this and this. This vertical 2 box is like the singlet or a trivial representation. It does not do anything the ladder operation; any group operation will remain keep the state itself.

So, this can be removed, the space is like attaching multiplying all your results with the identity or multiplying all your characters with a 1 representation; e cross a 1 is e; e cross a 2 is e. So, it is like an a 1 representation or a trivial singlet representation and it is not going to harm your you can just forget about it. It is like identity, you can remove it. So, the space can be removed and I can re draw these diagrams as if it is this is for this and similarly here ok.

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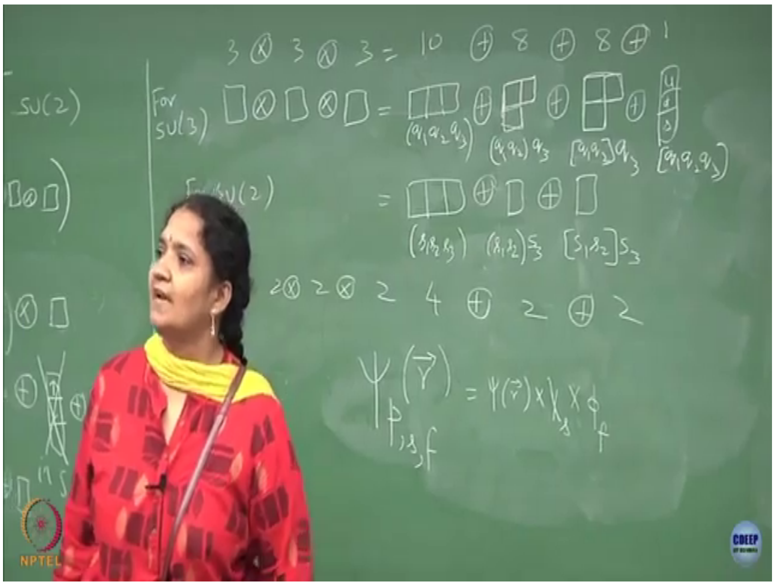

I have shown you that a composite bound state involving 3 spin half particles; finally, for the SU 2 can be withdrawn as. This is true in general, for SU 2 it will be ok. So, that you can say it to be for S U 3 if you want. Single box is repeating, where is it repeating? This one? Down?

Student: (Refer Time: 05:22).

Yeah good. So, that is a good point. This single box indirectly in has an information that there are 3 spin half particles, which are composed to give you a spin half right. You can do it in two ways, you can make. So, it is 3 objects right. So, this 1 I would say that it is actually going to be a spin of particle 1 and then, you can have spin of particle 2 and spin of particle 3 to have some symmetry combination.

Here, you can have, so you can call it spin 1 ok. So, there is some mixing where the 2 spins you take a symmetry combination and multiply with the third spin. Here you do antisymmetric combination; the square bracket means antisymmetric combination. So, there are two possibilities exist when you are combining 3 spins. You can have some different symmetric like this one is totally symmetric in all the three spins and this one has some mixed symmetry.

So, let me do it here. What will happen here? There will be a q 1, q 2, q 3 which is symmetric because the diagram forces you, it should be a symmetric. If you interchange the object which I put in this box and this box you do not get anything new. Clear?

So, similarly here you could view something like it is the symmetric object here and then, you add a piece here or you have an antisymmetric object here and add a piece here that is called mixed symmetry. I am sure you have done mixed tensors. Symmetric tensors, antisymmetric tensors, you can have mixed tensors where some indices well be symmetric, some indices will be antisymmetric and those are the mixed tensors right.

Similar thing can happen here; but has a diagram this is the meaning that there will be a subset which will be you know which will have this S 1 has no symmetry, S 2 and S 3 you can make it symmetric or S 2 and S 3 you can make it antisymmetric and writes the states ok.

Student: Ma'am.

Yeah.

Student: (Refer Time: 08:26).

Yes, both will be.

Student: (Refer Time: 08:32).

No, if you see here, if you see here when I compose this with this, the these 2 boxes are symmetric; when I compose this with this you see that these 2 boxes are symmetric. You can call S 1 and S 2 is symmetric, I just call this as 2 and 3 and that one has 1, but it is not the it is because of associative property. But technically you can call if you want maybe this is a better notation as per as the diagram is concerned, you have an S 1, S 3 S 2 symmetric and then, you add an S 3.

That is what you take here; S 1 and S 2 were symmetric and then, you added in S 3 ok. If you see here, it is S 1 and S 2 were antisymmetric and that to that you added this which is S. So, this is another way of seeing it. So, these two have different tensorial properties ok.

So, the next question you can ask is this proton belong to this or will belong to that or a linear combination? All this questions starts coming up, you know. One of the ways in which they fix all these things is by somehow getting the magnetic moment result experimentally seen out of which linear combination gives me the result, that is the way they fix it. We do not know why that linear combination. Nature seems to project 1 linear combination of these 2 pieces such that your magnetic moment in your experiment can be matched for proton and neutron ok.

So, I do not have any handle which one; I cannot say that this is proton, I cannot say this is proton, even though they are spin half particles. But a linear combination of it; what linear combination, what coefficients I should choose are all dictated by experiments. But it does not give me anything other than 3 by 2 and spin half.

So, I play around with a bound state of 3 quarks; three three fundamental objects. To understand spin I should take them to be fundamental objects to belong to SU 2 representation; to understand the quark content I need to see it has a SU 3 representation because I am just working with quark content which is only three objects which is u d s which is sometimes called as a flavors ok, yeah.

So, is this clear? Now, what are these dimensions; can someone tell me for SU 3; what are those dimensions? So, this one is dimension is 4 plus 2 plus 2, this is for SU 2. So, what about here? SU 3 by your hook cum numerator, this is any way I can see it to be a trivial representation. So, that is one.

Student: (Refer Time: 12:22).

Ha?

Student: 10 8 (Refer Time: 12:24).

10 8 8 and 1. Is that adding up to 27; 3 into 3 into 3 is 27? 10 18 26 27. So, the young diagram actually helps you without doing a projector how the breaking happens, you can do the project by writing the cliff code in matrices; even before you do that, you can say that you can find this quickly ok.

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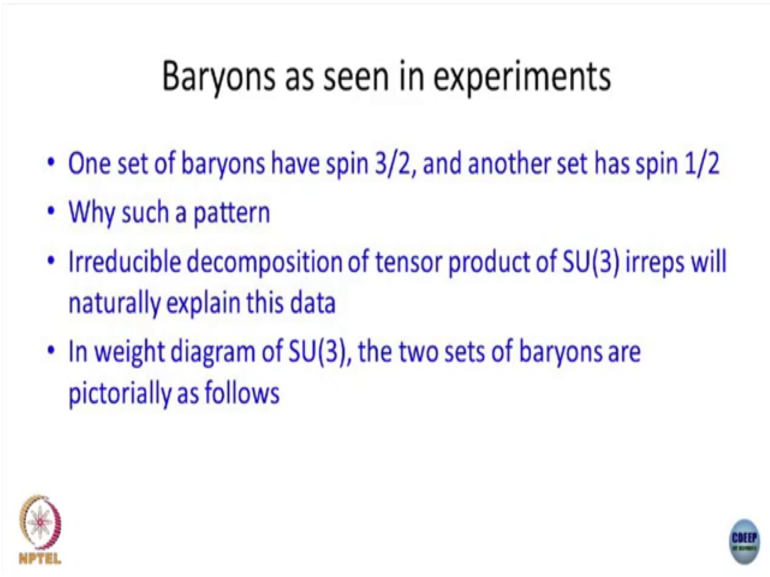

So, this was the crucial part which Gell-Mann did. The reason why he did, let me get to the slide. So, as I saying one set of baryons have spin 3 by 2, another has spin half; why such a pattern; then, the irreducible decompositions of the tensor products of S U 3 we will explain this ok.

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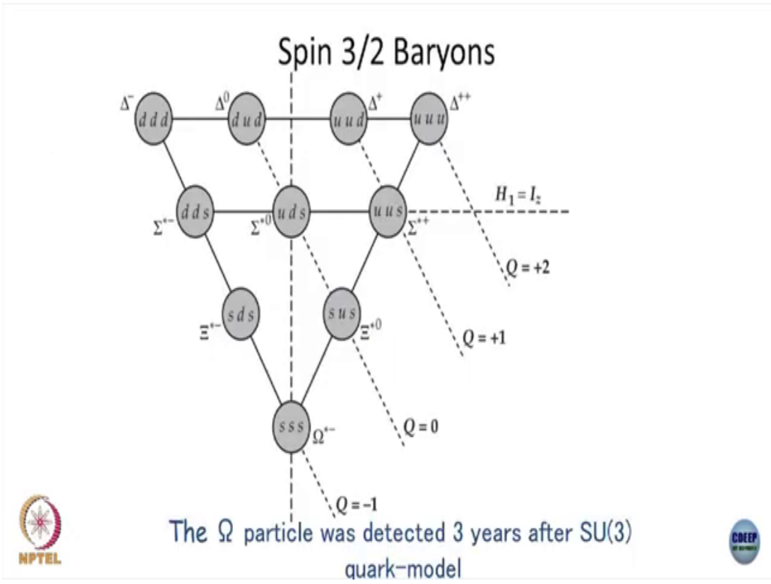

So, what he saw in the observation, in the experimental lab was that later on you can put it in the H 1 H 2 plain; what is this called? This is called the weight diagram. Weight diagram for an irrep and you have used fundamental objects as u d s. So, it is an irrep of S U 3.

So, what happened was that the charges along these lines are you know this one has charge plus 2, this one has charge plus 1 and so on ok, charge 0. There is some nature seems to have some kind of a pattern. See, you wanted to account this as a weight diagram of S U 3 irrep; interestingly or experiment did not have this particle.

This omega particle at that time when Gell-Mann was doing his SU 3 group to fit in this data all of these particle has spin 3 by 2. He wanted to fit this data by using S U 3 group. When he was trying to do it, he got 10 that is this 10 on the board. But he did not the experimentalists did not have this omega particle and he said that you have not detected it, you better go and check for it. It is beautiful actually.

Gell-Mann recently passed away. Did you did you see his profile? You should see his profile; it is very nice. Very simple way of looking at SU 3 and he accounted that there should be 10 particles which belongs to an irreducible representation of SU 3; whereas, only 9 particles or 9 states with spin 3 by 2 were seen in the lab and interesting after 3 years they detected it. They detected that particle, so that is why he got is Nobel prize.

So, it is very beautiful to see that you know completely it may look very in mathematical, whatever I was saying is mathematical, but ultimately nature seems to have some kind of a symmetry and he was able to show it was ok. Is that clear?

So, this diagram is sometimes called decimate diagram because there are 10 possible states; which is the highest weight state here? Highest weight will be this corner state and then, you can use all the possible ladder operation of SU 3 right? Alpha 1, alpha 2 and alpha 3 and you can generate all the 10 states by the SU 3 generators.

So, this is the irreducible representation of SU 3 whose young diagram is this and whose dimension can be computed to be ten-dimension, those 10 are the 10 states of the decimate of 3; 3 by 2 baryons. Why baryons? Baryons are the ones which are composites of 3 quarks. These are all baryons; proton is a baryon, neutron is a baryons.

So, all of them are baryons and you can explain from group theory as an irrep and you can also account for the 9 already present 9 states before when Gell-Mann proposed this and 1 more was seen after 3 years and validating that SU 3 quark model is the right approach to understand the set of particles which has spin 3 by 2; set of baryons which are symmetric. This is clear? Ok.

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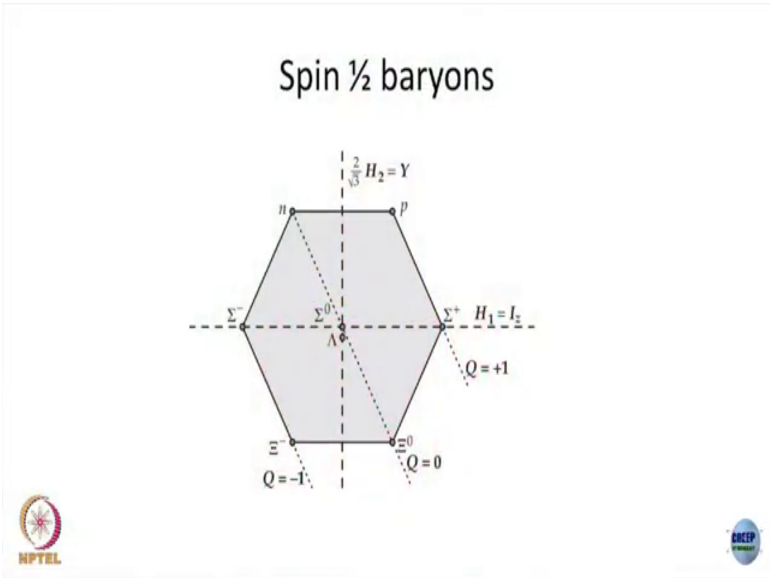

Similarly, the spin half baryons also had a this is very different from the earlier one, this is another irrep and this irrep had 8 particles and that is your eight-dimensions. The spin half baryons which includes protons, neutrons belong to a different irrep from this.

This irrep was totally symmetric. This irrep is partially symmetric ok. So, you can write this as if it is q 1, q 2 is symmetric and q 3 is has no symmetry. Similarly, here you can say q 1, q 2 is antisymmetric; antisymmetric, I am denoting it by a square box. Here, you all know by now what is antisymmetric, symmetric right.

If we interchange the 2 content, if it is antisymmetric the wave function will pick up a negative sign. Symmetric means if you interchange, it will remain the same. So, q 1, q 2 and then q 3 here all of them are antisymmetric. Clear? That is why it is a trivial representation of SU 3 and these are non-trivial representation.

As I said that whether the proton will belong to this or this, whether the proton in the SU 2 language will belong to this or this is dictated by some linear combinations. Because whenever we have to write a wave function, you have to write your wave function. Suppose, I want to write a wave function for a proton ok, this will depend on the position coordinates, it will depend on the spin, it will depend on the quark content ok; sometimes that is called as the flavor ok.

So, you will have a piece which is psi which is function of r, then you will have you have to multiply it with chi which is a function of spin, you will have another phi which is the function of flavor. You have to multiply them. Why multiplying? They are independent spaces; spin space, position space and the flavors space. They are independence space. Only thing is this is totally a fermionic object; the proton is a fermion right.

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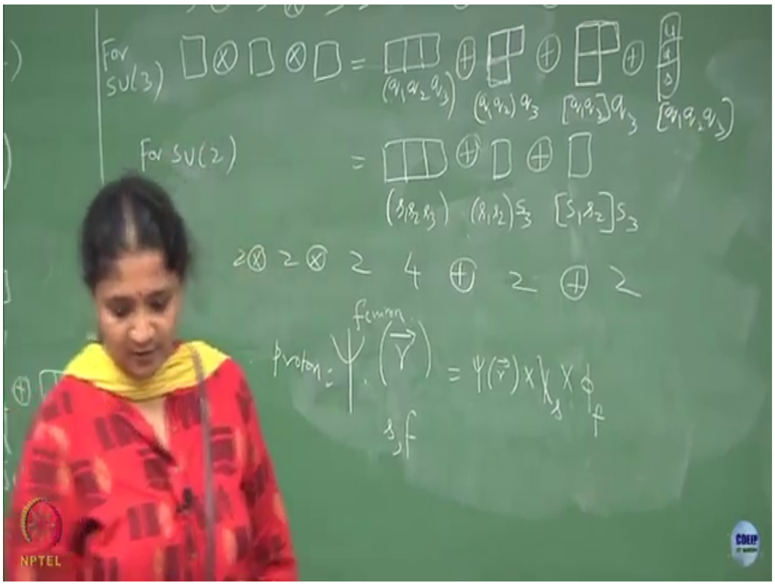

It is a fermion. So, what you have to remember is that a fermion which is a bound state of 3 objects should be totally antisymmetric. If you swap the spin content, flavor contents; so, something's could be symmetric, something's have to be antisymmetric so that the product remains antisymmetric. So, these are things which you will all do not to violate Pauli's exclusion principle.

So, what I am trying to say is that you could take the spin wave function to be symmetric; flavor wave function to be antisymmetric plus a linear combination where spin wave function is an antisymmetric, flavor wave function is symmetric. All these playing around you can do. Finally, find a linear combination of this total wave function for which you will find the magnetic moment which is unique. So, there is lot of mixing going to happen between this and this and so on. So, I am not going to get into it, but that way you can uniquely pick a combination.

So, as of theory group theory, it tells you that all the elementary particles which they call it as elementary is not exactly elementary; baryons are composites and to account for the composites with spin 3 by 2 and half, you can allow 3 fundamental objects and once you are allowed 3 fundamental objects, you can say that in the flavors space; it allows for 3 flavor states which is SU 3 and then, you start accounting for this eightfold path or the octet diagram.

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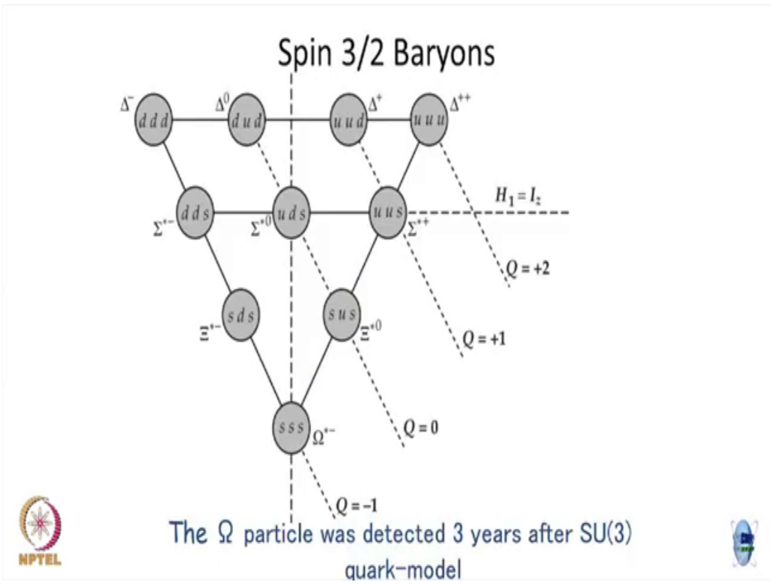

So, the eightfold path or the octet diagram, you can account and the previous diagram which I showed is a decimal diagram and you can account them by treating it has a SU 3 groups symmetry ok.

So, I hope I have given you some flavor of how your group theory language actually speaks, what you see in your physics.

Student: Ma'am, what is the (Refer Time: 23:45)?

Which one?

Student: (Refer Time: 23:48).

Proton, I just wanted to call that as a for a proton ok; so ok. So, I can say this for the proton, for proton the wave function ok. So, this p has nothing to do with momentum; maybe you confused it momentum. So, p is just a proton ok.

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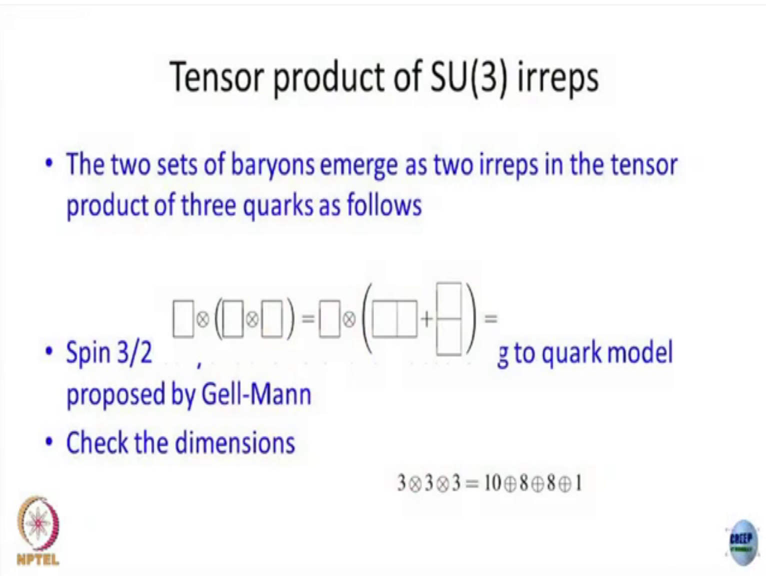

So, to summarize the two sets of baryons which I showed you, one as a decimate diagram minus the omega particle initially. Can be viewed as tensor product of 3 quarks as follows ok. So, this we have done it now.

Student: (Refer Time: 24:44)

Spin 3 by 2 baryons at 10 of them according to quark model proposed by Gell-Mann.

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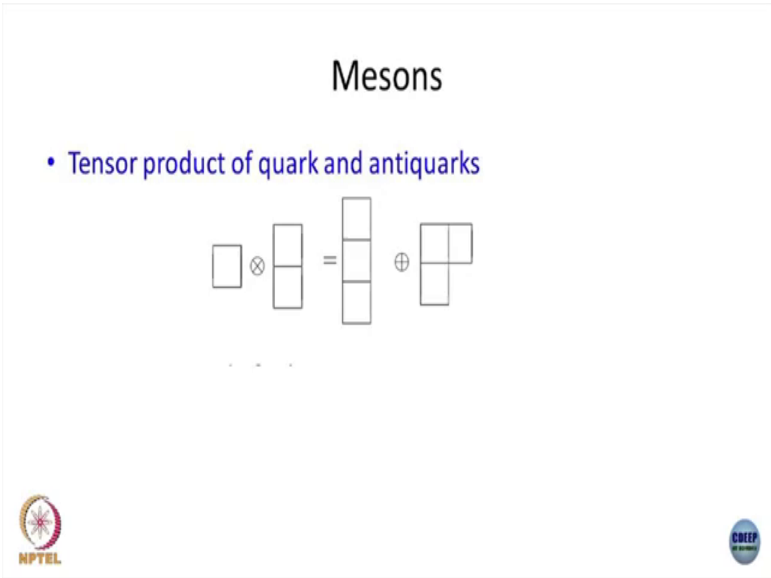

And you can check this dimension which we have already checked.

Student: (Refer Time: 24:56).

Similarly, Mesons are bound states of quarks and antiquarks. Mesons are baryonic or sorry fermionic or bosonic?

Student: Bosonic.

It's bosonic, it is a bound state of a particle with an antiparticle ok. Now, I will leave it you to see what should be happening and what is happening in the experimental scenario for mesons.