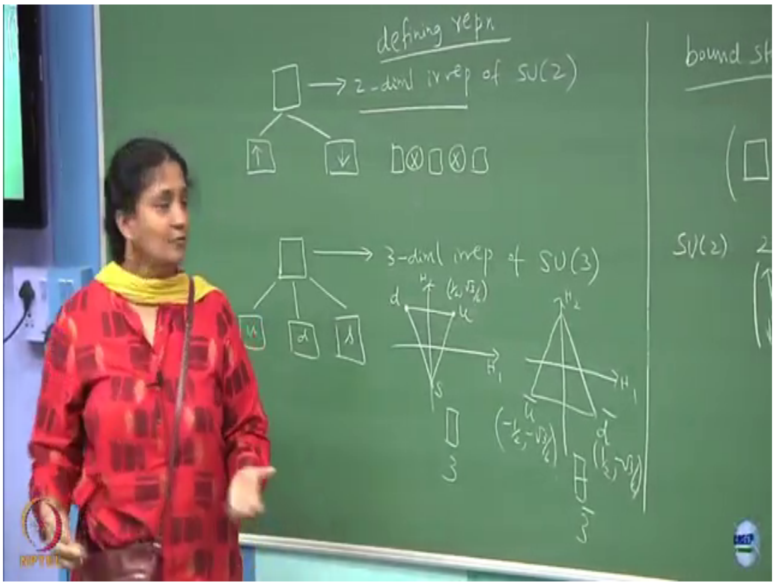
**Group Theory Methods in Physics**
**Prof. P. Ramadevi**
**Department of Physics**
**Indian Institute of Technology, Bombay**

**Lecture – 52**
**Tensor Product of Irreducible Representation – I: Composite Objects from Fundamental Particles**

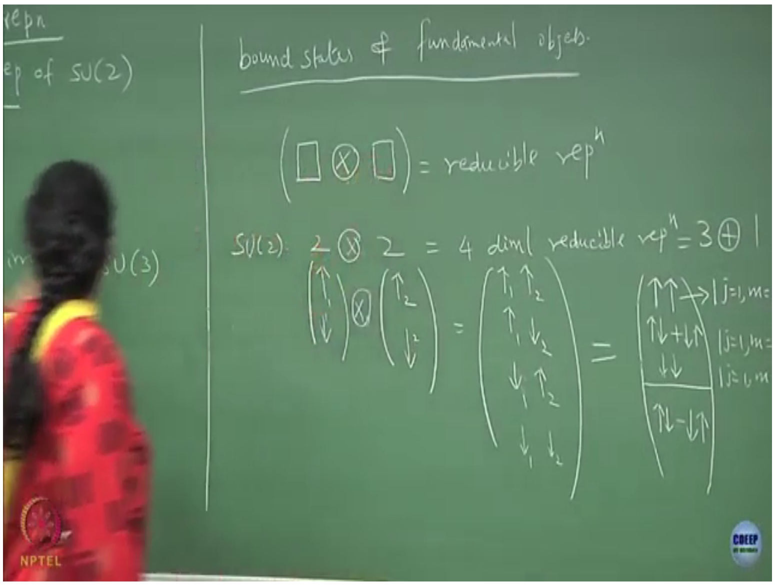
(Refer Slide Time: 00:20)



So, basically I said that this single box will be 2-dimensional irrep of SU 2. The reason is that you can put either inside this box you can put up, up spin or down spin, right. So, this is what was the notation. And this irrep, we call this irrep as defining representation or the lowest defining representation ok, the lowest non-trivial irreducible representation of SU 2 is 2-dimension.

If you go to SU 3 this can have inside and up, a down, and a strange. So, that is why we call this diagram, did we note 3-dimensional irrep of SU 3. The same diagram has different meanings for different groups, ok. So, this is the fundamental object you can put, sometimes it is called defining representation, all of them are defining representation. So, the defining representation is what I have shown as a single box defining representation is 2-dimensional for SU 2 and 3-dimensional for SU 3, ok.

(Refer Slide Time: 02:15)

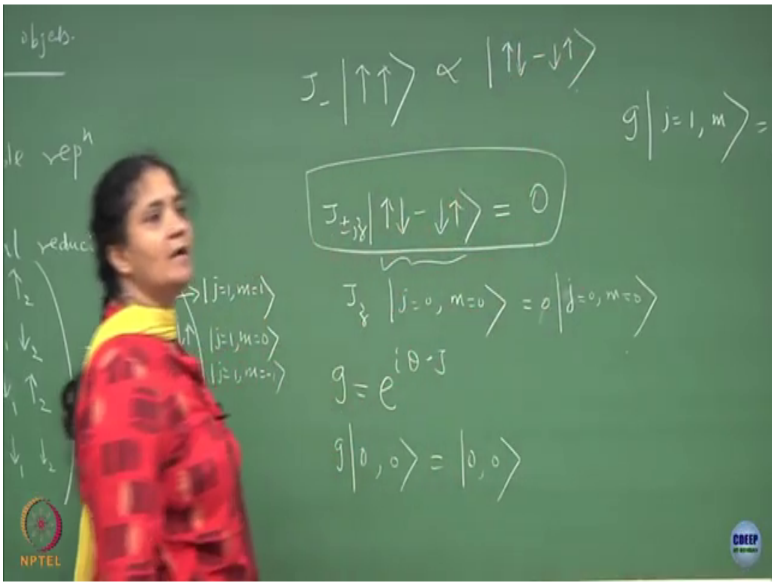



So, next thing what you can do is we can start doing bound states of fundamental objects, ok. You can have what do we mean by bound states you start taking a tensor product of the defining representation, ok.

This one is a reducible representation, ok. So, this one will have, this is the corresponding dimension of this for let us do SU 2, the corresponding dimension of this is 2, the dimension of this is 2, this is going to give me a 4-dimensional reducible representation, ok. This is 2-dimensional times 2-dimensional, 2 into 2 is 4. So, you can see this also as if I have an up and down state for this case and another one we can call with the subscript for the particle 1, and this one as particle 2. You could combine this and what you get is, this is on the vector space which is 2-dimensional vector space it becomes a 4-dimensional vector space.

Now, we need to decompose this, ok. You have to decompose this into a piece which involves from linear count it is like binary basis what you did in your discreet groups. Now, you have to find a projector such that this reducible representation becomes the irreducible representations of SU 2, ok. So, the irreducible representation should be such that whatever ladder operators you apply the generators of SU 2 should not take you out of this piece that is what is meant by an irreducible representation.

(Refer Slide Time: 05:35)

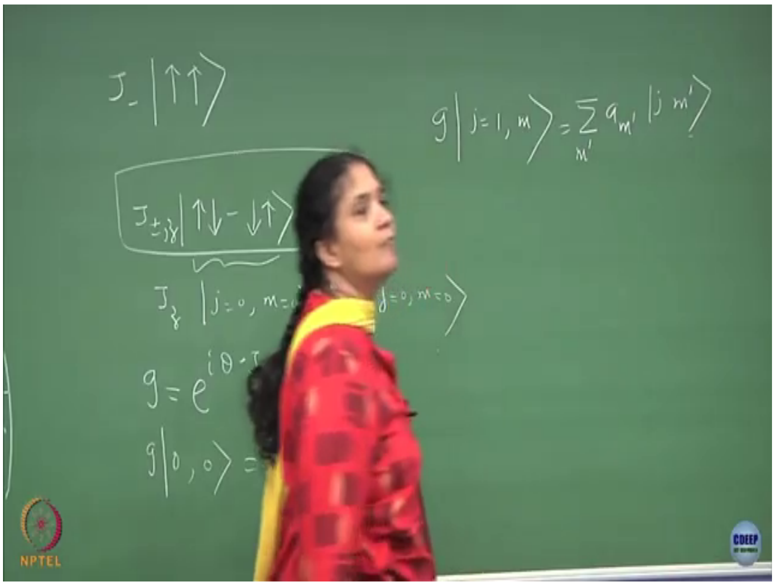


So, it will turn out that if you start with up and do a ladder operation, so do the ladder operation, ok, if you start doing the ladder operation on this you will not get, you will get only some linear combination and again a ladder operation will give you this, ok. So, this is the meaning of saying that you will get from one to the other by the generators of SU 2 on some linear combination subset, ok. So, there is a the other one will be the downer. Whatever ladder operation you want to do on this J plus or minus. So, now will be including z what will it be? Always 0, right. Why is it 0? This is like your unit representation, trivial representation. This representation can be written as j equal to 0 m equal to 0, right.

So, J z will give you m which is 0 times j equal to 0, m equal to 0. Ladder operator supposed to take m to plus or minus 1, but it cannot exceed 0, right, you cannot exceed this to 0. So, which means that all the generators acting on the state will remain inside the state only, cannot go anywhere, only identity operate j plus, j minus, J z because you have to exponentiate the generators to find a group operation, right. So, the group element which you write will be exponential of i theta dot J. If g operates on 0 0 what will this give me? 0 0.

Why it is that? Because the identity element will give you that the others are all going to give you 0. So, this is why the singlet this is what we call it as a singlet or a trivial representation or one-dimensional representation that will be a loner that is why it is called as singlet.

(Refer Slide Time: 08:56)



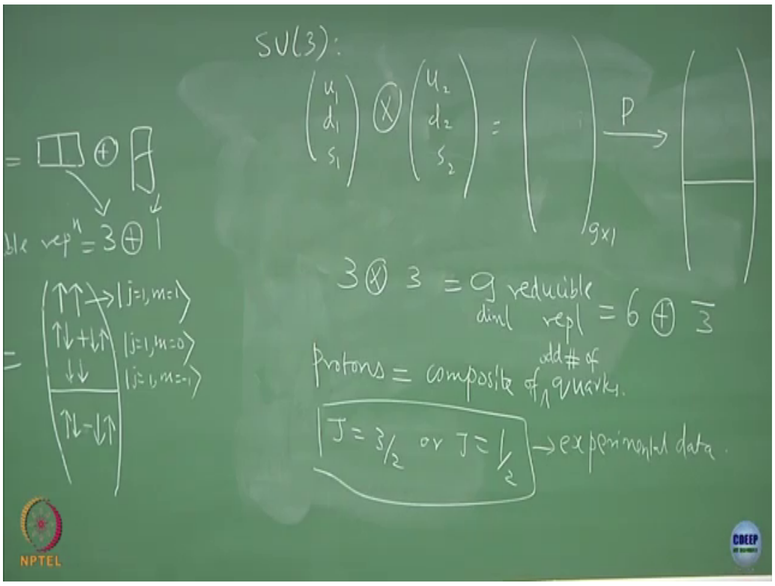
The rest of the things if you do a ladder operation you can go from one to the other and any group operation which you do if you do a group operation on an arbitrary state. What do you except? This should give you some linear combination over m, where m prime has to be, so this I am calling it as j equal to 1, m equal to 1, this I am calling it as j equal to 1, m equal to 0, j equal to 1, m equal to minus 1, ok.

So, this what will do is any group operation because the ladder operators will take you from one to the other. So, J minus will give you up down minus down up, right. All of you have done this in quantum mechanics course. I am not really doing the derivation here, but I am just trying to map what you know from group theory.

So, the group operation is going to mix only amongst these states, ok. So, this is what you would have fancily written in this notation that the 4-dimensional reducible representation is actually 3 plus 1. So, this one is tensor product of two 2-dimensional defining representation ends up getting you a 3-dimensional irreducible representation plus some one-dimensional irreducible representation, right. So, this is what you would have done.

So, for this course I am not expecting you to derive this projection coefficients which is called the Clebsch-Gordan coefficients. If required in the exam I will give the coefficients as a table, but if you knew the coefficients then you will appreciate why the selection rules off some matrix elements being 0 or nonzero is given by (Refer Time: 11:33), ok. So, students are not required to memorize or derive CG coefficients in the exam, ok, fine. So, whatever I have said in the vector space here the same things can go on for SU 3; whatever I have said here will go for SU 3. What is a modification in the context of SU 3?

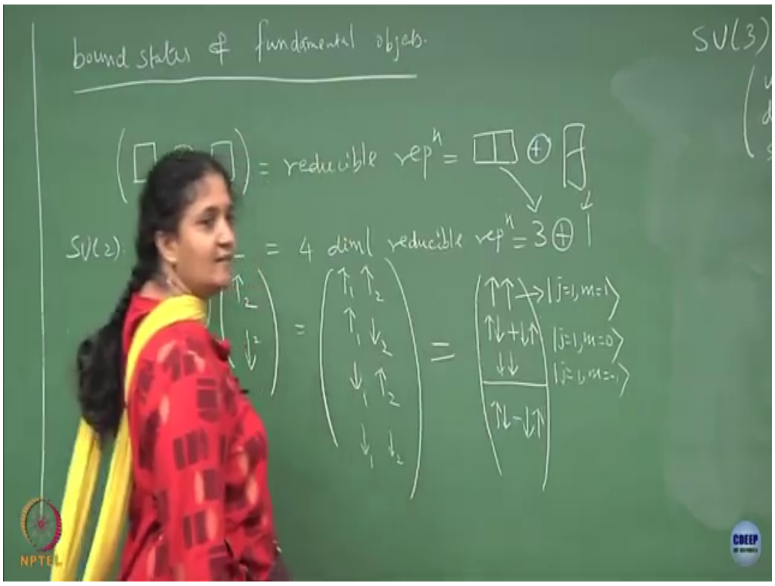
(Refer Slide Time: 12:26)



So, if you do a SU 3 the same thing which I have written here will turn out on the vector space as if it is u 1, d 1, s 1; s 2 and then this will give us I am not going to write it, but you know what I am saying. Exactly like what I did. And then you have again the projectors which are the Clebsch-Gordan coefficients for SU 3 which will break it up, ok; it will break it up into two pieces.

The reason why I am saying it is two pieces all this for any whether it is SU 2 or SU 3 or even SU n is that when I compose two fundamental objects in the diagram fashion, the diagrams have these meanings will always give you the diagram notation which is always this, ok.

(Refer Slide Time: 13:39)




So, the diagram notation is that the combination or composite object can be decomposed, irreducible decomposition into two irreducible objects always. What are the dimensions of them? That also we have worked it out. For SU 2 if you work out the dimension, it will be the first box should have numerator value to be 2, second box will be 3 and hoop number is the same as what you did in your symmetric groups. So, the dimension of this piece is what? It is 3. And similarly the dimension of this is 1 which is the trivial representation of the singlet.

Now, the same diagram will hold for SU 3 what will that give us this is 3-dimensional cross 3-dimensional these two, that will give you a 9-dimensional reducible representation, 9-dimensional, say 9-dimensional, ok. But using this diagrammatic young diagram language even without doing the projectors you can say that it should be 6 plus 3 is what we would have written as dimension, but if you remember I said that this diagram corresponds to the anti-quarks.

So, this sometimes people denote in many books by a bar there, but dimension is only 3, ok. So, some books denote the dimension with a bar to denote that it is a vertical. So, just to distinguish that it is not this, but it is this. This is 3 and this one is the anti quark representation which is 3 bar, right.

So, if you remember here I also drew the diagram which is this one for this one and here it will be this one that will be for this diagram. Basically, every quantum number electron will become a positron, so the charge electric charges become negative. So, this one if I call it as d, this is u, this is s, then the d which you had there will become d bar, then u bar because u will become go here with the negative completely diametrically opposite or you can call it inversion the whatever value you had here this point had a value which was half route 3 by 6, right. So, this one will have a minus half and minus root 3 by 6. So, that is why it is anti-particle of this u quark, ok. So, u quark anti-particle will have both the quantum numbers to be negative and this one will have half minus u 3 by 6, and so on.

So, this diagram is what we call it as a, this is defining representation which is a quark representation fundamental representation for quark. This is the representation for the anti-quark. People sometimes denote this by a 3 bar, this by a 3 and this are your H 1, H 2 axis, ok, fine.

So, this breaking up can also be done by using the CG coefficients, I not going into the CG coefficient part, but I did indicate how SU 3 can be seen as 3 SU 2 sub algebras and you can do the ladder operation and figure out the coefficients. Just like the way you did in your SU 2 in a quantum mechanics course, ok.

So, now this is the procedure. So, in the experimental situations people have seen that protons; protons they can be experimentally, so they measure magnetic moment and so on, they do not find like electron has this more a magnetron multiplying some integer number. Here they find some sought integers, ok. That is the first step showing that it is not an elementary particle.

If it was an elementary particle then it should always be suppose I say that you have an electron, then I know that the charge is some plus 1 time c if I had a composite of two electrons I can say plus 2 times c, but when you start doing the magnetic moment for it still you get some integer times, but that is not the case with proton or neutron. They find buying I think minus 0.81 to board magnet and all these complicated numbers are coming. So, that is first thing they think is that they should see that not as a fundamental object, but as a composite and they want to account for this composite which can explain many of these results.

So, protons turns out to be not elementary, but a composite of quarks, ok. Just like I talked about bound states of fundamental objects, this is going to be a bound state of some number of quarks, ok. How many number is not in our hand? What do you know is that protons have been shown to have spin there are spin half particles, right, there also spin half particle. This composite of quarks can it be even number or should it be odd number? If it is even number you can never get a spin half out of it, its fermionic if you want to get a fermionic object you have to have composite of odd number of quarks, right. That is a first step.

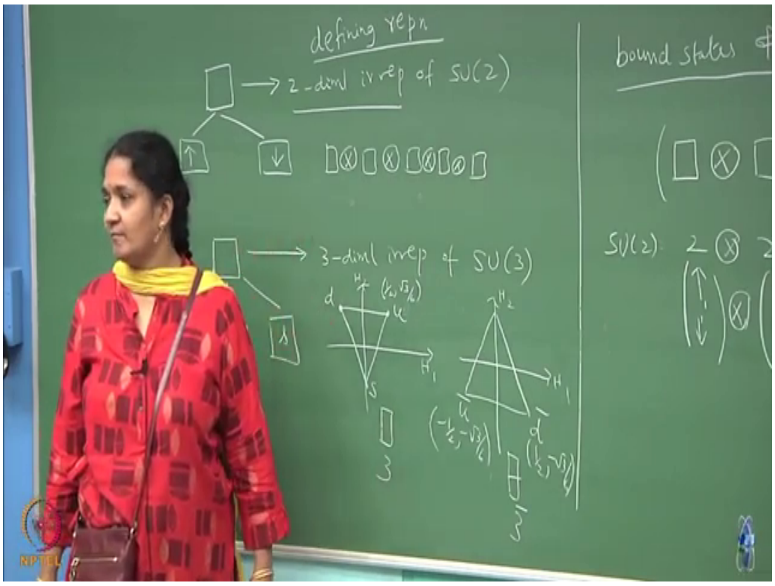
And then they do look at experiments and see what other particles are seeing in the experiment and they find particles with j equal to 3 by 2 or j equal to half. So, these are experimental information, ok. Particle zoo you can call like periodic table in chemistry;

youcan call that elementary particles which makes up the nucleus, right. You all know that items are further broken up and nucleus is made of protons and neutrons, but you could also have other possible states only thing is that those why we do not see like day to day proton, neutrons, but not other particles you can ask, but they may decay immediately, their lifetime maybe not very high that they can decay, ok.

So, protons are something which you always see, but you do have other particles which are seen in the lab and which could undergo some decay with a finite a very small lifetime. So, that is why we do not see in many books talking about other particle, but if you take a particle physics book you do see them listing out so many particles, ok.

So, you do have particles other than protons or neutrons and those particles some of them have can take this to be in the rest frame, so you do not need to worry about orbital angular momentum and take them to be spin. They do find that the spin out of two types, some of them have spin 3 by 2, some of them have spin half, ok. So, that is the experimentality. So, if I want to get spins. Spin means I need to remember that I am doing SU 2, right, they want to get spin to be 3 by 2 and 3 by half, sorry 3 by 2 and half, but nothing else what composition I should take here? Should I take only 3 or more than 3?

(Refer Slide Time: 23:56)




Suppose, I take more than 3 should be odd number; should be odd number. Suppose, I take 5 and do a tensor products in the SU 2 language just like I did here you can do it here and this is what I call it to be the composite object which could represent the proton which will have experimentally we needed to be 3 by 2 or half, right. How many such odd number I should choose? Any guess on this? This odd number is not arbitrary.

Once I put j equal to 3 by 2 or j equal to half, what should be that odd number? Can it be 5? Can it be 7 or it is forced to be only 3? 3 is the only one which can allow me to get 3 by 2 and half. Of course, 5 will also allow you 3 by 2 and half, but as a bonus you will also get 5 by 2, you do not know what to do with 5 by 2. So, indirectly we know that the composite objects in the particle physics is made off 3 fundamental quarks and these quarks have spin half nature, as far as SU 2 is concern. So, let us do the composition of yes.

Student: (Refer Time: 25:31) theoretically.

Theoretically, also you know we could do any number, but then you have to pick something. It is exactly like if you solve a the quadratic equation and find the roots, and if you say that these roots should have a physical meaning, then if you get complex roots you delete it. If you get negative roots if that is something which gives me a positive value in the experiments you say that is unphysic, right. This is what we do. Mathematics can give you certain things, but mathematics hand in hand when experiments will only tell us which one to be taken for physical reality, ok. So, that is not given by math.