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Quantum Information and Computing

**Prof. D.K.Ghosh
Department of Physics IIT Bombay**

Modul No.02

Lecture No.7

Quantum Gates

In the last lecture we had talked about the two qubit and multi qubit states in general, and then we had proceeded to just to introduce the elements of operators which would act on these states.

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Converting them to different types of states and we had said that if my input is a single qubit and the output is a single qubit this operation has to be done by a 2 x 2 matrix or an operator corresponding to 2 x 2 matrix and we had also pointed out that there is a NOT gate which even classically is a eversible operation and obviously the corresponding quantum operation it will also be reversible.

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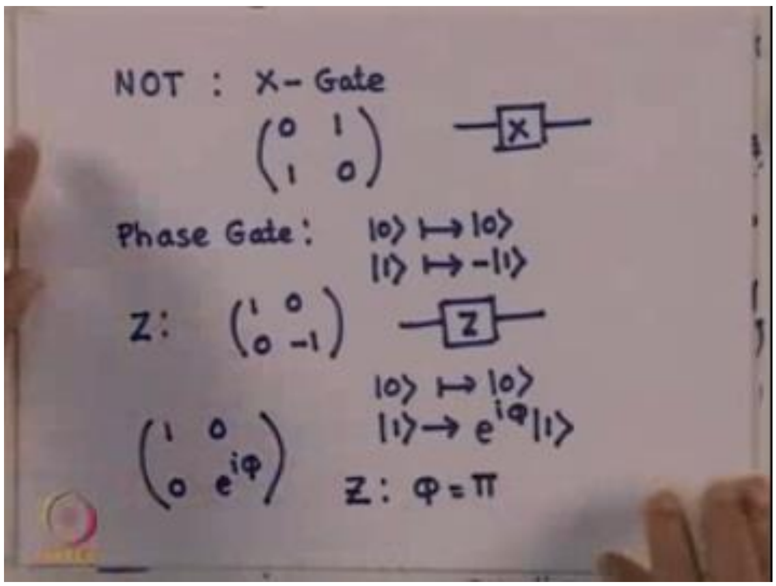

And though we had said that the NOT operation is taken care by what we call at them X-gate, an X-gate has a matrix representation which is the Pauli sigma X matrix which is 01, 10. The another one which is popularly known as a phase gate, now a phase gate what it does it something interesting it acting on a state 0 it leaves it as 0 so therefore |0> → |0> but when it acts on a state 1 it becomes -|1>. So in other words it selectively provides a phase to one of their bits.

So the corresponding operator is given by what is known as Pauli Z operator, so it is also called Z-gate, so NOT gate is pictorially represented as X and Z gate is pictorially represented as Z, the however in general, I could talk about a phase not which simply makes 1 to -1 but it could for example get be give a selective phase, so it could also be |0> → |0> but the state |1> → $e^{i\phi}$ |1>

and so on. So the corresponding matrix would be (100 $e^{i\phi}$). Of course, you can immediately see that z : $\phi = \Pi$ because cos Π + i sin Π and cos Π = -1 there are a few other phase gates which are important, one of them is known as a T-gate.

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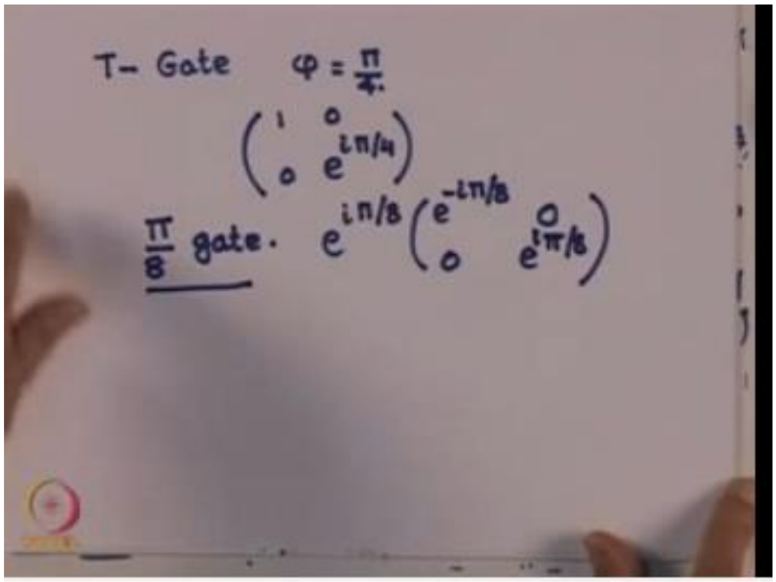

Where we take the value $\phi = \Pi/4$, so the corresponding if this is (1 0 0 $e^{i \Pi/4}$) this interestingly is also called a $\Pi/8$ gate. Now the reason for this funny nomenclature is that if you pull out a overall phase of $e^{i \Pi/8}$ then the diagonal elements have only $\Pi/8$ along the diagonals. So both the nomenclatures 8 gate and $\Pi/8$ gates are fairly common.

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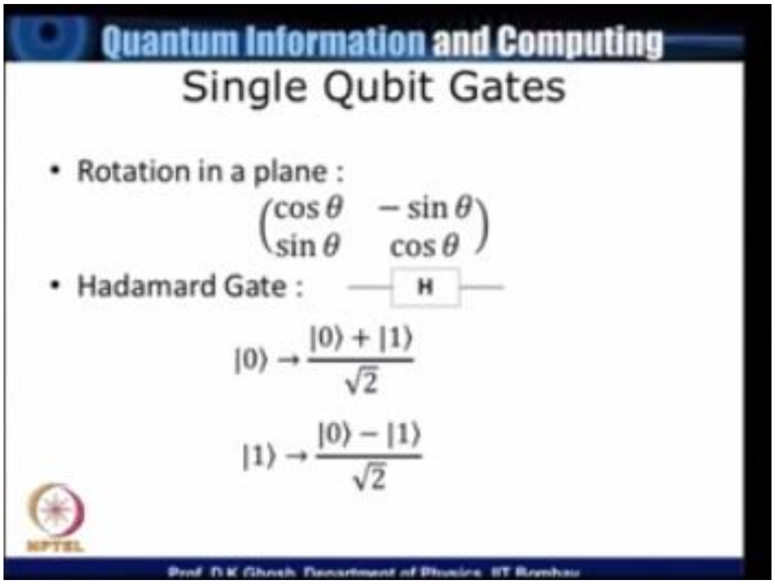

The next thing is what is possible is to have a rotation in a plane, the rotation in a plane is given by the rotation matrix which is cos θ, -sin θ, sin θ, cos θ which is rotating in a plane about the Z-axis which is obviously a possible rigid rotation, but one of the most important single qubits. Now remember in classical situation, the only one cubic or one bit gate that was possible was a NOT gate.

But we have already seen that several other possibilities exist in case of quantum operations. Now one of these operations is very popular.

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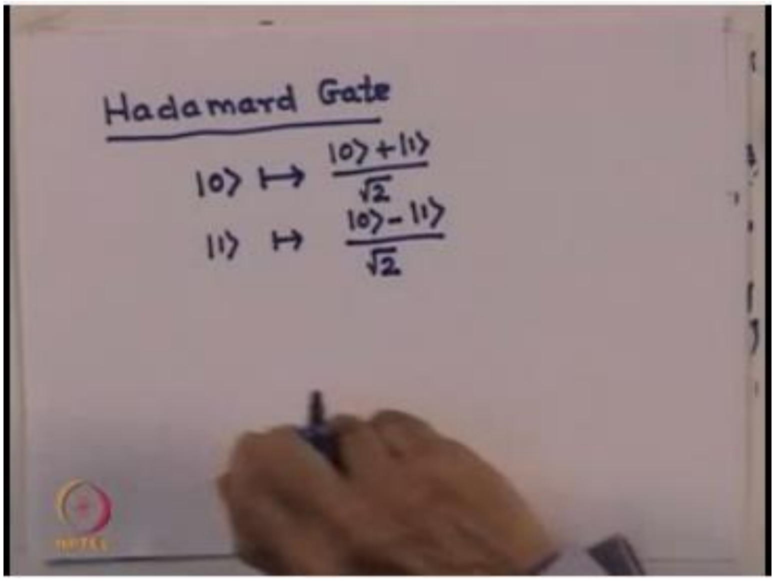

And is known as a Hadamard gate, now this is very specific to quantum computing from the simple reason that while in classical computing a 0 can either go to 0 or to 1 and similarly a 1 can go to either 0 or to 1 a in a quantum computing there are other possibilities that is a state 0 or a state 1 can go to a linear combination of 0 and 1 as well. So here is the situation where the state 0 goes to remember this state 0 is the correspond to bit 0 in the computational basis.

But it goes to |0> + |1> by square root of 2 which as, you know is the one of the bases in the diagonal bases and |1> → |0> - |1> divided by square root of 2. Now this is known as a Hadamard gate and let me sort of explain.

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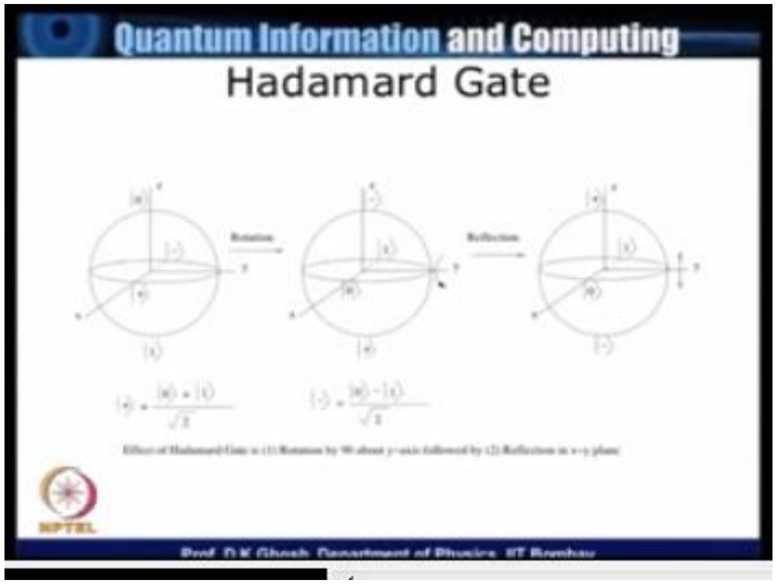

Pictorially what does Hadamard gate stand for? Now first thing you have to realize is this is there in the picture, but let us look at it. So the Hadamard gate corresponds to a rotation first a rotation by 90 degrees about the Y-axis followed by a reflection. But before I explain to you this particular diagram let us recall the our representation on a Bloch sphere.

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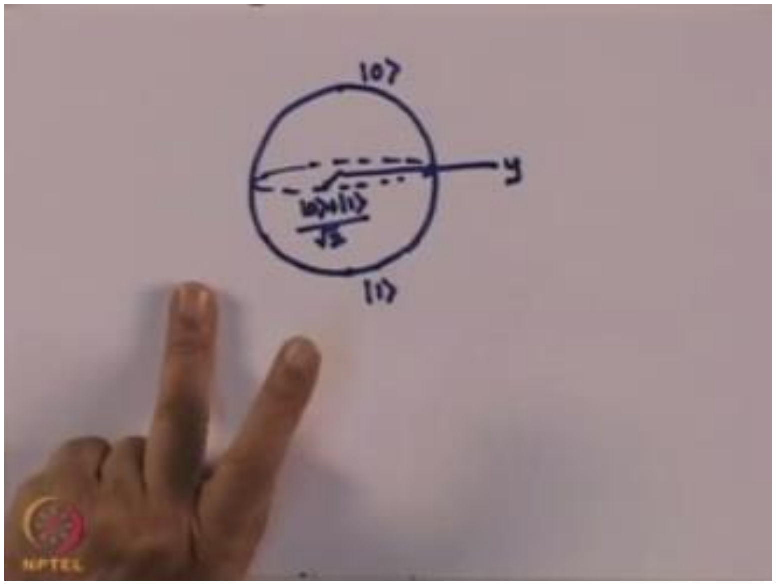

So remember my north pole was state 0, my south pole was state 1, and this is of course the equator, and we have said that the point where the positive is that X-axis meets the equator that is my 0+1/√2 and the opposite point, that is the point at which the negative X-axis meets the equator is 0-1/√2. So you notice now, now remember that this is my Y-axis, so what we are saying is this that the Hadamard gate which takes 0 to 0+1/√2 and 1 to 0-1/√2, now how does it work. The, so the first thing that you do is this have a 90^{o} rotation about the Y-axis.

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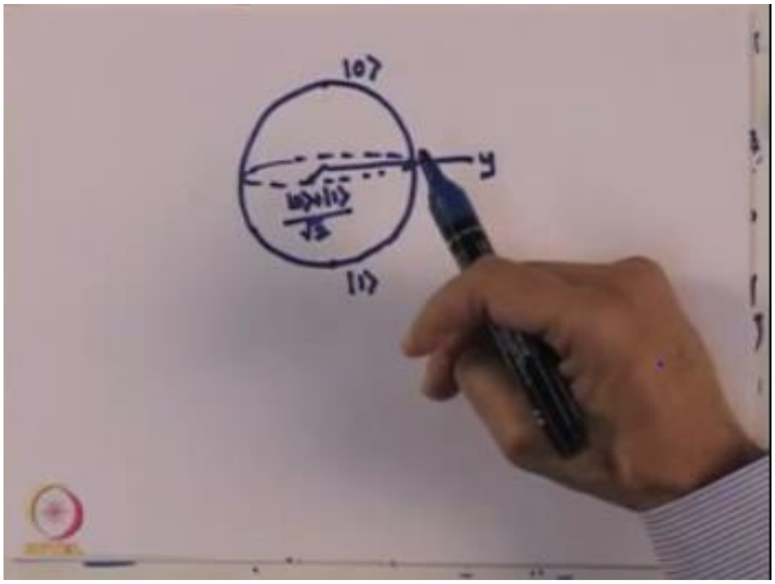

Now you notice what will happen in this picture, if you do a 90^{o} rotation now remember when we talk about a rotation the rotation is always considered to be along the counter clockwise direction as viewed towards the object from the axis. So therefore if you did that they did a 90^{o} rotation what will happen in the counter clockwise direction what will happen is that this 0. So let us look at what is happening now.

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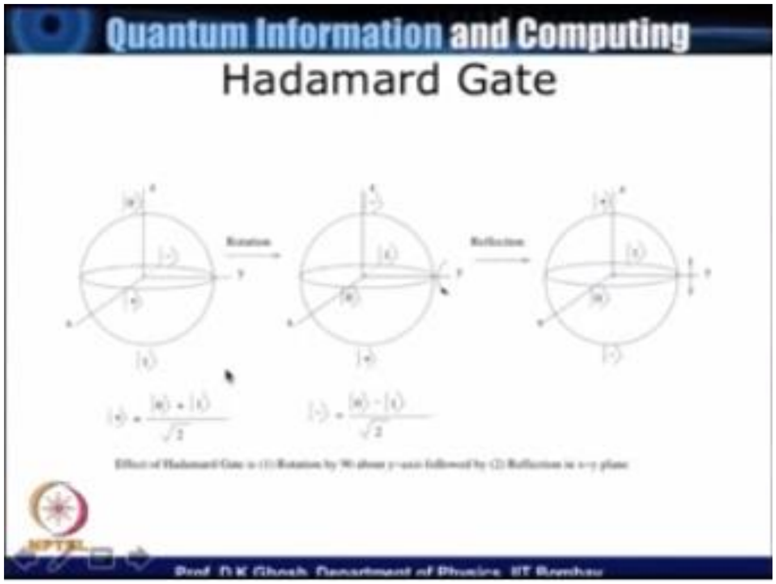

So come to this picture so I have this situation here, that there is a 0 here, there is a 1 there, okay. And if I now make a counter clockwise rotation the 0 has come to this point, the 1 has gone to that point. But you notice the state which is minus which was here because you are rotating long being counter clockwise direction that is now gone to the North Pole. But that is not the normal position on the North Pole, the 0 and the 1 are properly placed, but the minus state should not be on the North Pole but should go to the South Pole.

So therefore, what you require now is a transformation which in words this plus and minus while keeping the 0 and 1 this side, which is done by having a reflection about the x, y basis which is what so therefore this is the geometrical interpretation of a Hadamard gate.

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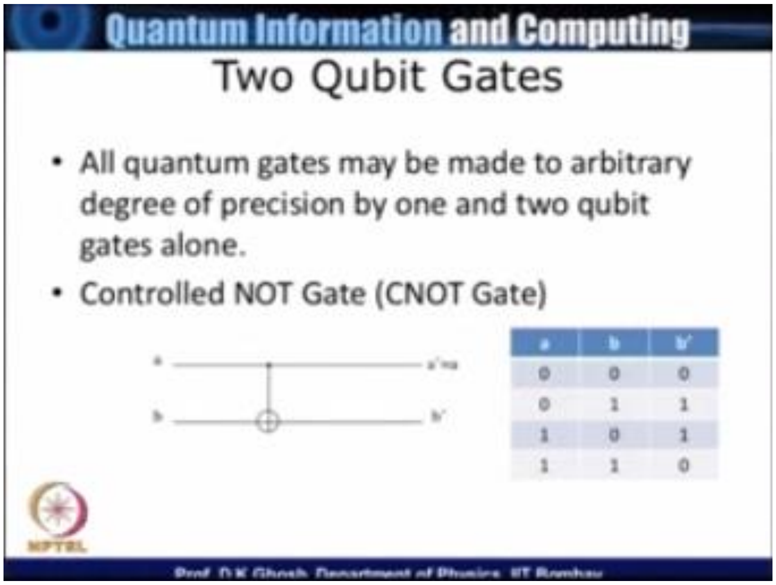

Now so these are the various single qubit gates, I need a few two qubit gates as well. The, now first thing that we realize is this.

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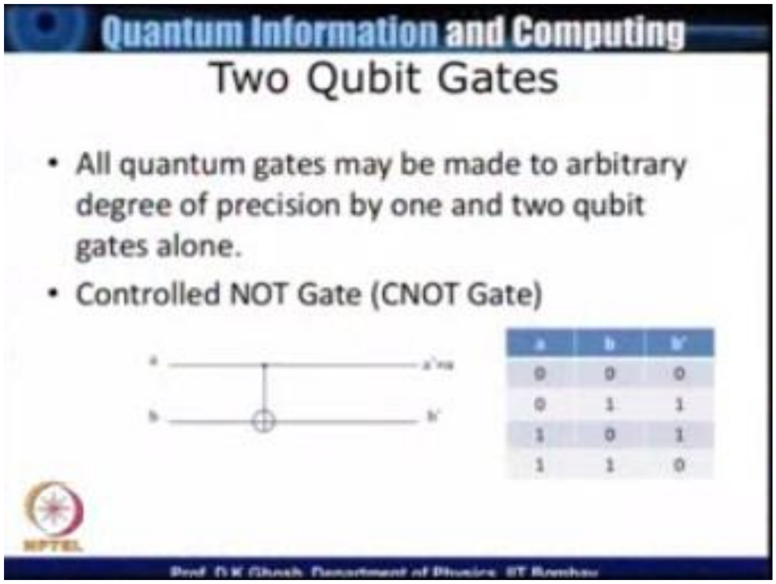

That all quantum gates may be made to arbitrary degree of precision by only having one and two qubit gates and we will that, we require certain other three qubit gates for a complete description, but on the other hand if you just want to have single qubit gates this is would enough one and two qubit gates. Now so first we talk about some two qubit gates, now one of the two qubit gates which is also very fundamental in quantum computing.

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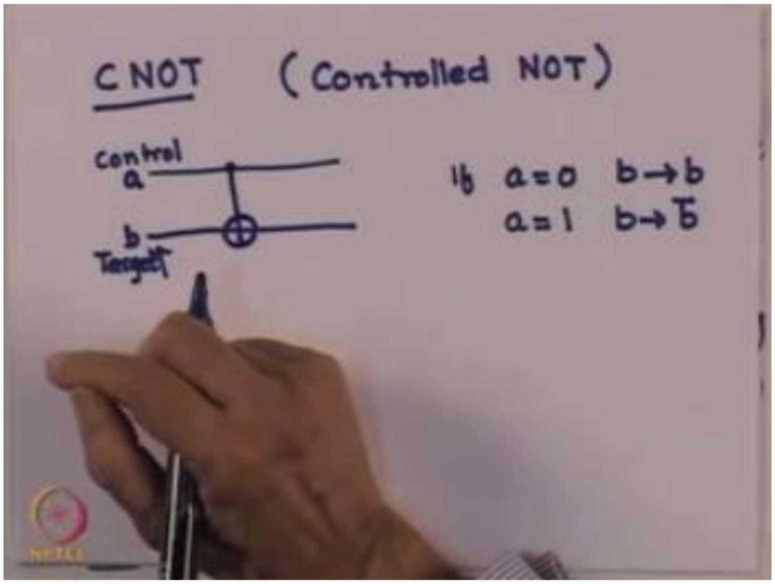

Is what is known as a CNOT gate is full form is actually controlled NOT gate. The controlled NOT gate works like this. Now suppose this is my input side I am essentially drawing something which is the direction in which the logic is showing, so this is my input side so there are two qubits on the input side one is what is known as the control so the upper one is control bit and this is known as the target bit. So supposing my control is A and target is B, now the rule for the controlled NOT gate is that whenever the control bit is 0 nothing happens to the target bit.

So if a=0, b remains b, but if a=1, if a=1 then the b, b becomes b bar that is script, that is 0 if b is 0 it becomes 1, if b is 1 it becomes 0. And the way the CNOT gate is represented in a quantum circuit is to have a control there with a small dot there and a symbol for the bit. Now what about its matrix representation, the matrix representation is shown in the slide.

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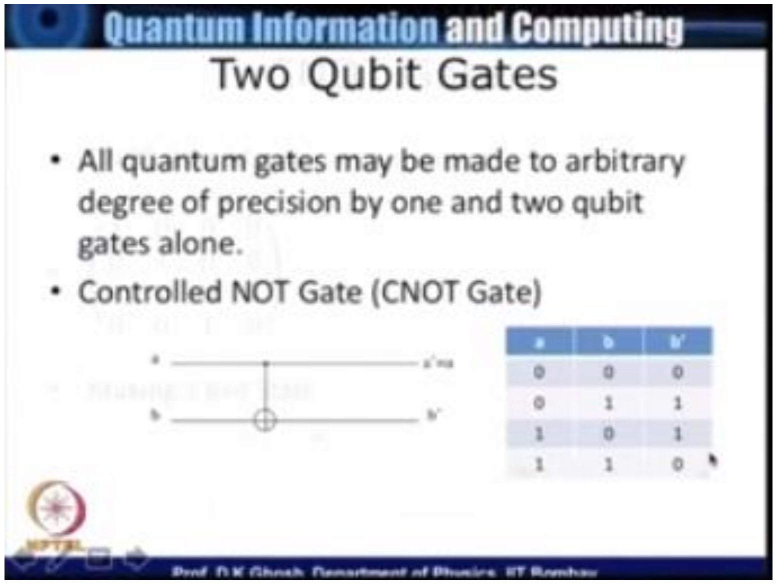

So look at it, if a is 0, b is 0 the b′ which is the output it remains 0 because the control bit is 0, second case also control bit is 0, target bit is 1, so the target bit is unchanged remains 1. But if a=1 in these two cases, if b is 0 b′ that is output is 1 if b is 1 output is 0.

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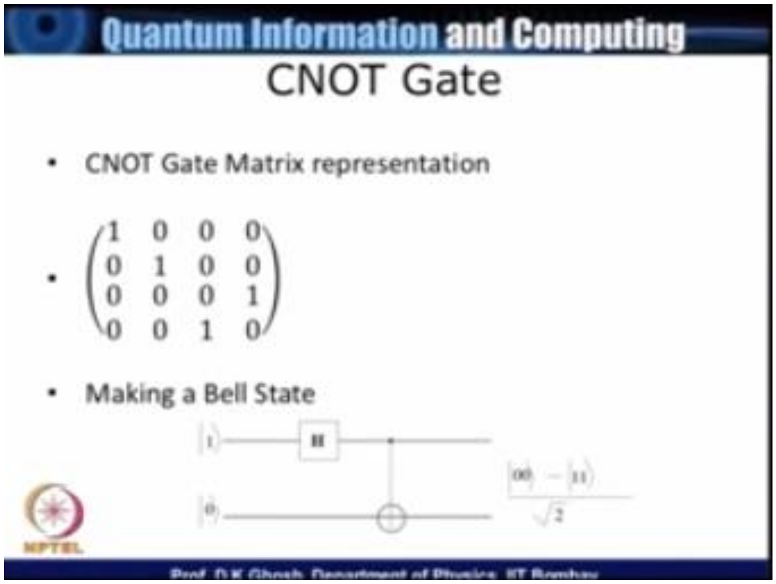

I can represent this remember I give you a basis for the two qubit which was 1000, 0100 etc., and this basis is written as the following.

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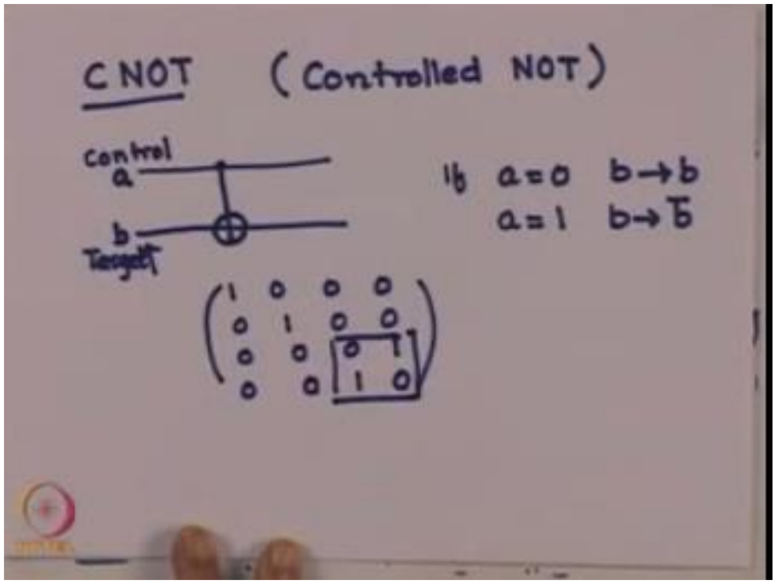

I have 1000 in this basis the CNOT matrix is 0100, 0001, 0010 so notice this that this block is essentially a σx matrix. And, and this can operate on one of the four basis states, right. So the four one of the four basis states where the first two entries correspond to control and the lower two entries correspond to the target.

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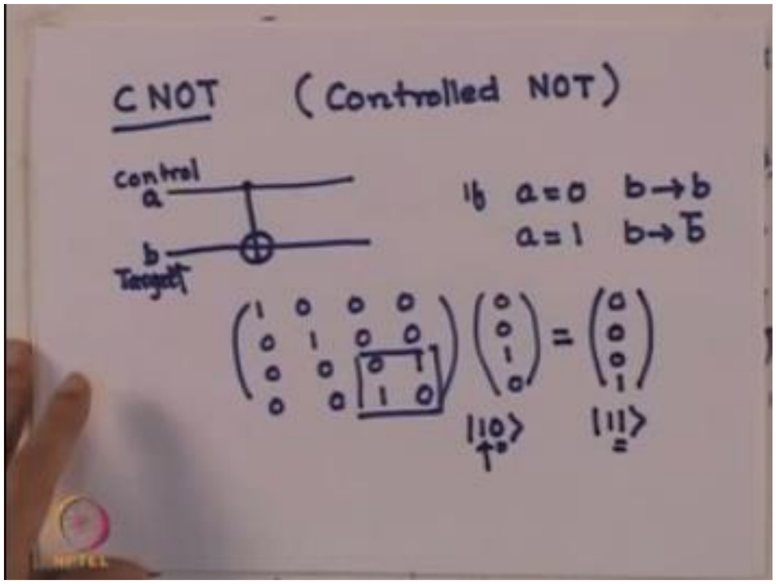

And you can sort of convince yourself look at it for example what happens when this is 0010. What does it mean, so this means basically my state is 1 0 because this is the representation for 10, now do the matrix multiplication since the control bit is 1 this means I would get the target bit equal to. Now you can multiply this you can see that this gives me 0, 0, okay and 000 so 01 which is just be the representation of the bit 11 which what I expect because the control bit of 1 so therefore that target is kept. So this is my CNOT gate.

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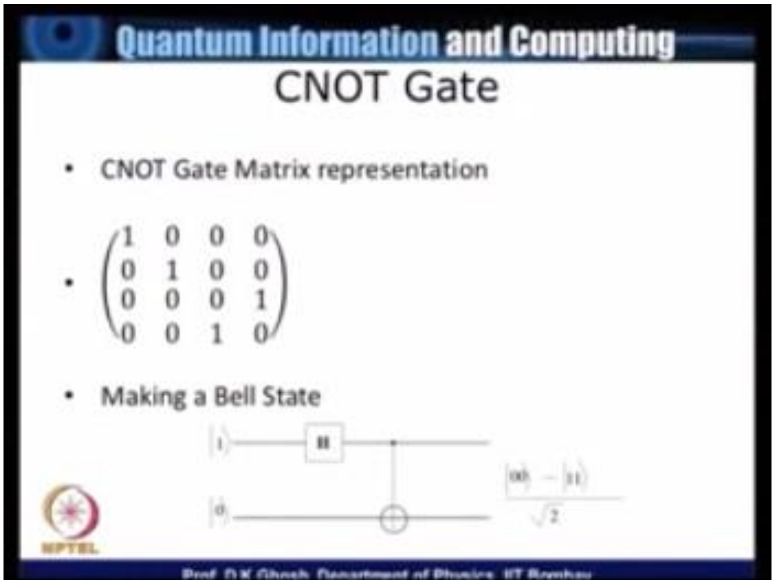

Now remember I talk to you regarding Bell state now in the two things that we have just introduced I can design a computing circuit quantum circuit which will give me a Bell state, now let see look at for example the bell state.

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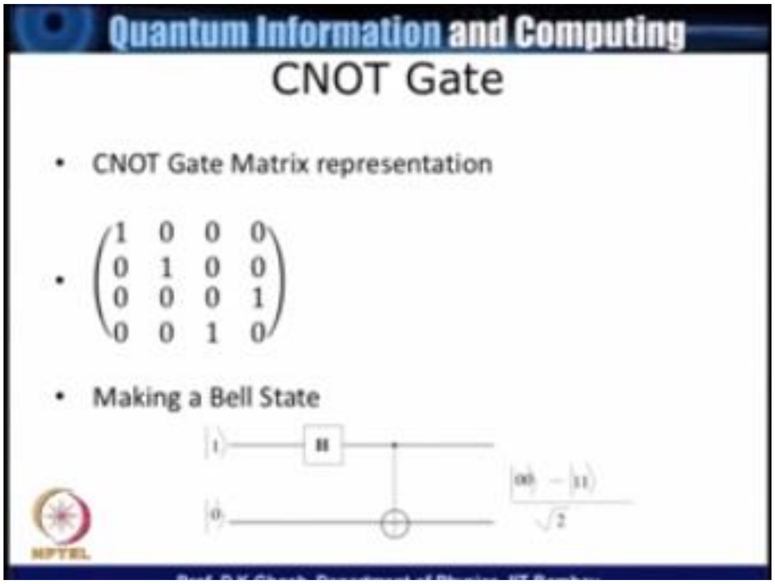

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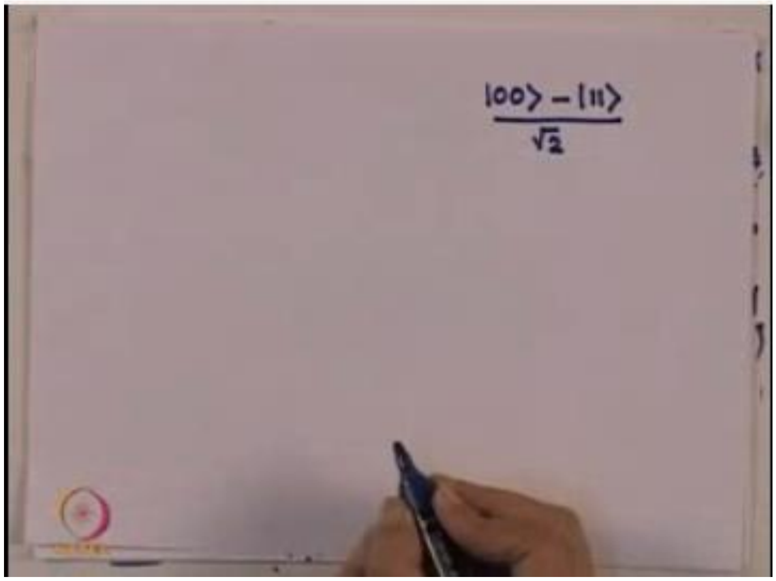

That I would try to get is given by 00 – 11/√2 let see how it works.

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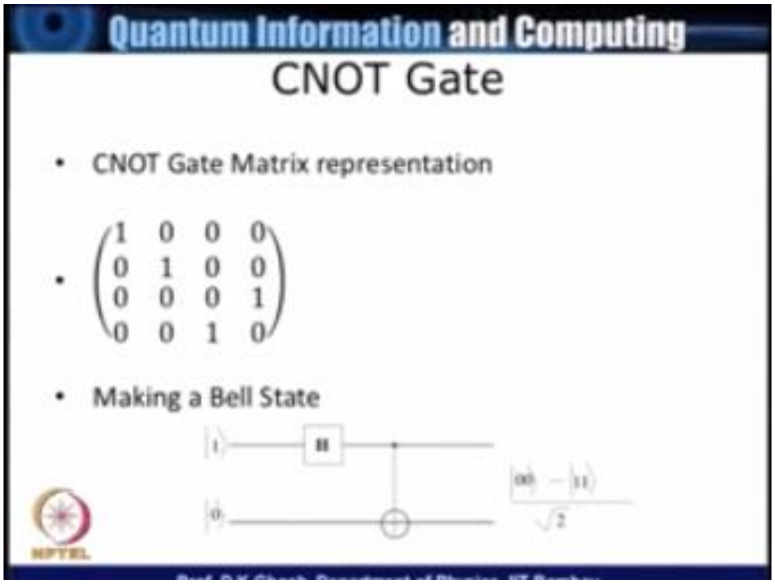

And what you can do as an exercise is try to form the four Bell state the remaining three bell states one of them I am explaining. So what I will do is this, I take essentially a CNOT gate.

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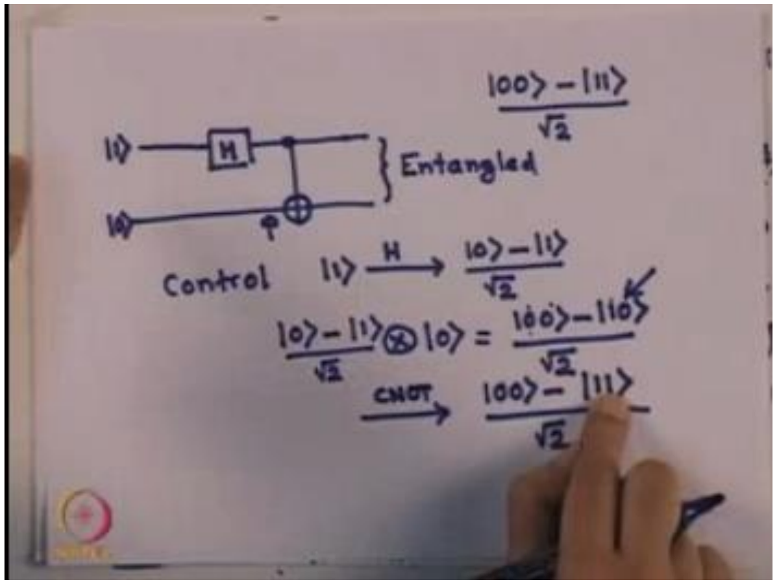

And my CNOT gate, but before I do that let me draw it out and then I will explain. Okay, so what I do is this I fix my control bit, the first line which is control bit to be 1 and this 1 I set it to be equal to 0 initially. Now the way this circuit is to be looked into is this is just the direction it means the logical straight. So because in time H is appearing earlier than this, so the first qubit would be subjected to Hadamard gate.

So my control bit become like this, so I had one, now this one is subjected to Hadamard gate. We had seen by definition of the Hadamard gate this gives me 0 – 1 /√2. So you notice this control which is there now is not just a single control either 0 or 1, but is a linear combination of 0 and 1. Now help this qubit is to become 0 this will leave this 0 the same. So in other words since my target initially was 0, my next state at this stage is 0 -1 /√2 direct product with 0.

So which is just 00 -10 /√2. Now when I subject this to a CNOT gate in this case the first cubit is 0 that is control bit is 0, so therefore nothing happens to be 0. In so when I ask this to a CNOT gate in the second case my control bit 1, so my 00 remains the same there is a – sign already there, in this case control bit is 1, so therefore the target bit 0 will be flipped 0 will become 1/√2.

Now so this state that you get here is see I cannot any longer write down the state here and the state there because this states, the two states here have become entangled, these are become entangled because there are no individual identifications here. And the output state is now 00-11 what will make a different is if I make a measurement. If I made a measurement of a first cubit and I get a 0 the second cubit automatically becomes 0 and if I get a 1 from the first cubit the second cubit collapse to 1 that the concept of measurement I would talk later.

But this is the way you can produce one of the four Bell states and I would argue to try to design simple circuits by which you can find the remaining or design the next three Bell states.

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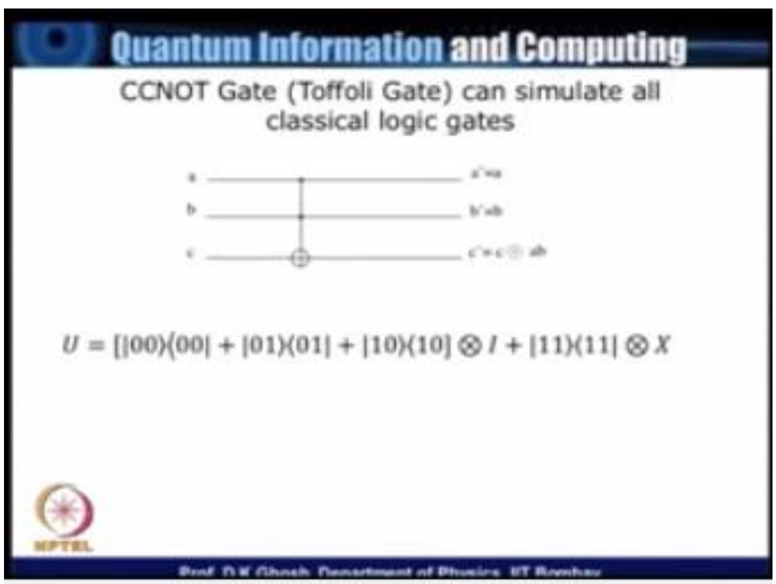

There is another get which a [indiscernible][00:19:51] gate. Now this can actually even simulate all classic logic gates it is not done like that because as I told you there is garbage problem there, but this gate is called as CCNOT gate. Control, control NOT also called as a Toffoli gate. The logic for Toffoli gate is very simple, it has three inputs let us call them ABC. Now there is a control double control in this and this is my target both the end we have control. The target bit is flipped that is 0 becomes 1 and 1 becomes 0, if both a & b both the control, become a good one, which means if the product of a and b is equal to 1 then only c will reflect.

Now look at the way a control bit action on the target bit can be written. Now recall that we had a normal CNOT gate where we said the following, if this a this is b we said this is always left without change, here also a and b will be left without any change. Here we said if a is equal to 1 then only b will change, the way of writing this is to say b′=b addition modulo two of a, this is the same as an EXOR because you notice if b is 0 a, b′ takes the same value as a which means if a is 0 b′ is 0.

If a is 1 b′ is 1, even the b was 0 which is what CNOT did. If a is 1 then b=0 okay. I still get bb′ = 1 which means b has been flipped. If a=1, b=1 this is 1+1 EXOR 1 which is equal to 0 okay. So this is the way it works, because it flips only if the control bit is 1. And this case I can write exactly the same way and I would say this is equal to C + addition modulo two with the product of a and b.

So that if this product is equal to 1 which means if either a=0 or b=0 it does not work but both a and b are 1 then I am adding 1 to C which will flip.

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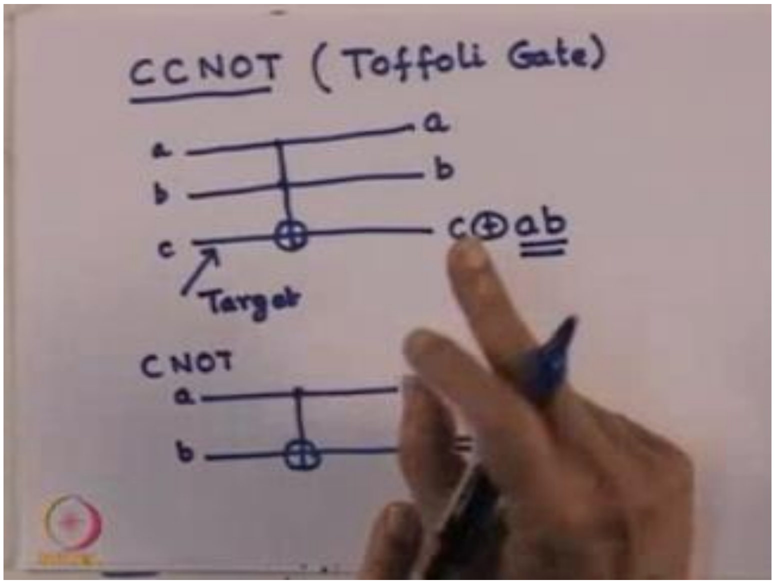

You can repeat this using the slide I gave you that the operator representation of the unitary operator.

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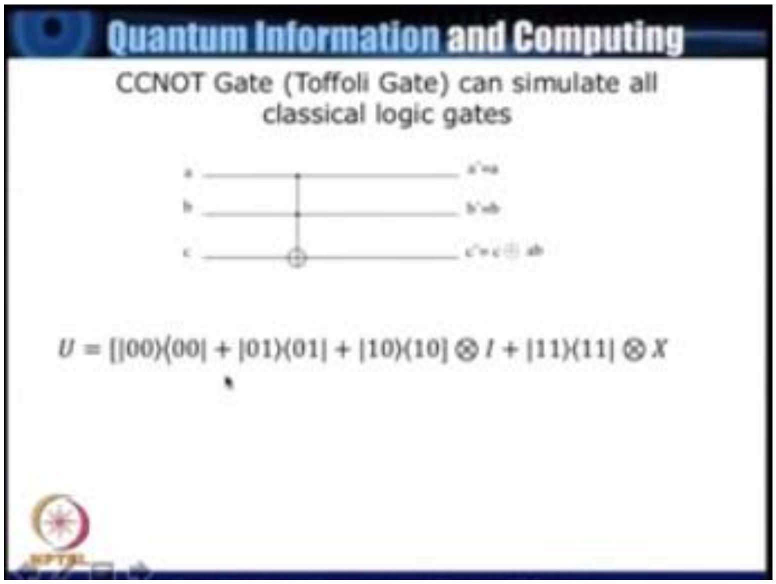

And you can see that is what it works.

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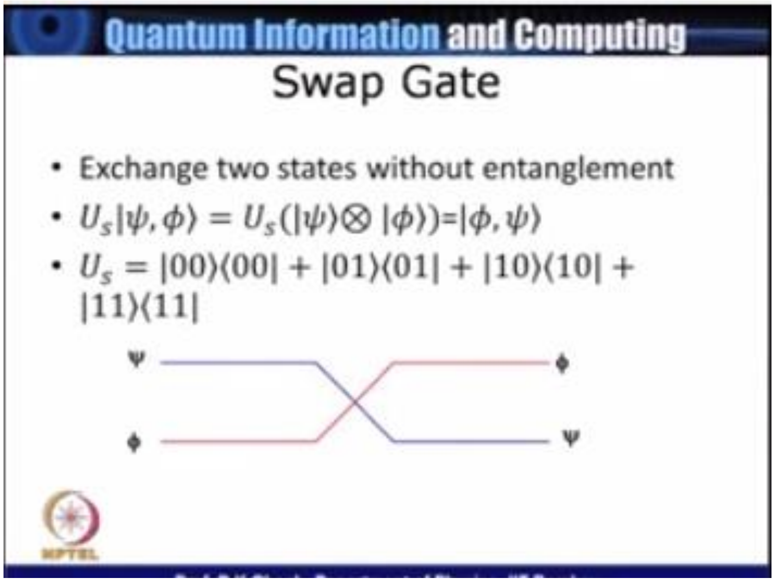

Let me talk about two more gates first is a swap gate, swap as the name suggests simply interchanges two states. So therefore, the operator u as working on ψϕ, ψ is the first bit ϕ is the second qubit which is same as u as acting on direct product of ψ and ϕ it gives me ψϕ. So the corresponding operator expression is this, so that the ψ becomes the first line was ψ now the first line become ϕ.

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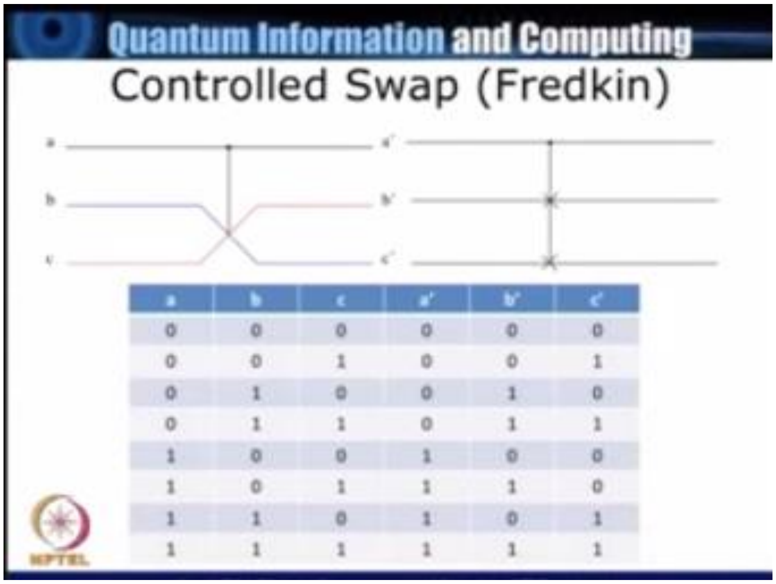

a	b	c	a'	b'	c'
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	0	1
1	1	1	1	1	1

There is a swap gate which is called a controlled swap gate, which is very useful in quantum computing and this is the same idea of a swap, this is, there are two pictures there represented either by this notation or by that notation, which says there is a controlled bit a going to be a′ number a′ as everyone say. Now this acts between b and c interchanging b and c making them like this.

If the control bit is equal to 1, now I have given you a table by which you get the entire prove table that is called of what happens, for example, if a=1 means control is equal to 1 okay, b=0, c=0 right. Then a′ of course will always remain 1, b and c which were 0 will be interchanged but it do not matter, because they were already 0 each so therefore it will remain 00. But if control bit is 0 as b is 1, c is 0 you notice b′ becomes 0, c becomes equal to 1. So this is also known as Fredkin gate.

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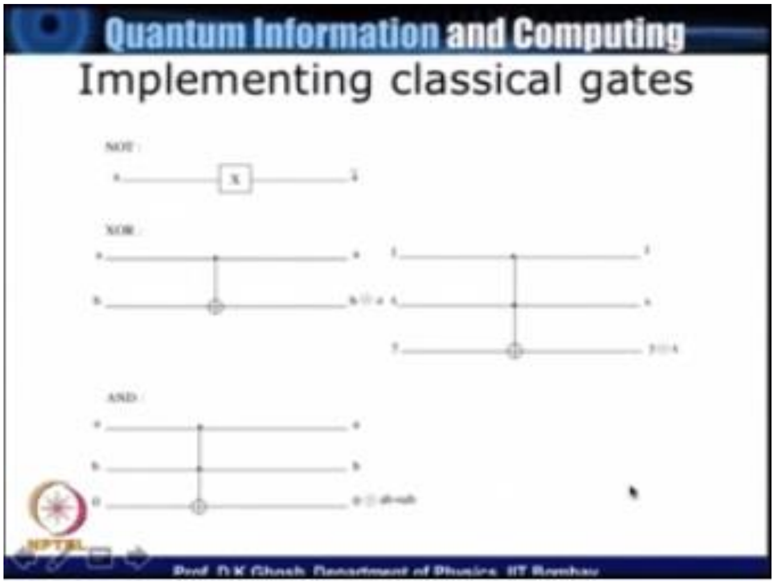

In the next lecture we will be talking about quantum circuits and how does one carry out or represent logic mathematical logic using these gates which we have talking about. As I have said to summarize we have elementary gates which operate on 1 qubit, 2 qubit, or request 3 qubits, you do not need any more to work even on n qubit states.

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NPTEL
Principal Investigator
IIT Bombay

Prof. R.K. Shevgaonkar

Head CDEEP

Prof. V.M. Gadre

Producer

Arun kalwankar

Online Editor
& Digital Video Editor

Tushar Deshpande

Digital Video Cameraman
& Graphic Designer

Amin B Shaikh

Jr. Technical Assistant

Vijay Kedare

Teaching Assistants

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Bhargav Sri Venkatesh M.

Sr. Web Designer

Bharati Sakpal

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Riya Surange

Sr. Web Designer

Bharati M. Sarang

Web Designer

Nisha Thakur

Project Attendant

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Vinayak Raut

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