Physics of Biological Systems

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Lecture - 18
Understanding reynolds number

So, now that I have the Navier Stokes equation that mean now precisely define what I mean by this Reynolds number.

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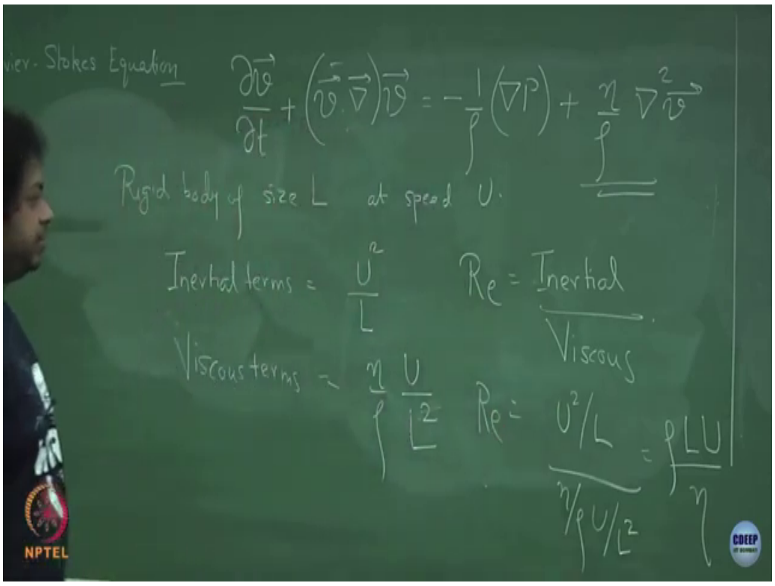

So, let me write it nicely del v, del t plus v dot del times v is equal to minus 1 by rho; gradient of p plus eta by rho; del square this is my Navier Stokes. Now, let us say I have some body of size L; let us say I have some rigid body, rigid body of size L which is moving through this fluid at a speed U.

The strength of the inertial terms will be given by this right. So, the inertial terms; inertial terms is this; v dot del of v which to an order of magnitude. So, this is two velocity terms right v into v which means if it is moving with the speed U with U square. It has a spatial derivative which means it is something like an U square over L right. The viscous terms are given over here; so the strength of the viscous terms; strength of the viscous terms is over there which is eta by rho, this is two derivatives with respect to space and one velocity; so one velocity and two derivatives L square L square.

So, this characterizes the strength of the inertial terms in this equation, this characterizes the strength of the viscous terms in this equation. If you take the ratio of these that gives me remember my Reynolds number which characterizes how important the inertial terms are compared to the viscous terms right. So, that is like inertial is U square by L, viscous is eta by rho; U by L square right which means it is rho L U by eta ok; this is what is called my Reynolds number.

So, Reynolds number for a body of length L or dimensions L moving with a velocity U in a fluid of density rho and viscosity eta is this rho times L times U divided by eta. So, this Reynolds number which determines the physics like we saw in these Taylor's experiments GI Taylor's experiments is not simply a function of the fluid properties only. It is not simply a property of the viscosity of the fluid, but it also depends on the body that is moving through this fluid, it depends on the dimensions of this body; it depends on the velocity of this body.

The dimensions of that dye nozzle or the velocity of that dye for example, which is why even if you take the same fluid for example, water and you move through it and you are like 6 feet tall or whatever nobody 6 feet tall. You are a 5 feet tall versus let us say e-coli which is a micron long that L term over here will change from 5 feet to 1 micron and that will bring down your Reynolds number even though the fluid is exactly the same.

The viscosity of the fluid is the same let us say the density of the fluid is the same because the same fluid will have completely different Reynolds number by orders of magnitude right; five orders of magnitude, six orders of magnitude depending on what body is moving through that fluid right.

So, depending on and depending on the velocity of course, so if you move very fast if the e-coli were to speed through, then it would not then through might increase its Reynolds number for a. But for a typical swimming speed which is again tens of microns typical lengths which is again microns; the Reynolds number for these objects are going to be very small.

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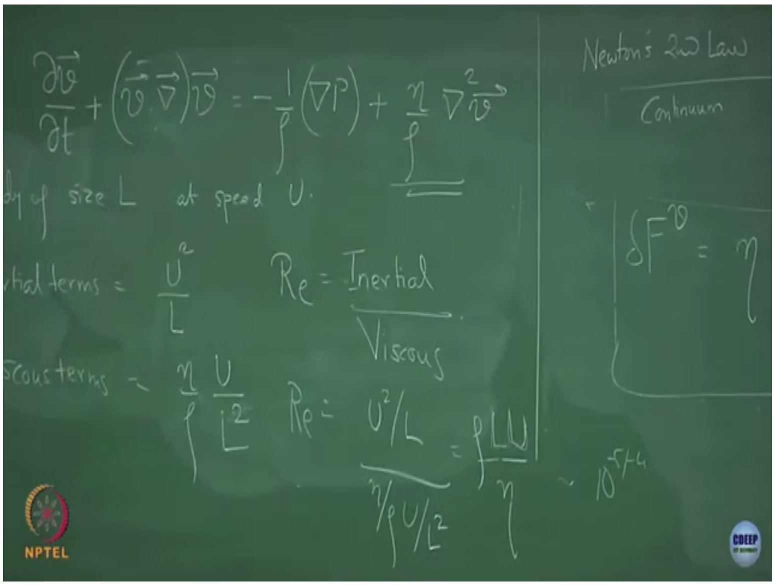

So, typically in biological fluids I think will if you put in the numbers, you will get Reynolds numbers so of the order of maybe 10 to the power of minus 5, minus 4 something like that. And for this macroscopic world that we are used to you will get Reynolds numbers in the regimes of thousands or tens of thousands ok. So, these are many many orders of magnitude apart and so depending on whether this term is small or this term is large; depends on whether this term will play an important role or this term will play an important role.

If the Reynolds number is very small which means that this inertial forces are very small; you can maybe neglect this inertial forces ok, neglect these inertial forces in the equation that you will have will comprise only of this pressure gradient force and this viscous force and that is what is called actually the stokes limit or the stokes equation.

So, we will do that; we will do the stokes flow at least in a simple case next class, but depending on whether you can neglect this inertial terms or not, the solutions of this equation will be very different ok. So, that is the whole idea that when you are looking at these biological systems; they live in a world which has very very low Reynolds number which means we can neglect these inertial terms.

And once we neglect these inertial terms, you get the solutions of this Navier Stokes equation that you get or the Stokes equation in that case are very different from the solutions of this full Navier Stokes equation ok. And in fact, the Navier Stokes equation; the difficulty in the Navier Stokes equation arises mainly from this term. This term as you can see is a non-linear term right; it has two velocities v square which is why it makes general solutions of this Navier Stokes very difficult and in fact, if you can solve it you get 1 million dollars I think; it is one of the clay millennial problems if you can show that a solution exists, you get 1 million dollars ok.

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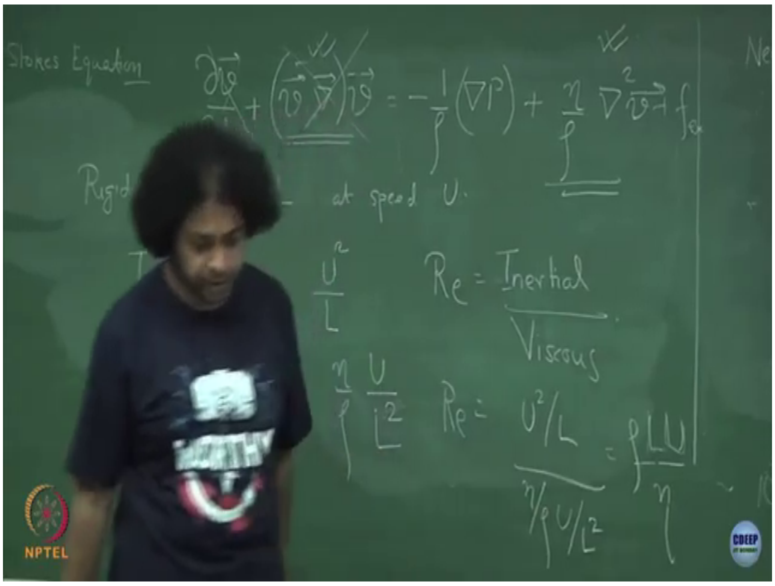

So, it is an open on the general solution as an open unsolved problem, but as long as you are talking of Stokes flows of these low Reynolds number flows; we do not need to worry about these inertial terms; you only need to worry about this terms plus any external forces like that all right.

So, we will continue on this next class; let me see; so since I have 5 minutes ah; let me just define for you, although we will not talk about it let me just define. These remember I derived all of these for Newtonian fluids, let me just define a few types of non Newtonian fluids and how the viscosity comes in that case.

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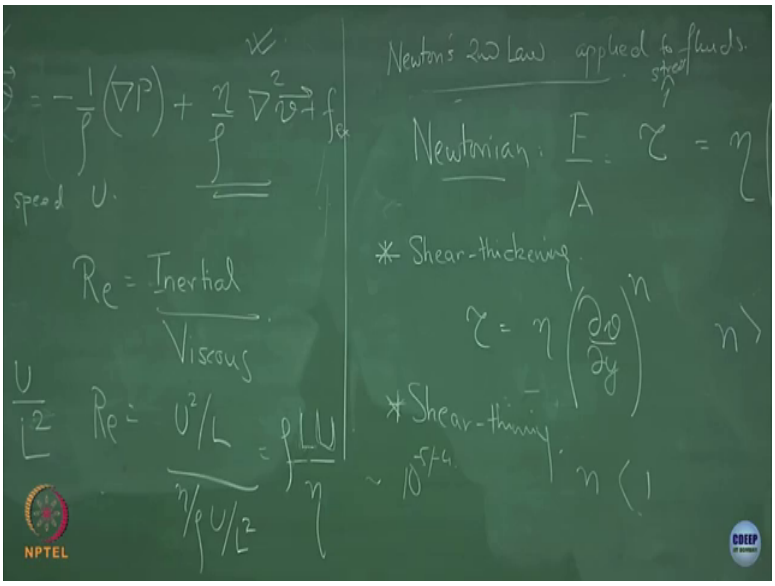

So, for a Newtonian fluid remember. So, for a Newtonian fluid you have force upon area which is the stress, let me call that tau that is equal to eta; the viscosity times del v, the gradient of the velocity which is like my strain rate alright. So, this is my stress, this is my stress that is my strain rate and I have a linear relation between them for a Newtonian fluid and with the constant being the viscosity.

If you have a non Newtonian fluid you can have different types. So, one for example, is called the shear thickening fluid; shear thickening fluid and in that case you get this tau is equal to eta del v; del y to the power of sum n; sum exponent, where this n is larger than 1 ok. So, the effective if you talk of some effective viscosity that sort of increases the more; more stress you put on it. Those are called shear thickening fluids for example; this corn starts in water as a shear thickening fluid.

On the other hand, if you can have a sheared thinning fluid; you can have shear thinning fluids the same expression, but now with n less than 1. So, here the effective the viscosity decreases with increasing stress and things like paint and blood and so on are shear thinning fluids alright. Paint for example, if you paint it as you apply the shear force on with your brush it flows; the moment you stop, it stops flowing. So, as you increase the shear force the viscosity decreases and it starts flowing; if there is you decrease the force it stops flowing; so it is a shear thinning fluid.

Same for blood; if you were a mass murderer you could try a an experiment like that with blood as a non Newtonian property. You could sort of try to apply a shear force and see whether it flows when it flows and when it does not flow.

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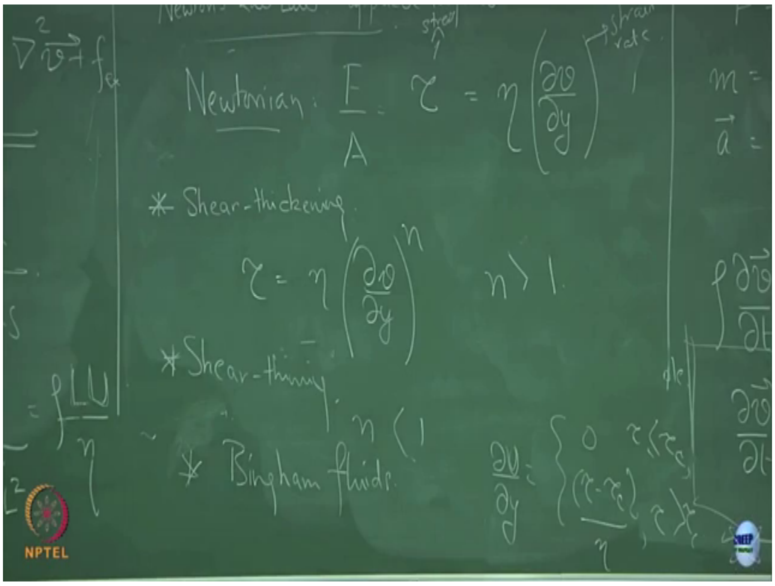

There is another for example, there is another class which are called Bingham fluids which are called Bingham fluids; where it does not flow until a certain critical stress and then starts flowing. So, del v del y is 0; if your stress is less than some critical stress and then its tau minus tau z by eta if your stress is greater than suitable stress ok.

So, this for example, I think is toothpaste as an example if this needs certain amount of critical force for it to self start flow; below that it does not flow. So, there are different kinds of non Newtonian fluids that you can get and for this Navier Stokes equation will not be valid, you need to make corrections in the derivation to account for this non Newtonian character. But we limit ourselves for the time being to this Newtonian fluids; we will assume that all fluids are Newtonian and we will try to look at what are the effects of this low Reynolds number.

So, what happens when these Reynolds numbers are very low? If I just write down this Stokes equation, then what sort of solutions can I come up with? And what does that basically tell me for these biological microorganisms that are swimming in these; in these sort of low Reynolds number environments ok. So, that is what we will do for the next couple of classes alright. So, I will see you.