Path Integral and Functional Methods in Quantum Field Theory
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Lecture – 18
Asymptotic Theory – II

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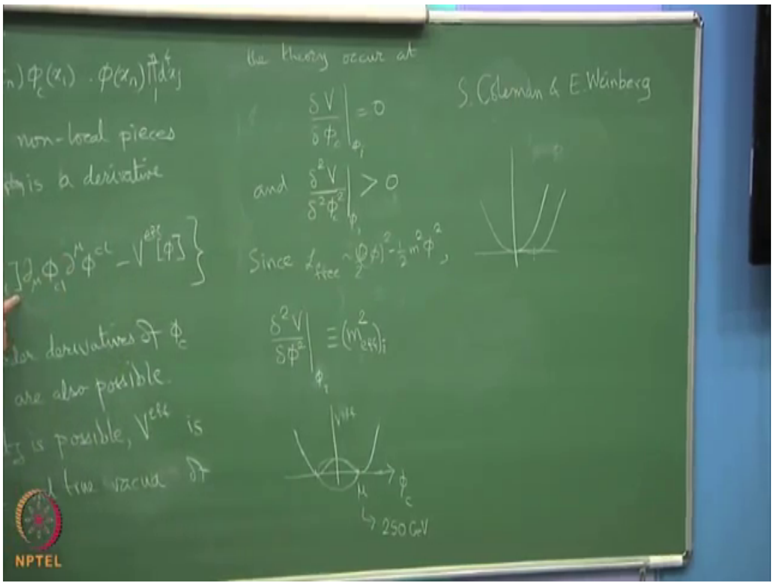

There is a interesting discovery that was made by Sidney Coleman and Erik Weinberg not Steven Weinberg which I have requested Vikram to cover, but it basically starts with a Lagrangian whose V[φ] = 0. So, whose minima of V are at φ equal to 0, but after you do the loop calculations and calculate the effective action they shift to nonzero. So, it is a very cute and clever example they constructed, you start with a V like this, but it is the quantum electrodynamics, so there is a coupling to a electromagnetic field. So, after it develops a logarithmic curvature like this and becomes like this.

So, if a charged field acquires a nonzero vacuum expectation value it means the vacuum is charged and then gauge invariance will be broken and the photon will acquire a small mass and etcetera.

But what was eventually realized for Standard model, we are actually living in a charged vacuum if you like, if you want to feel strained and stress you have a less because there is SU(2) change filling the vacuum and that is why the W bosons are so massive because that where is a 250 GeV ok. So, that is the real machinery behind and still perturbative.

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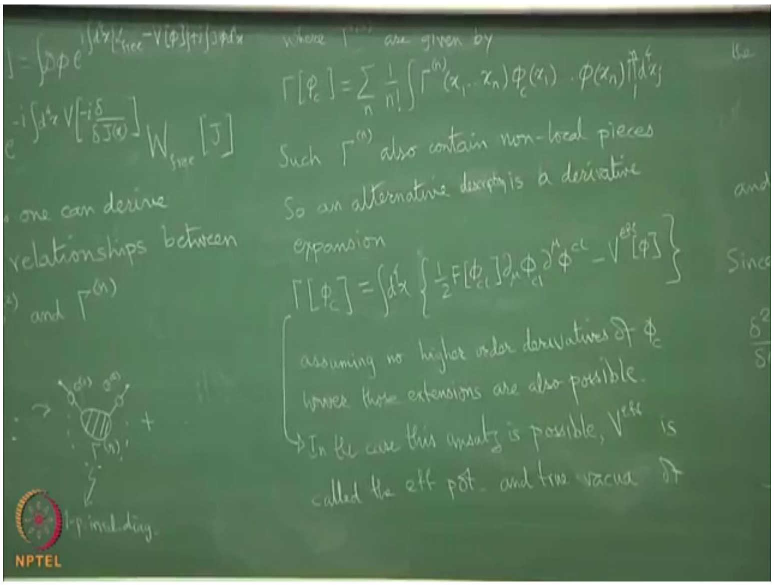

So, here in general we proposed that look what is the most general derivative expansion including only local fields I can make of Γ in which no higher than second derivative or square of the first derivative occurs, then this is the more general expansion. Except that you have to supply some overall factor which involves only φ and not derivatives, then you are still with that domain. So this F would be would come from the quantum corrections and sometimes you have to put that kind of a overall multiplication even otherwise this is what I will come to next later.

The other thing I want to tell you is that the real local expansion in terms of only local monomials of φ and $\partial^\mu\varphi$ we need not always be so civilized and in fact, there are things called effective field theories of pions and nucleons which is called chiral Lagrangian for hadronic physics.

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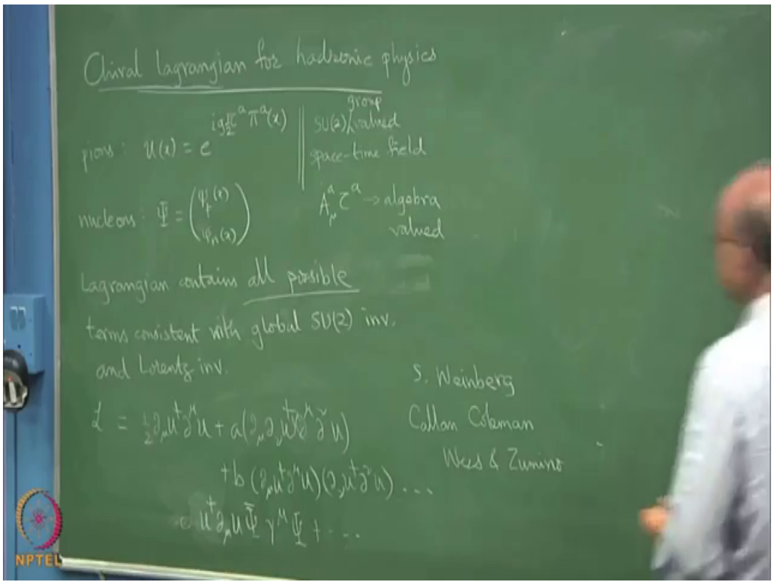

So, here we are actually living in a world where the strong force has shielded all the color charge and the only observed things are pions and nucleons. Then it turns out that those pions are Goldstone bosons we did not talk about it so far, but maybe next time we will do that before I sign of and the Vikram takes over.

Let me simply say that pions are described by

$$u(x) = \exp[i \frac{g}{2} \tau^a \pi^a] \quad .$$

So, this is a SU(2) valued space time field, SU(2) group valued. Often one writes algebra valued fields like $A_\mu^a \tau^a$, the gauge field is algebra valued, but here we actually write group valued fields.

So, where as for hadrons or nucleons we write a doublet

$$\Psi = \begin{bmatrix} \psi_p(x) \\ \psi_n(x) \end{bmatrix} \quad ;$$

so, this u is a 2×2 matrix because its exponent of taus, so it is a SU(2) matrix which is space time field, this is a doublet representation and then the Lagrangian is to be constructed from all possible terms consistent with global SU(2) invariance, there is no gauge field and Lorentz invariance.

So, if you do this then the Lagrangian begins looks like

$$L=\frac{1}{2}\partial_{\mu}u^{+}\partial^{\mu}u+a\left(\partial_{\mu}\partial_{\nu}u^{+}\partial^{\mu}\partial^{\nu}u\right)+b\left(\partial_{\mu}u^{+}\partial^{\mu}u\right)\left(\partial_{\nu}u^{+}\partial^{\nu}u\right)+u^{+}\partial_{\mu}u\overline{\Psi}\gamma^{\mu}\Psi+\ldots\ldots$$

So, you begin to develop all kinds of terms which are all consistent with SU(2) global symmetry and of course, contract Lorentz indices.

So, you can have quite a while Lagrangian like this which will not fit into this kind of simple form and in that case you do not expect to derive an effective potential because all kinds of derivatives appear, this is called chiral Lagrangian. Reason for chiral is long to explain that has to do with this hypothesis, this was all figured out by well Weinberg first but he did not put it in the formal way. And by Callan Coleman, Weiss and Zunino; so Weinberg pointed out that you have to write all possible terms and then gave prescription, but these people then figured out that what it amount it two was doing this ok. So, it is not always that you will have this you can have all kinds of Weyl Lagrangian, but you can still define effective action there is nothing against defining an effective action except that you have done have to include the higher derivatives as well with a front factors containing local products of fields ok. So, the method will remain valid, but this expansion will be completely useless because there are all kinds of nonlocal pieces you can then do a derivative expansion which is local.

So, before this approach was developed people were using what is called current algebra. So, because what you do observer is there is some kind of conserve charge I mean baryon number is conserved and isospin was conserved. So, you can write the currents

$$j_{\mu}^{a}=\overline{\Psi}\tau^{a}\gamma_{\mu}\Psi\quad.$$

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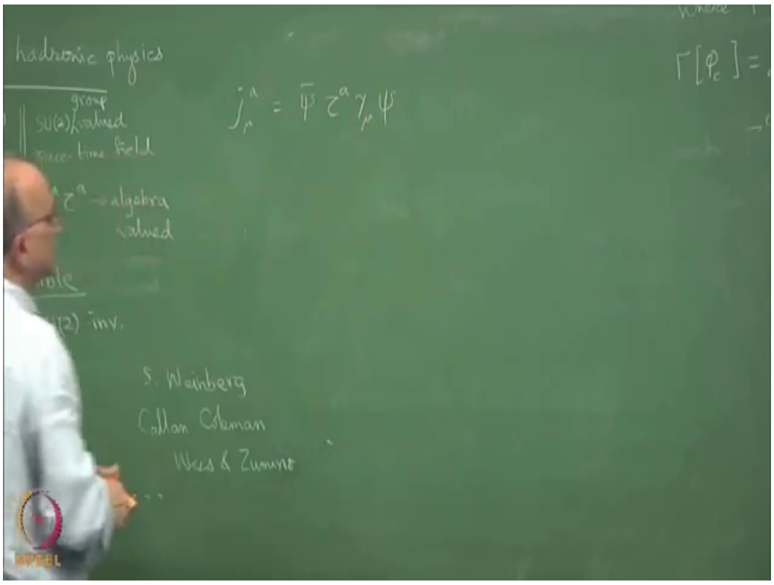

So, these currents would obey the algebra of SU(2) and you have to treat them as quantum fields. So, their local products are not well defined, then you can derive various relationships, but that was a very complicated method and this at least recast the theory into the theory of a Lagrangian.

I think next time I will try to do, one we said was try to prove this connected diagrams proper definition and why the exponential of connected diagrams to generates all possible diagrams or Green’s functions further and maybe the Goldstone theorem.

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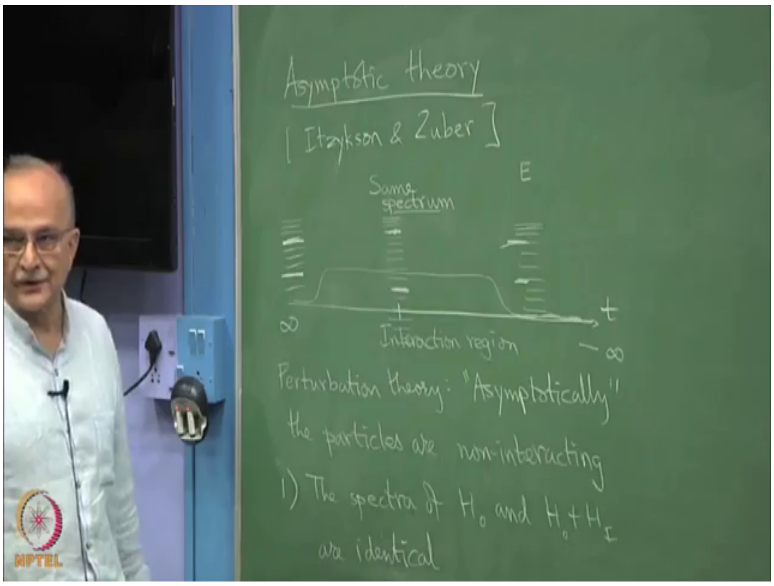

Today I just wanted to talk a little bit about Asymptotic theory and this is based on Itzykson and Zuber. So, the main kind of puzzle that peoples still date face is really what is meant by continuum was developed by Cantor and others in 19th century and that first even within mathematical community there are a lot of reaction to that, but then the actually realize the value of you know nailling down what is meant by the continuum.

But when quantum mechanisms came and to be described in unbounded space it really caused a lot of problem to understand everything and the problem still persist in a sense, but we kind of regulate them. So, the problem is like this. So, let us try to draw the energy spectrum, spectrum just means set of Eigen values and actually it is a continuum. So, drawing this is only symbolic it is a genuine continuum, but there are levels like this and we expect that the theory is free.

So, the whole assumption of perturbation theory is the particles are free, non interacting.

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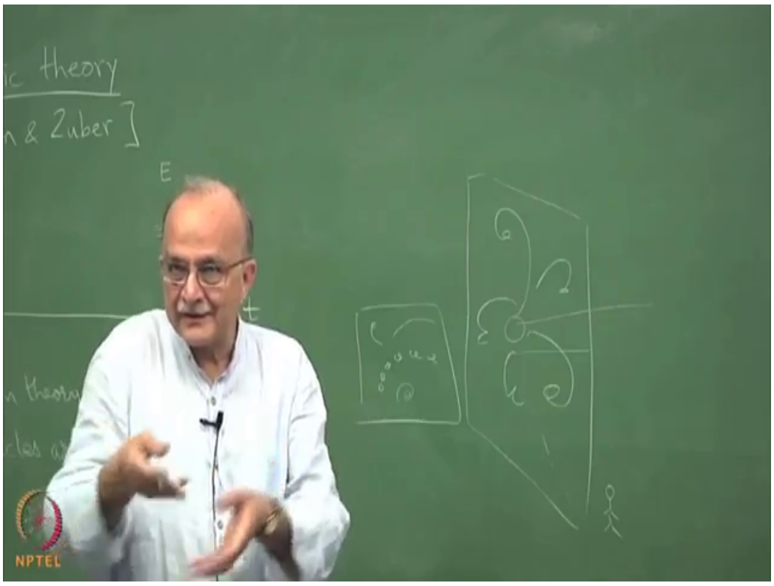

And if you really force me to say what this asymptotically means well finally, ultimately in the days of cosmic rays they use to observe this bubble chamber diagrams right, something crosses your plate, the some emulsion plate and you can see all kinds of nice curves if you see old cosmic ray plates. That is what a particle is ultimately that is what it means. It produces pretty consistent tracks and the story has not changed till date except that today we have gigantic detectors with a beam pipe running through them and human beings are something like this and they produce huge tracks. So, what is meant by particle is that.

So, what condense matter people tell you are electrons are not, they are really quasi particles and nobody isolated in electron from it just something you apply some voltage and something happens you do not know what is flowing inside, but yeah there is a good description in terms of almost free quanta which are fermionic and its a gauge invariance which enforces that they still have to carry unit charge. So, those quasi particles looks just like electrons, but the real electrons are observed in this emulsion they are just tracks.

So, we do know that there are these tracks and we do know that gluons and quarks do not produce such tracks. So, gluons and quarks do not exist in asymptotic states. So, the asymptotically there are particles and they are non interacting, but then you also want interaction because they have to do something, so in the region near $t=0$ interaction terms on.

So, in elementary quantum mechanics you know you are done Fermi is golden rule and all that. So, they say well you introduce a kind of theta function which turns on has a long plato and switches off, but the off part is going to be ∞. So, this is where it turns on and then turns off of course, nature does not wait to turn things on and off. So, what do we mean by this?

So, then people after lot of thinking have come up with the following prescriptions that keep our sanity and allows to use the mathematics we know. First is that the spectrum remains the same, this is a very big assumption. So, I send in particles they interact and why they are interacting of course, they have some interaction energies and, so on.

What will happen is that of one free particle and another free particle they may become something else here and develop some interaction energy, but the spectrum is the same, the list of eigenvalues is the same although they will occupy different entries in the list and then come back because ultimately they are again still electron and muon or whatever there. So, they will come back as free particles, in the intermediate region they will kicked off here and there. So hypothesis one is that the spectra of H_0 and H_0+H_I are identical.

Now, for one thing this precludes any bound states, because bound state would mean that there are some states below E=0. So, it actually does not allow you to have bounced states.

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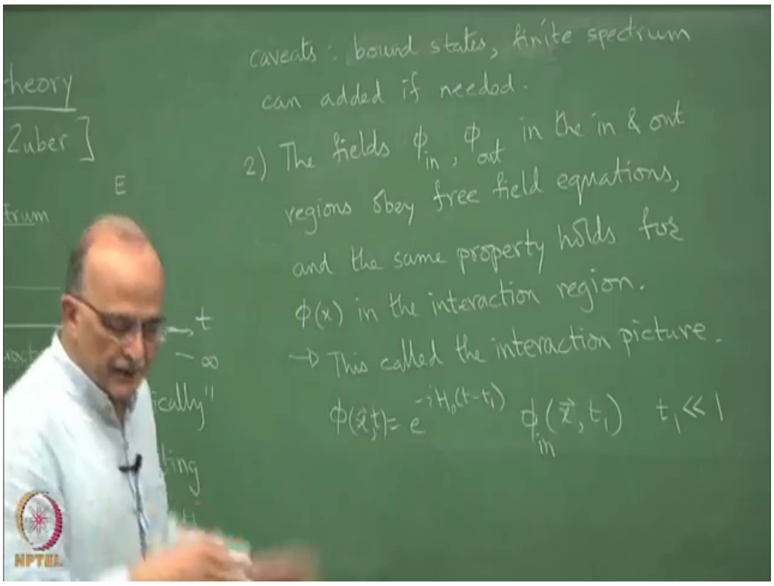

So, you add a caveats that bound states if any should be added by hand, which would be some finite spectrum can be added by hand, but it does not change the structure of the overall spectrum is just kind of a few things added at the bottom and you never really are able to calculate it, so it does not matter. So, but at the same time you want the field in this region to still base the free field.

So, the fields φ_{in}, φ_{out} in the in and out regions obey the free field equations and the same property holds for φ in the interaction region, this you know this we use already in quantum mechanics II because this is called the interaction picture. We evolve $\varphi(x,t) = \exp[-iH_0(t-t_1)]\ \varphi(x_1,t_1)$ and this could be at in the in region with t_1 vary with large and negative.

Now, because of this fact you can see beginning to see the contradiction because we want the asymptotic fields to obey canonical commutation relations which are equivalent to this evolution, but really the field which is in the interaction region has to obey this, but it has to somehow carry more information. So, the resolution of this is to propose that actually there is a normalization factor between the two.

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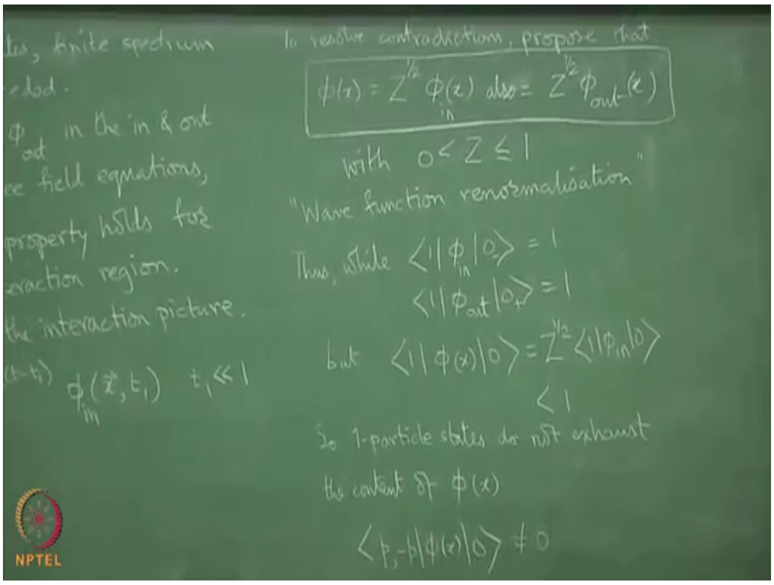

So, propose that $\varphi(x) = Z^{1/2}\,\varphi_{in}(x) = Z^{1/2}\,\varphi_{out}(x)$ with $0 < Z \leq 1$.

So, this is sometimes called wave function renormalization. What this means is that thus while see what is the effect of acting with φ on vacuum, φ has the creation and destruction operator. So, the destruction operator acting on 0 will give 0, the creation operator will connected to a 1 particle state and will be a exactly equal to 1.

So, these are just 1 because they produce one particle state, but $\langle 1|\varphi|0\rangle = Z^{1/2}\langle 1|\varphi_{in}|0\rangle < 1$. This means that phi has more content than just 1 particle states, it contains all the pair productions and things like that which are not terribly a part of the interaction, but their sort of kinematic redefinition of the vacuum ok.

So, I am now actually borrowing words from Itzykson and Zuber, 1 particle states do not exhaust the content of $\varphi(x)$. So, for example, you might get away with $\langle p, -p|\varphi(x)|0\rangle \neq 0$. So, conserve all charges moment everything that you have to conserve in the vacuum, but such states may be contributing to the actual φ.

Secondly, to avoid the contradiction with this we also have to propose that this statement is not an operator statement ok.

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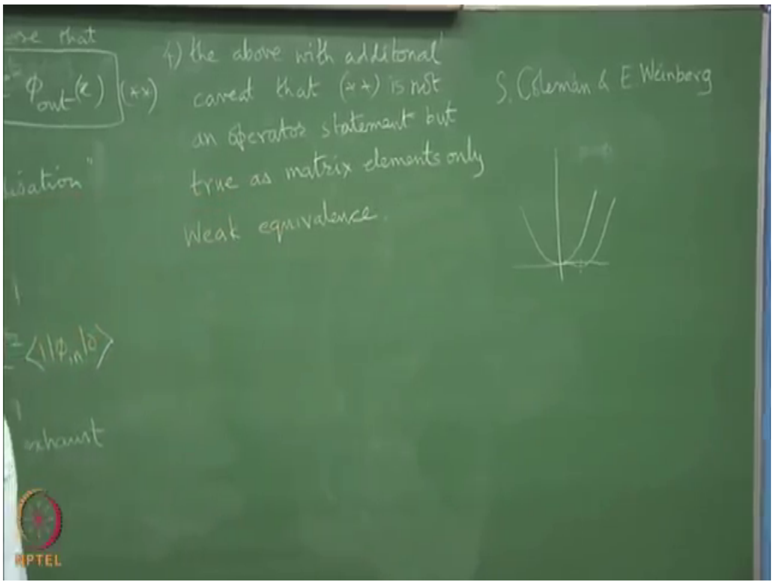

So, this is called weak equivalence, not an operator statement, it is true only by matrix element by matrix element, but not true for the whole operator.

Technically you have to keep the vacuum different because all you can do is that you see free particles in the in region you see free particles in the out region, but you have passed through this region of interaction and states in quantum mechanics are defined only up to an overall phase. So, you do not know what overall phase it may have picked up in going from in to out.

So, people technically keep it like that for most part it does not matter, but there are interesting experiments where for example, vacuum is not really stable and things like that. So, then actually even the out state is not true vacuum either, but there can be phases like this. In fact, one famous example is Barry’s phase, it does not have to do with asymptotic theory, but that is where you evolve a system through a series of parametrically change Hamiltonian.

You come back to the same value of the Hamiltonian, but the two states will differ by a phase, if there is somehow a topological obstruction for shrinking that path. So, the point is all though the initial and final Hamiltonians look identical, if you have either got a space or time region in between, so that you do not have way of directly comparing, then you should leave a relative phase between them.

So, if you send the things through LHC, do not expect that that electron is same as that one may have got a phase in it ok. So, the next steps how to do are rather technical. So, we will stopped today with just writing out this thing which you can take as homework which is as the preparation for the next main statement. So, what I do want to do is cover what is called the Kallen Lehmann representation.

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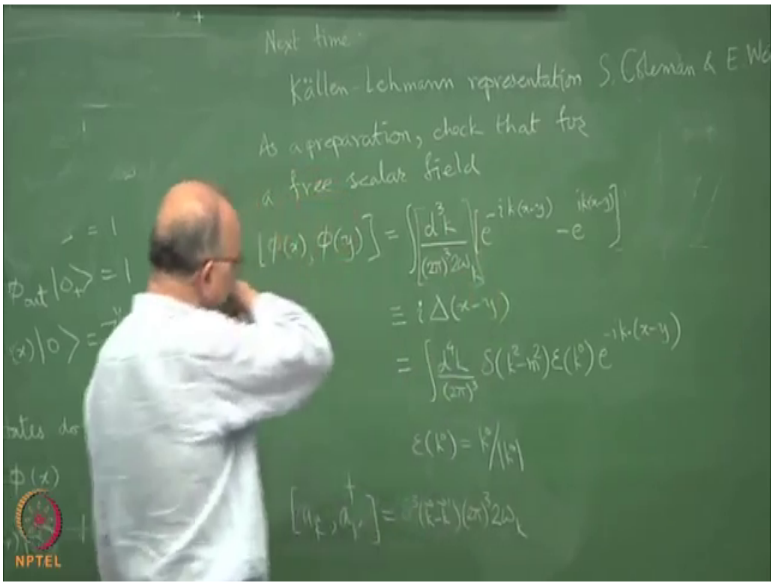

So, but as a preparation check that for a free field,

$$[\phi(x),\phi(y)]=\int \frac{d^3k}{(2\pi)^3(2\omega)}[\exp[-ik(x-y)]-\exp[ik(x-y)]]\equiv i\Delta(x-y)$$

Also the commutator can be written as,

$$\int \frac{d^4k}{(2\pi)^3}\delta(k^2-m^2)\epsilon(k^0)\exp[-ik.(x-y)]$$

So, you can try to do this, basically you know that delta function is when it has a polynomial inside is product of delta functions at the variants 0's divided by the norm of the derivative of the function at that point. So, you can carry out the dk^0 integration and you should recover the thing above ok. And nobody can change the canonical relations

$$[a_k, a_k^{+}]=\delta^3(k-K')(2\pi)^3 2\omega_k \quad .$$

So, if you do not put any other normalization then this is just like harmonic oscillator, it will just raise and lower the number operator, but if you put this then there are some advantages, what Itzykson and Zuber like about this is that this quantity together is Lorentz covariant.

So, their creation destruction operators are also Lorentz covariant and minor advantages nothing very great, but this is the normalization they use. So, in that normalization this will come out like this. So, you have to do a free field expansion also with

$$\int \frac{d^3k}{(2\pi)^3(2\omega)}$$

etcetera, then the a, a^+ you use this then you should get that.

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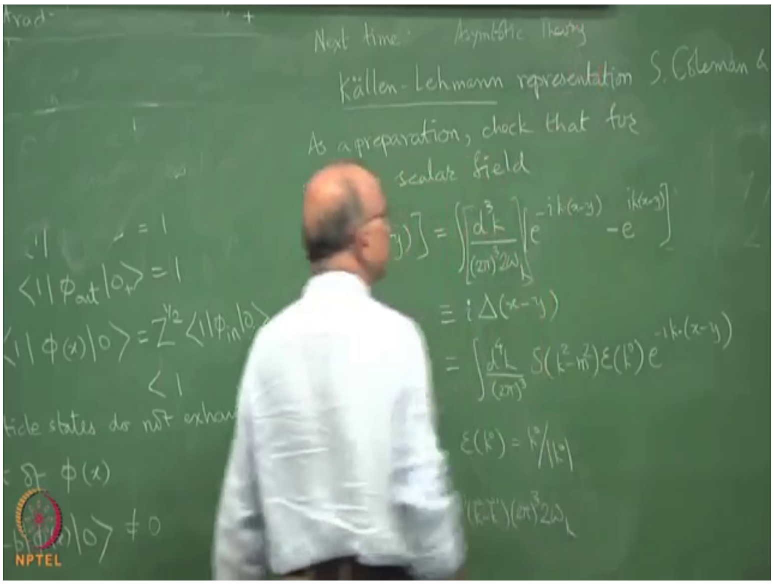

So, we will start with this next time, and yes if you would like to be prepared, then you use read this section from Itzykson and Zuber in the chapter called asymptotic theory which is in the chapter called the effect of external fields.