Group Theory Methods in Physics
Prof. P. Ramadevi
Department of Physics
Indian Institute of Technology, Bombay

Lecture - 17
Reducible and Irreducible Representation – I

The next thing is that you also see for the.

Student: (Refer Time: 00:22)

Ha.

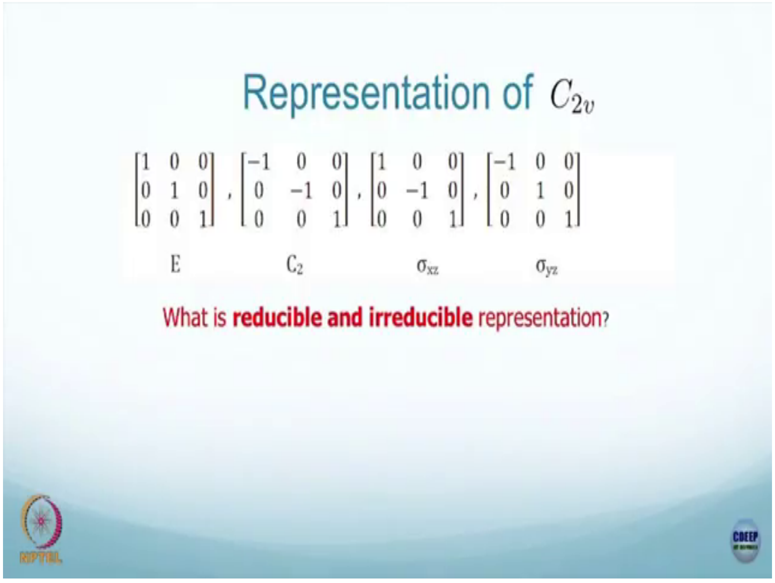
Student: All the elements become same?

All the let us to repeat your question all the elements become same?

Student: (Refer Time: 00:31) all of them are.

That is why there is no distinguishing whether it is identity element or C 2 and so on ok.

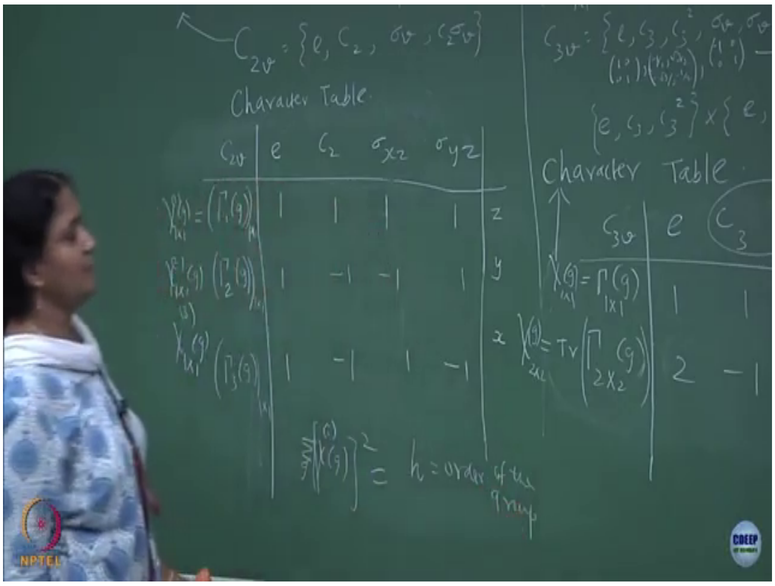
(Refer Slide Time 00:40)



So, basically like if you look at the z coordinate below; let us look at the z coordinate below. The z elements you know identity will have 1, C 2 will have 1, sigma xz will have 1 and this 1. So, that is there in the matrix representation if you see in the irreducible component ok. So, you can come to the other ones ok. So, they are all nontrivial in the sense that their entries are only plus and minus 1 for the C 2 v right.

But for other groups the entries are not going to be plus and minus 1. It just happens for C 2 v. The nice 3 by 3 matrix which I have written all the elements are in the completely diagonal form and each block, I can call it as an irreducible representation. So, lets write it out. Let me write it out the blocks.

(Refer Slide Time 01:52)

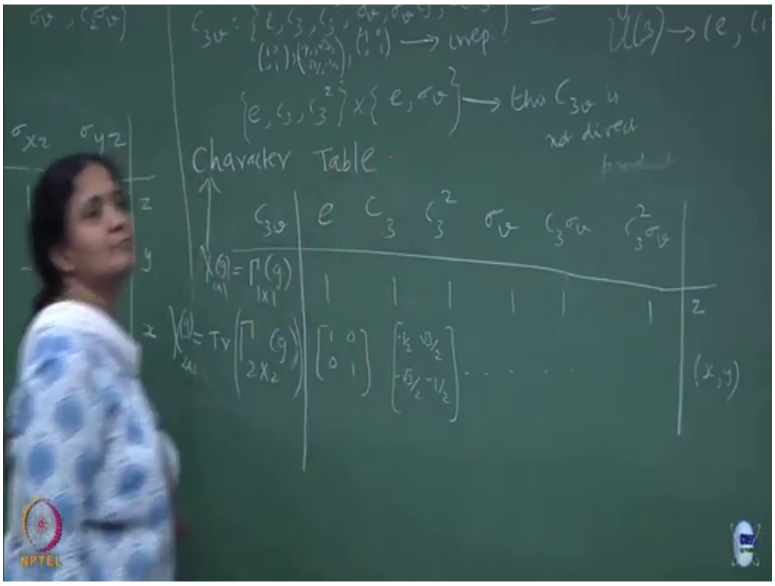


I will say it acts on the z axis. So, each one is a 1 cross 1 matrix. This is a 1 cross 1 matrix; 1 cross 1 matrix. What is the other one, if I say it acts on the y axis? Identity does not do anything. This 1 will change it to minus 1 and then, the first one if I call this to be in the sorry just to make this notation clear let me call this as x z and let me call this as sigma y z; then, what happens?.

Here, it will become minus 1 right and then, 1 and this sigma 3, it is going to operate it on the x axis and then what happens? You have 1 minus 1, 1 and? This has to be minus right.

So, what do you notice from this? There is something interesting happening. If you take the dot product of this row with this row, what happens? It is 0. Any 2 rows if you take dot product, it is 0. Just the property of those rotation matrices, nothing different it is just the rotation matrices have to be orthogonal right. What I have done here is that I am writing this 1 cross 1 matrices here.

(Refer Slide Time 04:31)



In the context of C 3 v, in the context of C 3 v, let us do the same thing. This writing the set you can do it in whichever order, but elements maybe different ok. If it is operating on the z axis, I have already said that you get a 1 cross 1 matrices which is. All agree?

Student: (Refer Time: 05:17).

Yeah, it can be possible right, what is everything is 1. It is a trivial representation that still satisfy any multiplication, does not matter.

Student: (Refer Time: 05:35).

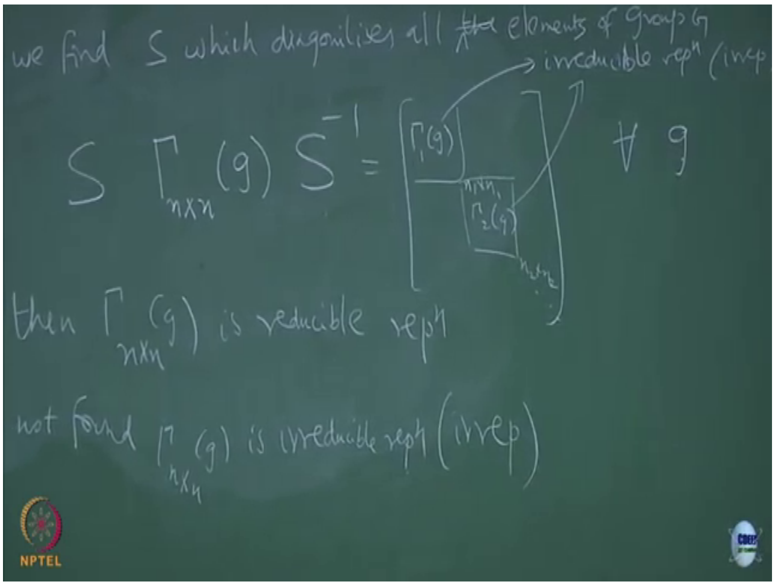
No, I agree with you, but I am saying it is a smallest nontrivial representation, trivial representation can satisfy the multiplication table. Multiplying 1 with 1 whichever way we you do abelian or non-abelian, the multiplication table will be satisfied; all the entries are 1. Does not care whether its abelian group.

Student: (Refer Time: 05:59).

Yeah. So, we will come to it. Yeah. So, what I am trying to say is that this is what we call it as a trivial representation ok. So, this is a trivial representation. The next thing which I said is gamma 2 cross 2 of g ok; gamma 2 cross 2 of g, if you try to start finding the trace.

So, write the matrices here. You can write those matrices and so on ok. You can write them and this is going to operate on x y basis ok. So, by this I mean it operates on the 2 by, the 2 by 2 matrices will act on x y quadrant that is what I mean by this.

(Refer Slide Time 07:15)



And as I said that if you do a similarity transformation ok, if you have these matrices and do a similarity transformation, the trace does not change at all ok. The trace will not change if you do similarity transformation or not do transformation. So, technically what people do is they take the trace of this and the trace of this is what is called as a character of g in a representation.

In this particular case, it is a 2 cross 2 representation. The reason why it is 2 cross 2 is the basis states on which it is acting is a two-dimensional vector space ok. Its acting on the x y coordinates. So, it is two-dimensional and you can write the 2 cross 2 matrices ok. So, once you do this. This is trivial right. This is also the trace for the 1 cross 1. There is no change. In all these cases, you get the same trace. let me call it as 1, 3.

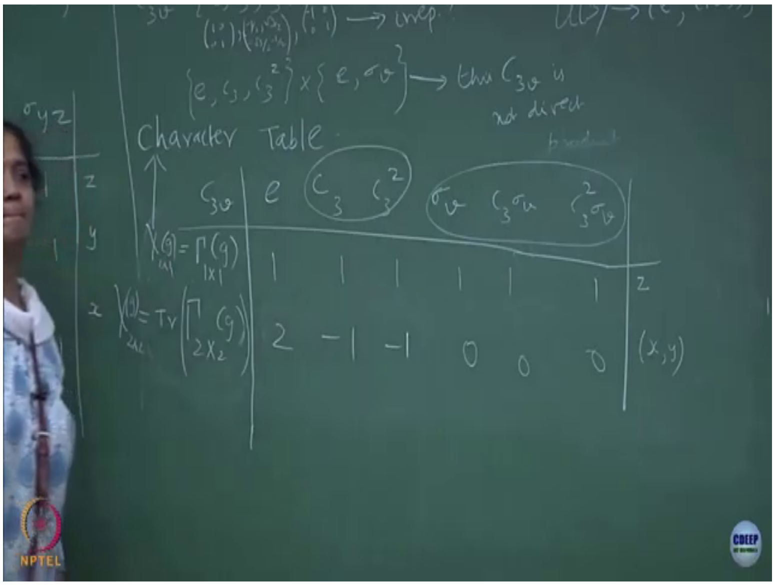
This is also the matrix representation of the irreducible components since they are 1 cross 1, whether you call them as matrix representation or the trace of the matrices, it does not really matter and this is what they call in the notation as characters trace of the matrices are called characters ok. And this table which I am writing is what is called as a character table ok.

Just like what you did for multiplication table, aim is to write a character table. I have not completed the character table, but I have started the character table giving you a matrix representation, explaining to you that some matrices will be irreducible; I said that you cannot find an s which further diagonalizes it.

Student: Ok.

So, this is irreducible. So, first irreducible on the z coordinate is a trivial representation, sometimes it is called as unit representation, but there is nontrivial representation. Instead of writing the matrices, I write only the trace of those matrices.

(Refer Slide Time 10:09)



So, the trace will be what? Someone? Ok and what will happen to the trace for C 3 2? Can somebody argue what it will be? C 3 2 and C 3 are in the same conjugacy class. So, which means the character which is the trace of the matrix here not changed. You all agree? And what about these characters? Sigma v, we have found sigma v was traces.

Student: 0.

0, what about the other two? Other two are in the same conjugacy class with sigma v right. So, because these two are in the same conjugacy class, the character is same. So, because these three are in the same conjugacy class, I am arguing that the characters will be same please. Check it ok. It has to be true, but please check it. What about here? It is an abelian group, how many conjugacy's class are there in an abelian group?

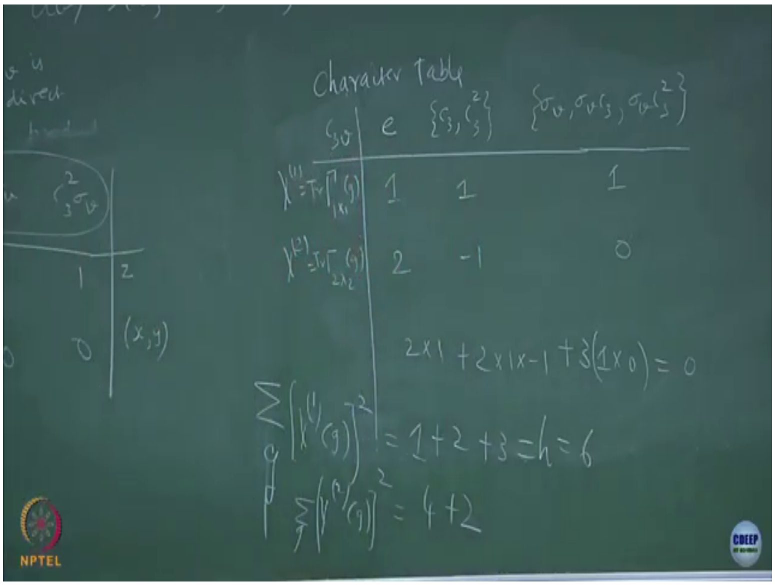
Student: Order of the group.

Order of the group; every element is a conjugacy class. So, there is no relation and you find that at least for this example on the xyz axis, you have found those nontrivial characters and we also observe that these characters are also matrix representation because they are 1 cross 1 matrices and you see that the they are actually only you find only 1 cross 1 matrix representations which are irreducible ok.

Here you find a 1 cross 1 as well as a 2 cross 2 which are irreducible. I have not proved it for you, but one way of seeing it, it is a non-abelian group; another way if you explicitly see by brute force whether you can find an s which diagonalizes C 3 as well as sigma v C 3 or CC 3 as well as CC 3 squared. It will not be possible. You can check it out. You can either diagonalize C 3, but not C 3 squared ok.

So, that is why you do not find an s. So, these are irreducible representations of the C 3 v group and we like to work with the characters; characters are defined as trace of the 2 cross 2 matrices and then, we can write down the characters and because the characters for conjugate elements have to be same, I do not even need to work out what is the character for this by writing out the matrix representation. But I can blindly write within that class. So, equivalently the way people write this.

(Refer Slide Time 13:23)



So, character table. They write e C 3, C 3 squared. This is trace of ok. This is another way of writing it. Something wrong. This has to be 2 right. This has to be 2 ok. Again, can you check whether the row if you try to do with these numbers, can you check whether this is orthogonal? So, what are the, what are our observations here? So, if suppose I take character of this irreducible representation and a character of this trivial unit representation, if we multiply; let us do that.

So, it is 2 into 1 ok. Here there are 2 elements. Remember 2 elements. I did not write it in this elaborate fashion here now. So, the number of elements in this class is 2 into 1 into minus 1 right and here, the number of elements is 3; 3 times 1 into 0. What is this adding up to? 0.

So, I am just trying to show you that when you start writing the character table, you will start seeing that each of the rows are orthogonal to the other rows. In this case, it was simple because every element is a class by itself. When you do it; two of the elements have to be negative to cancel with this. Not only with this row, it should also be with this row. So, you can check that this is 1 plus 1 is 2; these 2 are negative, it will cancel ok.

Other observation is if you take the characters of each of these rows. So, let us take this row; let us square these characters ok. So, let us take chi 1 of g square summation over g ok. What is the answer you are getting? It is 1 for the first one. There are 2 elements here. 2 and there are 3 elements here. What does it add up to? Adds up to give you order of the group which is 6 ok.

What about here? It is simple to see here. It is for any of these options, you can do the square ok. Can check for character for the first row, character for the second row, character for the third row; in the same way, you can do it for 1 or 2 and check out what happens. What happens tell me? The second row, what happens? So, first row we have checked; what about the second row? This is 4.

Student: Plus 2.

Plus 2 plus 3?

Student: 0

3 into 0 right. So, this adds up to give you six ok. So, these are some of these observations. Just keep this in mind, when we are looking at the formal aspects and you will get some more ideas on how to fix this ok. I somehow thought it should be 12, I do not know, well, hm?

Student: Ma'am, you did not square.

I did not square what? So, it should be summation over g right. So, it should be 1 plus 1 know, which is correct know?

Student: Whole squared.

Student: 1 plus 1.

Hm. 2 in 1 plus 1 right that is 2 right ok. So, the observation which you are seeing from all these things is that it is going to be related to the order of the group ok. So, I am just fixing some of the notations. If I use gamma n cross n, you know it is a matrix representation. If I write a chi, you know it is a trace of the matrix representation and then, some of the objects which we have done partially not complete character table is that from looking at how it operates on z x and y axis, I have tried to write the 1 cross 1 irreducible representations.

This is a trivial or unit representation. These are nontrivial ones and similarly, this will also will have trivial representation, but there is a nontrivial representation which we are familiar with our day to day life ok.