Physics of Biological Systems
Prof. Mithun Mitra
Department of Physics
Indian Institute of Technology, Bombay

Lecture - 20
Various phenomena at low reynolds number

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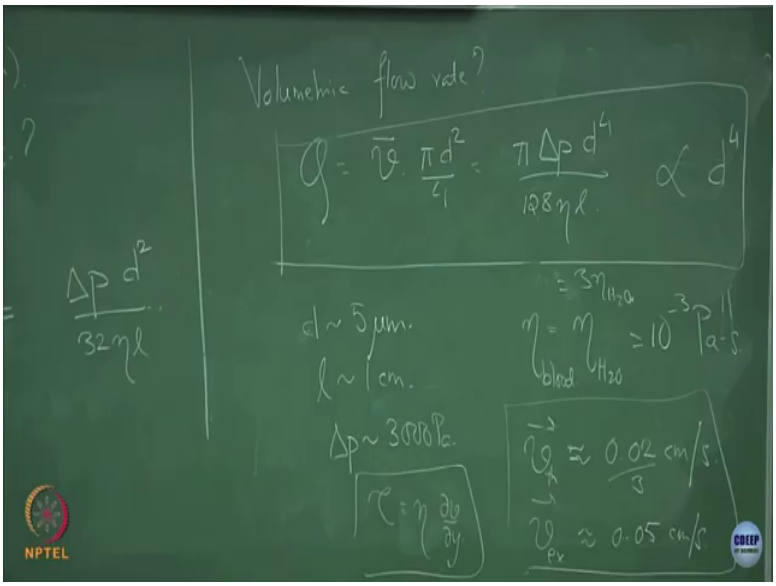

If you now want to go back; if you now want to increase it actually what you need to do is to take into account the non-Newtonian nature of blood. It does not follow this sort of a linear stress strain relationship that I assume for a Newtonian fluid; if you take that then you can recover back these sort of more accurate estimates.

But even within these approximations unless you are really precisely interested in that getting back to that number. So, that is what I wanted to show that the Newtonian approximation; you should keep it at the back of your mind that these are not Newtonian fluids, but often you

come across cases where the Newtonian approximation is not too bad, you can take that the fluid isn't; behaves like a Newtonian fluid and at least it is a order of magnitudes you will maybe get correct.

Student: Sir.

Student: The last one.

The last one; this one? This one? This is the stress strain relationship, this is the definition of my viscosity you remember. The stress force per unit area is proportional to the rate of change of the velocity and the constant of proportionality is my coefficient of viscosity.

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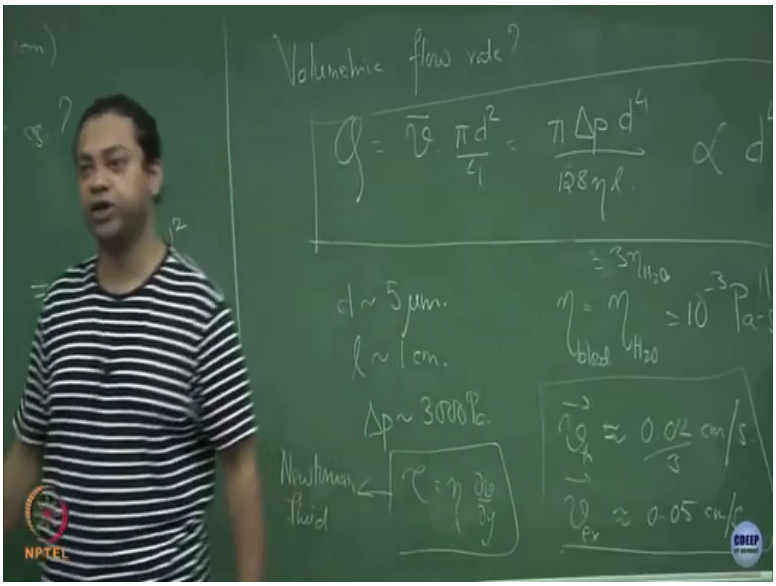

Remember, last class we talked about this and this was true or rather this was; this is what I used to define a Newtonian fluid that any fluid which is a stress strain relation stress strain relations like this is what I will call as my Newtonian fluid. If you had non Newtonian fluids like shear thinning or shear thickening, there would be an exponent n here which would be greater than or less than 1; depending on the type of fluid; yes so.

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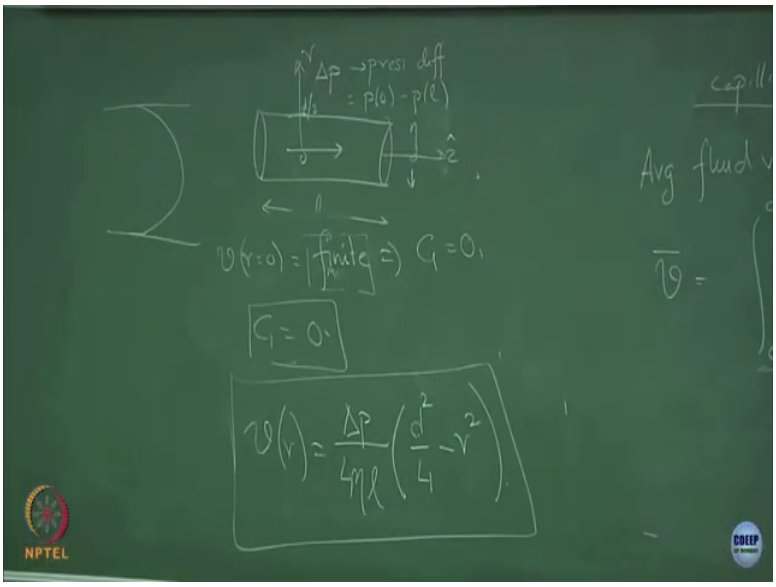

Student: (Refer Time: 01:58).

So, you are sort of; you are pumping blood through these vessels right, through an active like the heart is pumping or whatever that creates a pressure differential across the length of these tubes that number is sort of an estimate from experiments of what is the correct pressure differential to use for capillary roughly of that length.

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It is a mean; of course, it has its own dynamics and so on but let us the sort of rough estimate of the number. So, we were also talking about these Reynolds numbers; remember and the Reynolds number like I said we defined as this ratio of inertial forces to viscous forces which was this rho L U by eta. So, the same fluid with a given density in a given viscosity can behave like this sort of low; can if you put an object in that same fluid, you can have this low Reynolds number flows or high Reynolds number flows; depending on the size scale and the velocity scale of the object.

So, it is not just a property of the fluid; it is a composite property of this object moving through the fluid itself ok. There is a couple of other ways; so, just one to get it the same Reynolds number expression.

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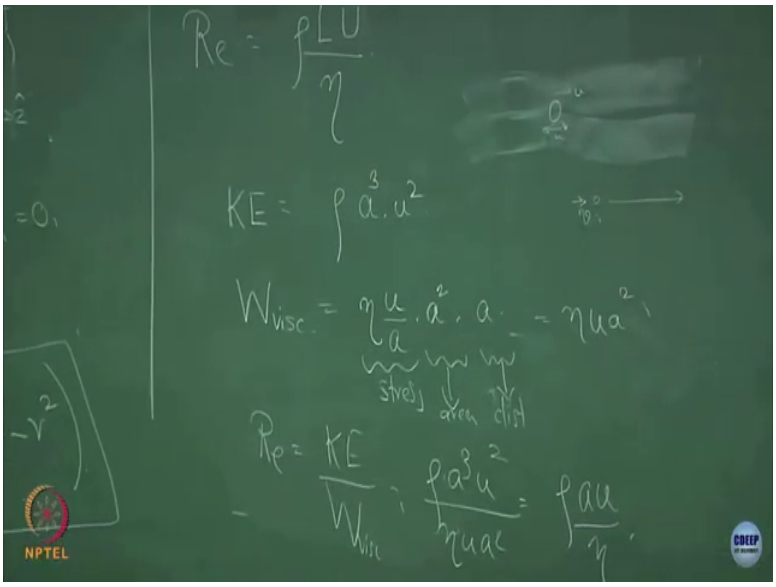

Reynolds number is rho L U by eta right. So, again so let us say you have this fluid, I have this fluid I just take a small parcel of this fluid of some size a which is travelling with the velocity u. You can estimate what is the kinetic energy that is contained in this fluid parcel. So, the kinetic energy of this fluid parcel is half mv square m is like the density of the fluid times the volume which is like a cubed and v squared right.

So, this is again I am not putting in coefficients and so on but this is the order of magnitude of the kinetic energy that is contained in this parcel of flow; its rho a cubed u square. You can also estimate what is the energy that is dissipated by the viscous forces as this fluid parcel is moving in this fluid. So, you can estimate the work done by the viscous forces what is work; it is; so work done by the viscous forces as it moves a distance comparable to its size again.

So, I take this as the size scale in my system. So, that is force into distance; forces stress; so that is my force per unit area. So, that is eta into my velocity scale by my length scale; there is my stress into the area which is a and then the distance that I move. So, this just to clarify; this is my stress from this stress strain relationship for a Newtonian fluid. This is a measure of my area; stress is force per unit area, so force per unit area into area is my force and into the distance moved is my work alright.

So, this is the work done by the viscous forces as this fluid parcel moves the length which is comparable to its own size and that is; so eta u a square alright. So, again if you take the ratio of these two the kinetic energy of the fluid parcel compared to the work done by the viscous forces as it moves the distance comparable to its length. So, if you take these two ratios K E by this W viscous again you will get; so rho a cubed; u squared by eta u a square. So, this is rho a u by eta which is again the same as this.

So, what it says is that if this; if this viscous work is much larger than this kinetic energy of the parcels. So, again Reynolds number is very small this viscous drag will quickly dissipate the kinetic energy. So, it will just move a little bit before this viscous drag has dissipated all the kinetic energy that was in this parcel.

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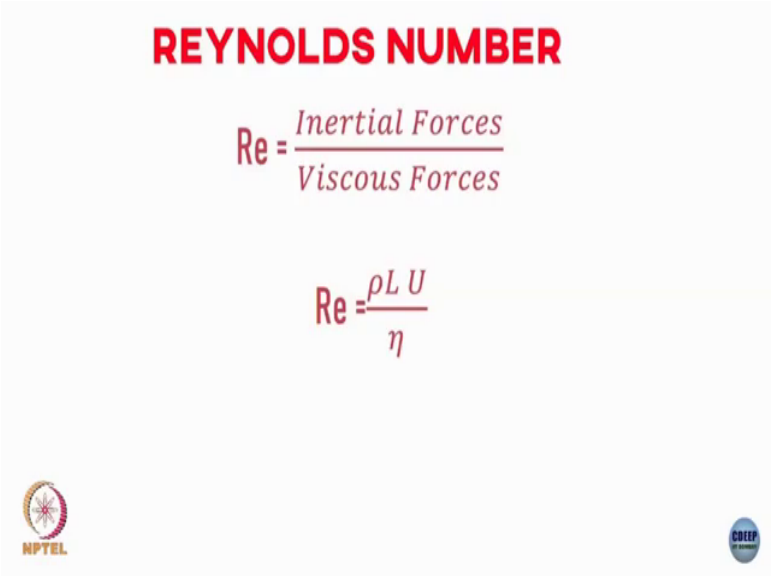

So, that is the limit of this low Reynolds number. So, it will not have this inertial term. So, something moving with velocity v will not continue to move with velocity v; this viscous drag will very quickly dissipate the energy and it will come to a stop. So, that is physically this regime of low Reynolds numbers ok.

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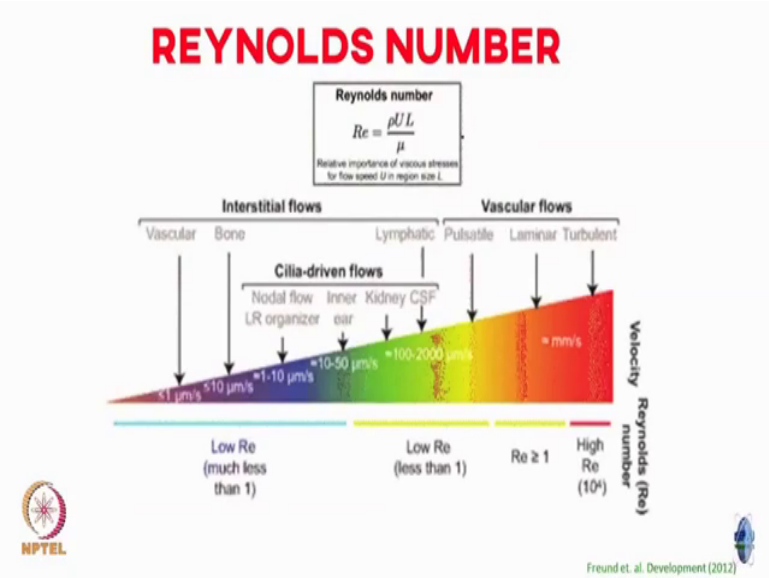

So, you just to get a sense of what these numbers look like; so these are some estimates from this paper in development. So, these are these interstitial flows are in the way the I do not know if you can see the figure, but these are interstitial flows are flows in between in spaces between the tissues and these are very low Reynolds number much much less than 1.

These are cilia driven flows like for example, cilia and your inner air ear; the cerebrospinal fluid these all fall in these low Reynolds number like less than 1 and then you can have this vascular flows which can be laminar or turbulent. So, laminar is in this roughly in around this 1 range or this turbulent flows would be this high Reynolds number range ok.

So, mostly at the scale of cells and so on the flows that we are interested in our fall within this low Reynolds number range 10 to the power of minus 3, minus 4 and so on.

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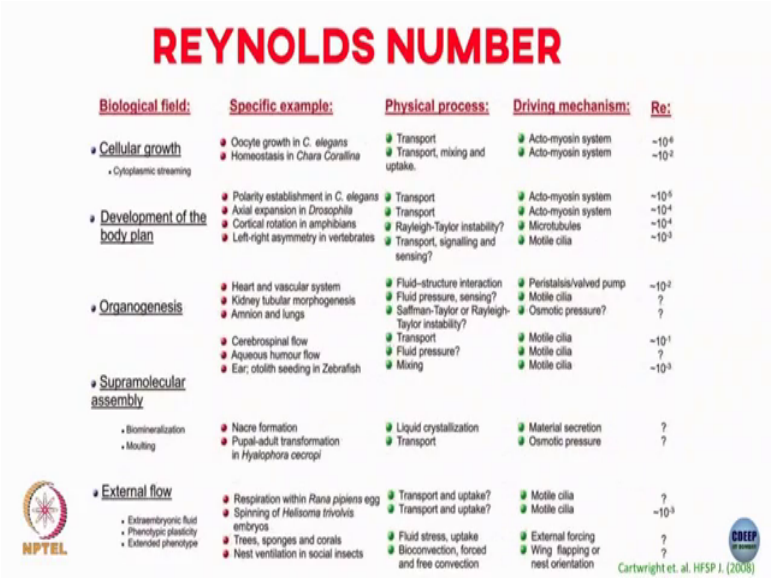

Actually, I have one more graph which might be better. So, for example, just to take a look; so the reference is cut off what whatever reason. So, here are Reynolds number estimates from different fields of biology is roughly related to development. So, for example in oocyte growing in C elegans the worm that has a Reynolds number of 10 to the power of 6. Then this development of this left right asymmetry in vertebrates. So, the fact that you left is different from your right that is driven by motile cilia I will actually show that.

The Reynolds number in that context is around 10 to the power of minus 3; the flow of cerebrospinal fluid again driven by cilia is 10 to the power of minus 1 and so on. So, most of these numbers you will see a very small numbers 10 to the power of minus 3, minus 1, minus 6 and so on; so these are very very these are very low Reynolds numbers which means that this assumption of using not using the full Navier-Stokes, but using only the Stokes equation.

So, neglecting all inertial terms will work very well if you are talking about fluid problems in this; in these sort of ranges; in this sort of Reynolds number ranges.

So, here is a couple of examples from this previous slide actually. So, this is an oocyte growth in C elegans.

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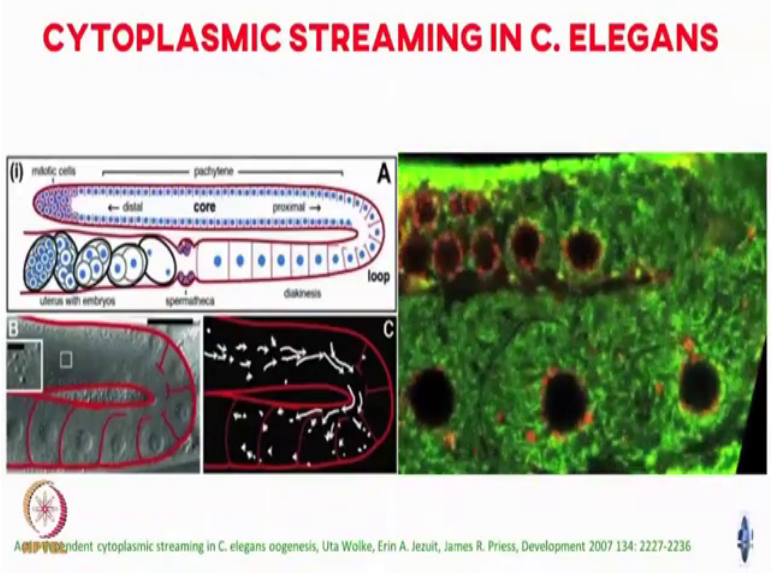

So, let me just explain this figure; this is the gonad of a C elegans; remember C elegans is this world right it is a nematode. This is one part of its body; the gonad here are the embryos of the C elegans. These; what is known is that these oocytes do not have any transcriptional machinery which means that these oocytes cannot produce their own MRNA or proteins; however, MRNA and proteins do get there because when these embryos are going to develop, they are going to need those MRNA and proteins.

So, what the C elegans does is that it pushes in this cytoplasm from these regions. So, from these distal regions of the gonad; they will push it into these developing oocytes until it has the amount that it means. So, these are experiments where it actually tracks the flow of these cytoplasms across this gonads; so if the gonad is this U like structure and if you look at this; so these are plots of particle trajectories roughly over a 2 minute period. So, these are tracks of a single particle over 2 minutes and you can see that they very nicely flow along this direction.

So, in fact if you see this movie you can see all this stuff literally. So, this is this portion of your of the gonad, stuff flows in like this goes around the U bend and goes into this developing oocytes; so these are the development oocytes. So, all the cytoplasm; the yolk particles they get sort of incorporated, coming in from here and then flowing into this region ok. And this sort of this is called cytoplasmic streaming in C elegans and this roughly has this sort of a Reynolds number 10 to the power of minus 6.

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You can actually; these are some very nice experiments, you can actually put outside. So, this is an oil drop which you put by hand just to show that these this transport happens because of a fluid flow; there are not any active mechanisms biologically specific mechanisms that are driving this flow. You just throw in a nonreactive oil drop and this oil drop is if you throw in it over there that oil drop is going to be carried along by the fluid flow and go again into the oocytes.

On the other hand, if you were to put in very large oil drop which is this movie the; so this is a very large oil drop which sort of impedes this flow; now stuff cannot flow in through it its true flow past it you sort of stop this cytoplasmic stream; the movies might be clearer there. So, stuff has stopped flowing and this oocytes do not develop as normally unlike the in this case. So, it is a problem in developmental biology which is driven by this sort of a fluid flow

a very rowan Reynolds number fluid flow which carries all the cytoplasm into the developing oocytes.

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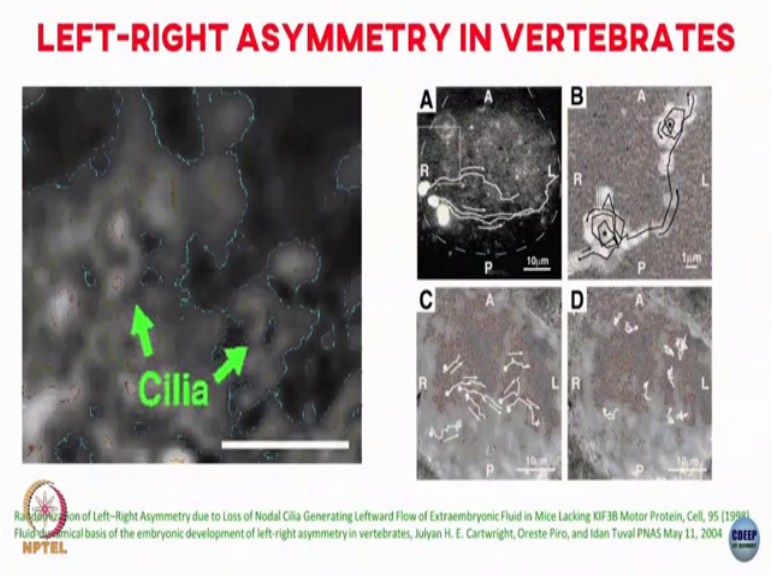

Similarly, this is another example this is in mice if I remember ok; I cannot see anything in this movie. So, in vertebrates we have this left right is symmetry right; on the left side of our body is not the same as the right and there is been lot of work and how this asymmetry initially develops. So, this is some work in mice where it says that these cilia sort of beat in a very coordinated counterclockwise fashion which leads to a flow towards the from the right up to the left of the embryo and that sets up this sort of a gradient which leads to this asymmetry between the left and the right. Let see if this were to play properly these were seniors which were beating.

But anyway, you can look at the still images; these are tracer particles that are put into the ciliary flow. So, the cilia beats in a counterclockwise way that generates a flow and you can tag these particles and show that these particles go from the right to the left; so they establish a sort of an axis this is similar sort of thing. This is a mutant where you have very mutator kinesin family protein that stops this counterclockwise beating of the cilia and in that case, you do not see any directional flow anymore; all these tracker particles that you put they sort of go ahead and do their own thing.

So, you can randomize this; so this from actually this paper, you can randomize this left right asymmetry by playing around with this motor protein; it is a Kinesin Family protein; KF 3 ok. So, again it is a process in development which is driven by these fluid flows generated by the beating of this cilia. And this if we go back to this table this left right asymmetry, this again has a Reynolds number its driven by these motile cilia and it again has a Reynolds number of around 10 to the power minus 3.

So, most Reynolds numbers that we will talk about in biology fall in this sort of a class where the Stokes equation becomes the right approach to use ok.

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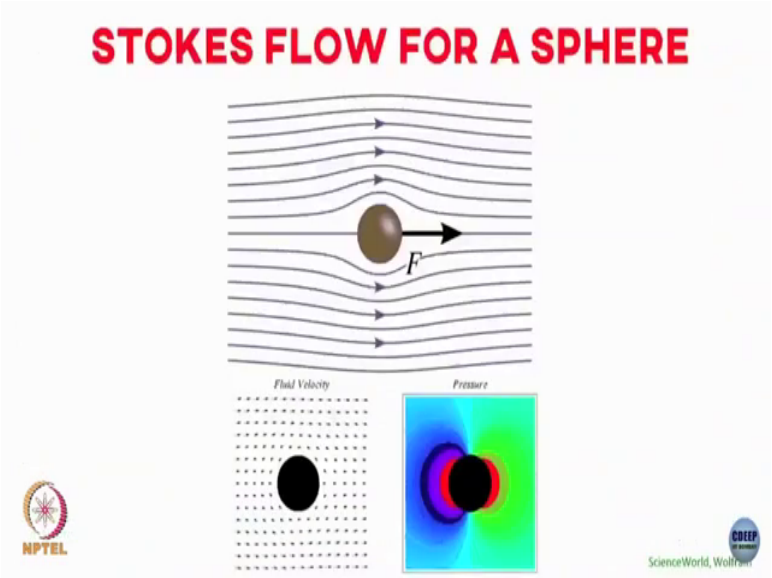

This I will not do this is the Stokes flow past a sphere I thought about doing it with them let it go.

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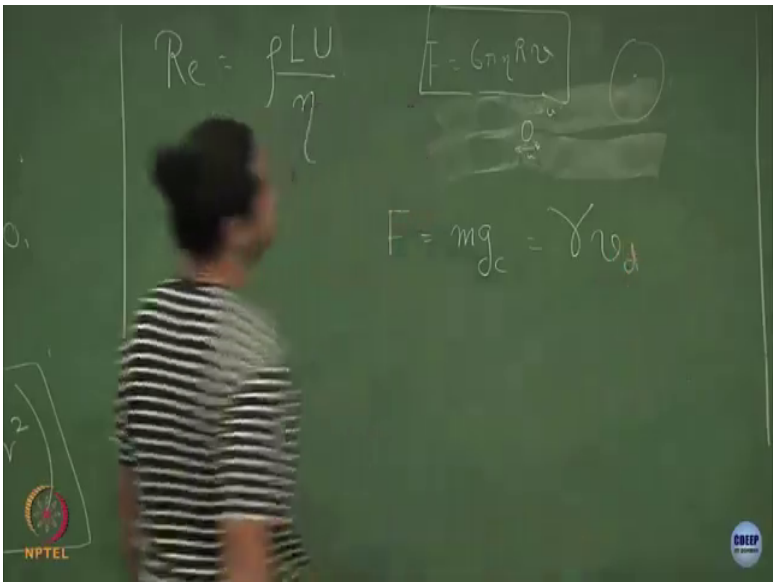

You can work out the solution of the Stokes equation past a sphere and you can show that this viscous drag is this famous 6 pi eta R v's formula. If you want, I can uploads if you an if you have not done it in a continuum mechanics courses. So, I can upload how to derive this 6 pi in your alright. For the final thing, I will just move on to a slightly different thing which is this process of centrifugation ok.

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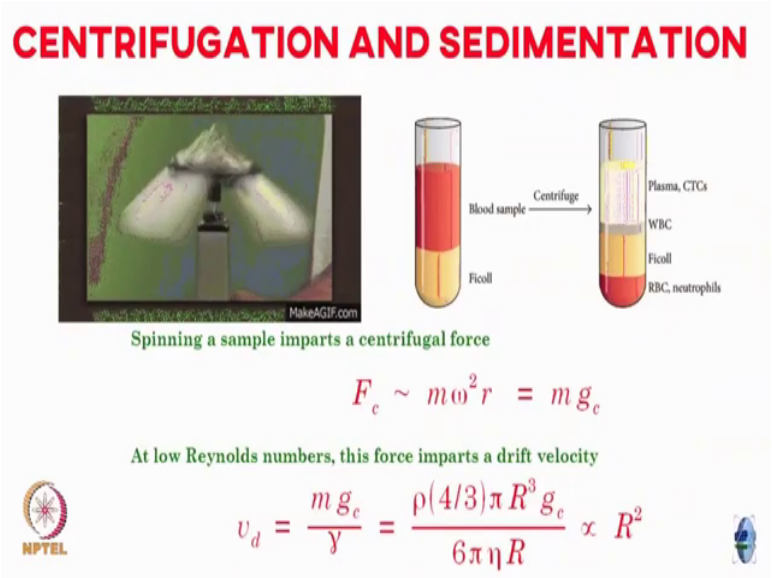

So, often you; so this is an experimental technique and again it relies on this low Reynolds number physics where the velocity is proportional to the force I will just thought I will discussed there. So, here is a process where your centrifuging some stuff in a test tube for example, you take a blood sample then you put it in a centrifuge; what it does is that it will separate the different components of the blood into different layers right. So, this is one layer of red blood cells, here is a layer of white blood cells, here is a layer of plasma and so on.

How does this generally work? How this works is that if you spin this sample; you generate a centrifugal force which is like your m omega square r right depending on with what angular velocity is spinning the centrifuge. So, let me call this as my centrifugal acceleration g c and if you are at low Reynolds numbers this force that you are generating through this rotation will impart a drift velocity to these particles right. So, the force that you generate; so you

have; you have some force which is m times g c; this is going to give rise to some drift velocity gamma times v d right.

So, the drift velocity is then going to be given by this. Mass is density in let us assume a spherical particle of radius R and the drag is 6 pi eta R right. So, the drift will the main thing is that the drift velocity is proportional to the square of the size of the particle ok. So, if we have the suspension which has particles of different sizes; different particles depending on their size will get a drift velocity depending on their R squared ok. So, you have this initial mixture; initial homogeneous mixture which has maybe some large particles, some small particles some medium particles.

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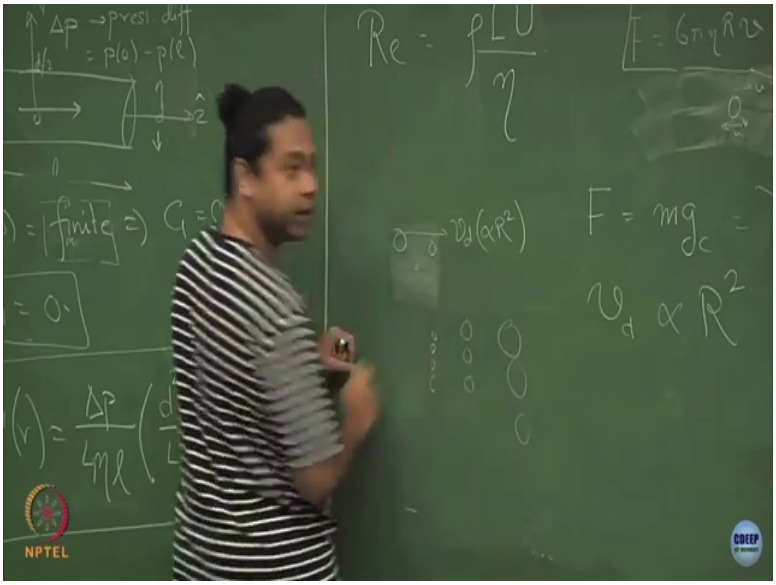

So, each of this is going to move with a different velocity v d which is proportional to their R square ok, but that is not all. So, you would you would expect that after some time you would

have one layer which are bigger particle, its one layer which are medium particles, one layer which is smaller particles right. On the other hand, these particles are also doing their own diffusive random walk which means that if you these layers will simply because of random diffusion will also tend to smear out right and again you can estimate what the smearing out with it.

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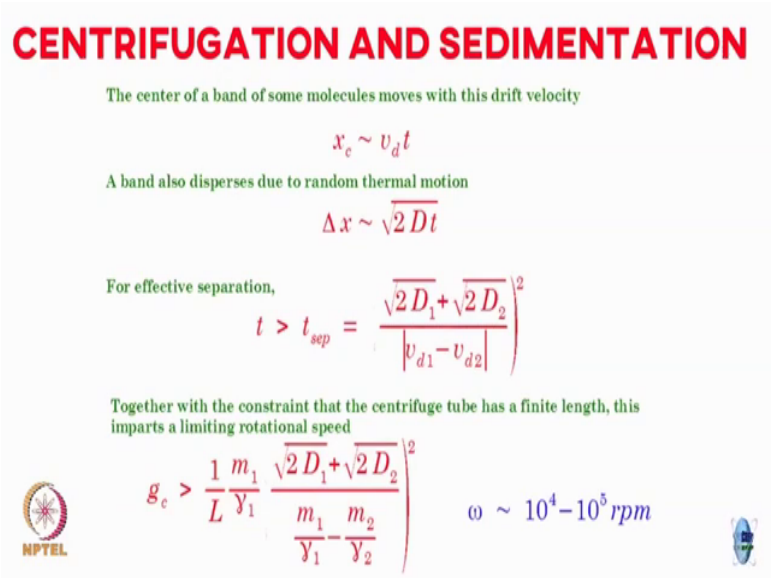

Like; so band of these molecules will move with this velocities. So, the center will move with this v d times t, but this band will also disperse due to diffusion and that is will go a square root of t right. So, if you have two bands of these two molecules. So, this idea that you have two bands; this one moves with some v 1, that one moves with some v 2 ok. But these bands also spread as time grows; these bands also spread across become wider because of diffusion is that let me.

So, this peak moves with a velocity, but it also spreads because of diffusion and in order to; in order to have effective separation in this sort of centrifugal systems centrifugation systems. The distance between the peaks should be greater than the width of these the sum of the width of these bands right.

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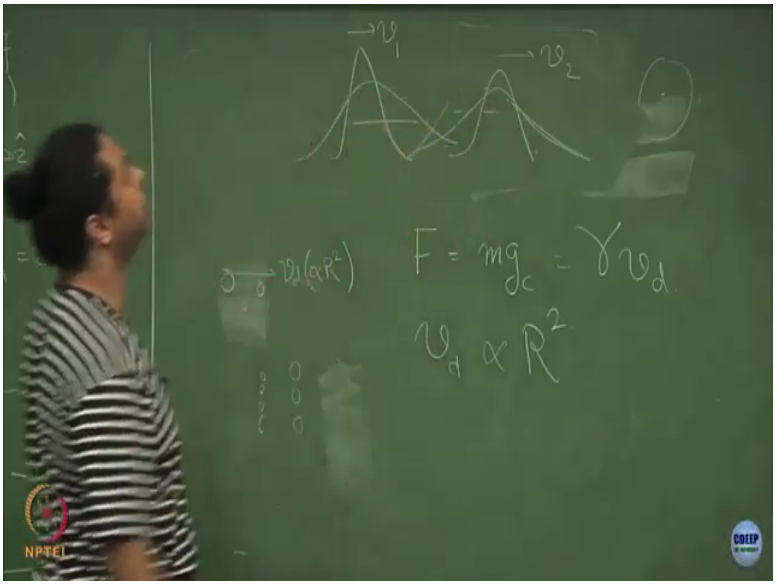

So, this is what is given over here; so you can solve for how long you need to wait in order to get an effective separation between these bands or more accurately; you can convert this equation to how fast do you need to spin such that you get this effective separation within; within a length which is the length of your test tube and that gives you an estimate of the rotational speed. So, let me say this during (Refer Time: 20:54) so these bands move with some velocity, these two bands move with some velocity.

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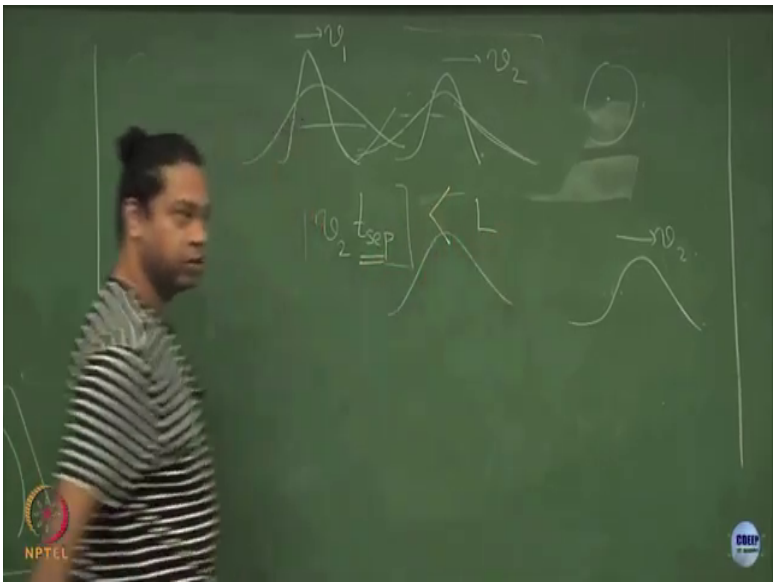

And you can calculate how long you need to wait in order for these two bands to be completely separate to be completely separate so that you can identify that ok, so that is given by this t separation. Now, in this time t separation; this band will have move let us say the band which moves with the largest velocity let us say this v 2 is larger than v 1 that will have travelled a distance some v 2 times t separation right.

What you need is that this distance that it has traveled needs to be less than the length of your test tube because it should by then it should have separated out; the centrifuge tube. So, then that gives you; so that you can convert into a constraint on the rotational speed; on this remember this v is a function of this g c which is a function of this omega. So, that you can convert into a constraint on this g c which tells you given the diffusion coefficients of the

particles and their masses and so on with what speed do you need to and the length of the test tube; with what speed you need to rotate in order to get effective separations.

And roughly for micron size particles; it comes to around 10 to the power of 4, 10 to the power of 5 revolutions per minute. So, it uses this concept of this drift velocity in this flow Reynolds number regime in order to achieve the sort of effective separation with examples ok.

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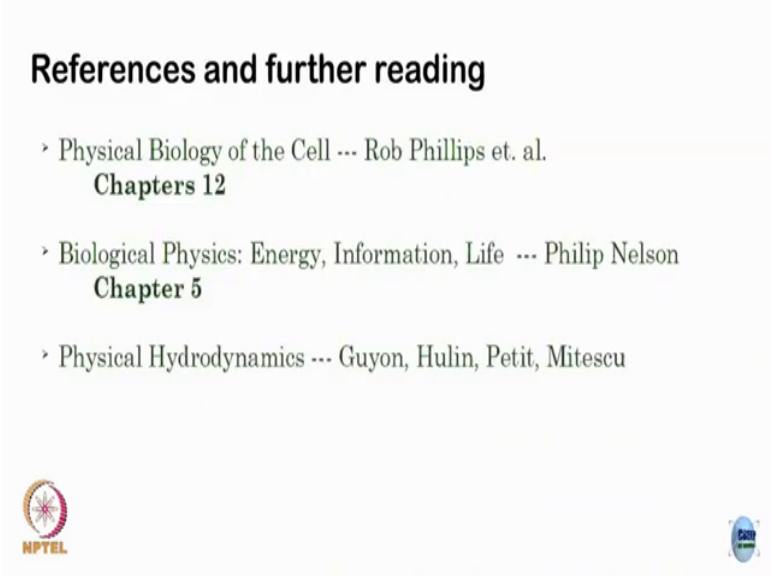

I think, I will stop here today because what I want to spend the next class doing is looking at bacterial locomotion. This is very famous scallop theorem by Purcell which tells you what are effective; last class we asked the question as to what are effective swimming strategies for a micron size bacteria as opposed to a fish and there is some very classic work by Purcell on that; that what swimming strategies work and what do not.

It is a little involves; so I will I think I will not put in a break in the middle. So, I will start that next we next class on Friday and hopefully finish next class as well ok. Again, if you are interested in some of them more details of these calculations is physical hydrodynamics book a good book to sort of go back and read ok. So, I will stop here today and we will continue in next class.