Path Integral and Functional Methods in Quantum Field Theory

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Lecture – 05
Path Integral Formulation- III

Last time we completed the Path Integral Formula and this time I will demonstrate some of its uses. And the whole direction of this is that we are going towards the functional formulation of quantum field theory. So we can now outline a bit of philosophy of the course.

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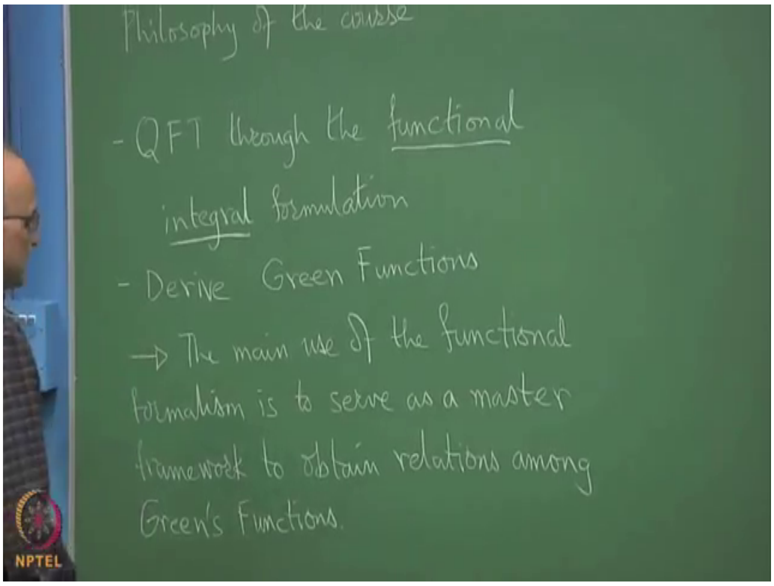

What we want to do is introduce QFT and in it show how to derive Green's functions. So, the most important point is that the most important use of path integral is really as a generating functional of the formally relating Green's functions. This is somewhat important because after investing so much time studying path integral you might ask what is the use? Because no chemists ever needs a path integral, no nuclear physicist uses a path integral, no particle physicist uses a path integral ok. Particle physicists can derive Feynman diagrams for what is called evolution operator the Dyson method of deriving. So, what does path integrals good for? The simple answer is they are good for amusement.

However, if you promote them to fields and instead of paths you are then dealing with integrals over configurations of all possible fields, then it becomes a very powerful tool and that is what we are going to call the functional integral. So, the functional integral is simply a generalization of the path integral and the main use of it is then it is a powerful method to derive relationships or results dealing with Green's functions ok. And we can derive one of the most important thing we can derive is effective potential.

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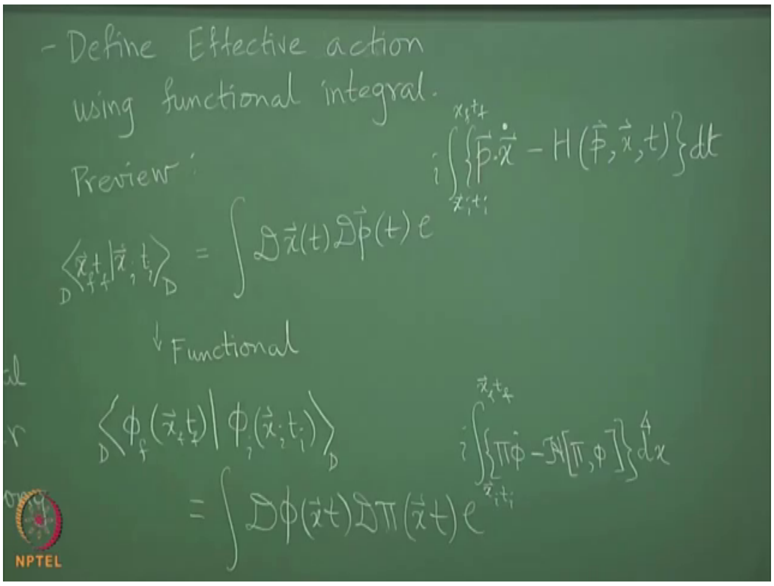

So, let me just a define effective action; so, these are the things for which the functional method is useful. And just as a preview like a trailer of things to come, we had this formula

$$\langle x_f t_f | x_i t_i \rangle = \int D\vec{x}(t) D\vec{p}(t) \exp[i \int_{x_i t_i}^{x_f t_f} (\vec{p}.\dot{\vec{x}} - H(\vec{x}, \vec{p}, t))]$$

And now we write the functional integral as,

$$\langle \phi(x_f t_f) | \phi(x_i t_i) \rangle = \int D\phi(\vec{x}, t) D\Pi(\vec{x}, t) \exp[i \int_{x_i t_i}^{x_f t_f} (\Pi.\dot{\phi} - H(\Pi, \phi))]$$

Here note that the integral over all the paths. So, all the paths that start at x_i t_i and terminate at x_f t_f, but then over the entire possible phase space. And you remember that all these curly things have all the legal definitions in terms of some $\sqrt{2\pi\hbar}$ and all that has gone below. So, it is defined like that and in the limit that n goes to infinity. Here we simply with a great flourish where this Π is conjugate variable of ϕ.

So, exactly same formula except that now it will be a density and it is d^4x and from x_i t_i to x_f t_f ok. So, we this is what we are going to do; we are going to extend the idea of things along paths, eventually it is integration over entire field configurations in the functional space. There is another thing I wanted to say which is the tying up of the Fock Dirac quantization for the weak field system.

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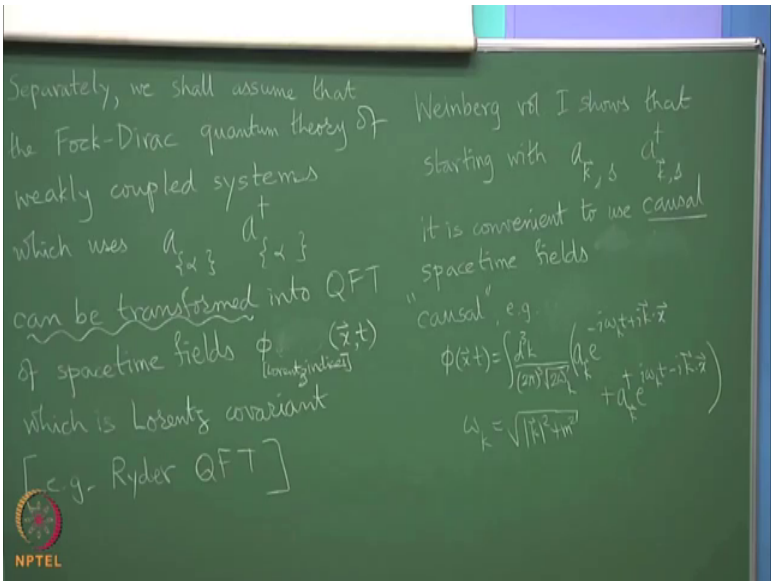

Fock Dirac quantum theory of weekly couple systems goes over to quantum field theory, a space time fields. So, here which uses $a_{\{\alpha\}}$, $a^\dagger_{\{\alpha\}}$,…... can be transformed into QFT of space time fields with some indices ok. So, let me just write Lorentz index because I will have to start talking μ,ν all that. So, for the time being it is not important what μ,ν you put there, but it is into QFT of space time fields which is Lorentz covariant.

So, what does all this mean? It means if you open a standard textbook of quantum field theory; one standard one and which is nice to read is book by Ryder which we have not referred to. He will start directly with these with this field theory, but the point is that there is a connection, but that approach is actually just tells you let us quantize fields.

Or as motivation it will say well put oscillators at every point in space time and now put some coupled by spring than in the continuum limit it looks like fields with derivatives because nearby springs are coupled, but then you ask why are there springs at every point in space time. So, there is no real abinitio motivation of while those fields come from.

The really the fields come from here which is simply as we proved a consequence of the Fermi or Dirac statistics. And if you try to build up the theories of those then you can introduce this a dagger operators because those states are labeled by purely number. It does not matter which particle is in which single particle quantum state just how many particles there are in a particular quantum number state and so, that allows you to introduce this commuting or anti commuting operators.

And then there is this can be transformed to is a slightly long story, which most people skip because it is somewhat technical. And then they simply write Lorentz covariant fields; scalar, spinor, the four vector field and so on, but actually this connection is the big one.

And from systemic point of view why we should construct Lorentz covariant fields and not just deal with a a dagger. We wrote a particular interaction potential you remember which some particles being created, some being destroyed and then some V; that thing is very cumbersome because you cannot guarantee that there is momentum conservation, energy conservation charge conservation and all the quantum numbers involved and whatever space time invariance. Therefore, it is much better to use these; now people just use these and find that it is a nice thing to do, but this connection is what is a rather a technical one.

And that is the content of Weinberg volume I. Starting with this it is convenient thing to use are the space time fields for. So, let me say this much it is convenient to use causal space time fields by causal is meant some linear combination of a and $a^\dagger$; you must have done in quantum III,

$$a=\exp[-i\omega t+i\vec{k}.\vec{x}];a^{dagger}=\exp[i\omega t-i\vec{k}.\vec{x}] \quad .$$

So, you split the positive frequency and negative frequency parts and so on that is what we call a causal field and if you construct this causal field then

$$\phi(\vec{x},t)=\int \frac{d^3k}{(2\pi)^3\sqrt{2\omega_k}}\left(a_k \exp[-i\omega_k t+i\vec{k}.\vec{x}]+a_k^{+¿}\exp[i\omega+kt-i\vec{k}.\vec{x}]\right)$$

that is what is meant by causal field similarly expansion for direct field for Maxwell field and so on.

So, starting with this it is convenient to use these and why is it convenient because this is what ensures and now get ready for the big content of that theorem.

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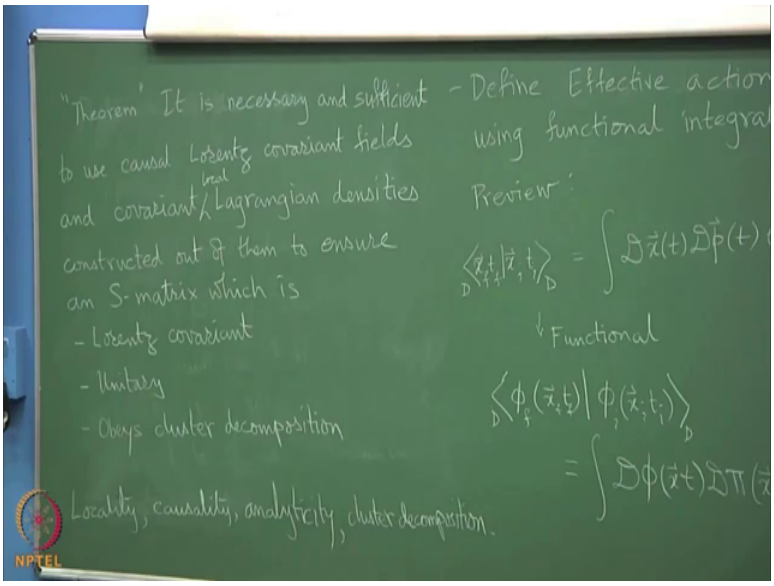

The statement is that if you use causal fields; so it obeys locality, causality, analyticity and cluster decomposition; this is the way I remember it, but to say it more simply one should. So, causality basically is this Lorentz covariance that influences propagate only inside the light cone. There are effects outside light cone also, but it is a Lorentz covariant description; locality means that you do not get actions at a distance.

And that would also violate Lorentz invariance. So, it should be local and unitarily is part of the causality statement in a way; analyticity is something else, but that has to do with also giving these causal answers. Causal and unitary answers the last thing is this cluster decomposition; which is a compliment to the locality part. Because cluster decomposition means that if I have some 5 particles interacting and 7 coming out. But if I begin to separate out a group of 2 of them; so 2 going into 3 and 3 going into 4 or whatever; if I separate out some clusters then effectively their S matrix elements will factorize that process becomes independent of this process.

So, that is sort of global implication of locality; that if you really move these things apart then they are not mutually interacting which then allows you to focus on local physics at a time; when you do experiment in Geneva, you do not have to worry about what is happening in Japan.

So, that has to do with this cluster decomposition that experiments that are far apart are not going to influence each other. So, all of these properties are necessarily and sufficiently obtained; if you use the causal fields. And the causal fields ultimately can be constructed out of these the Fock Dirac construction and this is the real physical basis of quantum field theory.

However, you will find people who say why worry about all these a's and $a^\dagger$s and so on just start with some space time fields. And in fact, here the S matrix becomes a crucial ingredient that we are only concerned about S matrix to be obtained from field theory; from this quantum physics.

But there are non as matrix things that we require from quantum field theory; there are for example, bound state. So, how do you compute bound states out of their ingredients? Well the answer is in relativistic physics we do not have very good methods at present. There are the so called goes by the name of beta ansatz and so on to obtain bound states, but I do not think they have been very successful. So, there was a nuclear bound states would have been calculated from quarks by now; they have not been.

So, there are questions that are not answered by the S matrix, but then they can still be described in terms of causal fields. So, the causal field framework seems to be more powerful than just this necessary sufficient requirement for the S matrix and this is why field theory remains of great interest.

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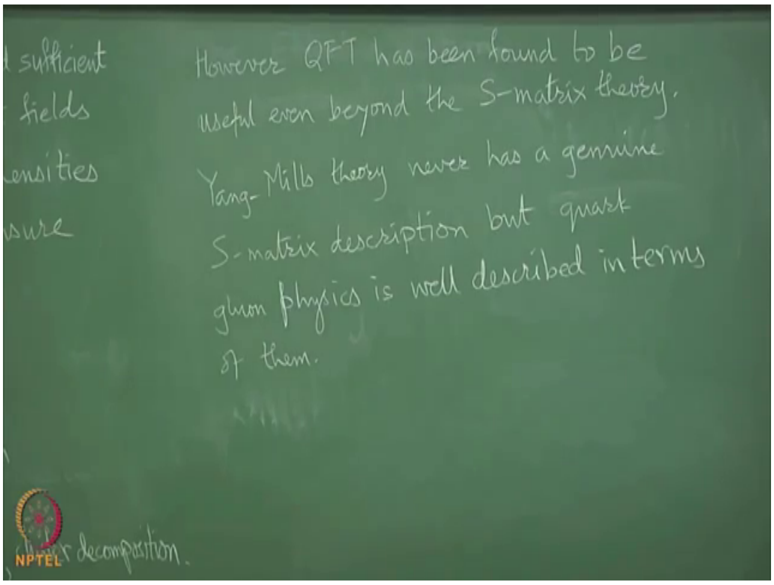

So, much by way of the philosophy of the course and I waited till the third lecture to lay this out because at least now we have seeing the path integral and the Fock Dirac quantization. So, you know what we are talking about and as you also know that its only after the initial credits rolled by that the movie starts rights. So, now, we plunge back into the path integral and learn how to do some calculation with it.