Group Theory Methods in Physics
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Lecture – 02
Introduction – II

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Ok. So, the next thing I want to say is generate. Let us take our dictionary ok. What is a dictionary? Dictionary if you bring a dictionary, it has all possible words. The words are made out of some fundamental objects, they are the alphabets. How many alphabets you need? 26 alphabets you need right. You can concatenate this 26 alphabets in whatever form you want and you a get a word, some words will be meaningful some words will be meaningless, but you can concatenate concatenation is like a group operation right. Does still dictionary form a group, yes or no? Closure property is there, empty space I can treat to be like identity.

Student: Inverse.

Inverse does not exist, I cannot find another word when I combine with a space it will or combine with some other word it will give you a space, it is not possible right. So, a dictionary they generate a 26 alphabets, but definitely it does not form a group. The same argument we can start looking at generators for sets which form a group right for example, this example which I took.

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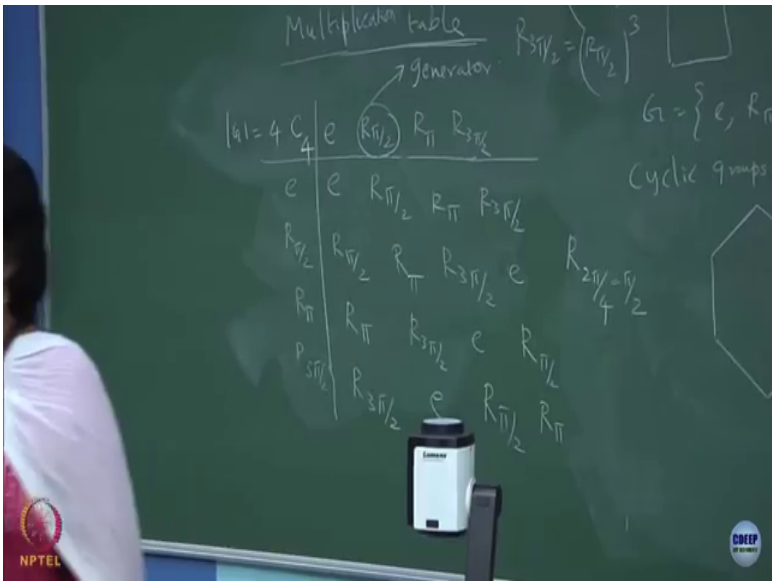

I can say that the fundamental operation is pi by 2 which I can call it as a generator, is that right? I can call that as a generator. What do I mean by calling it as a generator? I can stake the generator some number of times and you should generate all the elements which are there.

So, if you want to get R pi, I take R pi by 2 twice ok. Just like an alphabet 26 alphabets I can create a word ok, here I have only one alphabet which is R pi by 2.

We just R pi by 2 any number of times written, because if I had two different fundamental operation, I can do many things I just have one fundamental operation. I can do this many times, but also there is some limit this pi by 2 upto 360 degrees only I can go right. So, R 3 pi by 2 it will be, so in that sense for this particular group which is the symmetry for a square, I can say that R pi by 2 is the generator of a group ok.

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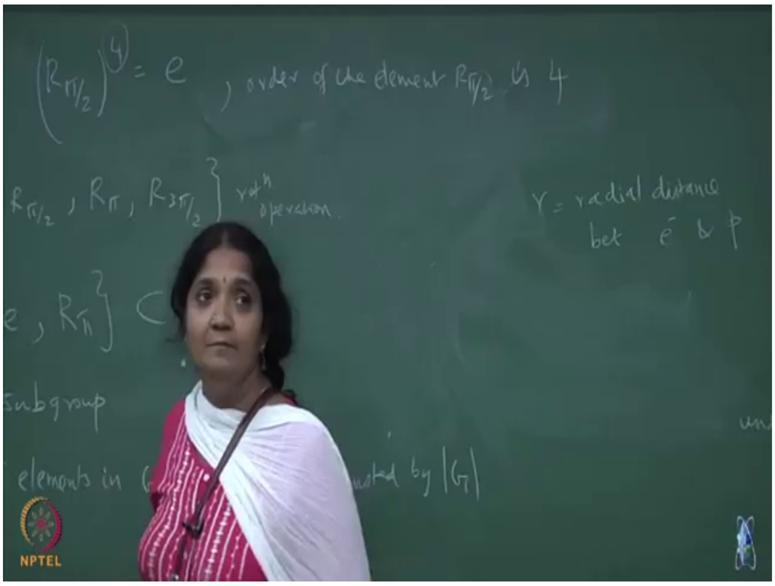

And since its generator also satisfies its identity ok. Sometimes we say order of the element R pi by 2 is 4 that 4 is because of this power, ok. So, let us take some simple example like cyclic groups ok, so let me give you some simple examples. So, whatever I did for a square I can do it for any polygon right, the rotation will be can you tell me what will be for a pentagon or a hexagon?

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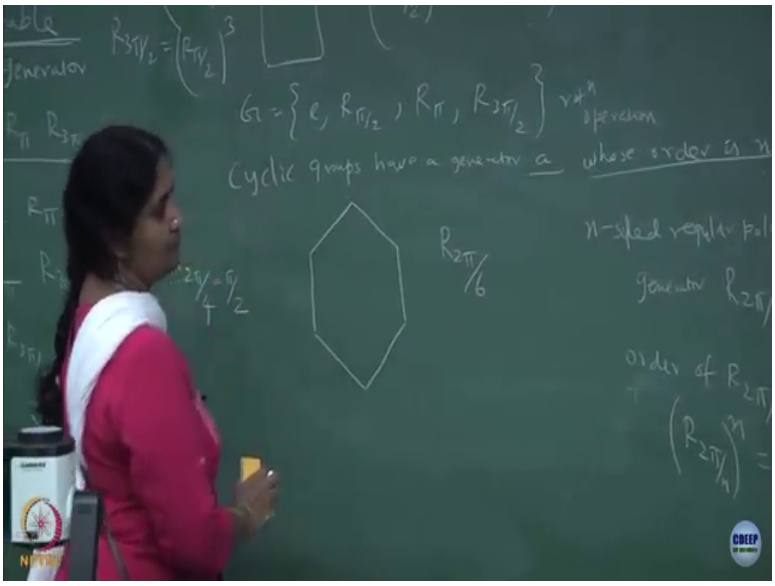

Let us take a hexagon, regular hexagon I am not drawn it very well take it to be a regular hexagon. What are the rotation operations, someone? 2 pi by 6, there it should have been 2 pi by 4 technically right. This one which I have written I could have said that there is a since it is an order 4, 4 vertices are there I could write that it is a generator is 2 pi by 4 which is nothing but pi by 2 here in this case.

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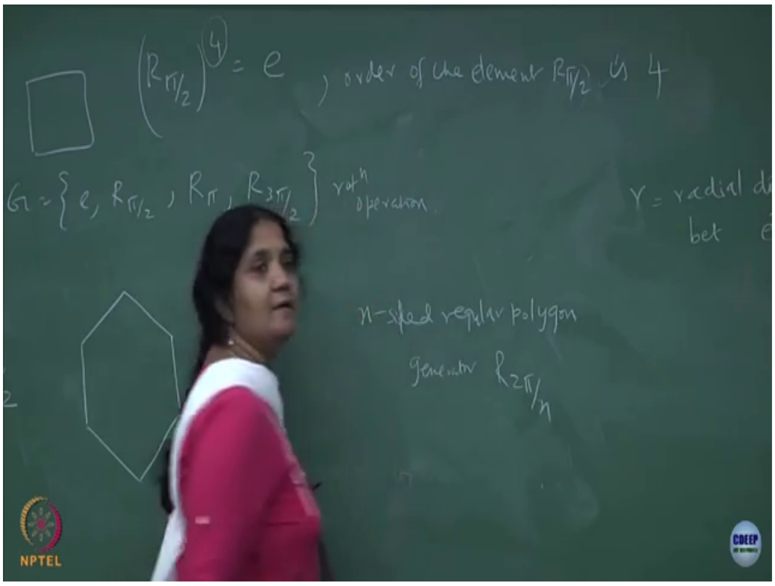

Here it is 2 pi by 6, in general (Refer Time: 05:17) if you take, what will that be? 2 pi by n for an n-sided regular polygon generator will be 2 pi by n that will be the generator.

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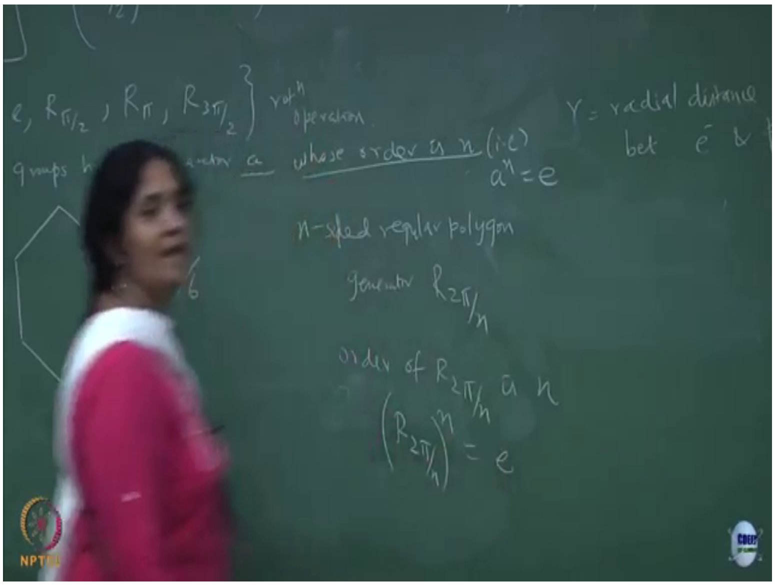

What is the order of that generator? Order of that element, why? If you take R to the 2 pi over n n times, it will be identity and this power is what is important ok. So, cyclic groups have a generator, let me call it as a, whose order is n. What do I mean by this? That is a to the n is?

Student: e.

e that is the meaning of this. So, what will be the group elements someone? Cyclic group elements of order n will be identity a, a is the generator all powers of it till.

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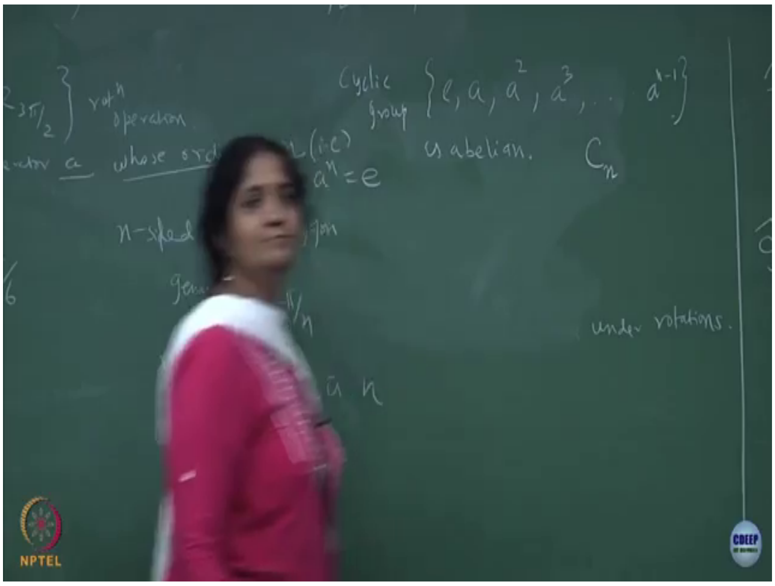

Student: n minus 1.

n minus 1. So, order of the group is also order of the generator and is it abelian not abelian, its abelian right, so cyclic group is ok. And because I am looking at cyclic group of order n sometimes a notation is cyclic group with a subscript n, so this group will be called C subscript 4.

Student: What is cyclic group?

Definition of a cyclic group it is just that it is given by one generator and the order of the generator, which is the order of that element defines the order of the group.

Student: (Refer Time: 08:09).

Yeah, you can permute it and start looking at any arbitrary element does not matter, because you can do that here also. You can take a square to be you know you can redefine things, but that should not really matter.

Student: (Refer Time: 08:28).

You call it as cyclic group if there is one generator and it is an abelian ok. So, just like alphabets let me give a group with two generators, let us start with let us do some examples.

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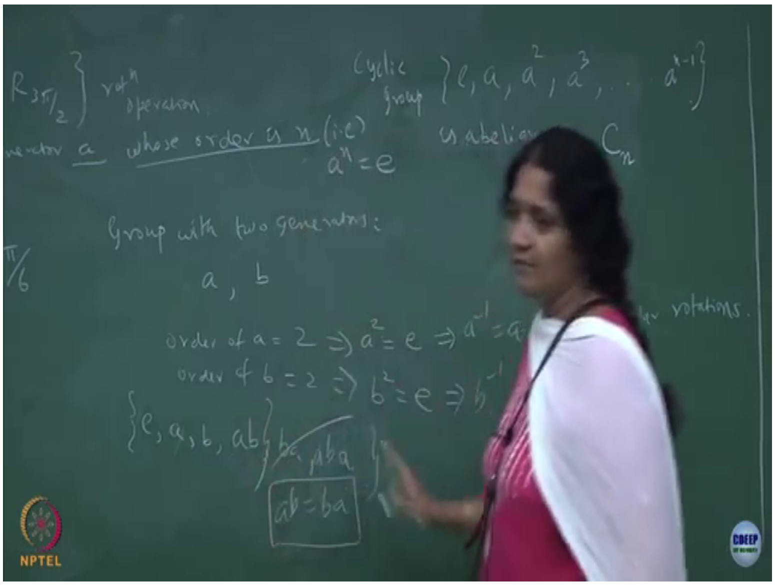

So, let us look at a group with two generators ok, let me call it as a and b; order of a is 2, what do I mean by that? a square is identity, order of b is 2 identity ok. Now, you want to construct just like using 26 alphabets all words, I want to construct all words using these two generators and powers. Let us start of so first is identity anyway a, am I right you can have a b. In general if it is different order you would have got a square, a cube and so on. Here it is a square is identity we do not need to do that, but you can have one more, what is that one more or two more?

Student: a b.

a b and b a right anything else that is it that you can even show that a b is equal to b a, how do you show that somebody? You have to first draw the multiplication table and see what happens ok. So, let us do that also in fact that group which is generated by a and b, its call that claim group.

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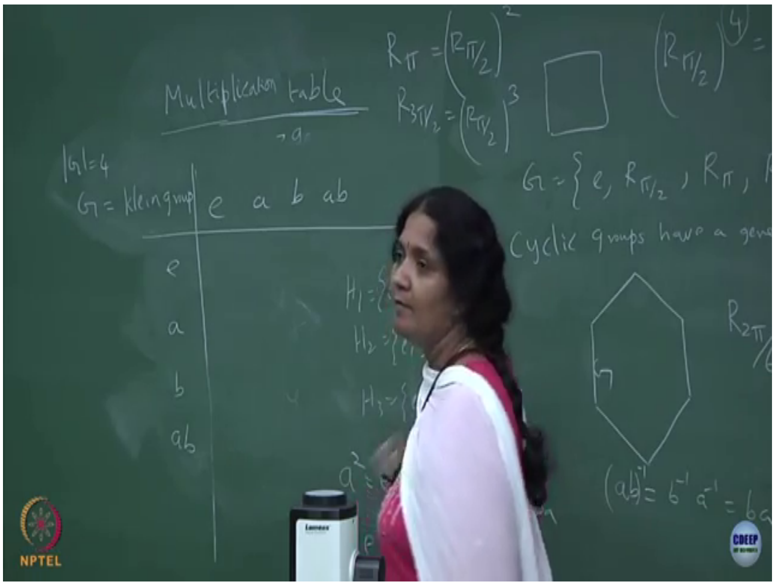

Student: (Refer Time: 11:13).

Good that is why I am saying a b will be b a. Yeah.

Student: (Refer Time: 11:17).

Inverse of a square is identity a is self inverse, b is self inverse. I am giving you some data, I am telling you form a group, generated by two different generators a and b, how many elements will be there in that group? Technically the set will have this right with the generator you have a and all powers of a, but I have given you additional condition that the order of a is 2, order of b is also 2 which means I cannot have in that set a square and b square; I can have an a, b and then I could also have a a b, I could also have a b a. What else can you have? Yeah.

So, how many powers you can go if the order of that a is only 2. When order was n, then you can go upto till a n minus 1; if order is 2, you can only go e and a right, that is why I do not have all powers of a. We will come to that variation, I have given only this specific case. So, this also implies a inverse is a, this also implies b inverse is b right. So, with this set I do not think I can have anything more than this, can I have anything more a b square is a, a b to the power of any power even power is does not matter, odd power will get back to the same set, so I cannot have anything more than this.

Student: (Refer Time: 13:15).

I agree; I agree it cannot be, but I am saying if I say the group is generated by just like the 26 alphabets creates all words, so it is not a single generator you can have many generators and you form the set.

Student: (Refer Time: 13:34).

No, no, no that is only for cyclic group which has one generate, but if you have more generators closure property tells you that combination of two generators should also be an element of the set right.

Student: (Refer Time: 13:53).

Generators are a and b in this case.

Student: (Refer Time: 14:00).

I am not able to hear you.

Student: (Refer Time: 14:04) what is the definition of generator?

Generators are basic fundamental objects by which you can construct all the elements of the group.

Student: Yeah, so all the elements (Refer Time: 14:14).

All the elements we have to make sure that this forms the set which is satisfying all the axioms of the group properties.

Student: (Refer Time: 14:23).

That is not required, that is not what is meant by that.

Student: (Refer Time: 14:32).

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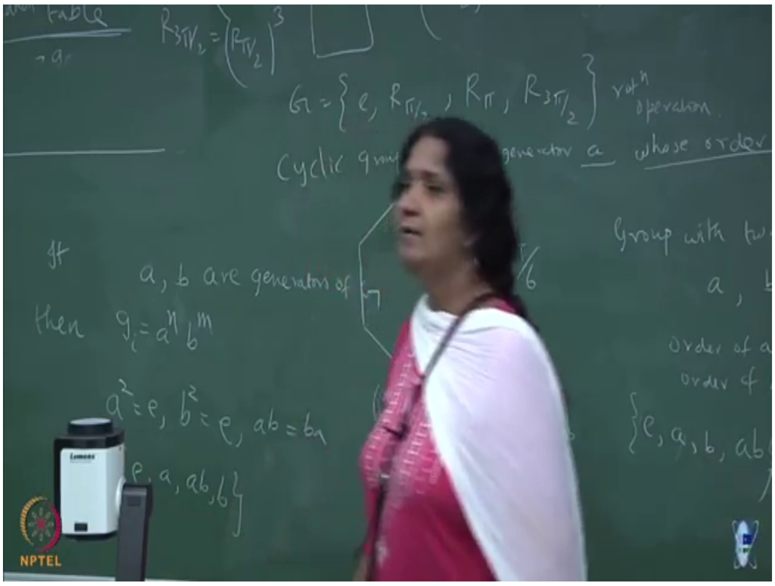

The general statement is if you have a and b are generators, any element g i of the group should be some power of n and b power n, this is all is ok. So, let me write it if a and b are generators of G, then g i should be some power of n and some power of m. If I on top of it I say that a square is identity and b square is identity, then the group set can have e, a, a b, b; it is not clear whether b a is independent or new, but right now as a set I can write this and then make sure whether a b is b a ok.

Student: Ma’am.

Yeah.

Student: (Refer Time: 15:36).

a b a will also be allowed good point I think, so what you are saying is that I could also have a b a. In principle if there is no condition on a b with b a, then this can I think go on like this yeah. So, this is possible, but let me also add one more condition a b equal to b a ok, so let me put that condition. Suppose I put this condition, then it truncates here I agree with you, so let us put that condition ok. So, I am going to take a square equal to e, b square equal to e and a b equal to b a; then generate the group if I say, then you know that otherwise you know it can go indefinitely I agree with you.

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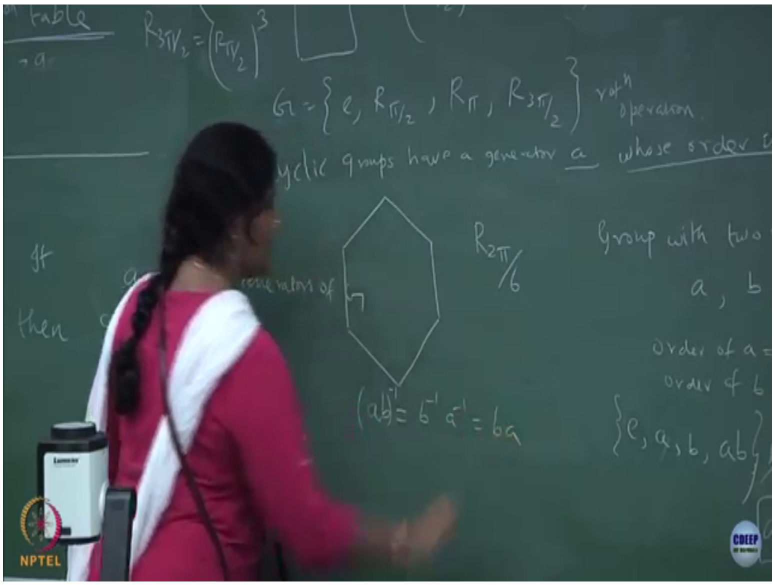

Student: (Refer Time: 16:35).

Inverse of yeah, so let us do that inverse of a b is what? b inverse is b a n, so that is also going to be a b by this property ok. So, everything is taken care of and this forms the group whose generators are a and b, a and b are of order two and a b is same as b a, it is what I have given as a condition.

Student: It was (Refer Time: 17:17) a b whole inverse is (Refer Time: 17:20).

a b inverse is b inverse.

Student: (Refer Time: 17:25).

Yeah, this is fine right. What is the confusion on this?

Student: He is asking whether we can apply that one?

This one?

Student: (Refer Time: 17:36), but a can be (Refer Time: 17:43).

No, but any operation when you do it as operators in an quantum mechanics, these properties we do not violate. If we do two operations by any group operation, if you want do the inverse of it, then you want to reverse it with the inverse that is always true not only for matrices. Those are the linear algebra properties right, we do not want to violate linear vector space, good.

So, let us write the multiplication table here for this group which I wanted that is it and this is what I call it as a Klein group, order of this group is again 4 ok. What are the subgroups here? Anyway I will leave it you to write this, what are the subgroups, I think I have to stop?

Student: (Refer Time: 19:02).

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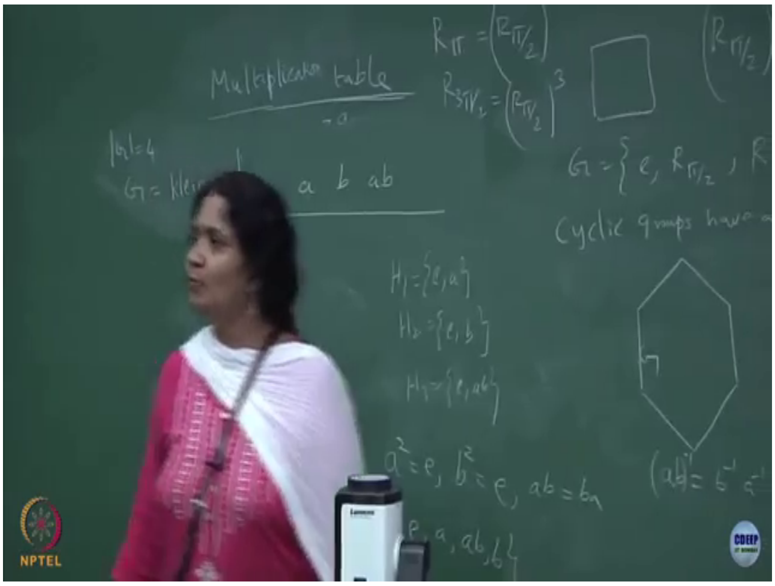

e, a is one subgroup H 1, H 2 is e, b is one subgroup. Can you also have e, a b?

Student: Yes.

So, this is a way to get some training on what is a subgroup how to write the multiplication table and so on. So, I have just put in the multiplication table for you for the Klein group, but you can yourself do it and check it out this will be there on the website. Let me stop here and on Monday, I will continue with conjugation, conjugacy class and so on ok, yeah.

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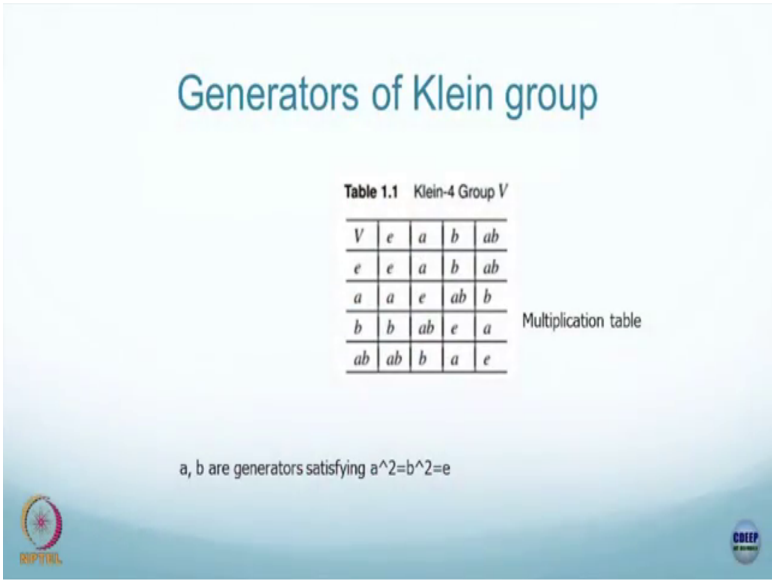

Student: (Refer Time: 19:55).

b, a it will in general it would have involved, but if I give this condition in the beginning itself, yeah inverse of this.

Student: That is because.

So, because a b inverse is a b that is why we had.