Path Integral and Functional Methods in Quantum Field Theory
Prof. Arjit A. Yajnik
Department of Physics
Indian Institute of Technology, Bombay

Lecture - 02
Quantum Theory Fundamental Quantization –II

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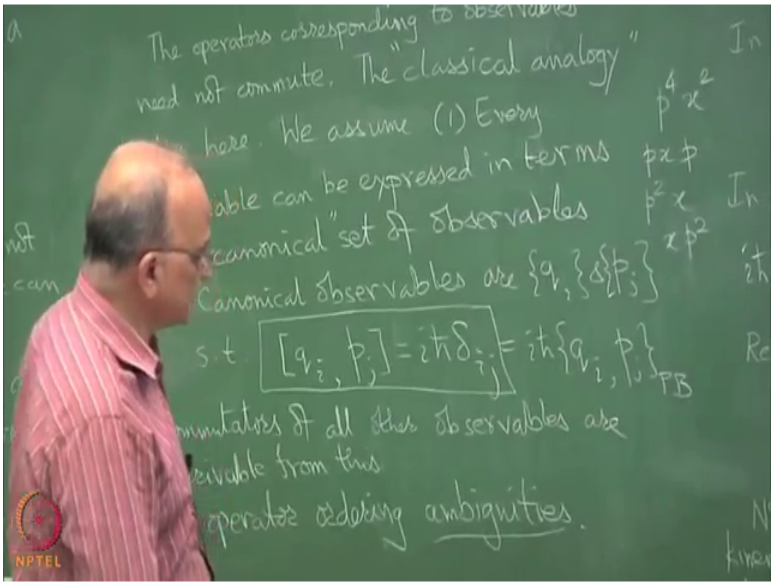

And there is actually another point here which is that this quantum kinematics hypothesis that every observable will be expressible in terms of q and p can fail and it fails royally with spin. Spin is a quantum number for which you do not have any space time representation and you do not have any classical analog. So, when you get to spin you have to add it by hand from your pocket. Amazingly enough it has commutation relations which look exactly like the angular momentum commutation relations which angular momentum can be made out of q and p.

And so they in fact mix as well. So, you can encounter when you go to quantum mechanics, new systems where you can not break it down like this to it may not have classical limit. So, any observable that does not have a classical limit like spin may not have a canonical representation like this. The other corresponding point being that in enumeration, the spin half you have to quantize by anti commutators instead of commutators which is very big jump. Because it has again no classical analog right the fermionic systems have to be quantize using anti commentators to account for Pauli principle.

So, there are divergences from this visible even in the very simpler examples of quantum field theory. But in modern times we are living in many many more complicated situations where in fact, quantum field theory itself looks in doubt ok. So, like this conformal field theories, you may not have a Hamiltonian description at all. So, you may not have so called canonical valuables, but the previous part the first part is correct. So, as you know this classical analogy was a simplification.

But the modern things do not affect the fundamentals of quantum mechanics because there all you have to do is list all the possible commutators between all the possible operators. And if you have a recursive way of stating it, then you are done you can state some ladder up ladder algebras between Virasoro algebra and so on. So you may have infinite number of independent of observables, but if you can state their commutation relations; then you have fix the quantum system.

So, not all systems will have this luxury, but we try to make do with this for all most everything that is known at present; except in the strongly correlated condense matter systems. So, super conductivity, super fluidity; all of these were very exotic states, they had to do with this bosonic enumeration of states and Bose condensation.

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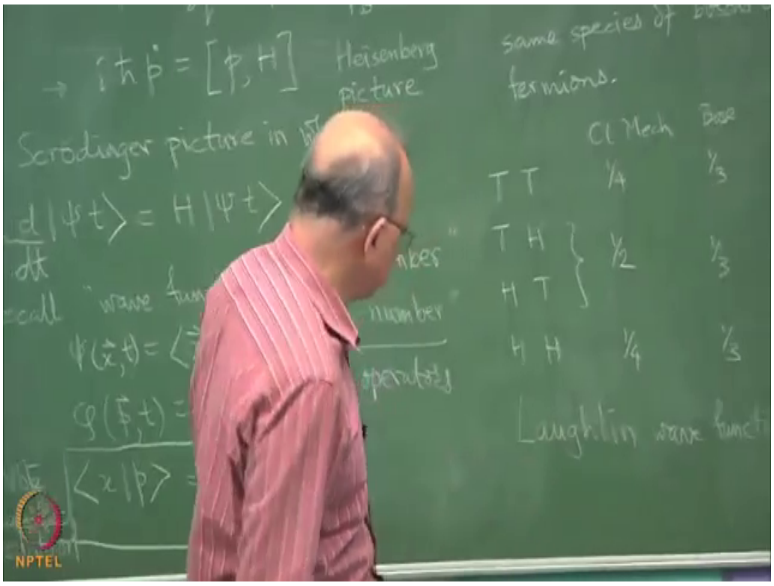

And also the fractional quantum Hall effect which has a wave function which goes by the name of the person who got the Nobel Prize for it Laughlin. Laughlin wave function, the wave function exists, but there is no Hamiltonian of which it is an eigenstate.

They do not know Hamiltonian system for which it is in the list of its spectrum. So, we are at that border line where this simply stated set of principles may need some extension, but there is nothing worried, I mean it will be nice and exotic if you we find newer quantum systems and just this simpler once, but there is nothing that threatens our civilization. So, the second thing I can do this is the issue of quantizing weakly coupled systems.

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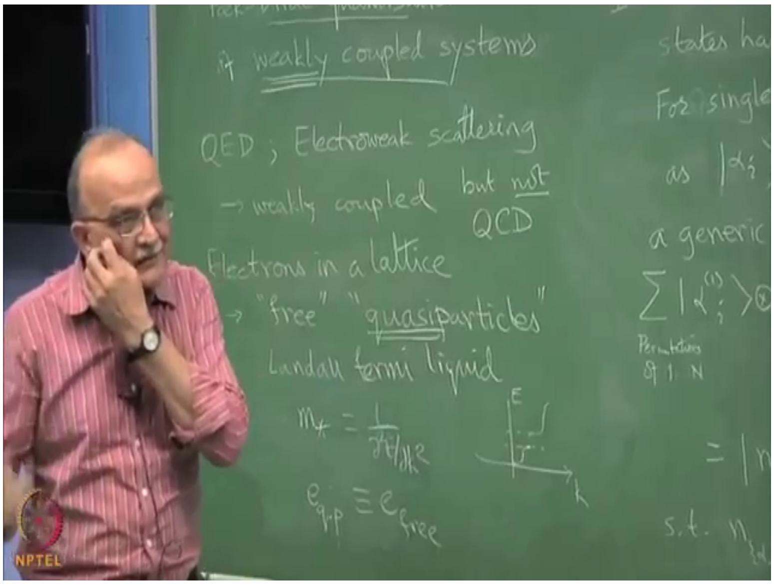

So, the next thing is Fock-Dirac quantization and it is important to say that it works only for weakly coupled systems. And if we talk about weakly coupled electrodynamics in scattering processes, QED scattering; QED as well as electro weak; all of this is weakly coupled. Electrons in a lattice miraculously displayed this weakly coupled kind of phenomenon because you can think of them as free gas of fermions in the lattice, but that there are really quasi particles.

So, both term should be put in separate; so you can continue to think as if there are electrons floating in the lattice. But actually its much more complicated situation to which in the situation where you are lucky you have that description available is often called the Landau fermi liquid. Think about this, you have the lattice which is positively charged and you have electrons which are all negatively charged and the coulomb forces infinite range force because it goes only as potential goes as 1/r.

So, they are all actually interacting with each other; every single electron is interacting with every single other one. But because somehow of this big positive charge that is given by the background and the way Fermi statistic forces them to stack up above each other, they just flow around as if they do not see any interaction ok, but their interaction is then hidden in their effective mass. So, $m_* = 1/\ (\partial^2 E/\partial k^2)$.

So, you are define it like this and you known that the dispersion relation energy as a function of k has this band structure with gaps and when you reach here; you have very large mass because the curvature becomes 0. So, as the curvature of d/dk curve become 0; the mass becomes infinity. So, here are the fermions do not move, here they cannot move things like that, but these are actually quasi particles.

So, what is beautiful is that a free electron has a charge and a mass and this complicated system just looks as if it is a collection of individual entities with charge and mass, except that the mass has to be redefined. So, this these entities we call quasi particles, mass has to be redefined; the charge cannot be redefined because of gauge in variance you cannot have charge changing in the system. The total charge maybe renormalized once and for all, but it cannot be a k dependent difference because then the gauge invariance operation will fail ok. Gauge invariance requires a phase in the exponent and the phases has to be single valued; so it can only go from 0 to 2π.

So, all charges have to be compatible which are integer multiplication. So, that says the charge of the quasi particle to remain exactly as the electron charge. But the mass gets renormalized and then you can continue to think as if you have some kind of fluid of particles. But it is just an amazing miracle and that breaks down when you get to systems where you know condensation of various kinds offer ok. So, getting back this procedure of quantizing quantum system correctly as and I said correctly means that you have to have postulate numbers 7 with you; without that you are not doing your correct job. So, this thing was thought up Fock and Dirac along the following lines.

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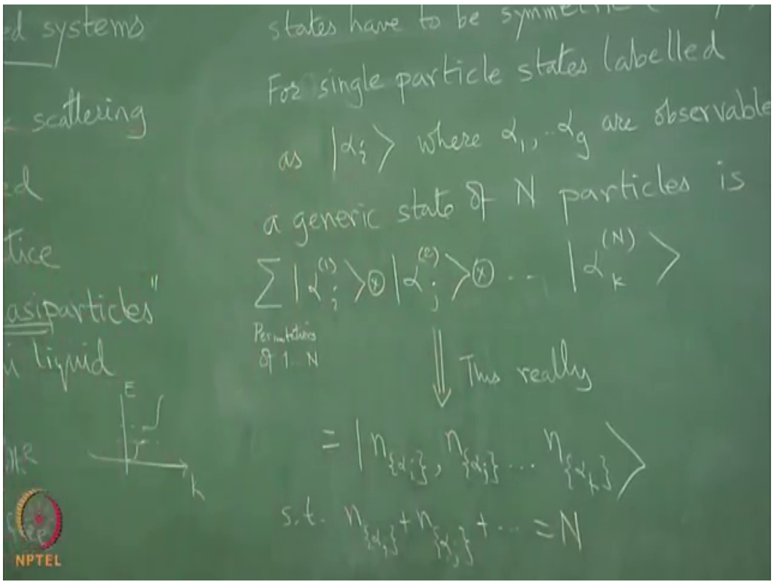

So, the first observation is that observe that the states many particles states have to be symmetric or anti symmetric. So, I will write the fermion part in the bracket. So, suppose we have a list of quantum numbers; so suppose I have single particles states labeled as some set of observable $|\alpha_i>$; then a general state $\sum |\alpha_i^{(1)}> \otimes |\alpha_j^{(2)}> \otimes \ldots\ldots |\alpha_k^{(N)}>$, but this is not correct because you have to symmetrize it. So, you really have to do of symmetry operation; sum over the permutations of this 1, 2, 3,….., n. So, the list $|\alpha_i>$ specifies one particular list, the list $|\alpha_j>$ is another list ok.

So, particle 1 could be carrying this and 2 carrying this that you also have to take the case when 2 is here and 1 is here and so on. So, you have to sum over all the permutations what this means is that thus really this is simply equal to $| n_{\{\alpha i\}}, n_{\{\alpha j\}}, \ldots\ldots\ldots, n_{\{\alpha k\}}>$; so think of the i and j is defining different sets of values right. In other words, the states are labeled by how many particles there are in a particular quantum numbers; set of quantum numbers. You cannot tell number 1 is with $|\alpha_i>$, but number 2 with $|\alpha_j>$; all you can do is count.

So, if $|\alpha_i>$ reccurs anywhere else, all you have to do is say while there are 5 of these that carries this quantum number, 7 of these carry this quantum number. So, the states are labeled by this such that of course, $n_{\{\alpha i\}} + n_{\{\alpha j\}} + \ldots\ldots\ldots + n_{\{\alpha k\}}$ that is all you have ok. Now if this is so, then we have a clever opportunity and this is where our; this psychological problem business is explained because we do have the number operator.

So, when people are worried that I have two electrons and then coming apart and then I observe spin here, then that spin gets determined. It is all to do with the fact that ultimately there is a quantum number associated with the number of fermions; it is 2 that state has 2. And that can be a observed because they do have a charge and the charge can be observed at a distance even without disturbing the system much.

So, you know for sure that there are 2; in the case of photons we do not have conserve number. So, there you have to be sure that you are not reducing or not relating a photon, but; so, I am saying this because there are lot of the entanglement experiments are done with photons, but I think they are able to control that part. So, the number has to make sense and the psychological conflict in this string has to do with the fact that it is 1 state, but the number operator eigenvalue in that state is 2 ok. So, the thing there are two different entities; this cross product where we take one particle states and then string them to gather to make more is really a mathematical device ok. If I did not have this summation, this has no physical meaning because if it is unsymmetrized then it has no meaning, it is not one of a physical state.

So, the fact that we can fall back on single particle states to string to gather a many particles state has to do with this weak coupling. And the famous thing that I have left out of here is QCD; in Quantum Chromo Dynamics, we have almost massless quarks and we have exactly massless gluons and there is no way you can count them individually.

So, they are never weakly coupled; they are weakly coupled only when they are being scattered at very high momenta or very high centre of mass energy which means that their weakly coupledness last only for fleeting seconds; fleeting movements. For most of the time they are blobs hadrons which are all in a strongly coupled state and you cannot pulled the quarks a part.

So, quarks are prime example of where this construction will fail. So, the first step is that due to postulate number 7; all the states are labeled only by number; number in each possible slot of eigenvalues that are available.

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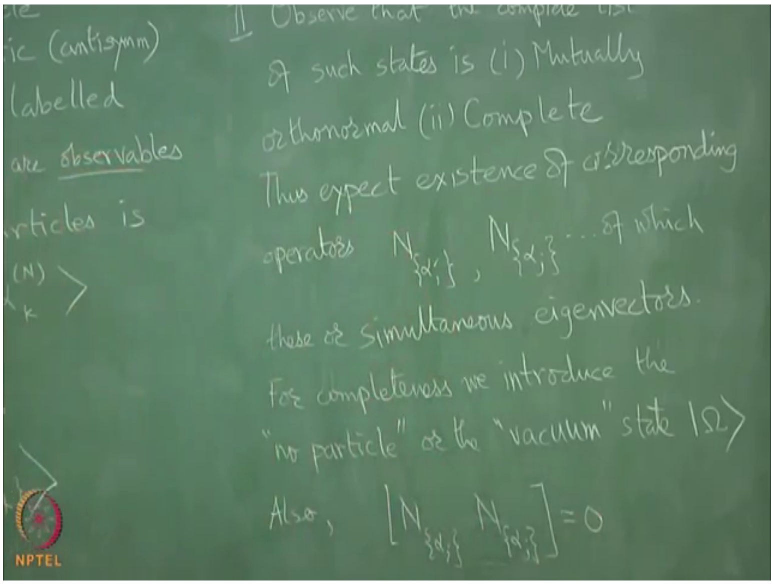

But if this is true we observed that the complete list for such states is mutually orthonormal and complete.

Because they are mutually orthogonal because if the number in any slot if I take expectation value of something else the number in any slot is different, then it will give 0. So, you have to an exactly same numbers in every slot then of course, you will get mod square of that state, but if any numbers differ then you immediately get 0. So, that is one property, the second property is that of course, every possible state of the system can be formed by linear combination of these. These and this is all there is if you list all the possible symmetrized states, then any possible state can be written as a linear combination of these; so, this is the complete set ok.

So, because of this fact that they are mutually orthonormal and complete; you can think and reverse that therefore, we are tempted to introduce Hermitian operators of which they are eigen sets right. Because it is a complete and mutually orthonormal suggest that there are some Hermitian operators. These are simultaneous eigenvectors; just occurs to me that how much damage you do to the quantum state in a observation process depends on the kind of observable. So, there are things called weak observation; so if you get in to that language then there is lot of discussion there right.

So, we can expect existence of corresponding operators which number operator corresponding to each of these; so there is eigenvalue is the $|n_{\{\alpha\}}>$. And one thing I have to add the to for completeness we will also need the vacuum state which is a no particle state. For a single particle there is nothing like a no particle state, but once you introduce this collective states; you also need for completeness a state that are no particles state. So, we also introduce | Ω> such that all N's are 0 in it.

So, then we can think of the spectrum to be fully complete with respect to all of these because even in because you will find that many of the ends are 0; let us say in some state. But then you have to consider the possibility that all the N's are 0 as well to make the whole spectrum of states complete. So, that is the ground state α; in modern times we know that much mischief's lies in fact this so called vacuum state.

Although, we think it is no particle it is not strictly no particle because it keeps doing something; it has quantum fluctuations in it. Now that you have understood that there are number operators and they are mutually commuting.

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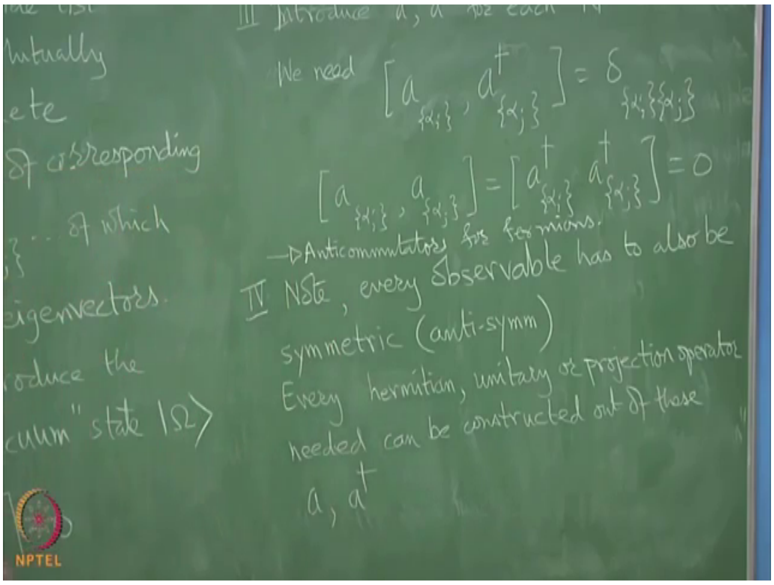

So, now we therefore, in reverse since we are so familiar in quantum mechanics, we are tempted if I have number operator to introduce a and $a^\dagger$; out of which I can get the numbers. So, you can always introduce this and that will reproduce this algebra. So, this expression in terms of a, $a^\dagger$ is a great calculation device now.

So, this gives you now new calculus if some time somebody ask me you know somebody was otherwise where you accomplished engineer and mathematics person. What is new mathematics of quantum mechanics? Well the new mathematics of quantum mechanics is that the a, $a^\dagger$ as the elementary operation then it allows you to calculate S matrix elements.

So, you generate the scattering processes by using these and using that commutators with the Hamiltonian. Note that the observable also is symmetric in the in terms of particles. So, the observables have to automatically be constructed out of a's and $a^\dagger$'s only. So, every Hermitian unitary or projection operator needed can be constructed out of a's and $a^\dagger$'s. And therefore, every calculation that you need to during this theory can be boiled down to computing commutators and anti commutators of a's and $a^\dagger$'s.

So far the system was free but we said weakly coupled. So, now, we can introduce some interaction; so I will end by writing that a generic interaction.

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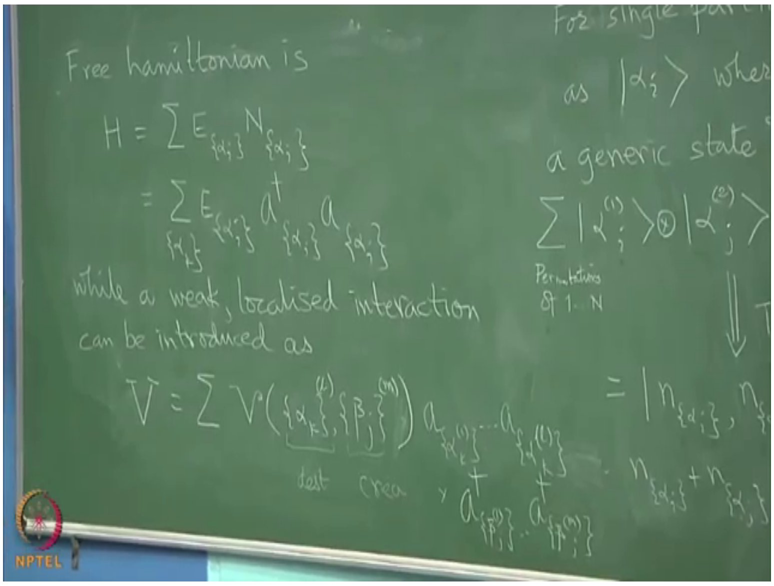

So, the free Hamiltonian $H = \sum E_{\{\alpha_i\}} N_{\{\alpha_i\}}$, while a interaction

$$V = \sum \mathscr{V}(\{\alpha_i\}^{(l)}, \{\beta_j\}^{(m)}).$$

So, I have some set αs and some set of βs which are l in number and m in number and I introduce; it just comes how many distraction and how many creation operators.

So, you can write out some expression like this where l number of particles coming and m number of particles go out. And we have to characterize the V correctly so that we get meaningful free theory. So, V can localized or properties that the V has to basically ensure what that means, is the content of all quantum field theory; how to construct those is what the quantum field theories about.

So, we will stop here; so incidentally if you want to read up this last this; Fock Dirac construction is given in Mursbakar's text book, with some variation in notation is the same physics the same things he talks about and very nicely moves on to the so called many body theory. But we will break from here onwards, we will go on to the field notation; of course, as you know fields can be recovered now by doing Fourier transform from the α space to a coordinate space. So, generically the α space is simply moment of k; the normal way of labeling free particles is their momentum in spin.

So, if your Fourier transform from momentum to coordinate space; you get the space time fields. Most books almost all the books tried to tell you that the field is some entity given by God and that we shall try to quantize it. And they say look do not be afraid of a field because you have seen coupled oscillators and then, but this is actually the construction. So, after they do it; they automatically get Bose and Fermi systems because they put commutators and anti commutators.

But why you did it has to do with this underlined postulate 7. So, this is the real construction of a weakly coupled field theory, but to lot of people it seems that if you start directly with the field, then you are doing some more advanced and may be you have you are accessing a non weakly coupled theory as well by making those postulates, but I do not think so.