**Group Theory Methods in Physics**
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**Lecture – 26**
# Binary Basis and Observables

Let us start as a warm up, we have gone through Tensor product.

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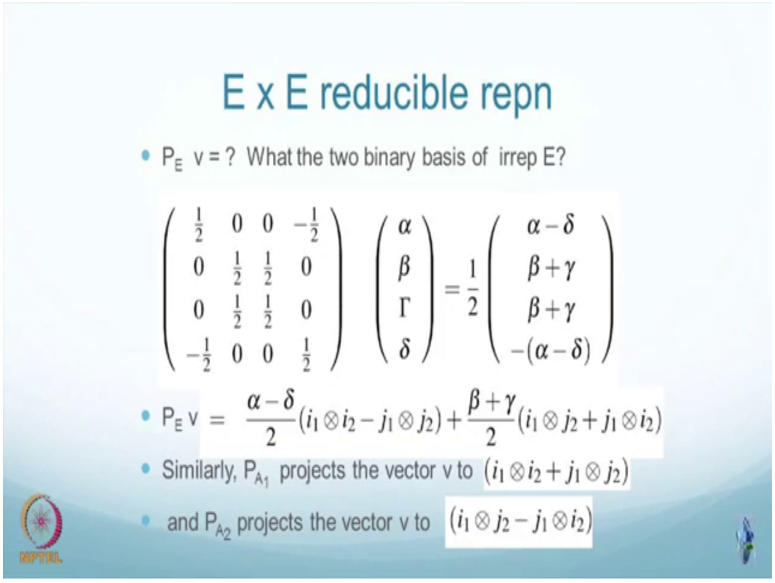


So, this should be the tensor product of E cross E by E cross E. What do we mean? E is a two-dimensional irreducible representation, I am taking a tensor product of two-dimensional irreducible representation with basis states i 1 and j 1 and this second E, with basis state i 2 and j 2.

So, the corresponding reducible representation will have a four-dimensional basis state and any arbitrary vector v, if I write it as a column as alpha, beta, gamma, delta, it should be understood as alpha times i 1 cross i 2, beta times i 1 cross j 2, gamma times tell me.

Student: (Refer Time: 01:19).

J 1 cross i 2 and then, delta will be j 1 cross j 2 and then, we have also understood how to write the projector for any of these irreps. It depends on the characters in the character table for the irrep E. I want to find what happens to an arbitrary vector. If I take the arbitrary vector to be this sorry, this gamma should be same as this gamma. If I take an arbitrary vector and the projection for the E dimensional irrep is this. If I do this on this, I end up getting this side which means I have 2 independent basis ok.

If alpha and delta are equal and opposite beta and gamma are 0, then I will get this basis state. If beta and gamma are nonzero, but alpha equal to delta, then you will get this basis state. So, there are 2 linearly independent basis states which is what you should get for a projector of a E dimensional irreducible representation. Because the rank of this projector has to be 2. There should be 2 independent basis. Is that clear?.

We did this, I am just repeating it. So, that you can recall what we did before the; last week what we did. So, that we can get going and similarly, you can also show that the projection operator corresponding to the irrep A 1. If it operates on the same vector v, it will give you a linear combination of this ok. Projection operator for A 2 will give you a different combination which is orthogonal to this and it is in fact, orthogonal to this and this ok.

So, the A 2 is also one-dimensional projector and you end up getting a projection operator. Projection operator gives you this basis state and what will this basis states called? These will be the basis states which are called binary basis right. If you are looking at just the basis primary basis, it will be either i 1 j 1 or i 2 j 2 for the E dimensional representation right. If you want to look at the A 1 basis binary basis, then the binary basis will be i 1 i 2 plus j 1 j 2.

If you try to put them as position coordinates and if these two are different vectors, you can call it as R 1 dotted with R 2 ok. So, that is like a scalar which is like the unit representation and if you have A 2 projector, then you get a cross product that is like your R z component. Is that clear?.

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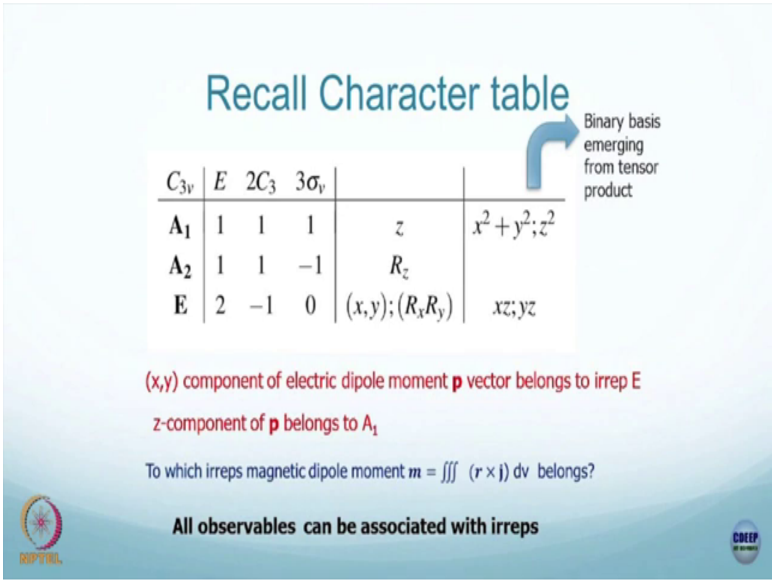


So, with this warmup, let us also recall how to since I have already said this now, you can see that the A 1 irrep in the binary basis will be a dot product of 2 vectors in the xy plane, which is x squared plus y squared, if both the vectors are one and the same and it could also be just z squared, that could also be a binary basis. These are the primary basis z Rz, it can be x and y.

It can also be pure rotations, does not distinguish for you whether it is a polar vector or an axial vector. Why? To see whether it is an axial vector, you have to do an inversion or a mirror transformation which is an improper transmission. If you are not doing that, you will not be able to distinguish it.

So, in this case you can say that x and y if it transforms like up a irrep which is two-dimensional E; Rx, Ry also will transform like A 2 dimensional irrep E. Another way of seeing is that this sigma v character is 0 ok. So, in some sense that you can have both are equivalent basis here.

If you got a binary basis, this we have I have also tried to ask you some product of what was the product? Product of A 1 times E, if you take a tensor product of A 1 times E, then the characters will get multiplied right. The characters get multiplied, they are all anyway 1. It does not really matter. You get back E and the basis for A 1 primary basis is z, primary basis for E is xy. So, you can get binary basis as xz comma yz ok. That is a two-dimensional basis.

But each one is a binary basis ok. Why are we doing all these things? That is a next question you can ask. Why are we spending so much time in understanding position space, primary basis position space, tensor product giving me binary basis right. So, you can ask looks like a routine monotonous thing we are doing. Is there some meaning to it? So, that is what I am trying to stress now that whatever you add review to the position space, you can look at physical observables in nature.

What are the physical observables? For example, if I take the electric dipole moment, you all know electric dipole moment of 2 charges equal and opposite charges separated by a distance r is q times the distance between them right. So, what is the nature of that electric dipole moment? Is it a polar vector or axial vector?

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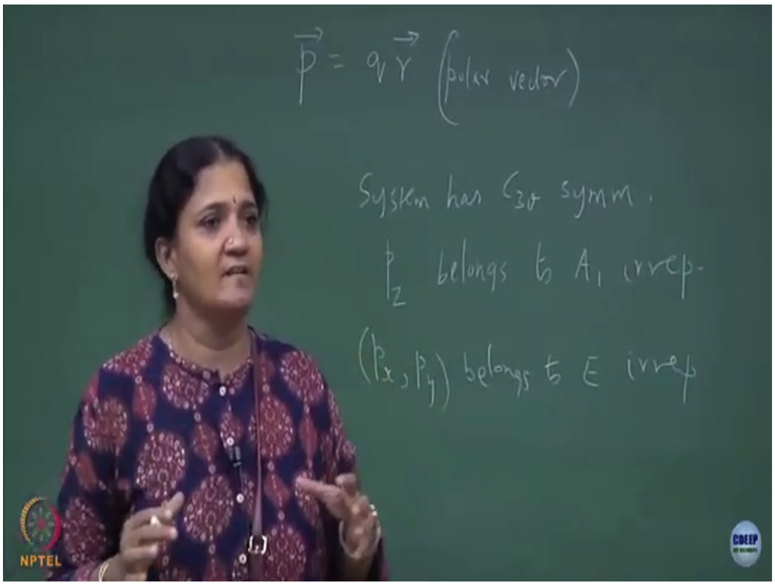


If I write electric dipole moment as q times r vector, is it polar or axial?

Student: Polar.

It is polar right; it is a polar vector. So, this is one nice observable which is which comes into play whenever we put in some interaction ok, in the energy with this electric dipole moment into picture, you do need them ok. So, these are observables and then, if I ask this observable belongs to which irrep? Suppose, I give you a system which has C 3 v symmetry ok, system has C 3 v symmetry ok. You can straight away say that p z observable belongs to A 1 irrep ok. Just looking at the z component, any polar vector z component will belong to the A 1 irrep.

Similarly, px, py belongs to E irrep ok. So, every observable which I am going to do I can start trying to associate whether this observer belongs to a particularly irrep given a system with some groups symmetry. I am going to confine myself to discrete symmetry ok.

So, for example, let me just confine to C 3 v; let me say that the symmetry of the system is C 3 v and then, observable one of the observable is your electric dipole moment vector. You can say that these components of the observable z component belongs to A 1 irrep; P x, P y belongs to E irrep.

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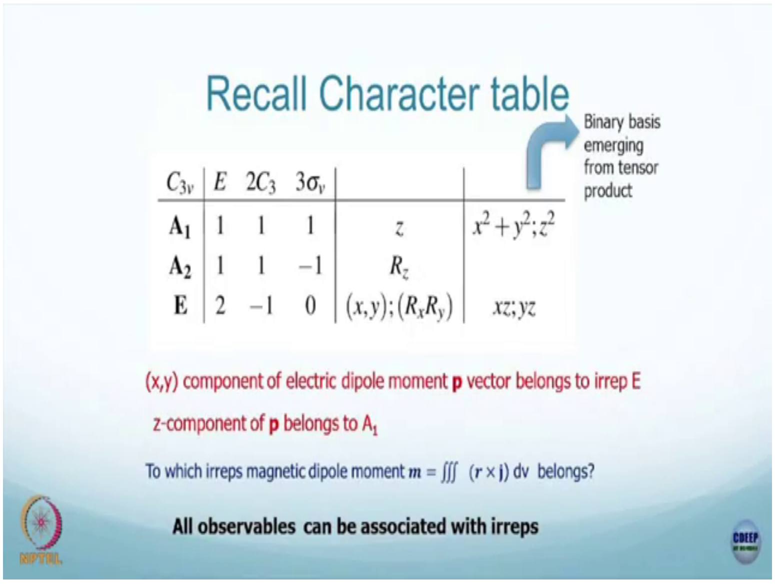


Similarly, you can also look at magnetic dipole moment, you all know magnetic dipole moment. So, magnetic dipole moment involves a cross product between current density and the position vector. Once it involves cross-product, what is it? It is an axial vector ok.

So, if you write magnetic dipole moment dv. This is going to be an axial vector. So, now tell me where the z component will belong? A 2 irrep. Of course, mx, my belongs to E irrep ok. Facts C 3 v does not seem to distinguish the x and y component whether it is polar or axial. Z component, it does distinguish clear.

So, every observable which I am going to write in nature, I can categorize which irreps it belongs to for a given system with the symmetry which I am looking at. If I am looking at a harmonic oscillator symmetry, then I have to look at it as a C 2 group symmetry and then, I have to see which one it will belong to and so on ok. So, now, I am doing a C 3 v and I am looking at the observance ok. Is this clear?

So, as of now these all belong to your primary basis; am I right? I have not really talked about binary basis, it is just primary basis whatever happens to the components of your position space vectors, will happen for any of these polar vectors, any of the axial components of the vectors in position space, you can again attribute it to the axial components of your other observables like magnetic dipole moment ok.

So, this is primary basis. So, I am just trying to justify the bottom line. All observables can be associated with irreps ok. So, why we did this elaborate exercise is because we want to look at interactions involving operators for these observables, whether it will trigger a particle in the ground state to go to the next higher state excited state and so on if you want to understand you need to associate irreducible representation to the observables. Is that clear?.

That is the main reason why I have brought in the basis states, not only basis states binary basis obtained from tensor products. After that you do a projector and get the binary basis. We have done this elaborately and the reason for it is that it will help us to understand what are the observables associated with binary basis, that is the next question you can ask right and that is coming from tensor products ok.

You can take 2 vectors; vector 1 which r 1 and then, you can also take r 2, but you can do a tensor product of this. So, this has 3 components; this has 3 components. Essentially, I can write something which is q i j, if I take the i th component of this and the j th component of this ok. So, this is something which I can do.

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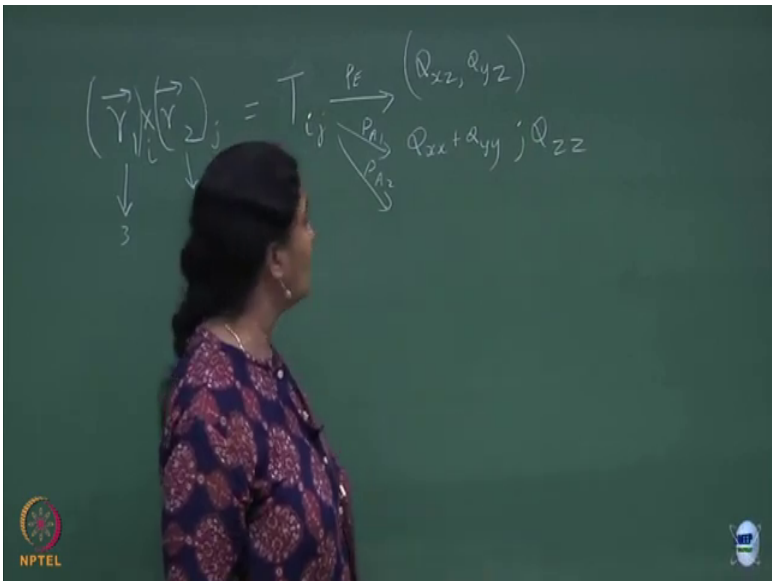


If you want to make it traceless, I have to subtract something; but let me just write it as up to some tensor. Let me not even call this to be q, let me call it as some tensor t i j. Is this reducible or irreducible?

Student: Reducible.

It is reducible. Whenever I take a tensor product the corresponding vector space will be nine-dimensional which I have written like a 3 cross 3, but it is reducible. And then, you will have to do the projectors; appropriate projectors. If you do P E, you will get something; if you do P A 1, you will get something; if you do projector A 2, you will get something and they will all be binary basis of the irreps which you have. Is that right? So, you will get those binary basis out of this.

So, one of the examples which I have just I am sure you have all seen quadrupole moment tensor, when you did this expansion in electromagnetic theory course right. Quadrupole moment tensor xx component if you try to write explicitly, you will get this answer; yy if you write it explicitly, you get this answer; zz is this answer.

It is traceless that is what I said the quadrupole moment tensor is traceless and you can also show by symmetry that this gives a 3 xy and xz component and yz component are like the xz in position space up to overall factor multiplying it which depends on the charges and so on.

Now, look at the basis binary basis which we went through. So, now if you want to say which component belongs to A 1? Can somebody help me out? If you look at this x squared plus y squared analog of it is Q xx plus Q yy ok. So, this combination will behave like a A 1 irrep in a binary basis and this binary basis is extracted by taking tensor product of 2 vectors and then, using a project. Are you all with me? So, now, tell me what will be the E representation?.

Is that right ok. So, that will be the binary basis representation which corresponds to your quadrupole moment tensor components. So, this belongs to irrep E. This belongs to the summation belongs to irrep A 1. You can also have separately Q is zz. This is a 1 d irrep and what about A 2? Someone? Is that right? Is that A 2 or no? Should be know? Yes or no?

Q xy will belong to your A 2 irreps. So, I have given you some kind of a feel for. So, you can still go further, you can find the tertiary basis. What does the tertiary basis be obtained from? You take binary basis, tensor product it with one more basis primary basis and then, start doing projectors and start finding the tertiary basis ok.

Student: Q zz, there will be 2 z square term (Refer Time: 19:21)?

I did not?

Student: X square plus y square.

Ha.

Student: There will be 2 z square term.

2 z squared, where is it 2 z square?

Student: Because z q z square by r square. R square will (Refer Time: 19:34) like what you want say there will be z square term (Refer Time: 19:36).

Which one?

Student: Q zz.

Q zz, there will be a z squared minus r squared. So, r squared is like a constant, I am taking it as a constant. So, then z squared is the basis anything shifted by a constant, I am not worried.

Student: Written that r squared z squared plus y squared?

Yeah, then you will get back this know.

Student: But there will be 2 z squared minus x square (Refer Time: 20:04).

So, in some sense you can also write a linear combination which is a one-dimensional basis, in some sense that is another way of doing it. But here, you cannot do that. Here, they are 2 independent basis; they are these are 2 basis, but then you can also play around with 2 linear combination of the basis.

Any other question? I am just taking that r squared is a universal constant or if you remove that then, Q xx plus Q yy will exactly transform like they A 1 irrep and Q zz will transform also like an A 1 irrep that is all I am trying to say. Even if you take a linear combination of two A 1 irrep, it should not really matter, any other question? Is this clear? Is the motivation clear? We had gone through the drill of doing character table, drill of finding why base states, why binary basis, why tensor product?

Now, you can associate every observable either to a primary basis or a binary basis and dot dot dot, it can go to tertiary basis you know; you can start looking at things.

Student: (Refer Time: 21:29).

I am I thought it is like a cross product like R z. So, I was thinking that it should be like.

Student: We do not have basis?

We do not have any basis here right. I am not written or it is not allowed; can we check it? If you take x y, maybe it is not right. What you are saying is probably not right. So, this is this is a question mark; whether it belongs to A 2 or not or it is something which has no meaning in this particular group. Some other group, it might have meaning. But what you are trying to say is that in the character table xy part, we never had.

I need to see y, but I think xy is not allowed right, if you do the C 3 operation on x and y, you get a linear combination and xy will not have eigenvalue is 1. So, I think xy is not allowed, I agree with. So, this one does not have any place to fit in the C 3 v character table, but this is a way to do it like you write it out, look at the binary basis table this table and see where the observables can fit in; which irrep it can fit in.

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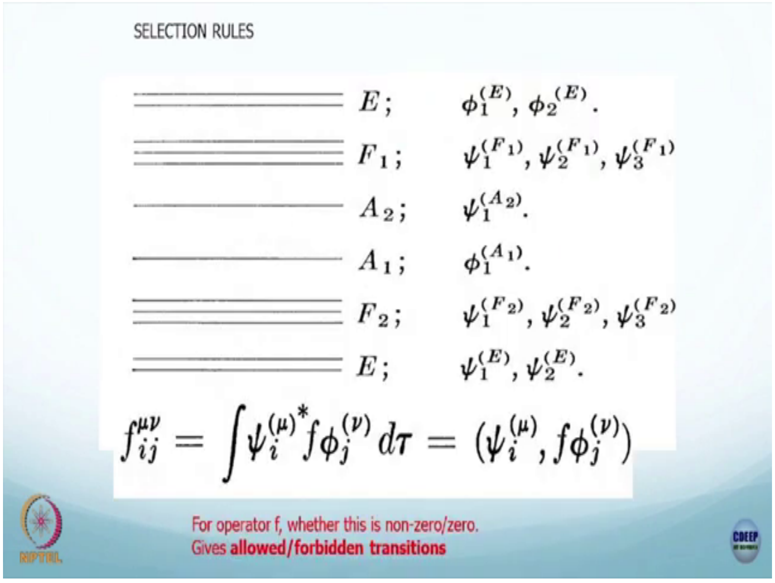


That is the theme of this, showing you. Yes, Q x y does not have any place to put it, I agree.