Group Theory Methods in Physics
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Lecture – 50
Young diagram and Tensor product

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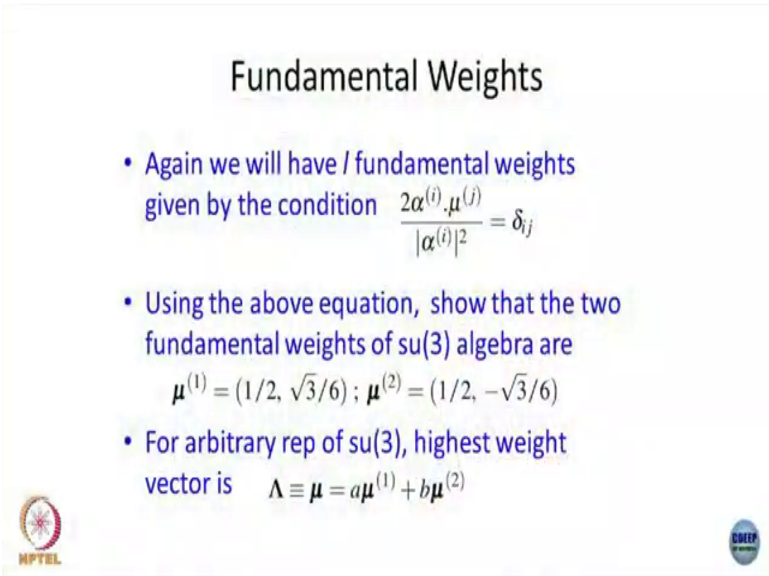

So first thing is that as I said there are l fundamental weights, how do I find the l fundamental weights? Basically it is out of those simple roots which you have you take the dot product of the simple roots with some arbitrary vector and that should satisfy this kind of an orthogonality condition ok.

So, using this and knowing the alphas you can actually fix what are the fundamental weights, there should be if there are l simple roots, there should be l fundamental weights and that is given by this expression which involves a dot product between the simple roots and the fundamental roots. So, you will it will turn out that the first fundamental weight is this highest weight and this I think we worked it out last time this turns out to be half.

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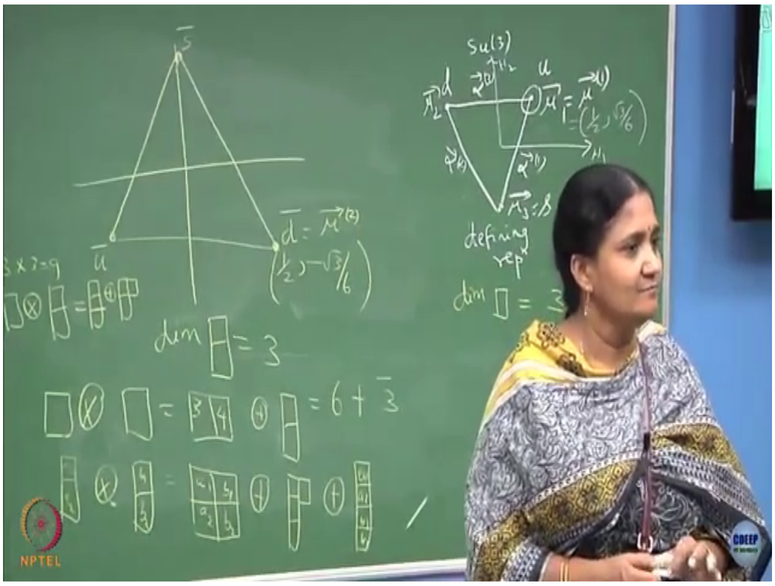

And root 3 by 6 in the normalizations of these Gell-Mann matrices. So, this was the highest weight that is the one option for the fundamental weight as shown in the slide and then you can find the other option which is half and minus root 3 by 6 ok. So, incidentally if you see this expression; if you see this expression you do see that it is the second eigen value which has become negative right, as a mirror reflection of this which you can draw; draw the mirror reflection of it anything downstairs will become upstairs anything upstairs will become downstairs. So, how does it look?

So, I will have a point here and 2 points here. So, this will be your half and minus and this will turn out to be your highest weight state and you can call this sometimes people call this as d bar, what is bar denotes? It is the antiparticle of the it is a antiquark diagram ok. So, this will turn out to be a d bar and then this will be a u bar and that will be an s bar.

So, the highest weight here which I will call it as mu 2 will be the fundamental weight. So, these two are the only both are 3 dimensional interestingly for Su 3, this is 3 dimensional, this one is also 3 dimensional and this I am going to denote it by this diagram. Dimension of this diagram is 3 this I am going to denote it by this diagram dimension of this is 3 and these two are the fundamental diagram to compose you can compose now this fundamental diagram with itself ok.

So, you can start doing composition or tensor product of 2 fundamental quarks you can do that you can do composition of 2 antiquarks, you can also do combinations, what is the combination? I can do a quark and an antique quark, this is quark, this is antiquark these are the 3 possibilities in the Su 3.

The Su 2 you do not have this diagram right you have only just spin half particle is a fundamental object up and down and you take tenser product of only spin half particles clear you just take multiplication of spin half particle in the case of Su 3 you have these combinations. That is why when I combine these two you have to make sure that it is an it is just adding things in such a way that the diagram formula which is here is not violated.

So, what will be the possibility for this it has to be allowed to be this and this one will be, anything else is possible as a diagram, respecting that kind of a situation. What about here, you can allow this is one possibility you can have all the 3 here and a single one and what else anything else diagram should be such that if I put a 1 a 2 b 1 b 2 a 1 and a 2 has to be anti symmetric b 1 and b 2 has to be anti symmetric, there is no relation between a 1 a 2 to b 1 b 2.

So, when you combine diagrams you can put the entries in such a way that here you see a 1 a 2 b 1 b 2, b 1 and b 2 are anti symmetric to a 1 a 2 here it is b 1 is symmetric to a 1, b 2 is symmetric to a 2 does not matter, because there is no symmetry between a 1 a 2 to b 1 b 2, but there is a symmetry amongst a 1 a 2 amongst b 1 b 2 I cannot violate that when I am putting those diagrams.

So, if you start drawing the diagrams these are the only 3 possibilities which I can get. What about this one, quark incidentally a quark and an antiquark together it is a composite object it is called meson. In nature you see a composite particle which is a bound pair of quarks and antiquarks which is called meson, to see that bound state you have to take a tensor product of fundamental objects one is a quark, one is an antiquark and they decompose into irreps.

How many will be the dimension here? It is 3 into 3 which is 9, this is 3 dimensional, this is also 3 dimension, this is 9 dimension, this 9 dimension is a reducible representation. Same here 3 dimensional quark, this is 3 dimensional quark, it will be a 9 dimensional diquark you have a 2 quark object, but it is reducible representation and that 2 quark reducible representation breaks up into irreducible representations, what are these dimensions now you can work it out, for Su 3 can you work out this anyways 3 this has to be.

Student: 6.

6 is that right. So, 4 3 4 and then 2 it will give you 6. So, this will give you in dimension language it is 6 plus 3 because it is vertical sometimes in books they put a 3 bar that it is associated with this diagram ok.

Student: Mam.

Yeah.

Student: There is a quark into antiquark (Refer Time: 10:02) 1 box.

So, if you have a quark into antiquark you will have one vertical and then one which is this, these are the 2 possibilities.

Student: Quark is represented by 1 box.

Student: (Refer Time: 10:19).

So, there is no meaning to this is just a diagrammatic way of representing it that any antiquark will be represented by a vertical column of n minus 1 boxes of Su n. So, for Su 2 there is no distinction, Su 3 that is an antiquark representation because both share the same dimension if you go to if you take this vertical 2 box and work it out the dimension for Su 4, you would not get 3 or you would not get 4 also.

So, it is not an antiquirk representation, antiquark representation will be always a vertical column with n minus 1 box. That is the you should not think of this diagram as if it is a composite of 2 quarks already that is what is your confusion right. Yes mathematically both are equivalent, but when you want to compare it with nature the way to see it is that this see you know when you take an electron if you make it into a positron which is an antiparticle your electric charge gets an opposite sign this you all know.

So, similarly if I take this u quark this should become a u bar with an opposite charge that is what will happen here ok. So, that is why this is like a diagram wise by doing this it could be interpreted as a bound state of 2 quarks, but this diagram conveys that it is an antiquark representation is that clear. So, just fall back only on the Su 3 when you are getting confused with the abstract notation ok.

So, let me get to the slide back again. So, for arbitrary representation of SU 3 any irrep the highest weight of an irrep will have number of single boxes a of them number of double vertical boxes b of them, clear ok.

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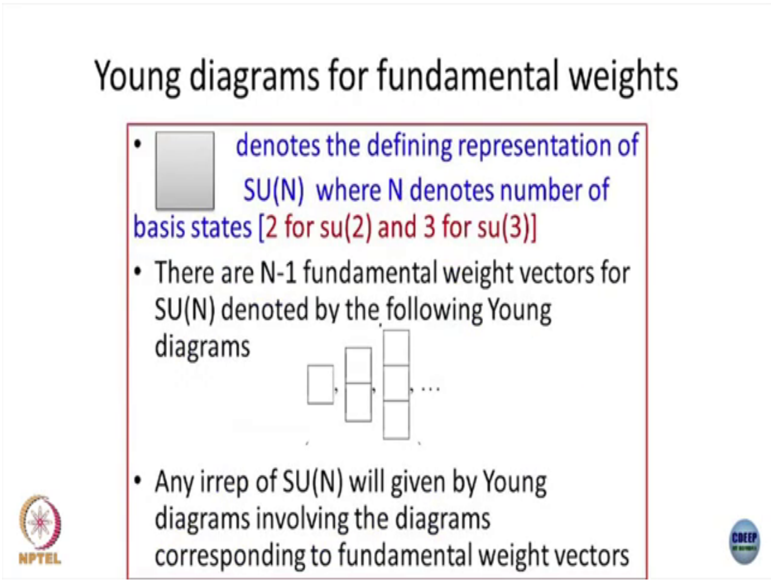

So, this is summarizing my young diagram way of looking at it, single box denotes defining representation of SU N, where N denotes the number of basis states that 1 0 0 you know that way of writing it you can have N of them 2 for su 2, 3 for su 3 and so on. There are N minus 1 fundamental weights I illustrated for su 3 that there are 2 fundamental weights in principle you can find for SU N N minus 1 fundamental weights N minus 1 is also the rank of the SU N group. So, you can have N minus 1 fundamental weight.

Pictorially as an Young diagram we denote this N minus 1 fundamental weights as single box, double vertical box, triple vertical box and so on, any irrep will be given by an Young diagram involving the corresponding diagrams corresponding to the fundamental weight matrix ok.

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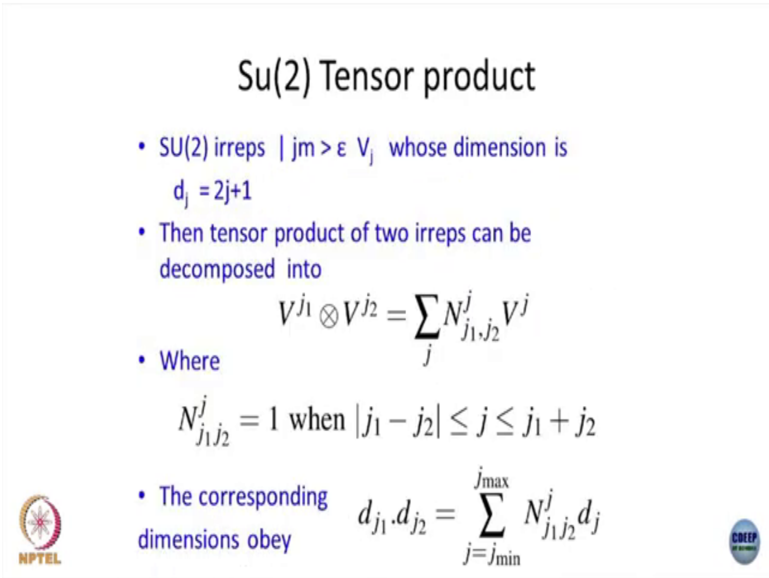

So, this is what I said and now I am going to get on to SU 2 for clarity on tensor products which you all know and how to see it from the group theory and also the Young diagram language. So, SU 2 irreps we denote it by j which is your highest magnetic quantum number which we call it as a highest weight and then the magnetic quantum number and these belong to a vector space V j that subscript j to denote that the dimension of that vector spaces to j plus 1 this is something which you all know.

And if you take tensor product of 2 irreps let us take spin j 1 and spin j 2 you will write it as a decomposed one involving mod j 1 minus j 2 in steps of 1 up to j 1 plus j 2 I am sure you all done this, but for the sake of completeness as a group theory I will take a tensor product of this is tensor product of 2 fundamental representation 2 spin half and that gives you 2 irreps and those 2 irreps have to be what are the, if you compose 2 spin half what do you have to get you have to get spin 1 and a spin 0, one of the diagram should be spin 1, another diagram should be spin 0.

So, that is the way to see the diagrams and the corresponding dimension should also multiply and add up that is what happened here 3 into 3 was 9, 6 plus 3 is 9 right. This is what you have done even in the earlier case I remember when you did c 3 v or c 4 v if you took a 2 dimensional irrep e cross e; e cross e is 4 dimensional reducible representation and then you broke it up into a 1 plus a 2 plus e which is also 4 dimension total, but it is 1 plus 1 plus 2 right.

So, such a decomposition in the context of lie groups can be written at least for the unitary groups in this Young diagram language and you can see that the number of irreps will be 2 for composing to defining representation ok. So, composing to defining representation is a reducible representation, but then diagrammatically you can break it up into irreducible representation to find the dimension you use the spot ok.

So, the dimensions also will match this something which you have done in SU 2, but this can be done for the SU 3 and higher groups. So, this is what I am explaining it here.

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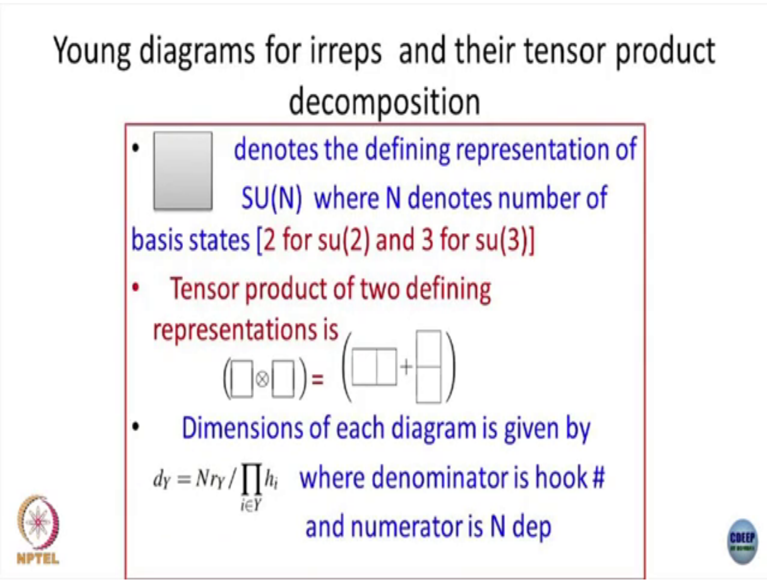

Take a tensor product of single box with single box this combination will be a reducible left hand side is a reducible representation and you can break it up into 2 irreducible representations. What do you have to check? The dimension of this times the dimension of this, what is dimension of this for SU N, it is N dimension for this is the N. So, N square you have to make sure that this plus this is also N square maybe you can check it by using the hook formula for an arbitrary N ok.

The hook formula also I have written here only modification is there is a non trivial numerator which I explained already.

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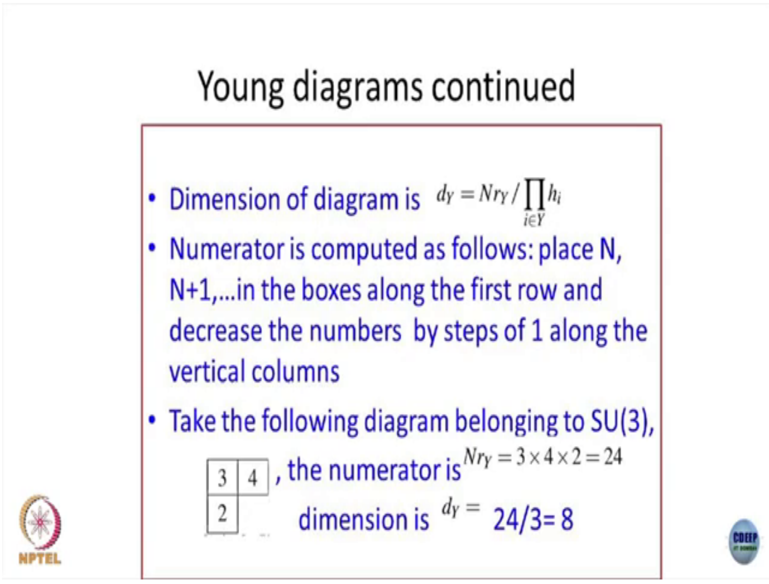

So, the non trivial numerator is that you place N, N plus 1 in the boxes along the first row and decrease the number by steps of one along the vertical columns and take the following diagram. So, for SU 3 you will have 3 4 along the row and column which I explained already and it gave you 8 dimensions.

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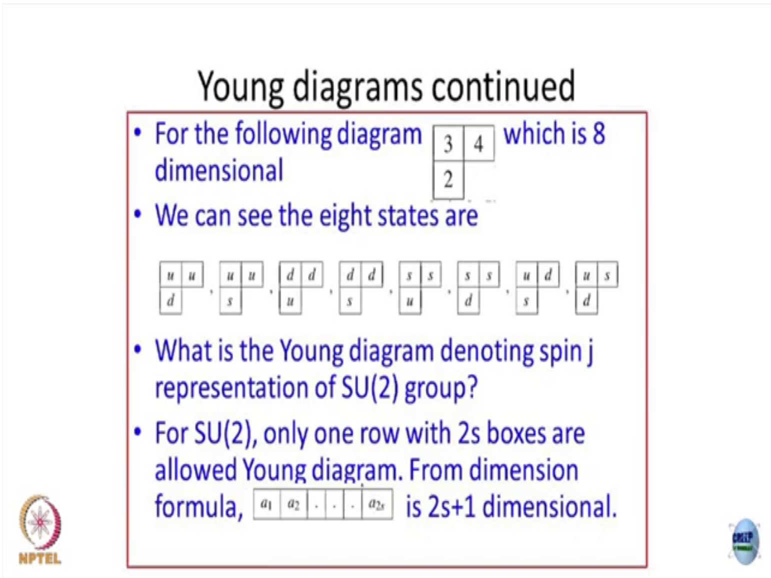

And that 8 dimensions are the 8 states which you can write it out using the 3 fundamental states of SU 3, keeping track that along a row you can have a symmetry, but along a column it should be anti symmetric. So, you cannot put the same state both you cannot put u and u here, if you put a u here the rest either it can be a d or an s that is all I am saying ok. So, now tell me what is the Young diagram denoting spin j representation of SU 2? What will be the diagram?.

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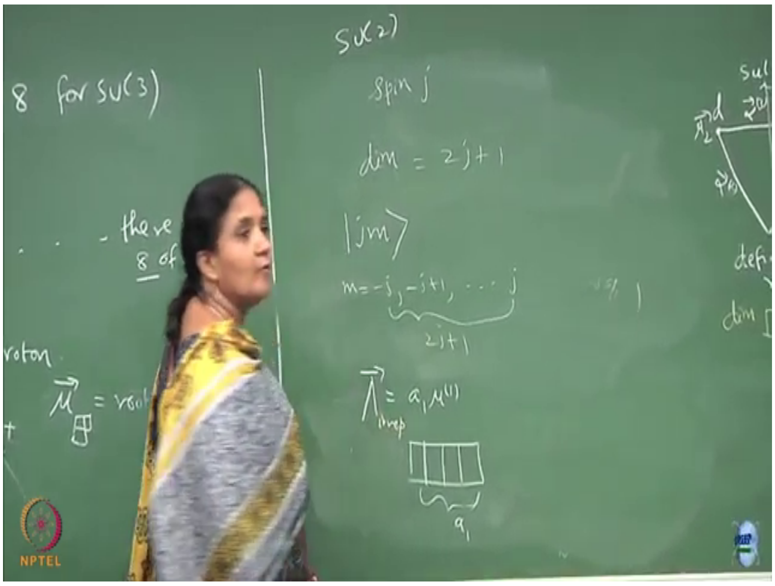

So, spin j dimension is what, 2 j plus 1 right because you write it as j m, m will take values minus j up to j. So, this number of states is 2 j plus 1, you all with me which diagram will give me 2 j plus 1 for the SU 2 group. SU 2 group you can only put highest weight as highest weight as some number of boxes times mu 1 right vector is also no meaning because it is just 1 component.

So, this can be written as a 1 boxes this is the diagram for SU 2 right it is just an irrep of SU 2 whose highest weight is a 1 number of fundamental weight there is only one fundamental weight and for that one fundamental weight I would have put a single box a 1 of the fundamental weights you will have a 1 box, what is the dimension of this, can you work it out for SU 2 somebody what will the last box be?

Student: a 1 (Refer Time: 20:52).

A 1 or a 1 plus 1.

First box is 2, second box is 3, a 1 th box has to be a 1 plus 1, what is a hook number for this?

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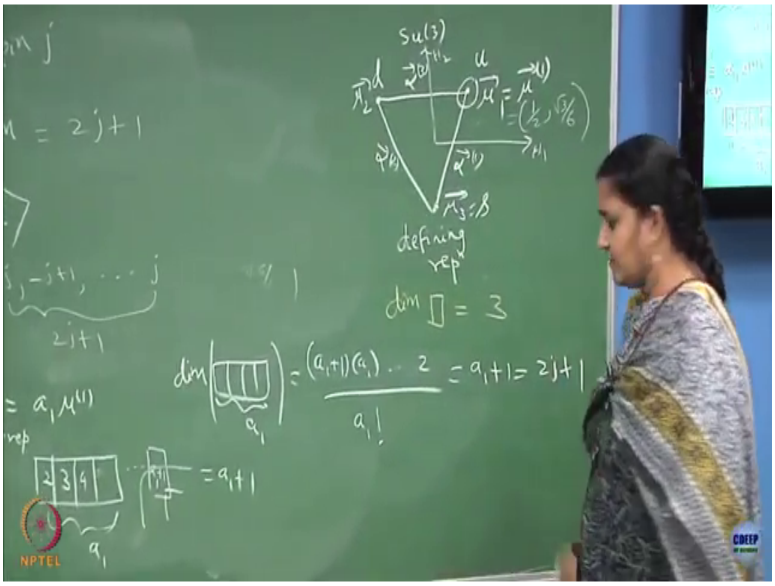

Student: (Refer Time: 21:10).

It is a 1 plus 1 a 1 goes on up till 2 in the numerator, denominator is.

Student: A 1.

A 1 factorial 1 2 are you all with me what is the dimension a 1 plus 1 right. So, for a spin j representation what is the diagram, you should draw a diagram with how many boxes? This dimension is a 1 plus 1 I want this to be 2 j plus 1, what is the diagram spin j diagram will be a single row with how many boxes clear.

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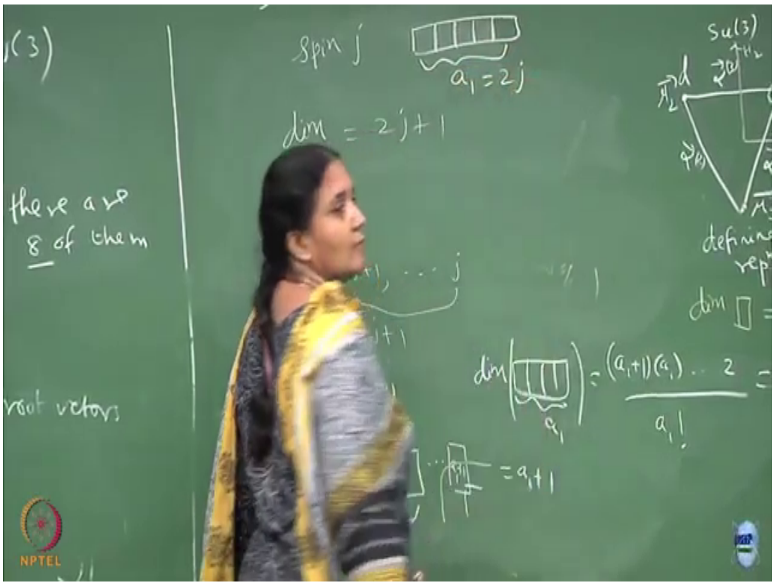

So, each of these diagram is a higher spin representation of SU 2 where the dimension tells you how many states are there and this one will be just a 1 plus 1 which is 2 j plus 1 the number of boxes has to be equal to 2 j clear.