NPTEL

**NATIONAL PROGRAMME ON
TECHNOLOGY ENHANCED LEARNING**

IIT BOMBAY

**CDEEP
IIT BOMBAY**

Quantum Information and Computing

**Prof. D.K. Ghosh
Department of Physics IIT Bombay**

Modul No. 02

Lecture No.9

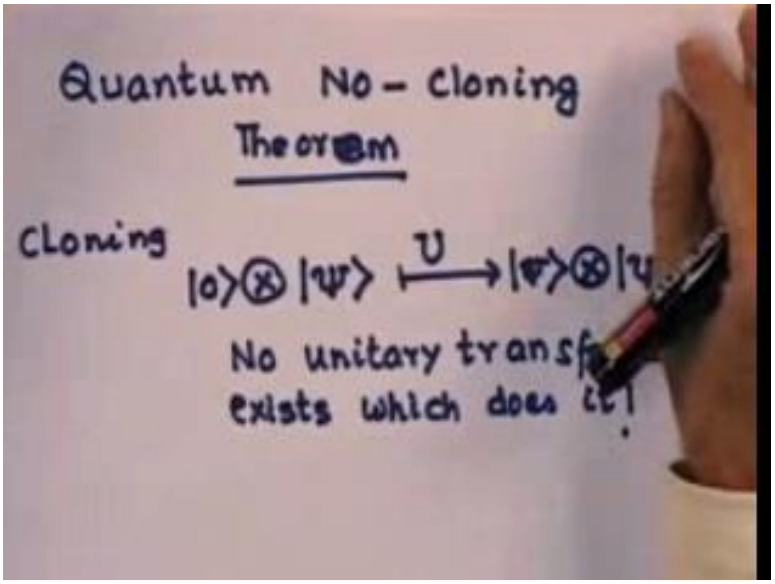
No Cloning Theorem & Teleportation

In the last lecture we introduced the idea of a quantum circuit what we said is that a quantum circuit is a representation in which we show how the quantum logic flows from left to right and it has components which are given by the gates or the operators which act on inputs or even on intermediate states. It has a very important component known as oracles which is used to compute specific functions which was as we said is like a black box computing very similar to the way, the subroutines are called for in classical computing programs.

And we also have a meter like thing which sort of tells you that you are actually making a measurement. Now what you also pointed out is that the process of measurement does not give you the output corresponding to each of the inputs which could be in a superposition of various states the information is already there before we make a measurement, but when you make a measurement out of the various possible outputs only one output would be determined with certain probability which depends upon what is the weight of each state in that.

Today we will be talking about the applications of a very different applications which will be quantum teleportation. But before we come to that let me first talk about one theorem which I have been talking about but not having proved it. And this is known as the quantum no cloning theorem.

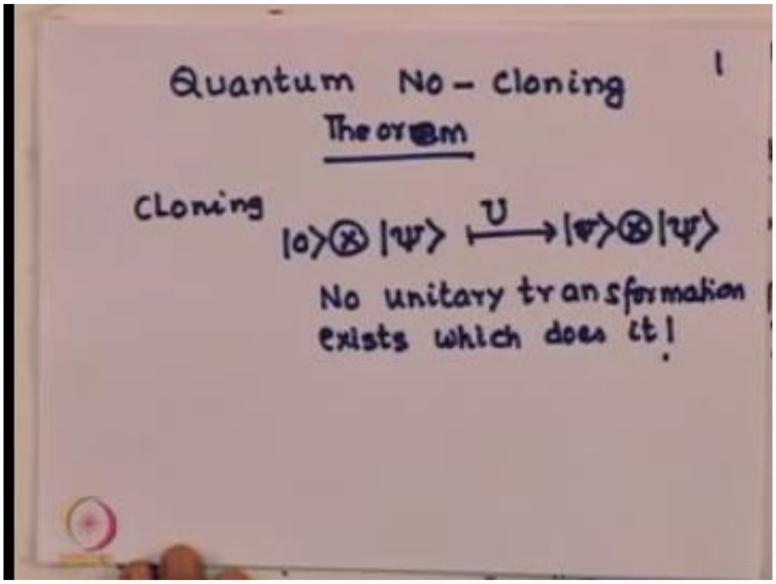
(Refer Slide Time: 02:21)



The principle is very simple the quantum no cloning theorem simply says, it is not possible to find the unitary transformation which will duplicate a given state and write it on to a blank state. Well basically what we are pointing out is that the Xeroxing of a quantum state is not possible. Now before you give the proof let us look at what the process of duplication does for example, in a Xeroxing.

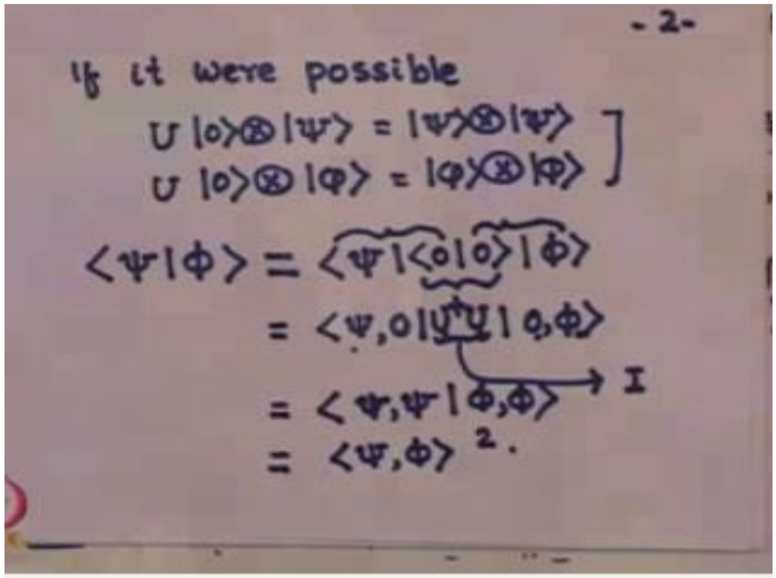
Now in a Xerox machine you start with two things, one is the original which you want to be copied and another is a blank page on which you want this copy to be made. So corresponding to the blank page I have what I will call as a standard state if it is a single key bit state or whatever I could take that as this state 0. And the original is the state ψ which I want to be copied.

(Refer Slide Time: 03:49)



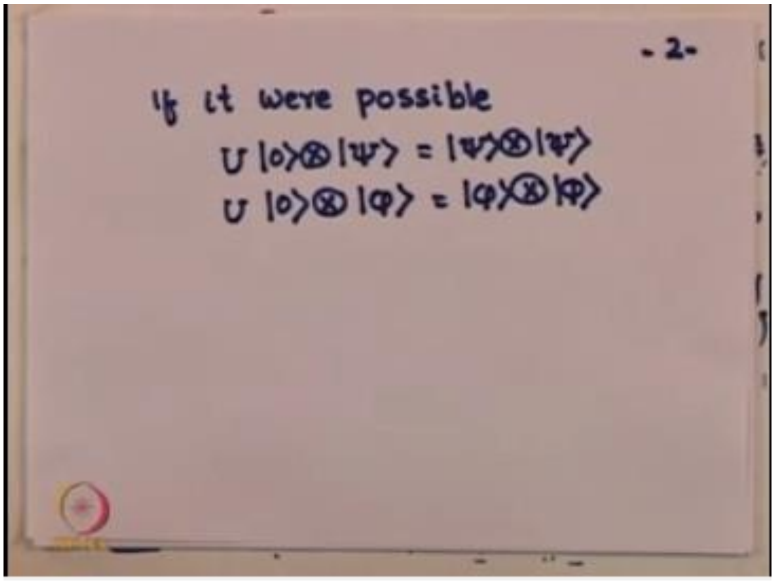
So basically a cloning would require that you take a blank state and a state ψ you find a unitary operation which gives me in place of this blank states ψ itself and of course, as you know in a Xeroxing the original is returned back. Now this is what cloning mean, now we are claiming no unitary transformation of this type exists which does it. Now let us see why not, now the proof is actually fairly trivial. So basically we will say supposing this were possible we will prove it by what is known as reductive [indiscernible][00:04:58].

(Refer Slide Time: 05:06)



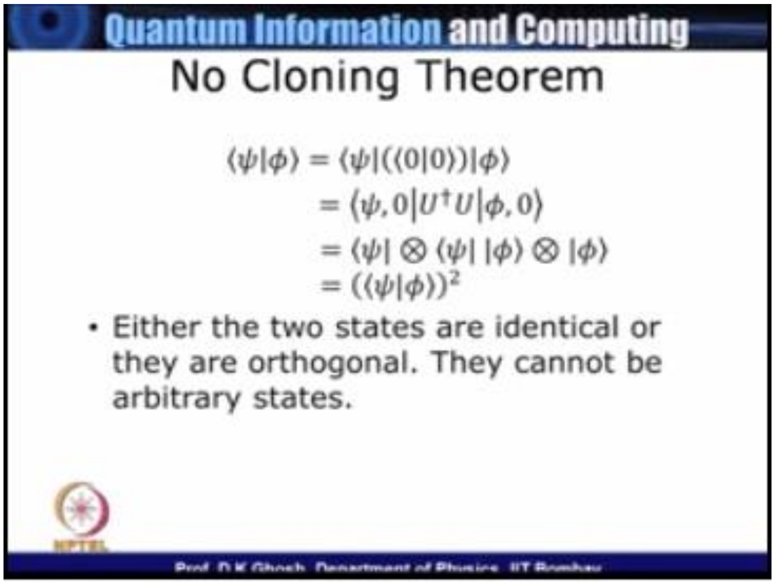
So supposing it were possible and there existed if it were possible and if there exists in you which did it, then U acting on zero direct product with ψ should give me ψψ. Now since I cannot talk about one type of gate specifically for duplicating a specific state, if such an unitary transformation exists in order that it is useful, I must have the same U duplicate a different state also. So let us suppose it duplicates some state ϕ as well.

(Refer Slide Time: 05:55)



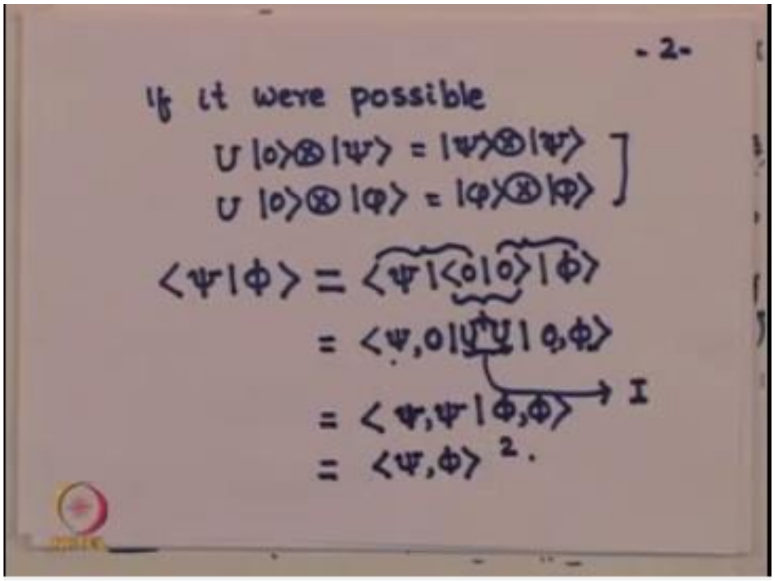
So I should get ϕ x ϕ the argument is very similar to my Xerox machine argument you do not have a separate Xerox machine for copying different originals. So if you have a Xerox machine it will copy both.

(Refer Slide Time: 06:09)



Now if this is true then notice what happens if I take the product of ψ with ϕ.

(Refer Slide Time: 06:20)

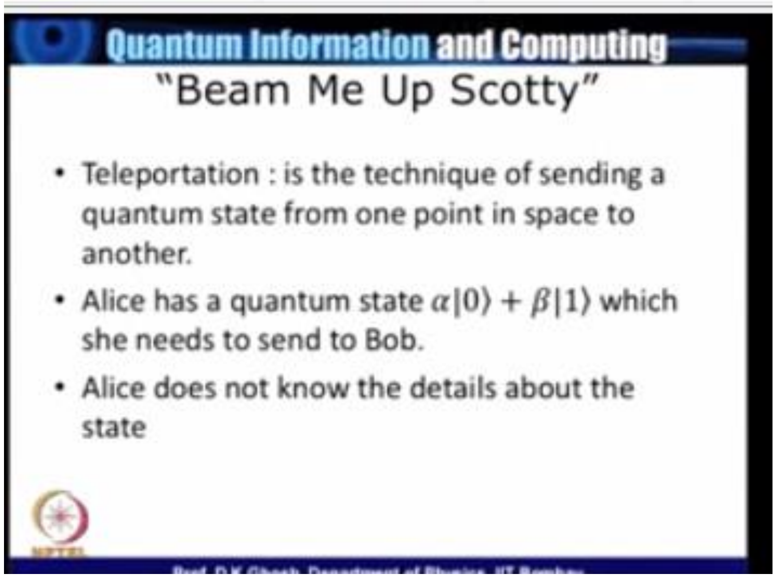


So we have seen that this then is ψ now I introduce a 00 notice that this scalar product is actually 1 so I am multiplying it with 1ϕ, but then this bra is ψ0 bra and this gate is 0ϕ gate. So therefore, this is, can be written as compactly a ψ0 okay. And of course, there is ϕ0 but let me come back to 0ϕ or ϕ0 does not matter. Now let me introduce here U^+U remember U^+U, U being an unitary matrix is identity.

So I can always introduce this, so U acts on 0ϕ U^+ acts on the bra of ψ0. So I must get this to be equal to ψψ and ϕϕ. So this is nothing, but $\psi\phi^2$, so this is what I want that this is possible this is possible only if ψ the state ψ and the state ϕ they are identical in which case both the sides we can go to on, or if ψ is orthogonal to the state ψ in which case the scalar product is 0 and 0=0, because scalar product of ψϕ equal to each square has the solution that it is either equal to 0 or equal to 1.

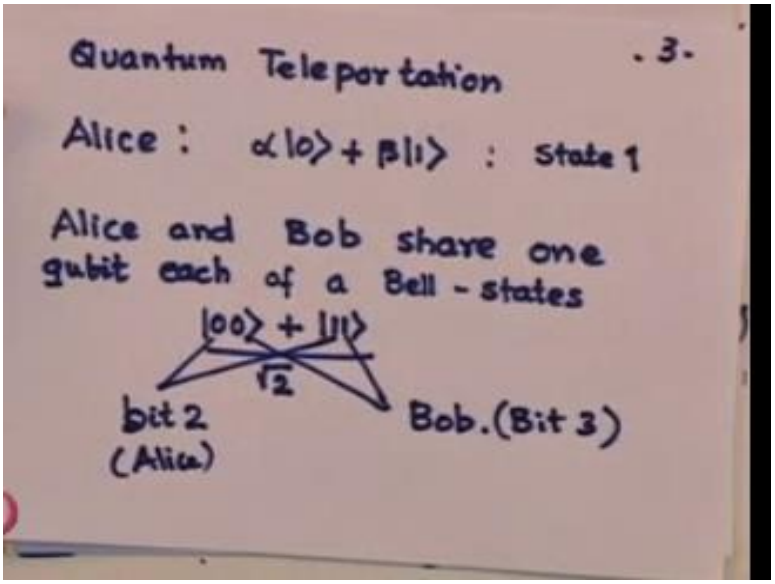
So therefore, ψ and ϕ cannot be arbitrary states. If cloning is possible if there is a machine which can clone a particular states ψ the same machine can act best clone a state which is orthogonal to it. And since we are looking for operators which would be cloning an arbitrary state this tells me that our original assertion must be wrong and cloning is not possible.

(Refer Slide Time: 09:02)

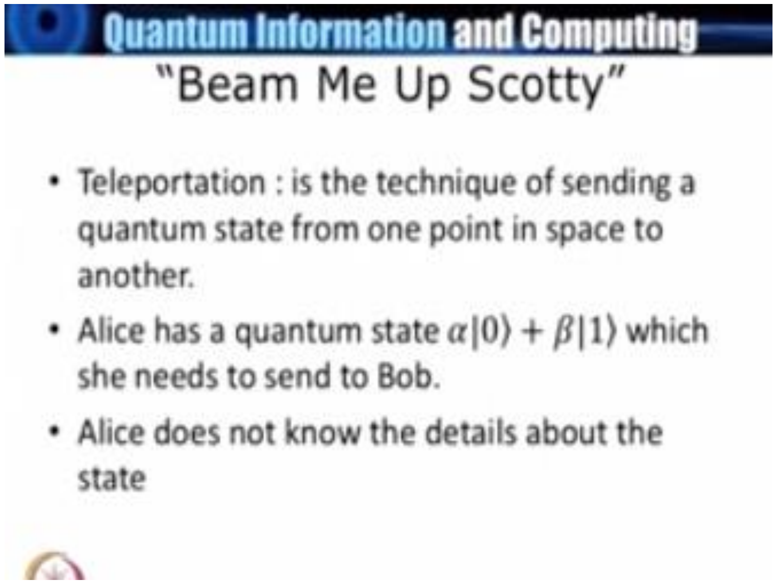


Now this is used in what is called as a quantum teleportation.

(Refer Slide Time: 09:08)



(Refer Slide Time: 09:22)

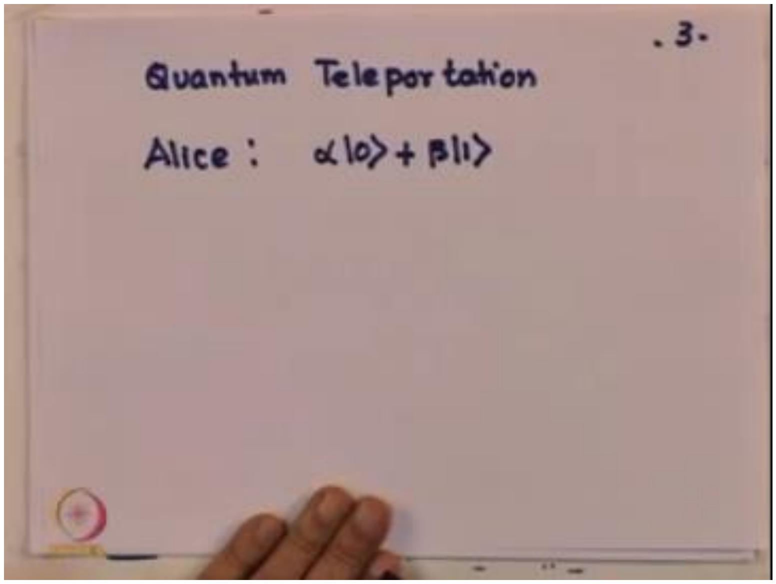


Refer to the slide. for those of you who have had at some time seen this old television serial called Star Trek you must have been familiar with this phrase "Beam me up Scotty" where the captain of the ship captain Kirk used to ask his engineer to energize him at that point and then reassemble the energized captain in another place. And this is in principle teleportation taking something from one place to another.

Now of course we are not talking about teleportation in this science friction language, but the principle is to send a quantum state from one point in space to another. Now the way we would do this is this is the pair of names which you will get accustomed as we go along in this course the, instead of just saying that I have two people one is called A another is called B in quantum computing it is traditional to give them names.

So we have a person called Alice and another person called Bob and Alice and Bob have become inseparable from the discussion of quantum computing as we will see later. Now Alice has a state which is written as, so Alice has let me state the requirement.

(Refer Slide Time: 11:00)

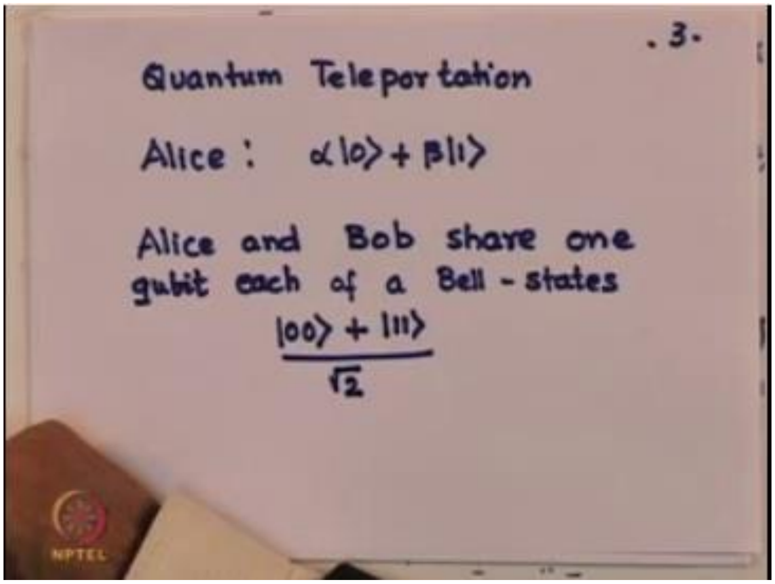


Alice has a state α 0 +β 1 and this is the state which he wants to send to Bob and Bob could be anywhere. So Bob is not at the same place so that Alice cannot simply hand over. Now notice either Alice does not know the details about this state, that is what are these α and β, so that she cannot simply call up for Bob and say that look this is α, this is β, why do not you just make a straight with this.

Then even if she knew the details, since α and β are complex numbers in order that Alice can give this precise information to Bob the requirement would be in principle that infinite number of places will be required for describing a quantity α or β, because it need not be a terminating expansion. So therefore this is not possible Alice cannot make a measurement, because if she did she would only get α or β with some probability.

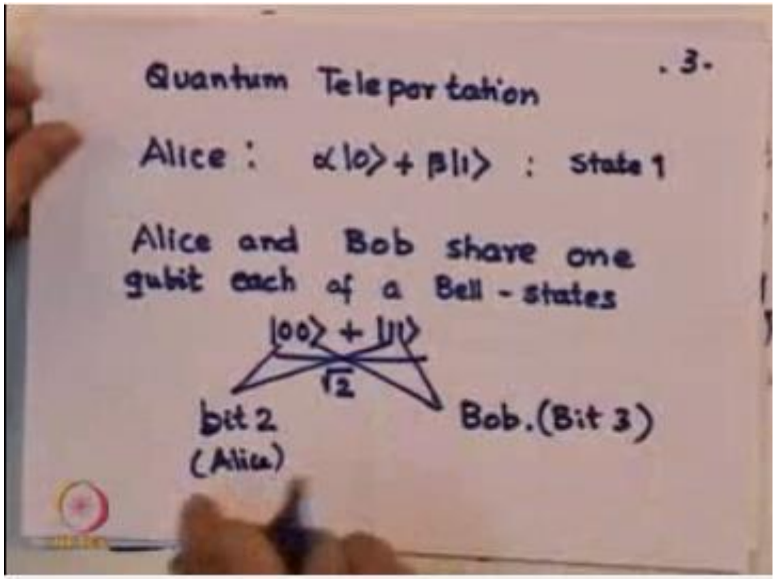
And in the process also destroy that original state, but the question is how does Alice then send that state to Bob. Now in order to, for them to do it there is some things are shaped. That Alice and Bob who have been, who have known each other for some time in the past they met and they decided to share one qubit each of a bell pair, remember that we talked about bell basis.

(Refer Slide Time: 13:13)



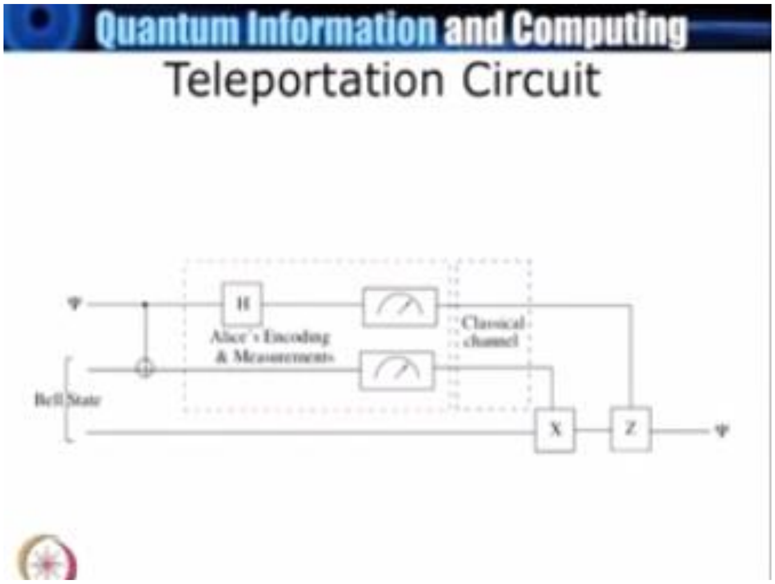
And let us be specific and say Alice and Bob share one qubit each for convenience I will assume Alice shares the first qubit, Bob has the second qubit of a bell state and like the, there are follow for bell states you can do teleportation which any one of them that you like and this state that I take here for illustration purpose is 00 + 11 by square root of 2. Now this is an entangled state, so it is not a state which you can write as a state with Alice multiplied with a state with Bob. This is an entangled state.

(Refer Slide Time: 14:08)



Now so with this notation what I will do is assume that this is Alice estate this I will call state 1, the first qubit of Alice I will call as the bit 2, so Alice has bit 1 and bit 2, and Bob has bit 3. Now let us look at how this process is carried out.

(Refer Slide Time: 14:46)

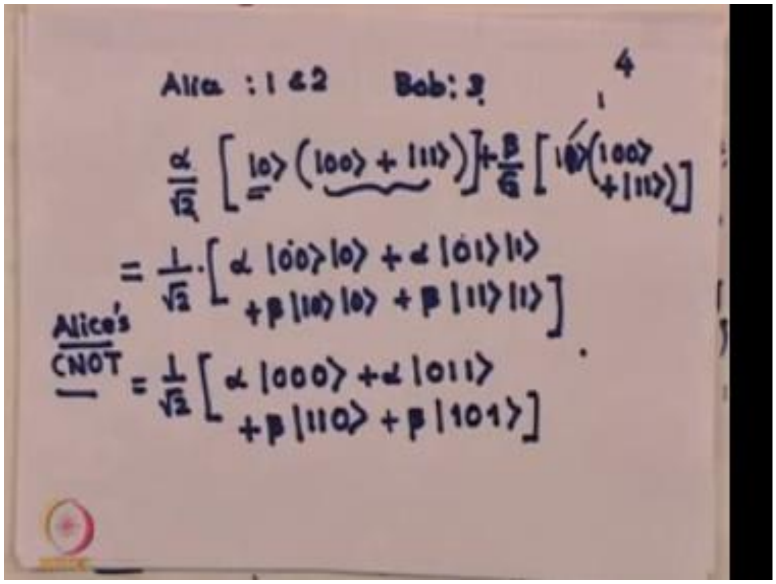


I will try to explain each part of the circuit this is a standard teleportation circuit. Let us look at that and then I will tell you how exactly it is able to do the teleportation. So notice this first understand what is happening in this circuit. So there is the state ψ which is equal to α0 + β1 which is with Alice this and this are entangled bell state and out of it the first qubit with Alice and the second qubit is with Bob, I will call this at the first, the second, and the third.

So what Alice does is to interact this ψ with her part of the bell state. Now having done that, Alice does certain operations so this is what the red rectangle here is telling you what Alice does. So Alice will pass the first qubit through Hadamard gate and then we will make a measurement.. And Alice will make a measurement of the second qubit as well, but in terms of time, the first Hadamard gate precedes the second Hadamard.

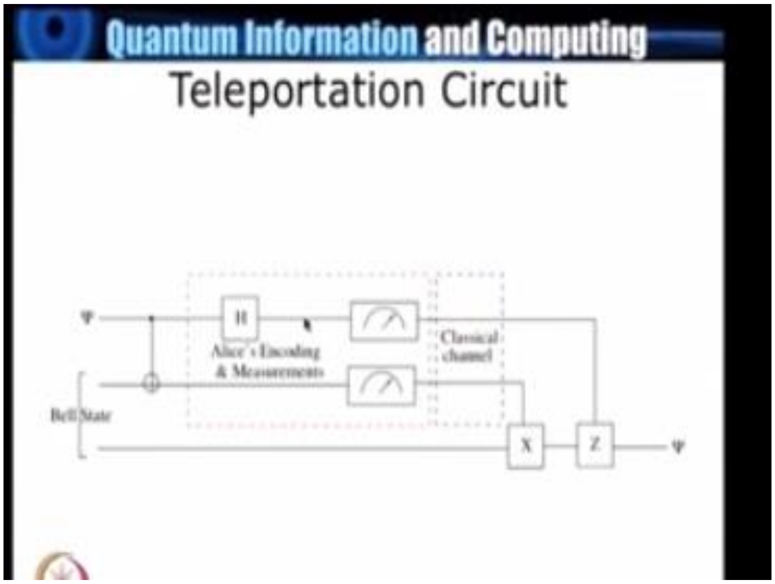
So one Hadamard gate on the first qubit and a measurement of the second first and the second qubits is what Alice does okay. So let us before coming to the second part of this operation let us try to understand what will happen. So we will write down remember that what we had we said that Alice has qubit number one and two.

(Refer Slide Time: 16:37)



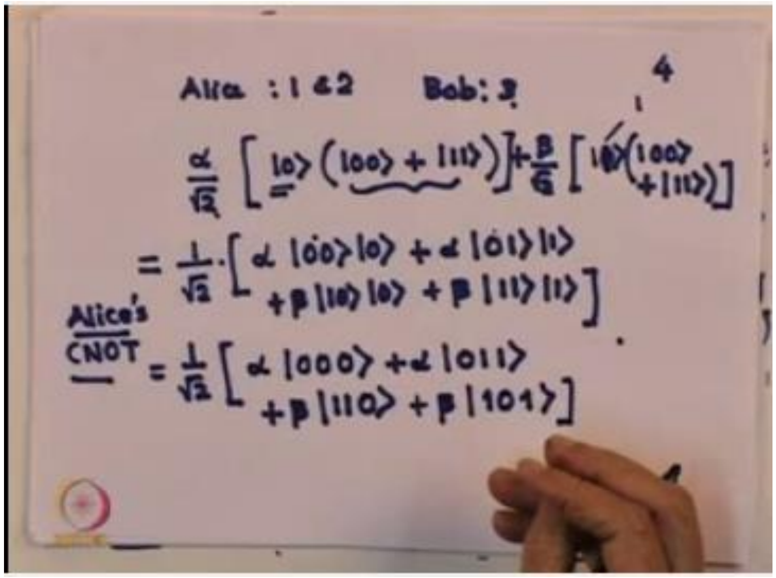
And Bob has qubit number three, so to start with what I have is the following state. We have α by root 2 I am rewriting this o let me write it down and then we will see how this has been written. There is a one wrongly written there, so look at what has happened so my original state ψ is α0 + β1 and this is the integral state 00 +11. Now I can rewrite them in a slightly different way I can write this as 1 over square root of 2 α 000 explicitly separated out Alice involved +α 0 11 + β100 + β111 okay. With this now let us go back to the circuit what is happening.

(Refer Slide Time: 18:30)



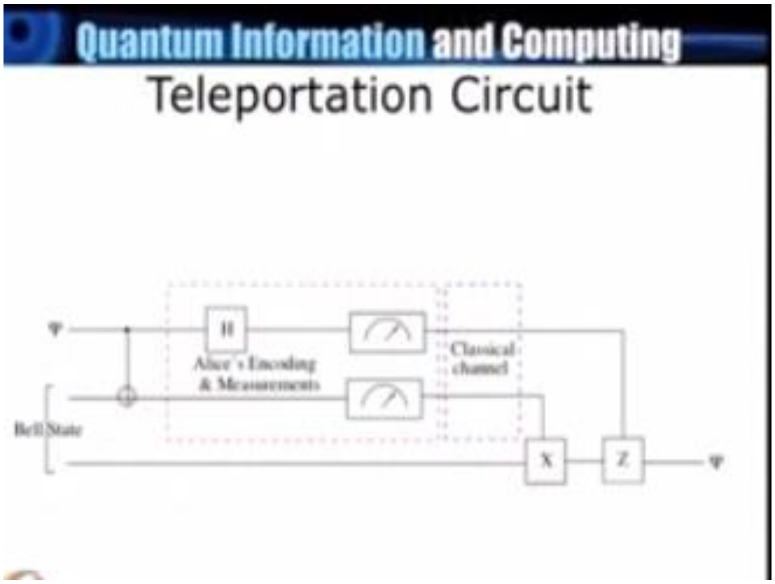
Look at the slide again so this is what we have got and the first step is Alice applies a CNOT taking the first qubit as the control and the second qubit as the target. You remember the definition of CNOT. So in the CNOT gate so Alice has CNOT.

(Refer Slide Time: 18:58)



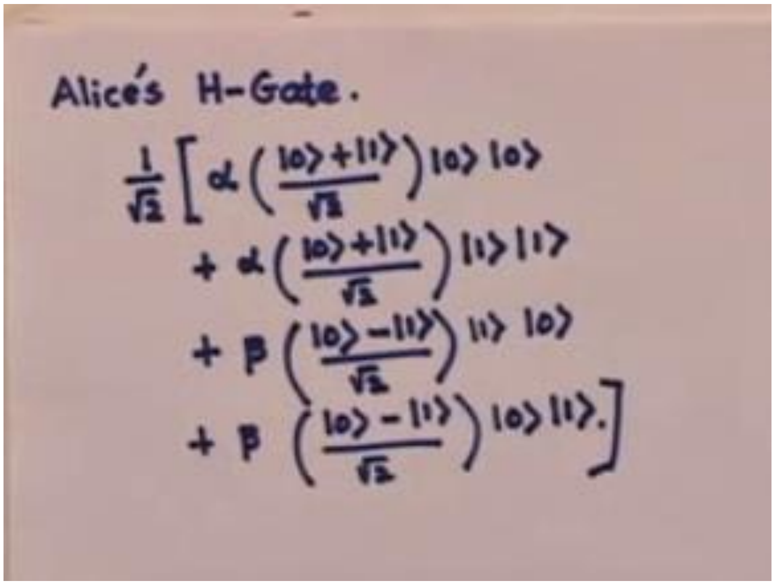
So Alice has CNOT does the following I have got 1 over root 2, so I only concentrate on Alice section, so since the control bit is 0 here nothing happens. So this state remains as it is, so does this state because here also the control bit is 0, here the control bit is 1, so this 0 will become one here again the control bit is 1 so this 1 will become 0. So how will you write it, we will write this is α let us return back to our three qubit representation this will be 000, the third qubit belongs to Bob + α011 + β110 because this is one so this device become 1 + β101 so this would be Bob.

(Refer Slide Time: 20:02)



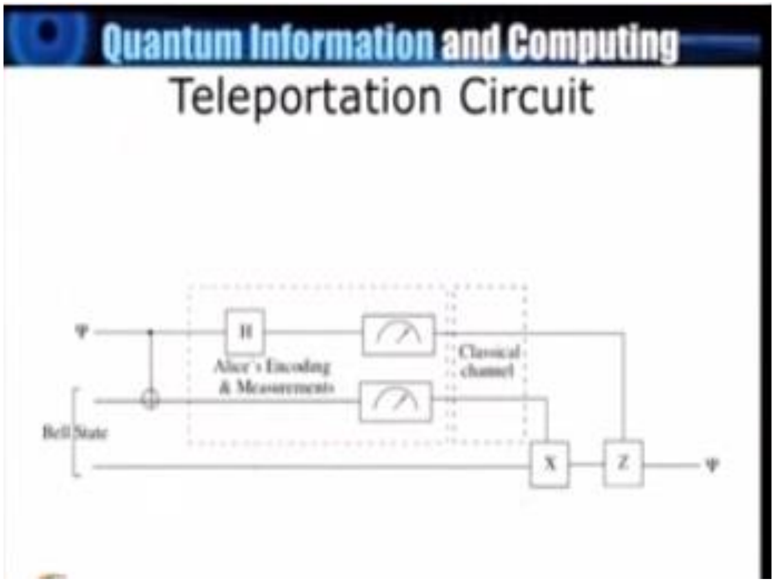
Look at the circuit again the slide again. The next step is Alice applies a Hadamard gate to her first of it. Now remember what the Hadamard gate does, the Hadamard gate converts a 0 to a 0+1 by root 2 and converts a 1 to 0 -1 by root 2. So if you look at this picture again the state now becomes a frame.

(Refer Slide Time: 20:32)



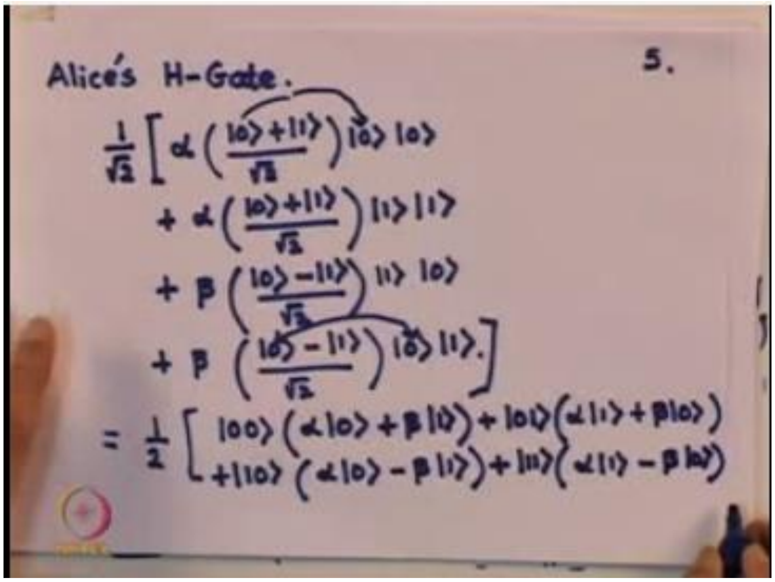
So after Alice has Hadamard gate the state will get 1 over root 2 α, Hadamard gate is applied on the first qubit. So therefore, what I would get would the, it is also 0 so I will get 0+1 by root 2, and the other two are unaffected so they were 00. The second one again is the same because the first qubit was 0 so α 0+1 by square root of 2 and this was 11+β so this was 1 so it will become 0-1 by square root of 2 and this is 10+β once again 0-1 by square root of 2 and of course this is 01.

(Refer Slide Time: 21:59)



Now what we will do is before proceeding further because now two measurements are coming up, two measurements are coming up because bit 1 and the bit 2 will have to be measured, because now the Hadamard gate has taken place. So let me rewrite this by combining terms in a particular way. So if you look at this the slides again for a moment.

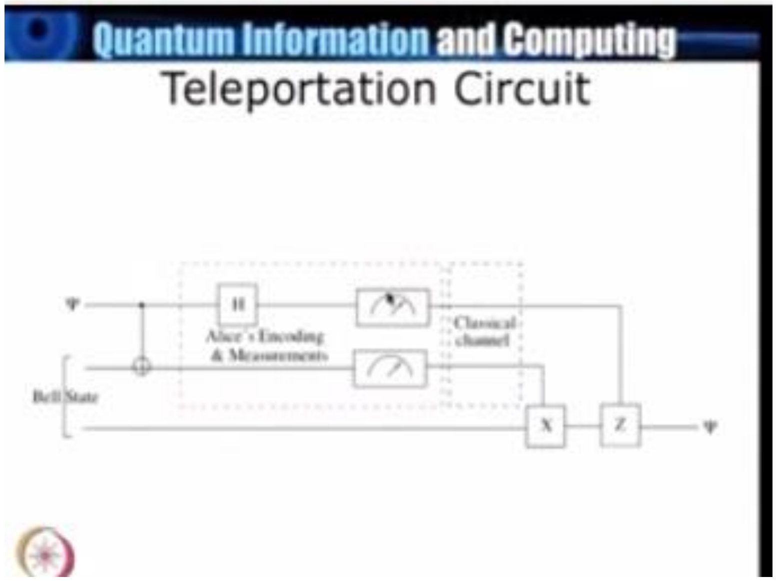
(Refer Slide Time: 22.25)



So the thing is look at this, so I have here α000 see what I am going to do is this. I have so far as Alice has concerned I have four types of state at 00, 01, 10 and 11. So I will put this thing together and there is another 00 coming up here. So I will put this + this so I can rewrite these in the following way. So let me just illustrate it here there is 1 over root 2 and 1 over root 2, so there is 1 over 2. So I get 00 this is for Alice, now look at what is here even 00 I got α0 + β1 so I will write this as α0+ β1.

And likewise you can combine the other states write it as 01 a little bit of inspection is required it is α1 + β0 + 10 with α0 - β1 +11 with α1 – β0.

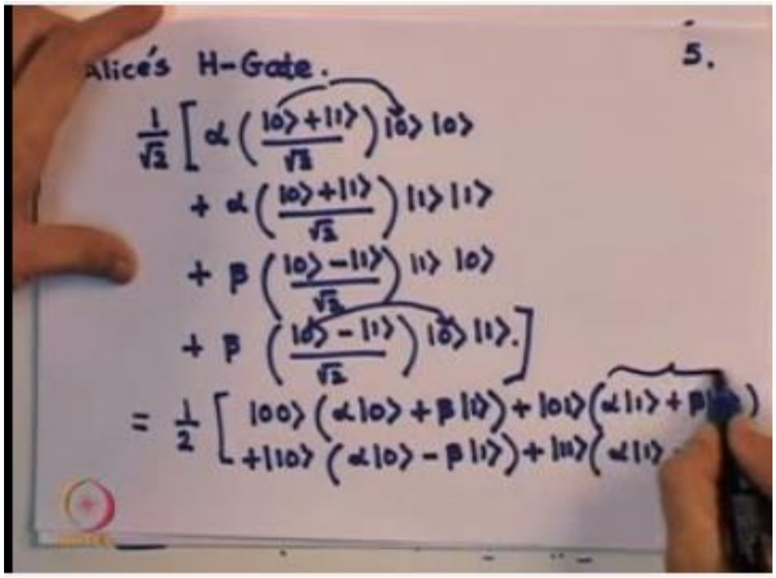
(Refer Slide Time: 23:59)



Now so if you look at the slide again I have got four different types of measurements that Alice can make. So Alice actually makes the measurement here, now when Alice makes a measurement she gets either 00, 01, 10, or 11. Now depending on what she gets, depending on what she gets she will now pick up a classical channel like a telephone and tell Bob that what did I get.

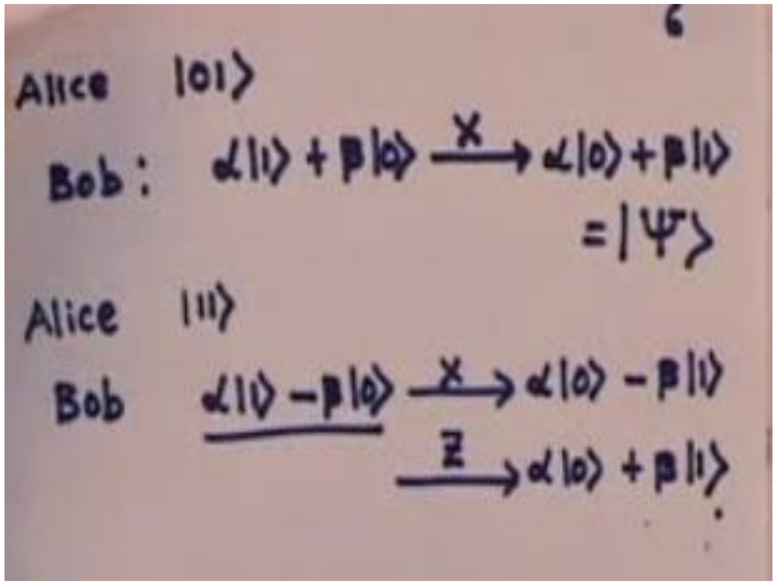
So Bob would, I insist again that this is a classical information exactly qubits of classical information that Alice is sending which is easy. So therefore, now let us look at what can we do, so come back here again.

(Refer Slide Time: 24:55)



Supposing Alice tells Bob that I got 00, now notice Bob does not have to do anything he already has α0 + β1 which is the state ψ. Now suppose Alice told Bob I have got 01, now look at this is Bob state.

(Refer Slide Time: 25:16)



So Alice has measurement gives you 01, so both state with Bob is α1 + β0 on hearing from Alice that she has got 01 Bob knows this is the state he has. So what he does is at least and he applies an X operator which will convert him to α0 + β1 which is nothing but my state ψ. Now suppose I will just illustrate once more, suppose Alice got the last one 11 then both have the following state, Bob has α1 - β0.

Now this does not quite look the same state as we started. So therefore what Bob does is to first change it to apply an X gate change it to α0 - β1 at that stage he applies a Z gate which will make it α0 + β1. And you can work out what happens the first case you do not do anything, Bob already has ψ I have given two more and the last case you can work out yourself. So at the end of this process what we have achieved is that Alice had started with a state α0 + β1 Alice and Bob both share a welfare and by doing certain measurements and operations on by Alice.

And then a classical communication channel the Bob is able to get that state that is the process of teleportation. There are two questions which we arise in this situation one is that we never said how far is Bob from Alice, in other words they could be separated by space like distances. So are we able to transmit a state with a speed greater than that of light, because that would, if that will show it would be against the principle of relativity.

Remember I said one part which is essential in this operation is the classical channel, there is a classical information which Alice must informed Bob, and so therefore that must take place with following the specialty of reality, and then finally the situation is that have we been able to copy as a thing, in other words has cloning being done. Once again the answer is if you look at what Alice has at her end after these operations are done you will find the original state she has become either 0 or 1, and Bob has simply recreated the state which Alice had from the bits that he had originally, he has now also lost the Hills part of the qubit. So this is the process of teleportation.

NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING (NPTEL)

NPTEL Principal Investigator IIT Bombay

Prof. R.K. Shevgaonkar

Head CDEEP

Prof. V.M. Gadre

Producer

Arun kalwankar

Online Editor & Digital Video Editor

Tushar Deshpande

Digital Video Cameraman & Graphic Designer

Amin B Shaikh

Jr. Technical Assistant

Vijay Kedare

Teaching Assistants

Pratik Sathe
Bhargav Sri Venkatesh M.

Sr. Web Designer

Bharati Sakpal

Research Assistant

Riya Surange

Sr. Web Designer

Bharati M. Sarang

Web Designer

Nisha Thakur

Project Attendant

Ravi Paswan
Vinayak Raut

NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING (NPTEL)