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Quantum Information and Computing

Prof. D. K. Ghosh
Department of Physics IIT Bombay

Modul No.01

Lecture No.5

Qubit- Bloch Sphere representation

In the last lecture we introduced the smallest unit of a quantum state. And we said that this is called a Qubit which is essentially a corresponds to our classical Cbits of 0 and 1 the difference between the quantum bit and the classical bit was while a classical bit can only stay in either the value zero take either the value zero or the value one. A quantum bit which is a Qubit can be simultaneously in a linear combination of the state 0 and 1 we also saw that while that is true now if you are making a measurement let us suppose that you make the measurement in a computational basis.

Then you would get the either this state 0 or the state 1with the probability which is given by mod a^2/ mod a^2+b^2 or now mod b^2/ mod a^2b^2 respectively. I also gave you some physical realizations of Qubits saying that they could be representing the spin half particle the it is spin projection along some arbitrary chosen direction like Z direction or it could be the polarization direction associated with a photon or even the ground state and the excited states of an atomic system.

Now we talked about the poly matrices and we are what we are going to do is this that the well-known poly matrices are σx, σy and σz high are 0 1 1 0 σy = 0-i i0 and σz=1 0 0-1 each one of this has an Eigen value + 1 or -1 for example σz has an Eigen value +1 corresponding to the state 1 0 and an Eigen value -1 corresponding to state 0 1 you can yourself find out what are the Eigen states of σx and σy.

Let me find out here an arbitrary direction the poly matrix in an arbitrary direction let us just call it σn as you are aware that an arbitrator direction and space the unit vector has components which are x component is equal to rsin θ, cos Φ y component is equal to r sin θsin Φ and z component is r cos θ or if I take r=1 that makes the σn that is the poly matrix along the arbitrary direction n as σz times cosθ which makes these diagonal elements cos θ- cos θ + σx times sin θ cos Φ which makes it sinθ cosΦ let me just write down here and then I will write it more compactly here.

So I have got sin θcos Φ + σy times sin θ sin Φ but you notice there is a –i in this so therefore it will be - i times sin θ sin Φ but you can use cos sin Φ - i sin Φ equal to e-$^{i\ \Phi}$ and then write this as e-$^{i\ \Phi}$ times sin θ so this term is e-$^{i\ \Phi}$ times sin θ and correspondingly this one is e+$^{i\Phi}$ 5 times sin θ.

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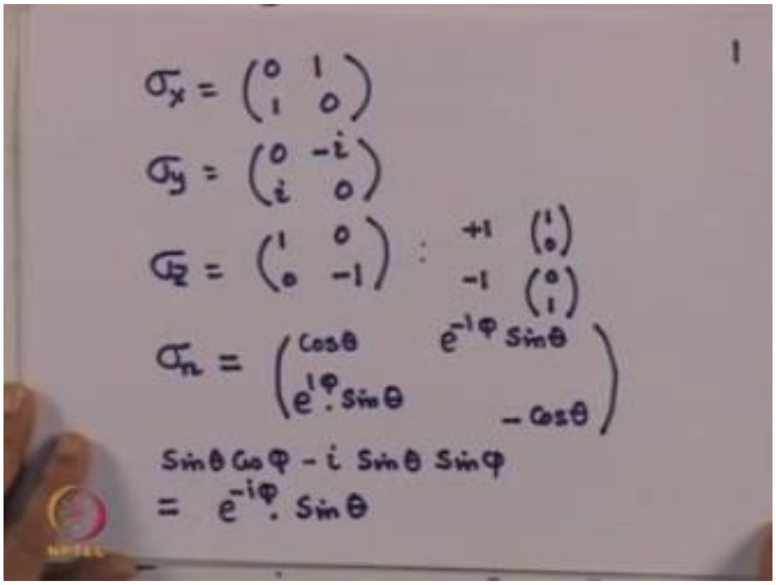

Now it is very trivial to find out what are the eigen values and the eigen vectors of this matrix eigen value as you know you, have to simply find out the secular matrix which is λ - cos θ here λ+ cos θ there and of course these two elements just become negative and then you find out the determinant put it to equal to zero if you do that you get the value of λ that is the Eigen value to be equal to + or - 1. Now which is not surprising actually the Eigen values λ.

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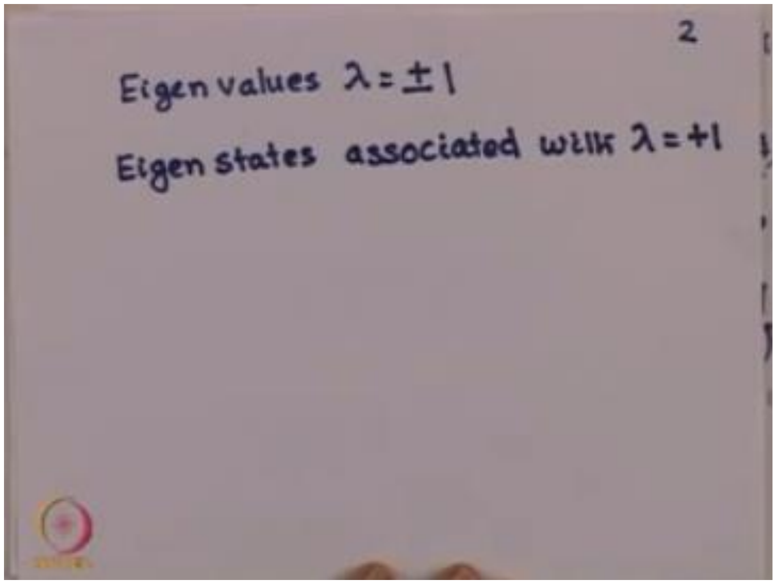

The reason why it is not surprising is because we have said that the Eigen value along the z direction which is an arbitrary direction there is no god given direction called z is +1 or - 1 so, if I take in any direction that is also should be + 1or - now you can use the same matrix in my m to find out what are the eigen states the eigen states associated with either of these Eigen values but I am going to be interested only in the eigen states corresponding to λ equal to + 1 because that is what is the standard convention for the pictorial representation that we talked about.

And, if you just do a little bit of an algebra I would argue to do that forgetting some practice in to Eigen values and Eigen vectors you will find that this Eigen vector.

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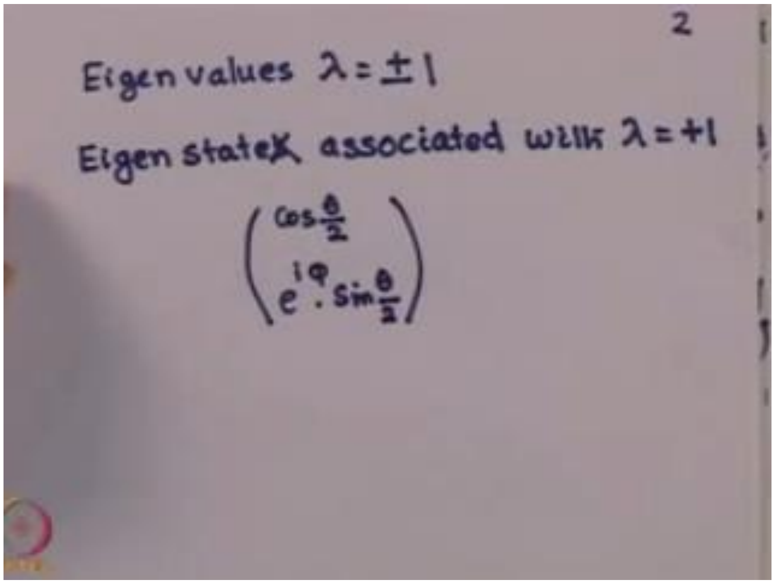

Is given by cosin $\theta/2$ and $e^{i\,\Phi}$ sin $\theta/2$.

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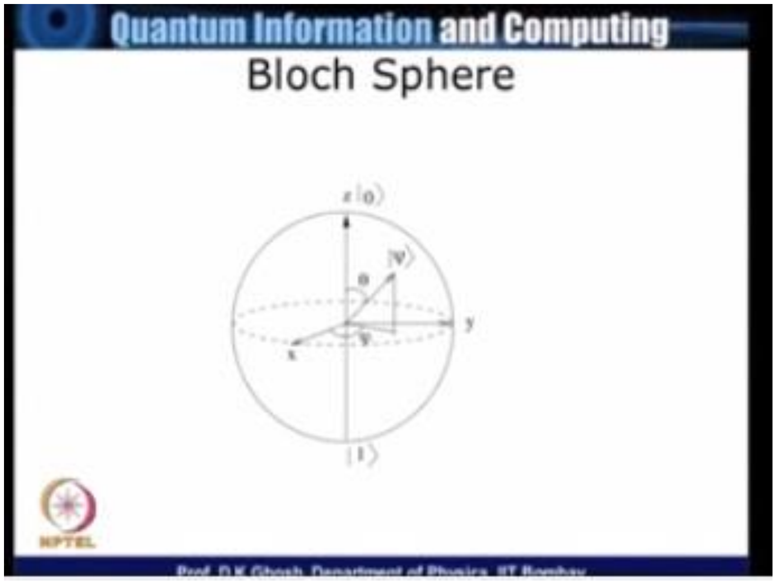

If you look at the slide here now see what I have done now is this, I now take this state which I wrote down that is the column vector cos $\theta/2$ and $e^{i\phi \sin\theta}$.

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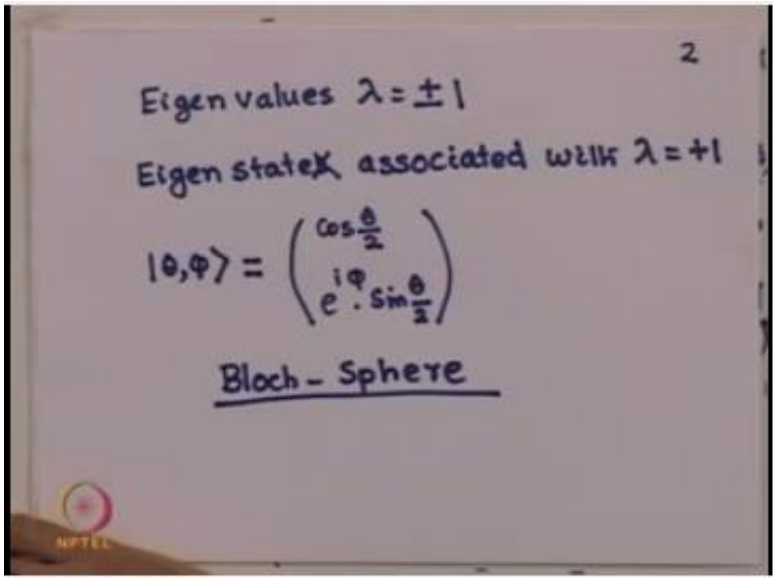

So this is my state which I will write as state ΘΦ pictorial representation that I am talking about is known as a Bloch sphere representation returning back to the slide.

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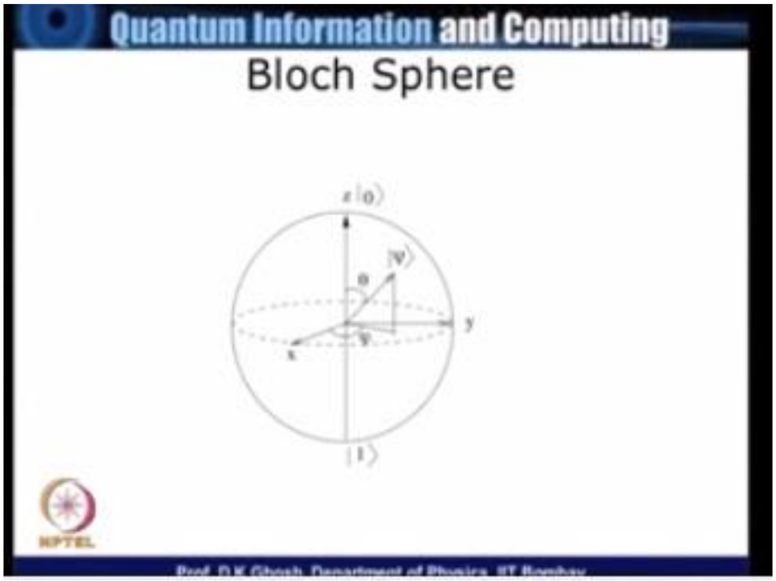

Let us look at the directions I said that now this is a sphere of does not quite look it on the on the slide it looks a little overlised but on the other hand it is a sphere of unit radius. So it is a unit sphere and what I am going to do is to associate every point on the unit sphere where a unique state with a unique state having a value θY, now let us look at what it means. Now supposing I take the point, take a position well remember r is equal to 1so I take let us say θ=0, Φ=0. Now what is θ=0, Φ=0 okay let me let me rewrite this state θΦ in a slightly different way. So this is cos θ/2 times 1 0 + $e^{i\phi \sin\theta/2}$ 0 1 or in our vector notation cat notation this is cos θ/2 0 state + $e^{i\phi \sin\theta/2}$ the state 1.

Now supposing I take the state which is θ =0, ϕ=0 put it here you notice because θ=0 this term becomes equal to 0. And since cos 0 = 1 so θ=0, Φ=0 which is nothing but the North Pole in the slide that I showed you.

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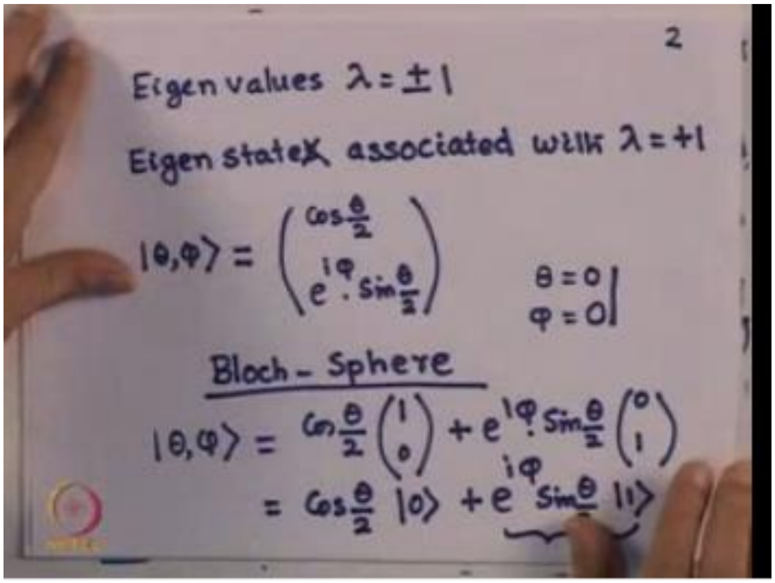

The, so North Pole corresponds to the state 0 look at that picture there so this is the picture on the slide.

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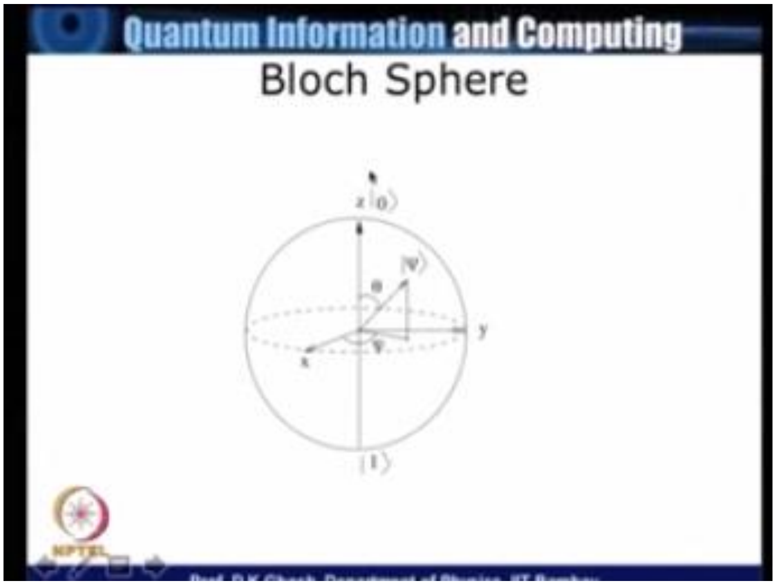

This is the point which stands for the point 0 that is the North Pole. Now let us go to the South Pole, now if you go to South Pole.

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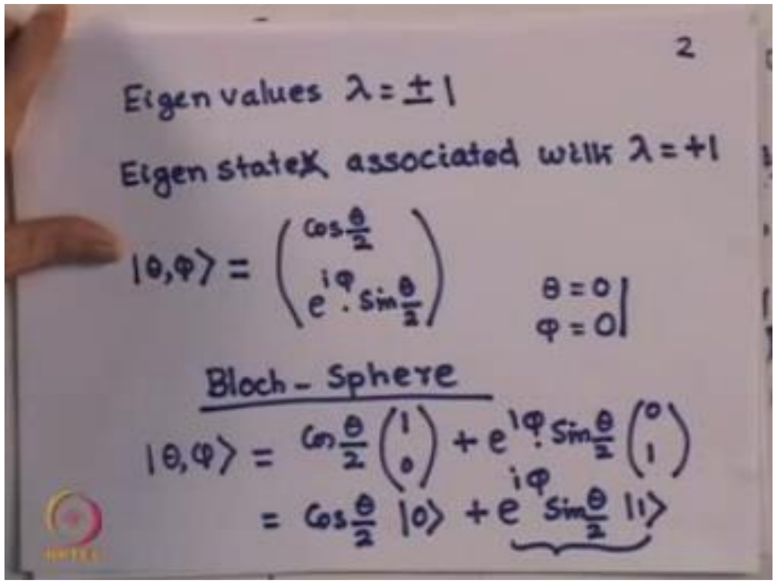

So I have got remember that so North Pole corresponds to the state 0, now let us look as South pole, the South pole has $\theta=\phi$ and ϕ =0. Now that obviously stands for the state 1, remember that I had written this format so if I have $\theta=\phi$ $\cos\phi/2 = 0$ so this term goes away. $\phi=0$ so $e^0=1$ and this is $\sin\phi/2 = 1$ so the state will be 1.

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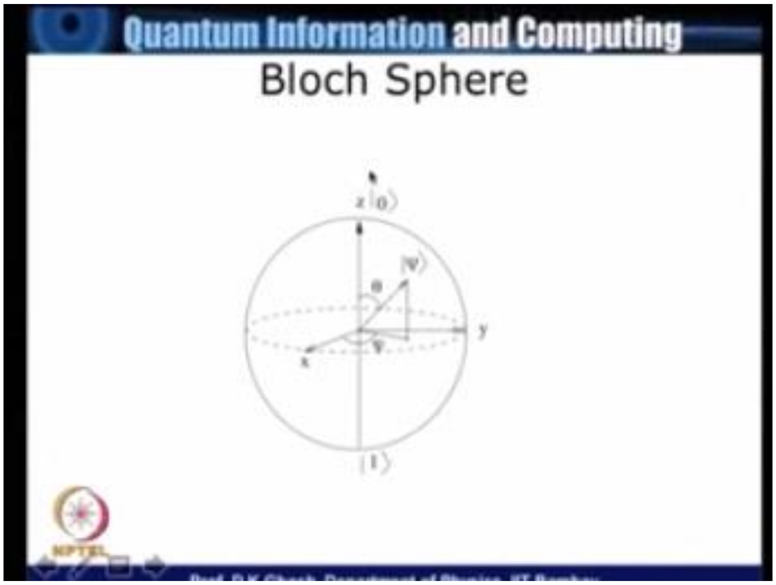

So if you go back to the slide you find that this is your state 1 okay. Let us go to a slightly different point. Let us consider the point where the positive x-axis meets the equator, the point where the positive x-axis meets the equator.

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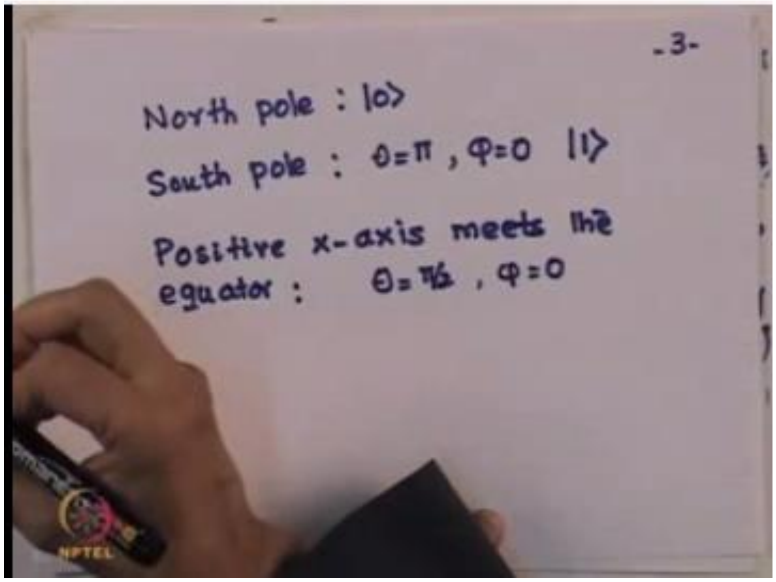

Now since it is on the equator θ=ϕ/2 and Φ=0.

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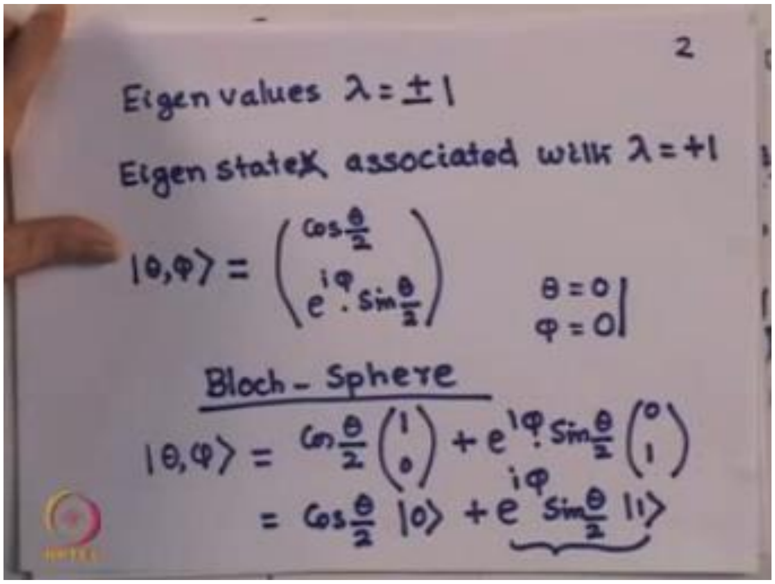

Now you can immediately go back there θ=ϕ/2 means cosϕ/4 which is 1 over square root of 2, e^o=1, sinϕ/4 which is again 1 over square root of 2.

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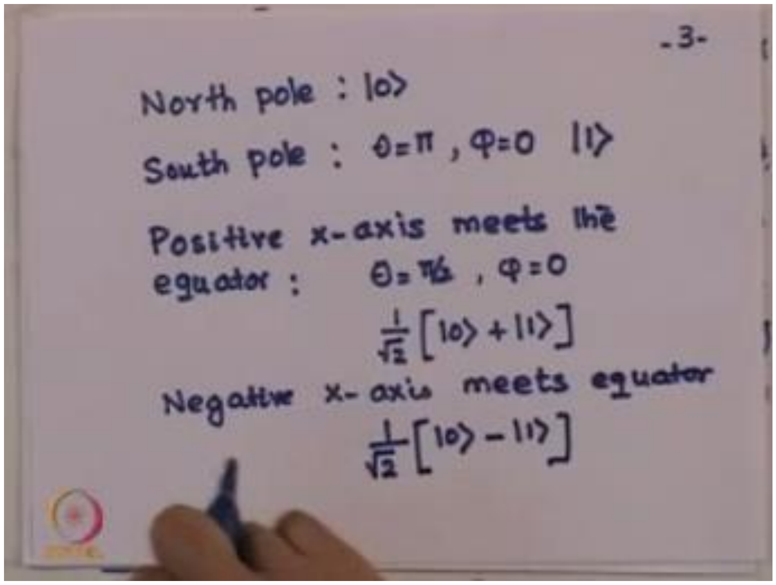

So this state is 1 over square root of 2 0 + 1 remember that I had called this the diagonal state one of the basis of the diagonals. And likewise you can convince yourself that the point where the negative x-axis meets the equator that is simply the other bases in the diagonal states that is 0-1. Like this every point on the Bloch sphere stands for a unique quantum state and the θΦ value can be okay.

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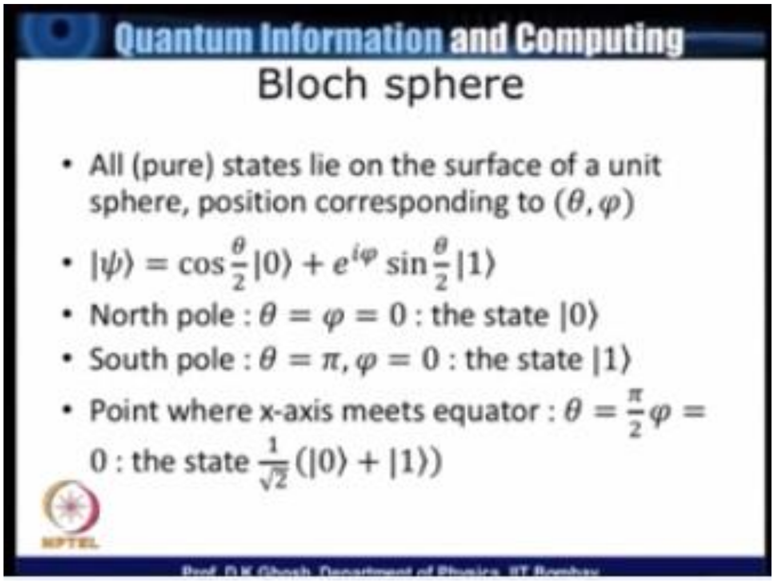

I will later on talk to you these states which lie on a bloch sphere they all are what are known as a pure state what is pure about it, we will be talking about as we go along.

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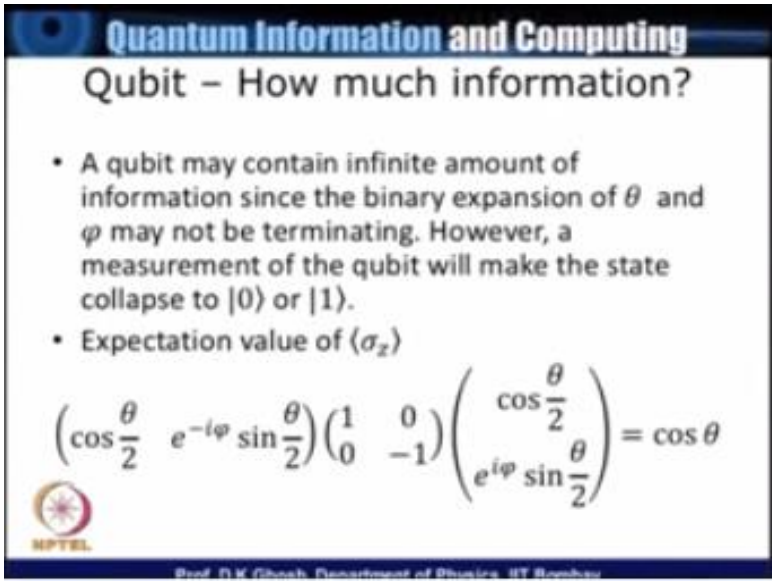

The next questions is that how much of information is there in a Qubit, now remember the in principle a cubit contains infinite amount of information. Now what is the reason, the reason is that remember my θ and Φ are the parameters in that and θ and Φ expansion in terms of binary digits may be essentially non terminating. But we have told you that in spite of the fact there is huge amount of information contain in that field much of this information is not really available to you because when you measure it you either get a state 0 or you get a state 1.

You can calculate what is the value of σz in this contest you can easily see it by finding out the way the expectation value of a quantity is calculated we have seen that the state is given by cos $\theta/2\ e^{1\phi\sin\theta/2}$ so, my σz operator is 1, 0, 0, -1 so the expectation value of σz is given by the row vector cos $\theta/2\ e^{1\phi\sin\theta/2}$ multiplied with this trivial matrix calculation which will give you cosθ as the expectation value of σz.

And likewise we can determine σx. Now remember that I have mentioned to you earlier though I have not yet proved it a quantum state cannot be exactly copied there is a theorem which will be proving as we go along that there is a no cloning theorem of quantum state. But then you imagine somehow we have prepared identical copies of quantum states and we are able to make a large number of mathematics.

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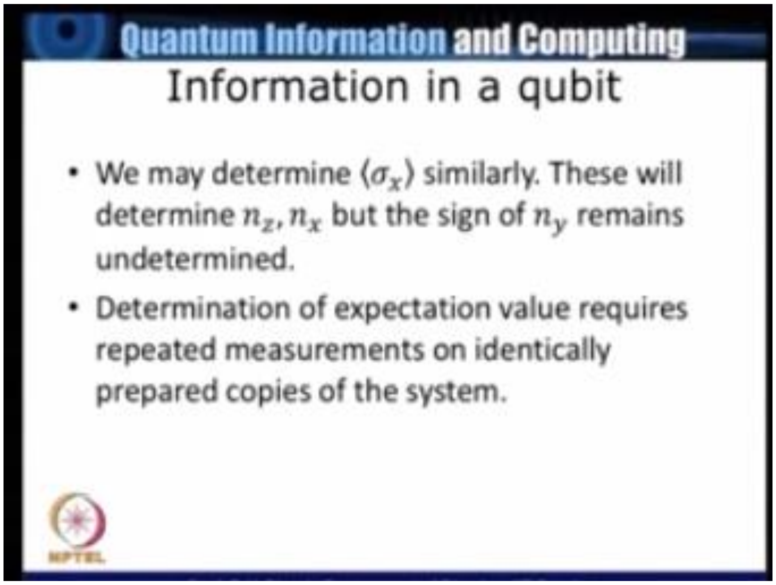

So I can do the calculate the expectation value of σx by that I can find out what are the directions n_z and n_x and you remember that I have $n_x^2+n_y^2+n_z^2=1$ so by these calculations I can find the values of n_z and n_x but the signature or the sign of ny will remain undetermined. So this is what we talk about.

Now the point then is this that we have so far said that when you make a measurement, you get either a state 0 or a state 1 with a probability. Now this probability will allow you to determine so the.

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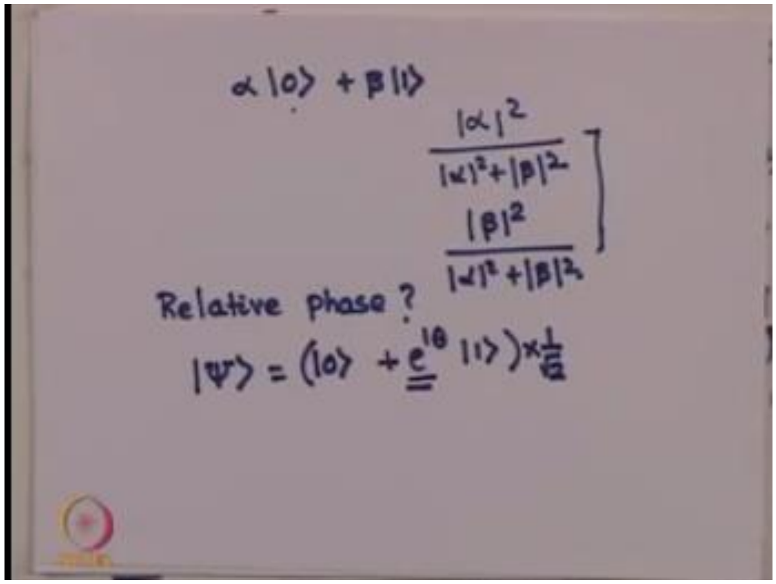

For example, if I have a state $\alpha 0 + \beta 1$ now, if I make a repeated measurement on identically prepared copies I will get state 0 with a probability $\alpha^2/\alpha^2+\beta^2$ and this state 1 with the probability β2/ $\alpha^2+\beta^2$. Now these are the probabilities which will get which will enable us to determine the magnitudes of α and β, is there a way of finding out the relative phase, supposing I consider a state for convenience $\psi = 0 + 1$ but I have a $e^{1\theta}$ and I normalize this with a factor 1 over square root of 2.

Now I know even if this state if I measure the states in a computational basis I would get the state 0 or state 1 each with the probability of ½ this is what comes from here, but how about the information on θ can I get it. Now if you are making a measurement in the computational basis you cannot, but I will just not show you that if you make a measurement in a diagonal basis okay, you will be in a position to find out the information about phase.

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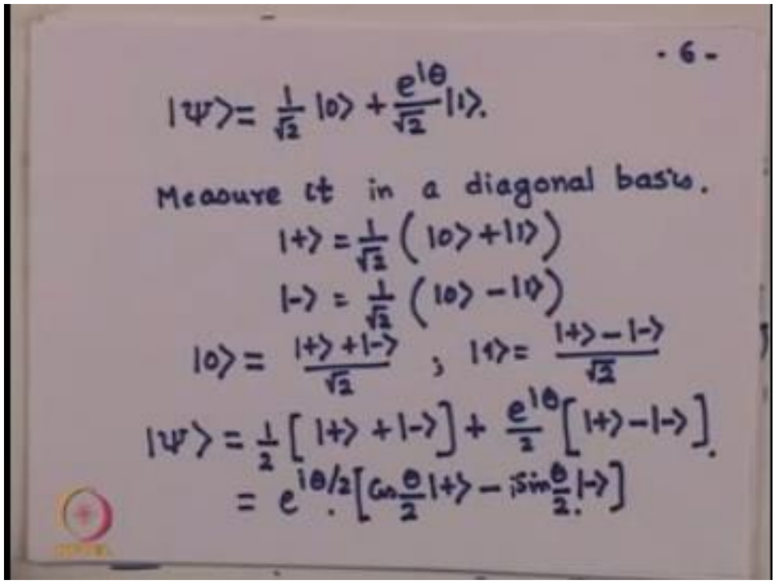

So let us suppose I have said is 1 over square root of 2 0+ $e^{i\theta}$ θ is the relative phase by square root of 2, 1. And let us measure it not in a computational basis but in a diagonal basis. Now in order to make the measurement in a diagonal basis I must first express the state 0 and 1 in the diagonal basis. Now recall that my diagonal basis is + = 1 over square root of 2 0 + 1 and - = 1 over square root of 2 0-1.

If you do a little bit of algebra this will tell you that 0 can be written as +, + - by square root of 2 and 1 can be written as + -, - by square root of 2. So this ψ that I wrote down just now can be now expressed as this 0 there is another 1 square root of 2 here so I get 1 over 2 + + - + $e^{i\theta/2}$ + - -. Now if you rearrange these terms you find that this is given by here, I will write it as ½ or + ½ of so let us look at this, let me write it down that becomes clear.

This is fairly straight forward algebra but let me write it down. See basically what I have done is this I have taken out an overall $e^{i\theta/2}$ out so that this term now has a factor $e^{-i\theta/2}$ and this term has $e^{+i\theta/2}$. Now if I now collect the + then I get $e^{-i\theta/2}$ + $e^{+i\theta/2}$/2 which is cos θ/2 and likewise the cosin in term – becomes -sinθ/2. Remember that this is the overall phase factor and in a quantum state as we have stated earlier and overall phase factor does not manage because a state ψ and another state which is C times ψ they have no physical development. So therefore, this is not going to what it is.

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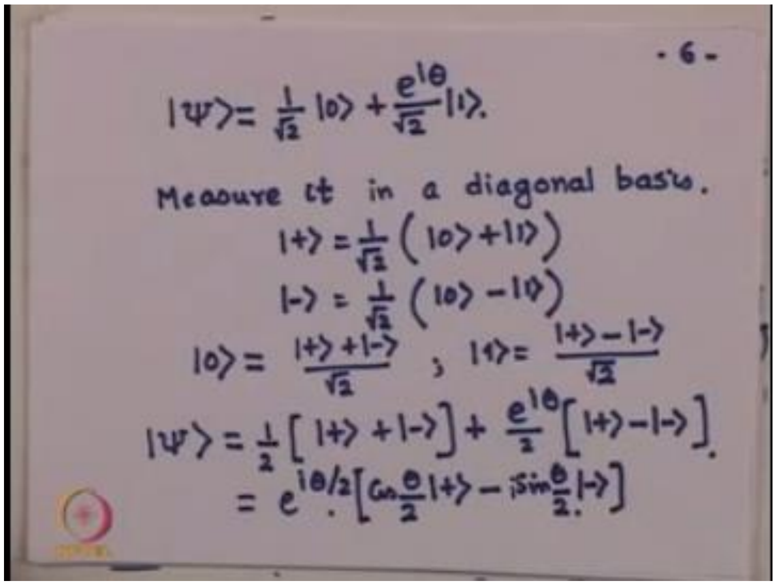

But suppose I am now making a measurement in the +, - basis. The probability with which in the diagonal basis I will get the value to be + is cosin$^2\theta$/2 and the probability with which I will get the – will be sin$^2\theta$/2. So therefore, this measurement in the diagonal basis gives us some information about the phase. So so far we have been talking about single Qubit, but notice that we all realize that a single Qubit is just one bit of a model.

It may be with linear combination it may be lot more informative or may contain lot more information that make classical bit, but nevertheless we are still talking about 1 Qubit. Now in order to do computation we need to go to more and more Qubits as you are aware in classical bits we talk about situations like where let us say 00 now suppose I talk about two bits what do I have there, I have 00, 01, 10, and 11.

Likewise in, if you take three Qubits, three classical bits and you have eight different combinations. Now in that case how do we handle this situation, now what we are looking for is the Qubits which come from a composite system looking at this slide.

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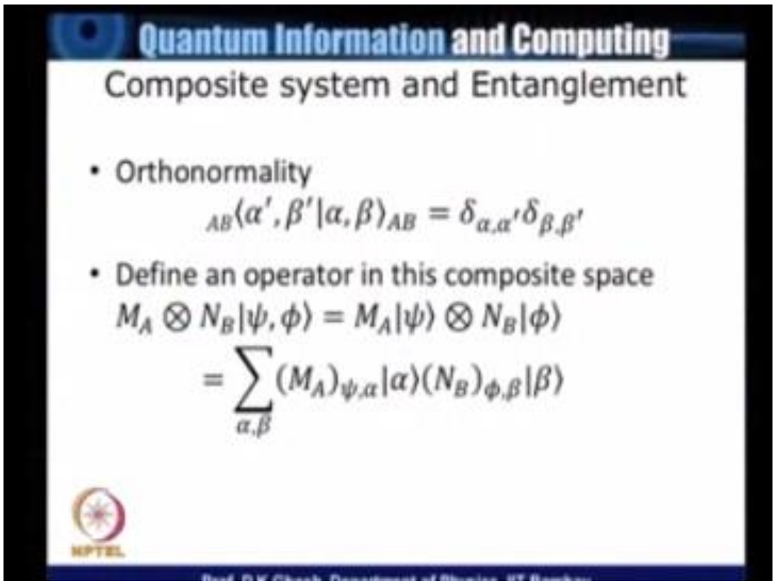

Suppose I have a composite system let me call it A and B they, and let us suppose α is a basis in the subsystem A and β is represents a basis in the subsystem B. Now I define my composite system like this that is αβ AB and if I take vectors in the subsystem A and subsystem B as α′, β′ then the corresponding bra multiplied with this ket gives me. Now analogously I can now define an operator in this composite space.

Suppose I talk about this operator this is a direct product sign M_A x N_B now it acts on state ψϕ so what happens actually is the state M_A acts on the state ψ, N_B acts on the state Φ and I know that an operator acting on the state gives me a linear combination of vectors in that subspace and likewise. So I would get a situation like this it is getting more and more complicated.

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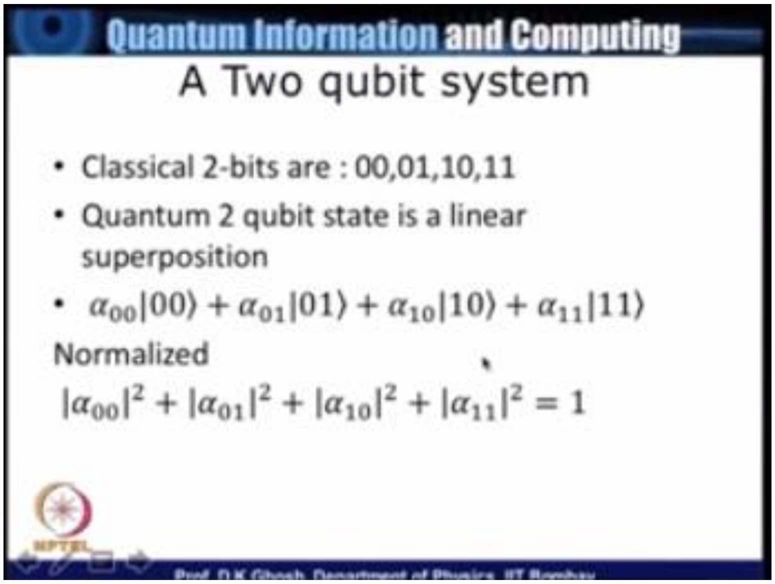

So as I said that the classical bits referring to classical bits are 0, 0, 0, 1, 10 and 11 the quantum two Qubit state will then be a linear superposition of states α00, 00 at something. The same normalized what is called α00's absolute square up to α11 absolute square is equal to 1. Now we will talk about two cubit system and its expansion to n cubit system in the next lecture.

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