Theory of Group for Physics Applications
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Lecture - 47
Fundamental Symmetries of Physics - I

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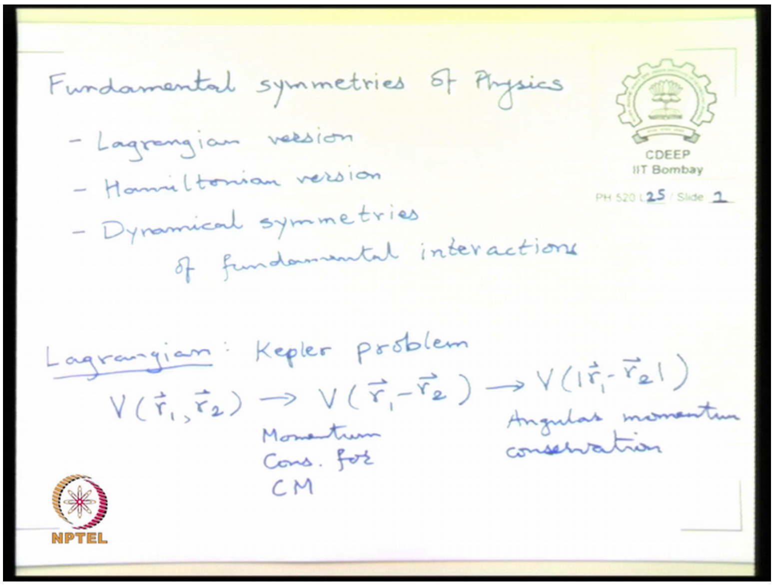

So, called Dynamical symmetries of the fundamental interactions so, in the lagrangian version what happens usually that; you discover and the standard example is Kepler problem, here the symmetry arises from the fact that the force is purely central. So, V of r 1 r 2 reduces to V of V 1 minus r 2 and further it is actually V only of modulus of r 1 minus r 2.

So, what happens is that when V is function of only the difference it needs to momentum conservation which is mostly further center of mars? So, it does not give you much detail of the problem just say is the whole, but it is an important outcome of the fact that it depends only on this, but this leads to angular momentum conservation.

So, as a result the symmetry of the problem that it depends on certain coordinates leads to conserved quantities.

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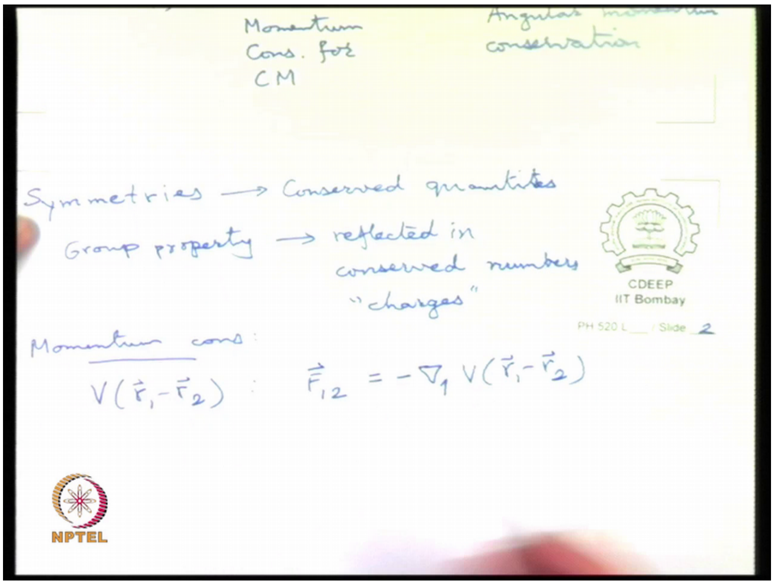

And whatever the group of symmetries, that is reflected in the conserved quantum numbers or sometimes we call them charges. So, for example, the conserved quantity angular momentum their components will mix the same way that the group property dictates. I am not going to into too much detail of proving things because we want to say some more things, but the first thing we can check very quickly this statement of momentum conservation, because it is such a cube little proof.

So, once we that V s function only of r 1 minus r 2, that implies that F 1 2 which we might say is the force on particle 1 due to particle 2. So, we call this minus gradient with respect to coordinate 1 of partic of r 1 minus r 2.

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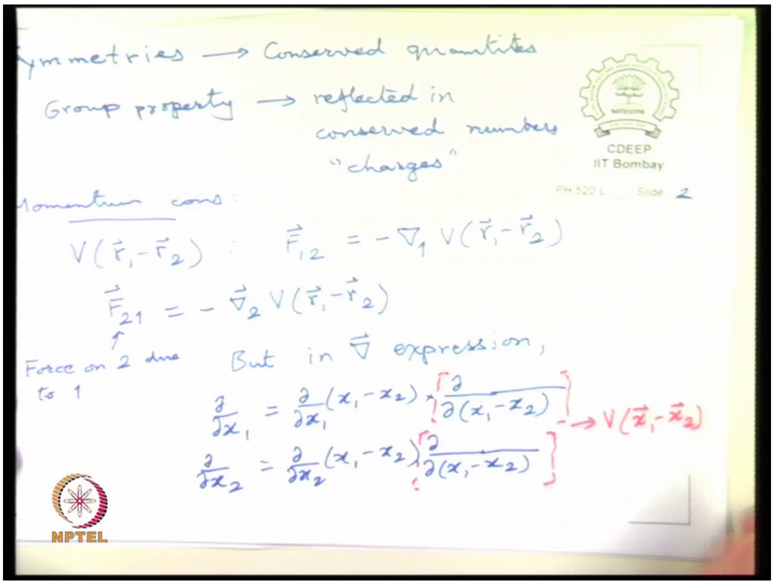

Whereas F 2 1 force and particle 2 due to particle 1, would be equal to the gradient at the location of 2, but clearly we can write the in gradient expression wherever I have something like d by d x 1 I can write it as equal to d by d x 1 of x 1 minus x 2 times d by d x 1 minus x 2 right by chain rule, but similarly I can write d by d x 2 to be equal to d by d x 2 of x 1 minus x 2 partial x 1 minus x 2.

Now, what happens is that this operator and actually (Refer Time: 06:32) writing the detail of a proof, this part is the same in both this goes and act on V of x 1 minus x 2 right. So, it is going to give the same thing in both V is intrinsically function of x 1 minus x 2. And, this gradient is with respect to that coordinate, but these 2 have opposite sign d by d x 1 of x 1 minus x 2 is minus of d by d x 2. So, that makes F 1 2 equal to minus F 2 1.

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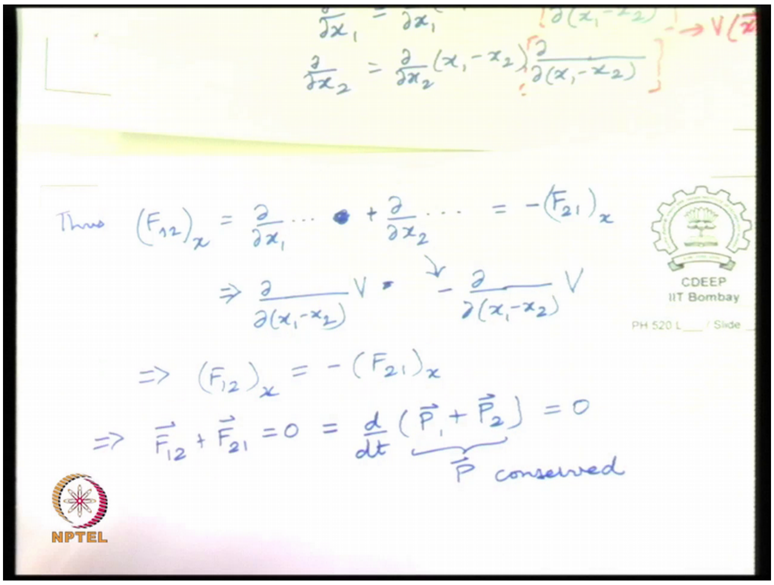

So, basically this gives a plus sign and I think what we should write is the detail of it is that this is just a plus sign and this is just a minus sign ok. So, it is not as if this is so, let me just keep this sign same, but this is equal to minus sign x 2 V whereas so, let us leave this out for the time being.

Whereas, this is equal to minus of d by d x 1 minus x 2 of V and so, that implies that F 1 2 of x x component is same as F 2 1 x component or we have the so, called Newton's third law that F 1 2 is equal to minus F 2 1, but that also means that d by d t of P 1 plus P 2, because F cause F 1 1 causes rate of change of momentum of 1 F 2 causes rate of change of momentum of 2. So, this is also equal to 0. So, we get that this total momentum is conserved.

Now, newton imposed this by hand saying that that and that is how we you can think of it in reverse because Newton's third if Newton's third law holds then it should be like this, but this is not always necessarily true Newton overreach a bit. Actually Newton's laws are not like Euclid's laws their mod like I think the real principle is the second law and the third law is essentially putting a (Refer Time: 10:04) on what the how the law is used, because otherwise the system is not closed, but we also get in electromagnetism.

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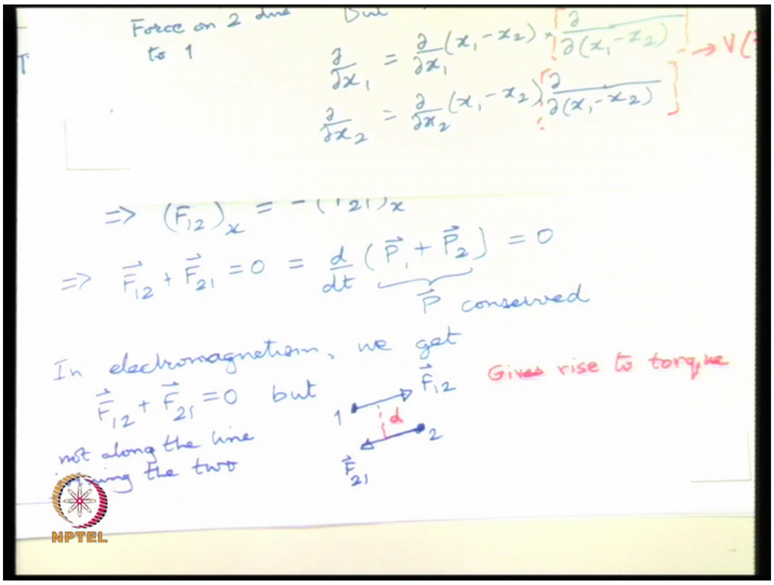

We get F 1 2 equal to plus F 2 1 equal to 0, but their sque. So, F 1 2 is like this and F 2 1 like this ok. So, they are not along the line joining the 2 points this particle is here, this particle is here.

So, it causes a torque so, but not along the line joining the tube ok. So, give rise to torque and thank god for it because that is all the motors run because if really it was so, rigid that it could not be sque like this. So, there is a separation between the 2 d and torque is you know the cup torque and couples are calculated from that. So, this is 2 this is 3, but the point is that when it is when both are satisfied in the more simpler problem.

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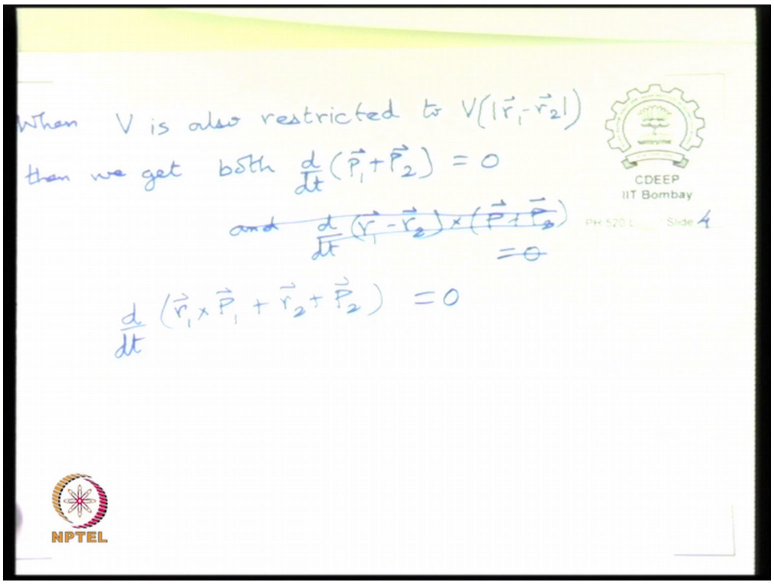

When V is also restricted such by the way this has to do it Biot-Savart's Law, it is not going to happen in coulombs law it is only when you have the currents and mutual so, you can check it and it is there in Jacksons book. Now when we is also restricted to being V of the modulus of r 1 minus r 2, that is a real restriction now it depends only on separation not on the vectorial separation. Then we get in addition to both d by d t of P 1 plus P 2 equal to 0 and that d by d t of r 1 minus r 2 cross P 1 plus P 2 equal to 0. So, then r cross P is equal to 0 ok. So, I am sorry. So, we have to write with respect to some center.

So, r 1 so, let us not this is not correct we have to write r 1 cross P 1 plus r 2 cross P 2 the way I am think ok.

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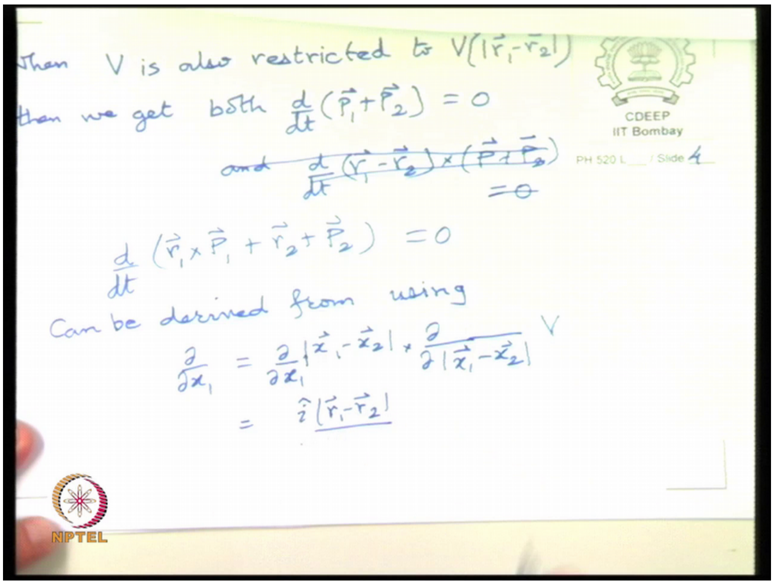

So, it is this that is equal to 0 and this can be actually derived from using the fact that d by d x 1 is going to be equal to d by d x 1 of modulus x 1 minus x 2 times d by d modulus x 1 minus x 2 of V, but this modulus derivative means that you take square root of x 1 minus x 2 etcetera. So, you will get x i cap times so, this raise to half and then you differentiate you get half this raise to 3 by 2 and eventually you are left with r 1 minus r 2 over r 1 minus r 2 cube.

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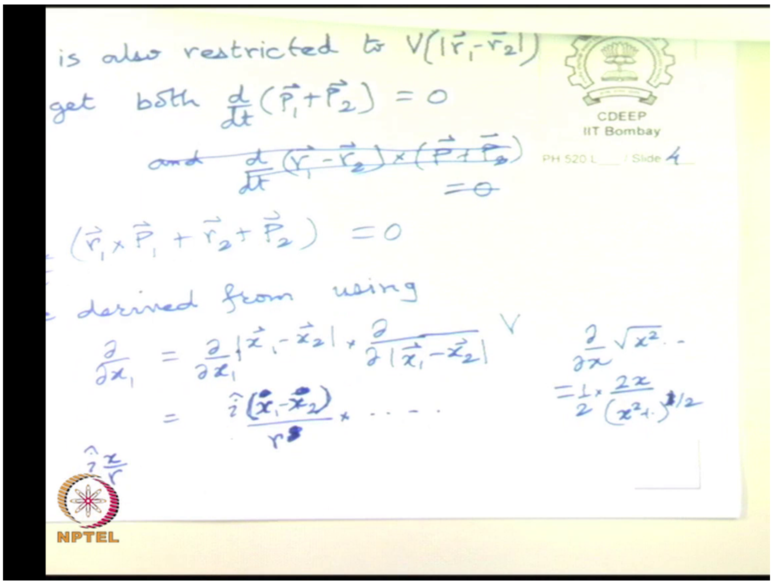

So, sorry trying to do things to first d d by dx of x square etcetera will be equal to half 2 x over and this was raise to half. So, it becomes x square etcetera raised to 3 by 2 right correct.

So, not r, but x this is what you will get and so, you get r cross the original this will have magnitude of P and combine with I you get r cross P. So, good that is right so in fact, you get you get I cap out of it, because it is it will cancel so, it will be over r. So, i cap x over r ok. So, you can check that this will work and give you angular momentum conservation.

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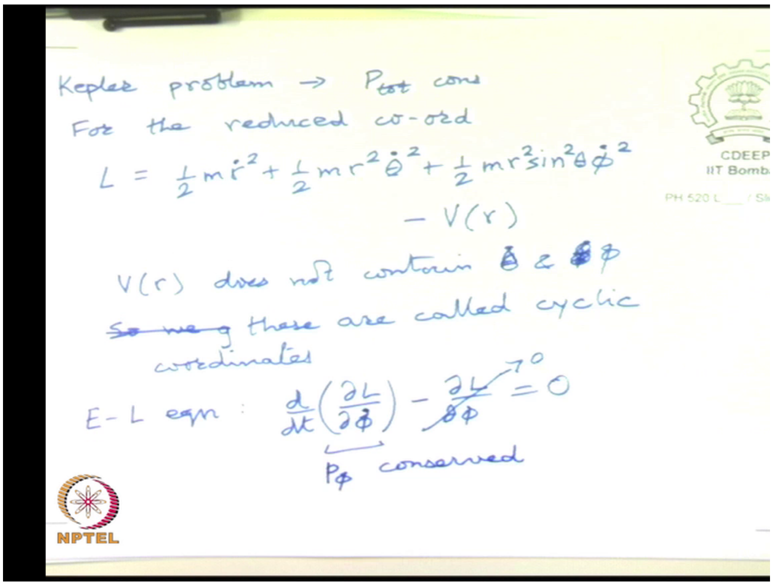

The more interesting things start when you get to Lagrangian for Kepler problem, here you write for the reduced coordinate. So, after removing the total P total is of course, concern for the reduced coordinate we get Lagrangian in spherical coordinates, because we want to write, plus V of plus 1 half m r square sign square theta phi dot square, and minus V of only r.

And, this is how the Lagrangian of a single pers single coordinates it is a reduce coordinate looks. Now, the point is that the V of r does not contain 5 dot does not contain the velocities. So, we get what are called so, these called cyclic coordinates. Because, what we find is that we if we write Euler Lagrange equation then something like d by d t of d L by d phi dot minus d L by d phi dot d L by d phi equal to 0, but this is already equal to 0. So, you find that sorry did I am very sorry I am being. So, careless do not contain theta and phi not the velocities.

So, this is then the P corresponding to phi is conserved also the P corresponding to theta is conserved, which actually allows you to choose the plane of the problem, of the relative coordinate perpendicular to total angular momentum.

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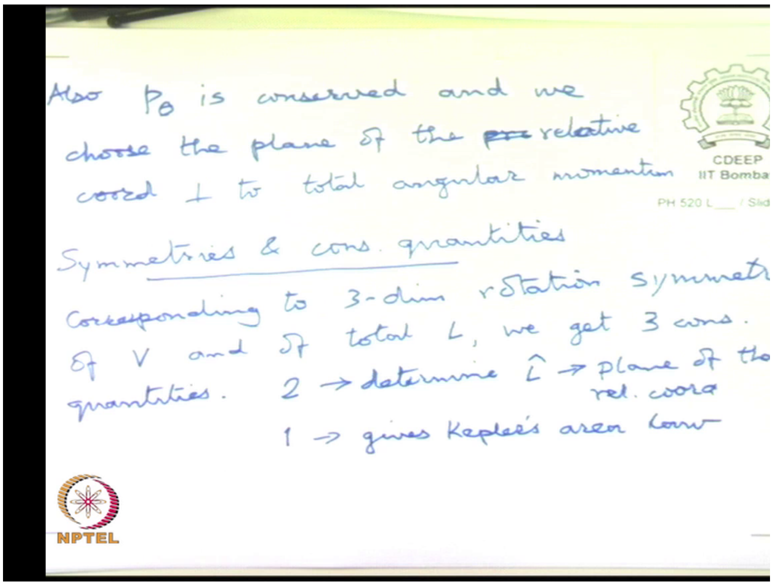

So, the P theta and P phi although we are calling them phi the generalize momenta and they are actually the components of angular momentum. So, this is how it goes? So, I want to just say that corresponding to the fact that. So, return to symmetries and conserved quantities.

So, corresponding to rotation symmetry of V and therefore, of total L, we get 3 conserved quantities 2 determine L cap, which is the plane of the coordinate because l cap unite vector as. So, fixes the plane and the magnitude determines the gives you the Kepler's area law, which is about equal area being swept out in equal time.

So, corresponding to the 3 symmetries you get 3 conserved quantities, there is additionally what is called for all dynamical systems that have independence from time you get energy conservation.

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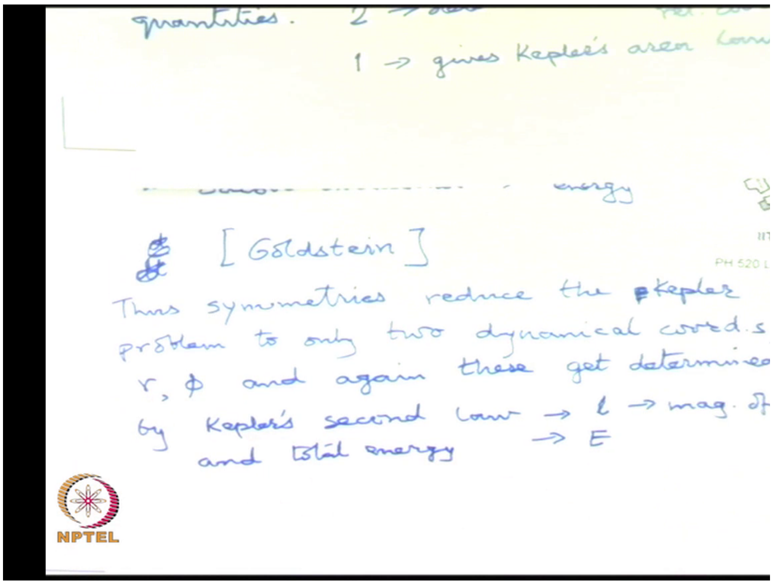

Which is plane languages energy for most of the time and the statement of the statement of Jacobian invariant is that if you do d by d t of L. So, I will not attempt a proof ok, but it is called Jacobian variant you can you will do it in classical mechanics or you can read it in it is there in Jackson in Goldstein's book as well. So, the point is that the Kepler problem then reduces to only knowing the angular momentum in the plane and the total energy.

So, which gives real angular momentum L so, the direction of L we used up in using the plane and magnitude of L is what remains and total energy, which you might call E.

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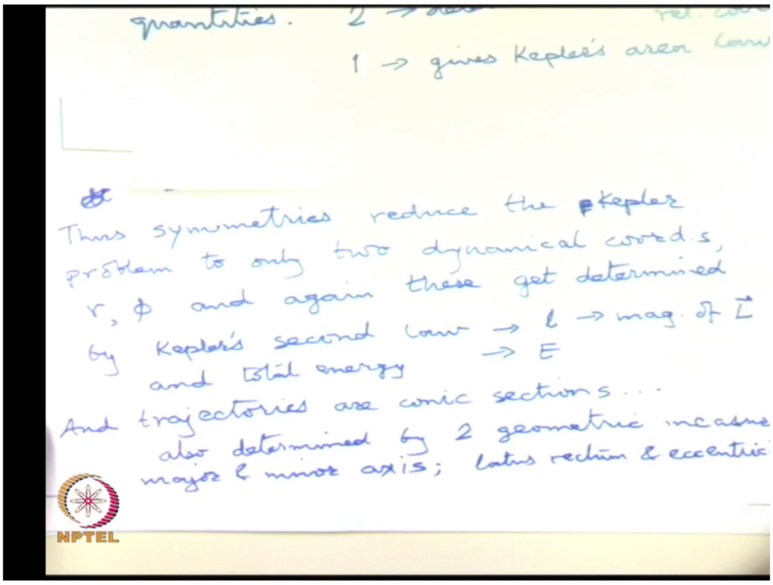

So, r n phi get traded for L and E the trajectory, [Noise] the trajectories are conic sections circle or ellipse or parabola or hyperbola, which also have 2 quantities that fix the geometry of the trajectory which is the major and minor axis or let us rectum and focus location of focus etcetera, major and minor axis are, but it is called x intercity and let us rectum ok.

So, these are the consequences eventually the whole idea form mechanics actually the advance mechanics is to you know completely formal way without referring to too much of the detail of the problem identify what are the invariants? Because, then you do not have to solve for them and then have more sophisticated methods to solve them and for the more sophisticated methods they reduce them to these oscillator problems, so, so called Hamilton Jacobi theory.

So, all of mechanics essentially tries is to do this identify invariants and for the remaining coordinates reduce it to oscillation problem called Hamilton Jacobi theory.