Group Theory Methods in Physics
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Lecture – 15
Matrix Representation of Groups – I

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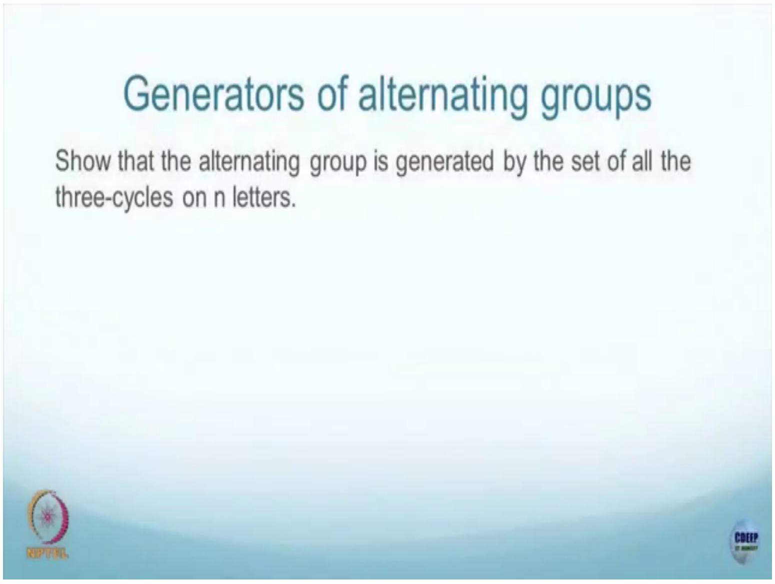

The other problem was show that alternating group is generated; alternating group was only symmetric; sorry even number of permutations, which means you cannot get just by transposition. If you have a generator all powers of the generators should be elements of the group. If you put transposition as a generator you are stuck right, all powers includes even powers and odd powers.

So, better thing to start with the generator for a alternating group is minimal one is only a three-cycle, is that clear. So, your alternating group u n will involve identity and all possible three cycles.

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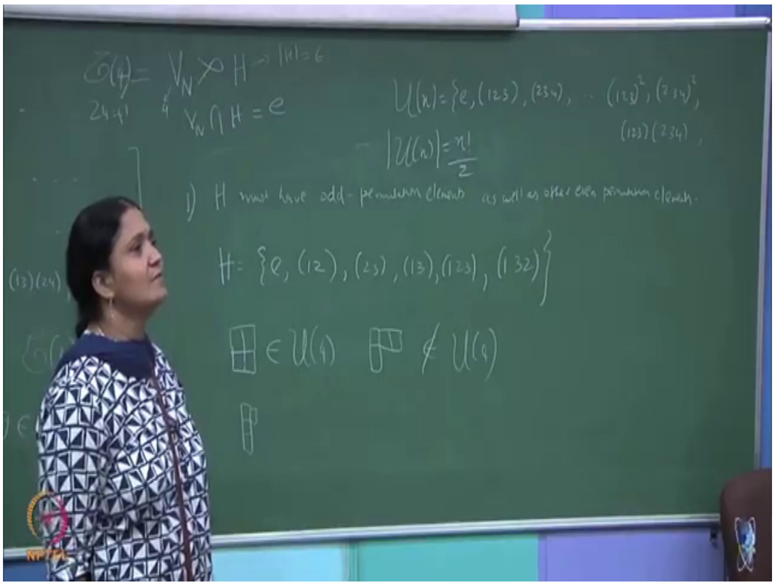

You know I am not writing and then you will have powers of them and then you can also multiply them ok, you can also have situations where you have 1 2 3, some power you know you can have this, where 2 3 4 and so on ok.

So, this is the meaning of saying it is a all possible three-cycles are the generators with those generators you can create the whole. How many, what is the order of this group? Order of this group, someone? n factorial by 2 half the number ok. Conjugacy classes another question which you have to worry here. In the symmetry group of degree n let us look at only the even permutation elements and see whether if they fall into a conjugacy class, do they still fall into the conjugacy class here, all these questions you need to check at least for permutation group of degree 4 ok.

Student: Yes.

If not for higher permutation at least up to degree-four you should have a feel of what happens. Conjugacy class we worked out for in the class for permutation of five objects right. We drew the partitioning for four objects you can write it out and see what are the conjugacy classes and then which other classes which are going to be elements of the alternating group, not all of the elements right.

Like for example, if I have a conjugacy class which is, if you have this it is two; two cycles its cycle is the transposition it is still allowed. It is an element of alternating group also. But, if you had something like this is this allowed? This is 1 3 cycle and 1 1 cycle, that is a cycle structure. So, this is what? Even permutation or odd permutation?

Student: (Refer Time: 04:00).

Three cycle will have two transpositions, one cycle no transposition so it is?

Student: Even.

Even. So, you can start figuring it out which are the even transposition, which are the odd transpositions and so on. So, this one is probably not allowed right this is not an element of is that right. So, this way you can figure it out and also look at the number of elements.

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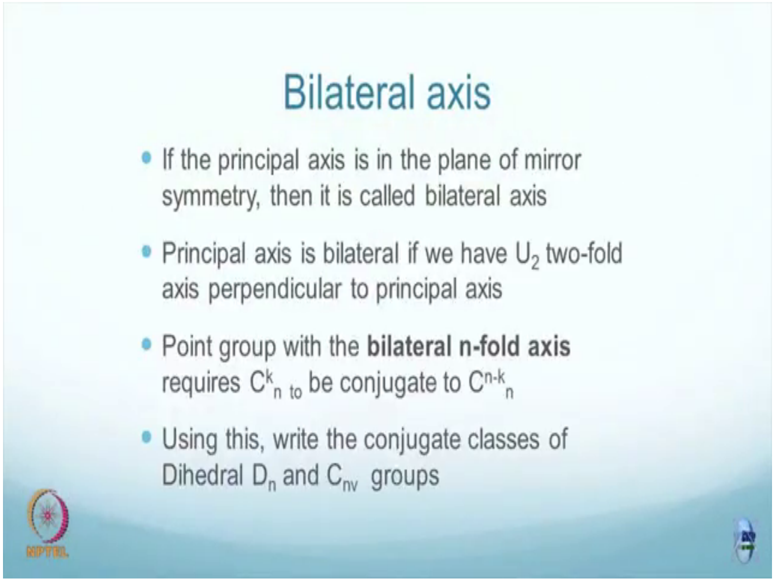

So, this is also passingly said bilateral axis if the principal axis is in the plane of mirror symmetry, that is if it had a sigma v plane that is the meaning then the axis is called as a bilateral axis. Similarly, if you had a two fold axis which I denoted in the last lecture as U subscript 2 U 2, if it is perpendicular the U 2 axis is perpendicular to the principal axis. Then also you call the principal axis as bilateral.

Student: Yes mam.

Yeah, a mirror plane basically mirror plane is plane will have two directions, but both the directions at least one of them should coincide with the principal axis. Which one coincides I do not care ok. The mirror plane has to x z or y z I do not worry. But it should have the z axis which is the principal axis direction, then you call it to be a bilateral axis ok. x y plane if you had as a mirror symmetry and z axis as the principal axis then you do not call it bilateral, is that clear.

Student: Yeah mam.

C nv has a bilateral axis n fold symmetry axis is that right? C nv and C nh.

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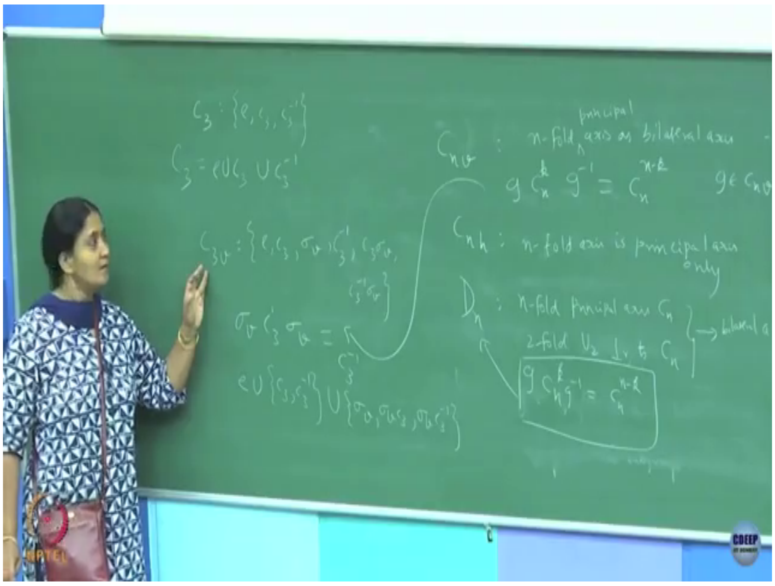

Here you can call the n fold axis as bilateral ok. Here n fold axis is principal axis only, not bilateral. So, if this n fold axis is a principal axis and it is known as bilateral axis, but this one is not a bilateral axis. Similarly, if you take D n groups; D n groups they have principal axis right, you also have a two-fold U 2 which is perpendicular to let me call this the principal axis C n ok.

So, if this is in this case also you say that the principal axis is a bilateral axis in both the cases the n fold principal axis will be called bilateral axis if there is an additional two fold which is perpendicular to the principal axis or if it has a mirror containing the principal axis.

What is the advantage of defining a bilateral axis? Once you have a bilateral axis, then you can you can check of course, you can verify, but you can also say that if you do C k n ok. So, if you take the principal axis do it for C n is the generator for rotation n fold 2 pi by n rotation. If you do it k times you can always find some element in the group C n v g inverse which will be this you can always find, where g is an element of for example, ok.

Student: Mam.

Yeah?

Student: (Refer Time: 09:04) difference between a (Refer Time: 09:08).

No no no; difference or anything I am just saying the principal axis has another name which is a bilateral axis for both the hydral group and C n v group, but not for C n h group. That is all I am saying, no difference. Once you have a bilateral axis even here you can show that; so this is true for the D n this is true for the C n. Once you have a bilateral axis you can show that the elements are conjugates to each other which elements are conjugate C n k is conjugate to.

Student: C n n.

C n n minus k. If you did not have the mirror plane. So, C 3 for example, what are the elements identity C 3, C 3 inverse. It is an abelian group right and you can show that each element is a class by itself right. C 3 is a class, identity is a class and C 3 inverse is a class, that is the C 3 group right.

Suppose, I go to C 3 v what are the elements generators are C 3 and sigma v right and then you have anything else that is it. And you can show that sigma v C 3 sigma v sigma v inverse is same as sigma v, we can show that this according to that property; what is it? It is C 3 1, will be C 3 2, C 3 2 is nothing, but C 3 inverse.

So, what happens now to the structure here? Here is you have an identity union C 3 C 3 inverse, they form a conjugacy class by this or this gets the three fold axis gets promoted to a bilateral axis. So, you have C 3 and C 3 inverse is the conjugate element; and what else? These are the in the planes of symmetry you started with one mirror plane, the C 3 started generating for you the three mirror planes right.

Take an ammonia molecule I showed you picture wise there were three mirror planes all these three mirror planes can also be shown to be conjugate to each other. So, essentially your C 3 v still has three classes like your C 3, but there are classes with two elements, there are classes with three elements and if you see it like a permutation group you will call this as a three cycle, you will call this as a transposition or two sides by isomorphism to symmetry ok.

So, I am just trying to justify for you for C n v, I also want you to check for D 3 what happens I will leave it you to check ok. So, I have tried to give you some flavor for C 3 v here, but I will leave it you to check what are the conjugacy classes breaking like this for C 4 v C 5 v in general a C n v and also to get some feel on dihedral groups. I have not done D 3, but you can do it and generalize it to dihedral groups with an n fold principal axis, that is the meaning of the D n ok.

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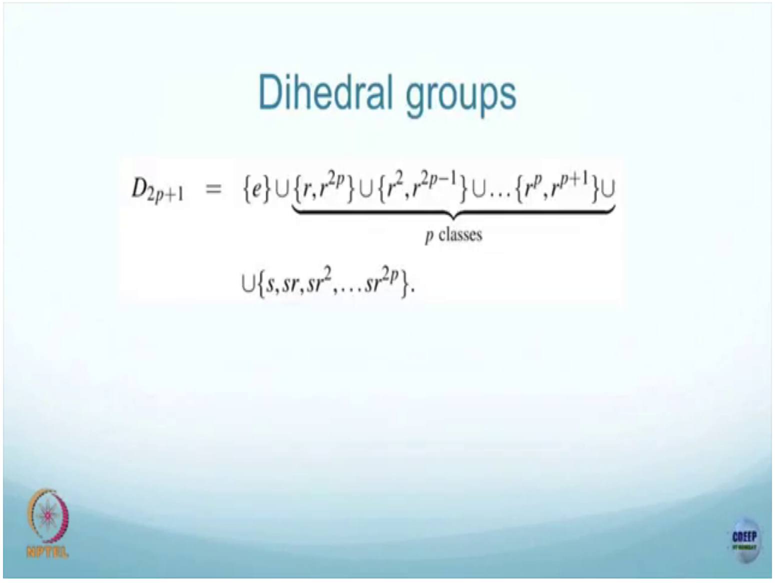

So, just to give as a result dihedral groups distinguish between odd principal axis if the n fold principal axis is odd, then you find there will be p plus 2 1, then you have this one by using this property which I was talking now. Similarly, for c n 2 and c n 2 p plus 1 minus 2 and so on.

So, there will be p classes for the principal axis symmetry which is 2 p plus 1-fold symmetry in it. And you can see that these are like your various twofold axes which are generated s is just like two-fold operation. You see that those set forms a class together ok.

So, do similar thing for D 2 p, you will see some difference there. The midpoint rotation if you take D 2 p sorry D if you had D 2 p clearly one side I D 2 p the 2 p fold axis is a bilateral axis right.

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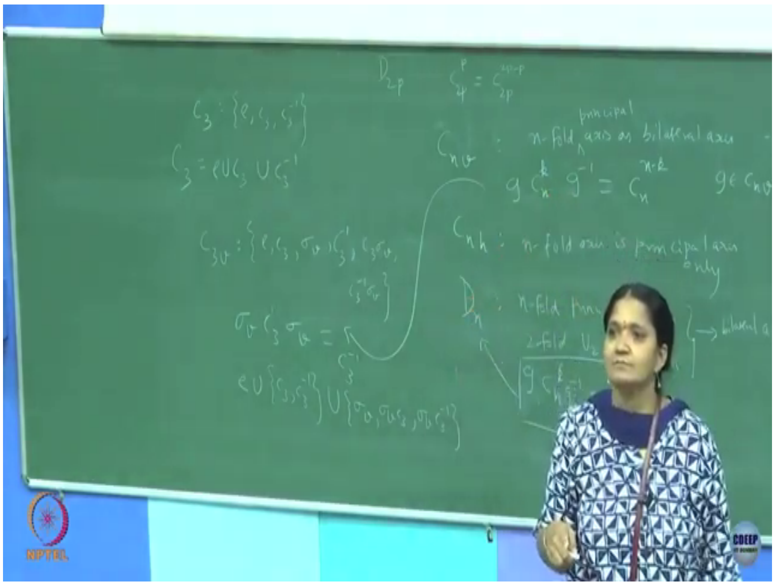

But what you can show is that C 2 p p is same as that you will not find in the odd case. So, that is why there will be at least one midpoint element which is a class by itself. I take C 4 v C 4 v will have a C 2; it is a class by itself. So, there is slight difference between an odd and the even.

So, the number of classes will increase by one or decrease by one. There will be at least you know I want you to check these parts, there will be some class it is not only identity element is an element which has one element you will also have a midpoint element which you will be a element which belongs, which is self-conjugated that is what I mean ok.

These elements r as conjugate t r 2 p identity is self conjugate, but if p was even 2 p was, if you did not have 2 p plus 1, if you just had 2 p then you will also find r p 2, be a class by itself. So, those are the distinction.

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So, the other things which I want you to look at it is that I did show you some slides on cube symmetries which are having twofold, fourfold and threefold. This comes under the classification of octahedral symmetry; they denote the point group symbol as O for it. And similarly proper symmetry, proper symmetry I mean only rotational symmetry, improper will involve reflection inversion all those combination.

So, regular tetrahedron the methane molecule has a similar symmetry and you can you denote it in the point group literature by a capital T; capital T is the tetrahedron symmetry. And then you can add improper reflection planes and you have O subscript h, T subscript d.

Student: Yes mam.

D is because you can generate diagonals mirrors. So, you can write the conjugacy class and so on here.

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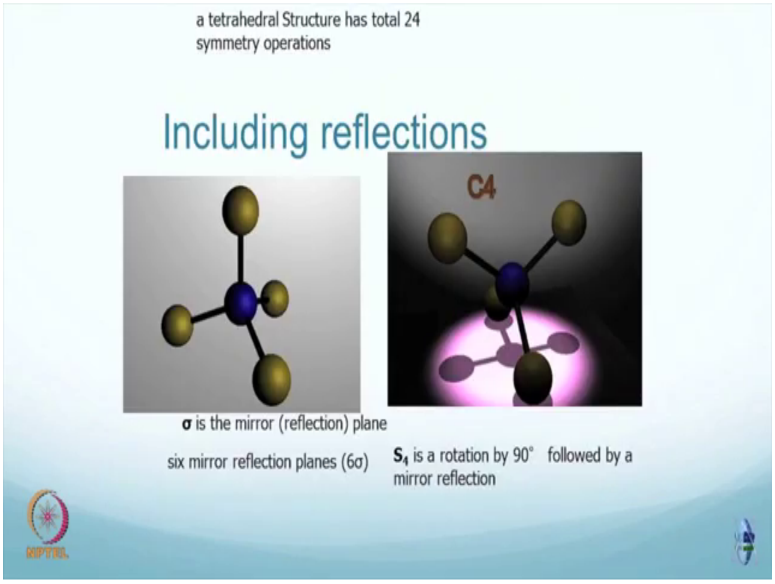

So, these are the symmetries and you can start generating the conjugacy class for the tetrahedral symmetry and T d is when you have the mirror planes and you will have roto reflections. So, I will leave it to you in fact, even in the auto bind website you can start playing around on the methane molecule and get a feel of what is the T d group symmetric ok.

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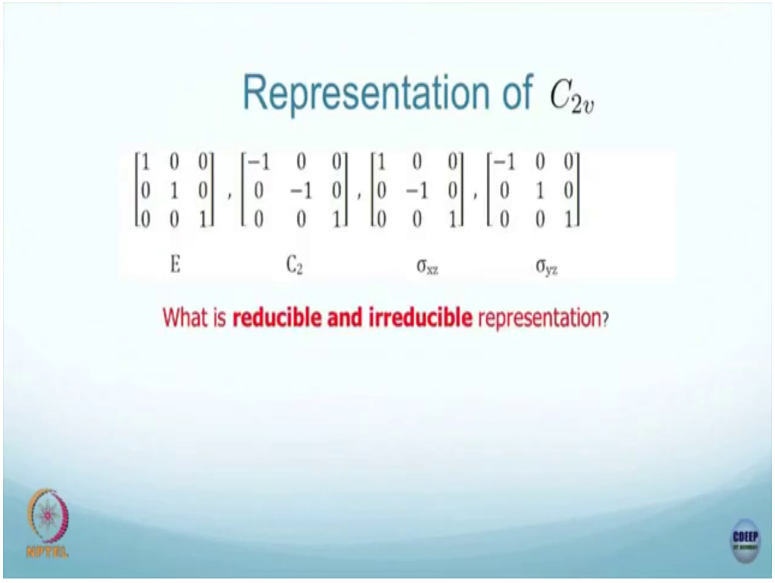

So, this is where I left last time, that you can give a matrix representations. So, matrices which you can write and then when you multiply C 2 with sigma x z you should get sigma y z right.

Student: Yes mam.

Is that right C 2 times sigma should give you this element is that happening if you do the multiplication it becomes minus, this becomes minus no this becomes plus and this is plus right. So, it is having the group properties and you have a matrix representation.

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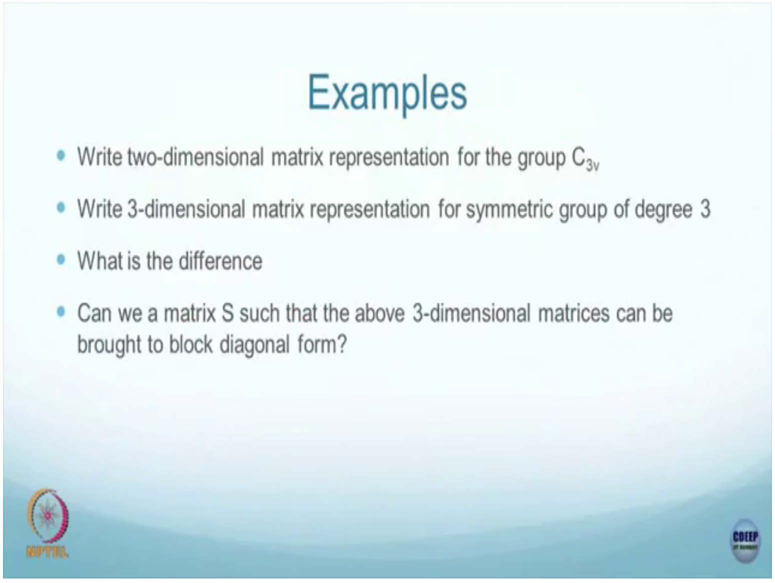

Can we write a matrix representations? Similarly, so we did this also I wrote it for the. So, suppose I have an object a 1, a 2, a 3 and I do an operation it goes to a 2, a 3, a 1.

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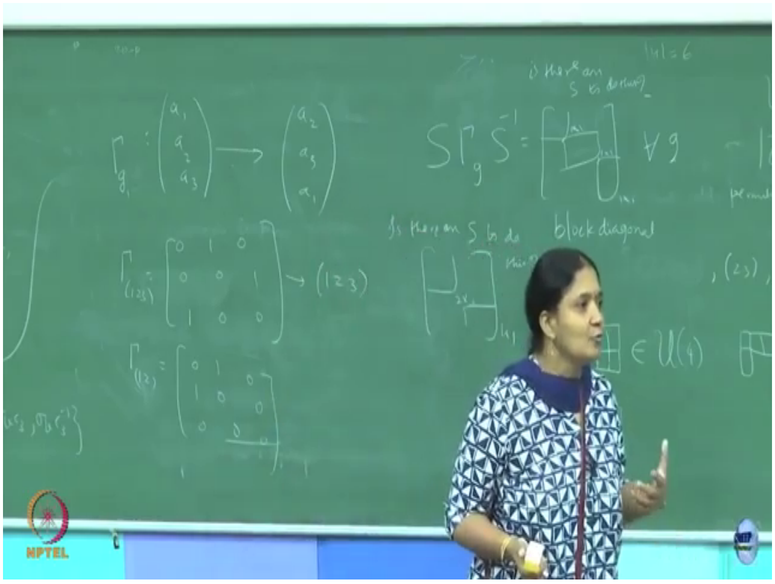

So, can we write this element? What will be this element somebody, is this right am I right. So, these are the matrix representations you can write which this is corresponding to like a 1, 2, 3 element in your permutation room right.

Suppose, I want to write gamma 1 2. So, let me call this here as 1 2 3. What will this be? Only 1 and 2 will get interchanged. So, clearly when I write all the elements some elements will be diagonal, some elements are off diagonal. In this particular example of C 2 v everything was diagonal, but sometimes you can take representations these are also good representations for the symmetric group of degree three ok.

The question is that when you have these representations do we need the fundamental minimal dimension of the matrices to be 3 by 3 or can it be lower? So, that is the question I am asking, do we need the representations or equivalently can I find a matrix S; some matrix S on I should be able to write it in a block diagonal form.

So, this is what I am calling it as a block diagonal, this is for all g same s S should be one S which breaks all the elements identity element is anyway in the block diagonal form and S r S inverse will cancel right, that is not enough.

I have to find an S such that every element can be completely brought into diagonals this is one possibility. It may not be possible to make it completely diagonal, you may be able to break it up into a 2 cross 2 and a 1 cross 2 is there an S to do this; this is 1 cross 1. 1 cross 1. 1 cross 1. Is there an S to do this, one possibilities you can give it to the computer and ask it to figure it out. Another possibility is that we should be able to rigorously figure by some kind of a methodology, just like I did permutation group and now you are comfortable with permutation group of arbitrary degree.

If I write a 50 by 50 matrix whether it is can be brought to this form or this form completely irreducible completely reducible as block diagonals. These are questions which one should be able to understand from a set of theorems and how to go about it. So, this similarity transformation will not alter the trace, that also you remember. If this trace is 0 by doing this transformation to make it into a diagonal or a block diagonal you will not change the trace, trace of the matrix will remain same.

This operation will only tell us whether we can break this 3 by 3 into further fundamental blocks which is what we call it as a irreducible blocks. So, this is the aim ok. So, let me stop here.