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Quantum Information and Computing

Prof. Sai Vinjanampathy
Department of Physics IIT Bombay

Modul No.08

Lecture No.45

Experimental Aspects of Quantum Computing - I

Welcome everyone to these lectures on quantum computing and quantum information so today what we are going to do is discuss a little bit about experimental quantum computing right so to do that what we should first do is ask ourselves what are the requirements for quantum computing right if I want to experimentally implement quantum computing what are the requirements to do this right. So this question actually leads us to something called the DiVincenzo's criteria.

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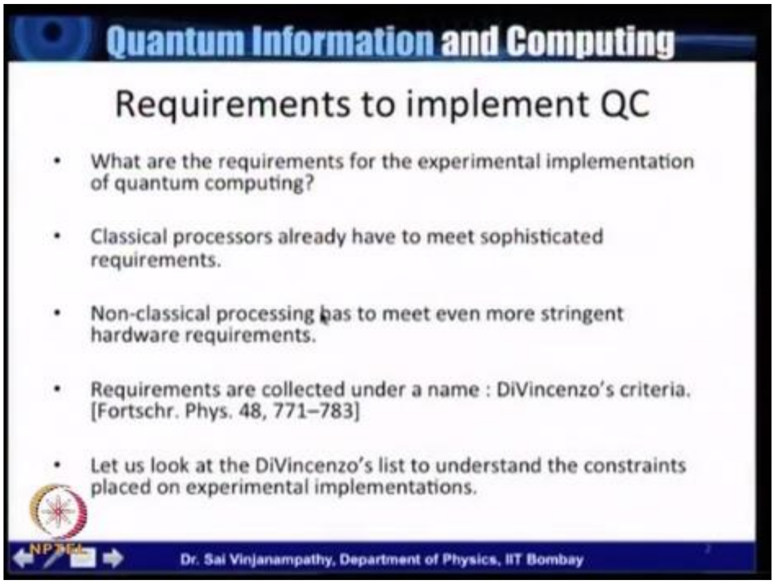

Right when one discusses the experimental implementation of quantum computing one has to realize that quantum computing has is only possible if a stringent list of requirements are met right let us remember for a second that even classical processors have to obey a list of very sophisticated requirements before they can be manufactured right inexactly the same way and non classical processing has to meet an even more stringent list of hardware requirements these requirements are collected under a name and this name is the DIVincenzo's criterion it is named after the person who wrote the paper.

So let us look at DeVincenzo's list to understand the constraints that are placed on quantum on quantum computing and the experimental implementations of quantum computer.

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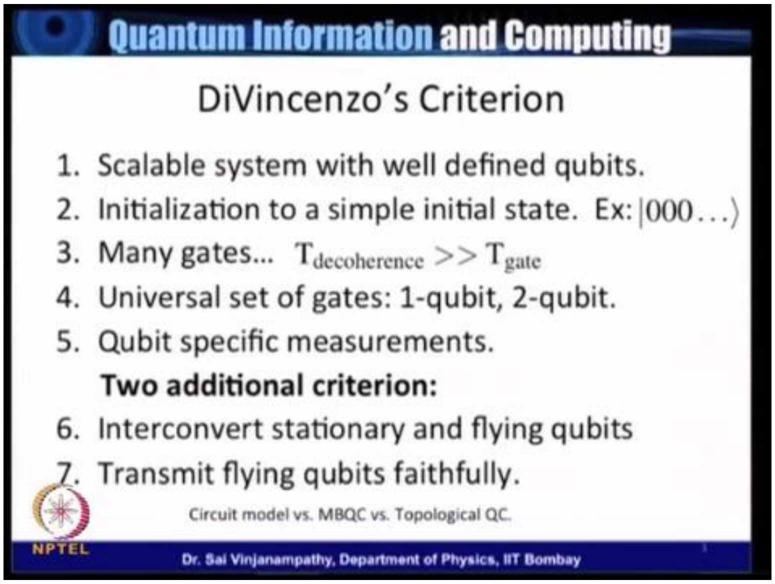

Right DiVincenzo's criterion is a list of five primary goals and two additional criteria the primary goals are the following the first is that DiVincenzo's demands that that for quantum computing to be possible there must be a scalable system with well-defined qubits right now what this means is that the individual physical qubits underneath must be must behave well and when several qubits are put together this system should behave should behave quite well right so this is the scalability requirement.

The second requirement is that the initialization of this system to a simple quantum statements be possible before any state preparation is possible the first step that that is often that is often demanded is that you must be able to initialize this quantum system in a given initial state right the for example the state can just be the ground state of let us say a spin quantum computer if it has many spins in it.

It is just the ground state of these of these spins right of the Hamiltonian and that describes these spins the third of these criteria is that many gates must be implementable before a characteristic decoherence time is elapses right this is a slightly elaborate requirement and so we will look at this requirement separately moving on the fourth requirement is that a universal set of gates should be possible this is of course very easy to understand one qubit and two qubit gates which are which compose the universal set of gates must be implementable on our quantum computer right.

The fifth requirement is that you must have qubit specific measurements right, so imagine that you have a quantum computer that has let us say 100 qubits in it and you encode the information in the first hundred qubits in the hundred qubits and then you perform the quantum computation going forward then what one must demand is that is that wherever the answer is or whatever part of this hundred qubit chain that you want to measure out to get the answer such measurements must be possible in general right.

Remember that we are discussing here the circuit model of quantum computing but if you consider alternative models of quantum computing such as measurement based quantum computing or topological quantum computing the requirements look fairly similar but with small differences so for instance measurement based quantum computing would demand that the initial the initial state of the system be a cluster state right but once you prepared this state the requirements that measurement based quantum computing has to perform Universal quantum computing are somewhat less stringent.

Likewise topological quantum computing comes with its own set of requirements there are two additional criterion that are also part of the DiVincenzo's list the first of these is inter convertibility between stationary qubits and flying qubits right if you try and imagine what the sentence means it is simply the requirement that if my quantum computer is composed of a large number of qubits like I just described then you can think of the computers having subsystems right subsystem over here which does something and then maybe I want to pass this information on to the subsystem over there for you to do something else.

And this passing of quantum information must both be possible this is the demand that you have inter-convertibility between stationary and flying qubits and the and the second demand the last demand is basically that such a transmission of quantum information be faithful right so this is the dimensions or list so to understand the DiVincenzo list what is left for us to understand is requirement number three which is basically the demand that the decoherence times be much larger than the gate part is right so to understand that let us try and understand a little bit about decoherence law right to understand decoherence.

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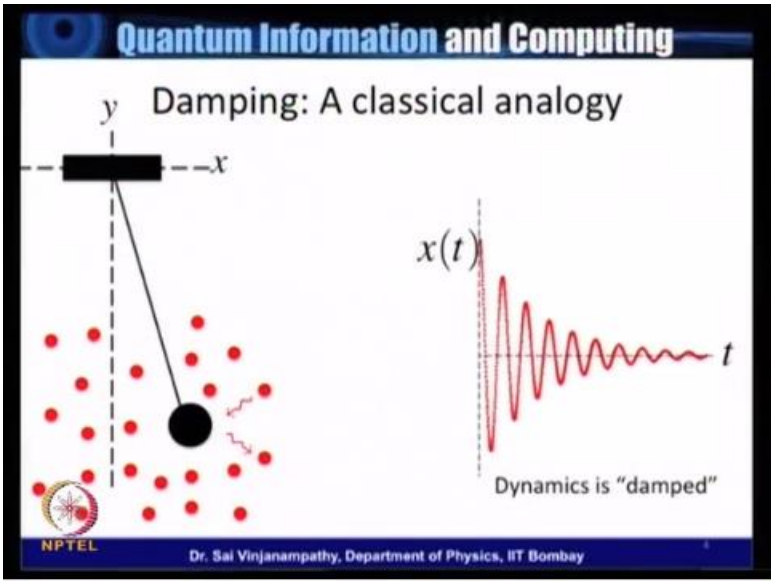

Here is a classical analogy so consider a simple pendulum that is that is that is suspended at some point and let us assume that we would like to track its dynamics let us see the x-coordinate is what we are interested in then what we would like to do is model basically what happens to this to the simple pendulum keeping in mind that this simple pendulum keeping in mind that this simple pendulum is, is suspended in air what that means is that not only is this simple pendulum simply pendulums oscillation dictated by its own motion. But there are all of these collisions with the, these atoms that I have represented here is these red point.

These atoms collide and, and carry energy and momentum away from, from the pendulum. So what we would like is to describe the dynamics of the pendulum and to do that there is the hard way and the easier the hard ways to actually write out Newton's equations for every one of these particles here the pendulum and all of the red particles that represent the air molecules and then solve this couple set of differential equations.

And then try and arrive it at what happens to just our pendulum, right. On the other hand we have developed a much better way of describing what happens to this pendulum in terms of a damping parameter, right. So when we write the differential equation for a damped harmonic oscillator or damned pendulum what we are describing is the effect that all of these molecules have on this pendulum.

And hence we see and we see a solution that looks like the one that I have plotted on the right, right. So the, the position as a function of time simply oscillates as you would expect a pendulum to do but also fades away which represents the fact that energy and momentum are getting carried away into the environment, right. Such a dynamics is called damp but the most important thing consider here is the fact that we have done two things.

One is that we have identified to begin with what the relevant degrees of freedom are for our for our pendulum, right. In this case they are the x-coordinate is what we are interested in, right. And the second thing that we have identified is that there is an effective model for the environmental interactions in this case air molecules colliding in terms of a damping rate, right. And once we put both of these things together we get a very simple differential equation which is the damped harmonic oscillator.

The solution to which is the effective dynamics from the subsystem on the system without considering the environment, right. So this is the basic plan that we would like to follow also in the quantum veggie, right?

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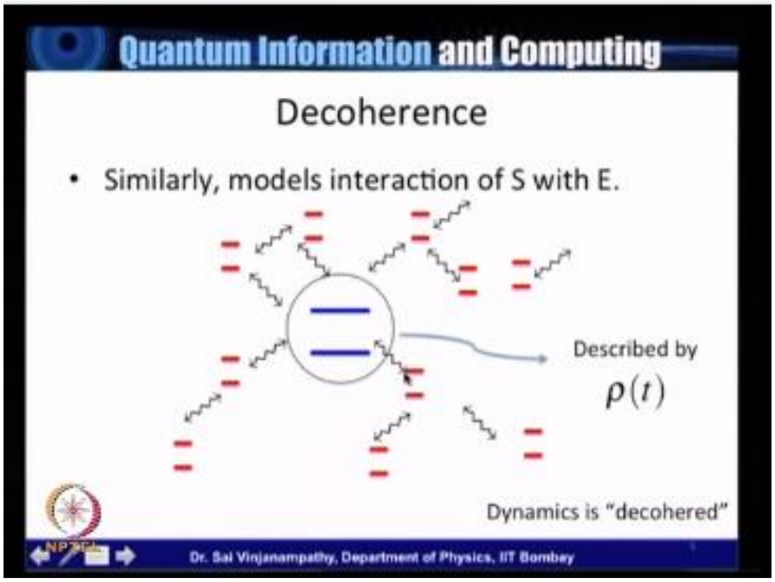

Damping the quantum analogue of damping also follows an exact exactly the same story, so here what we have is we have a qubit of our interest which is this blue qubit and there are several red qubits around it which represent an environment for instance an electron sitting in a sitting in a bath where there are nuclear ion, right? And what we want to do is just describe the effective dynamics of the system, right?

So the first thing to do is to is to identify how to describe the system and this is of course described by the density matrix $\rho(t)$, right? And then to, to discuss what happens to the to the environment we need a good model and effective model of what is happening in the environment, right.

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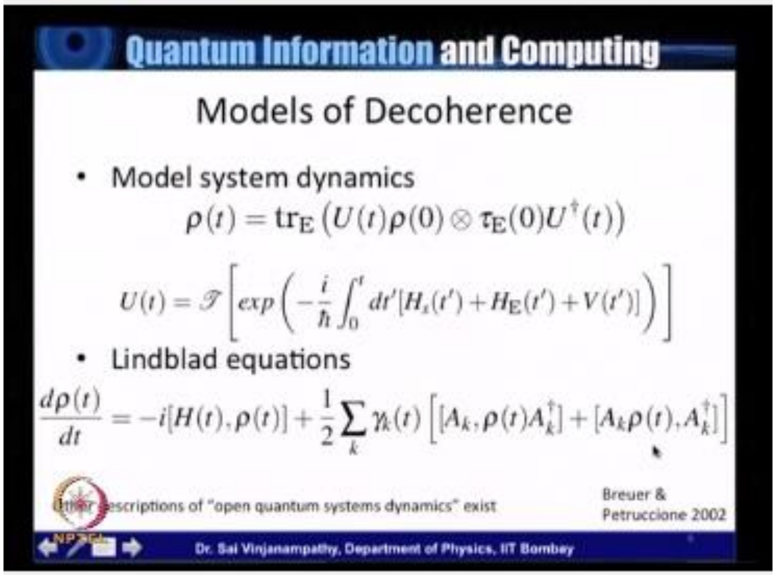

Such a model is given by considering the following so we start with the density matrix of the system at the initial time the environmental density matrix of the initial time and a joint unitary interaction which is composed of a system Hamiltonian environmental Hamiltonian and the coupling, right? And then what we do is we basically evolve this, this joint system for a while and we trace out the environment.

Because we do not we do not wish to know the dynamics of the environment, right? Once we do that under certain other assumptions which are very which are simple to stay one obtains so set of equations called the Lindblad equations, the Lindblad equations can be described thusly the time derivative of the density matrix has the usual Hamiltonian part this is just showed in this equation in the density matrix picture.

And then there are a set of terms that accompany, right. These terms are in written in terms of these damping rates gamma K(t) and corresponding operators A_K (t) what I will show you now are several examples of, of these damping rates and awe and these operators to develop some intuition about this model, right. What I would like to emphasize is we have done the exact same thing here that we did previously for the classical damped harmonic oscillator, right?

Which is we have the system dynamics being described in terms of some operators that act only on the system the environment is no where here, right. It appears indirectly through these operators A_K and $\gamma_{k,}$ right. Let me point out that other models of so called open quantum system dynamics exist and here is a standard reference, right.

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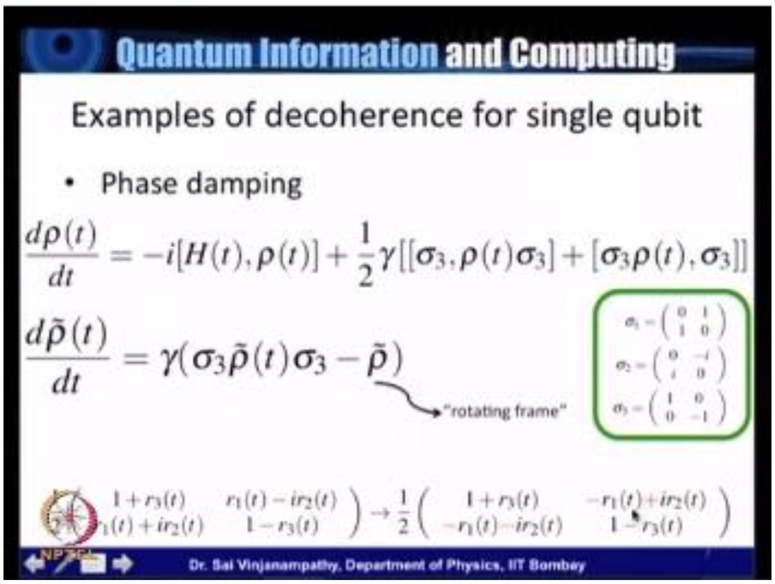

So here is the first example this is called phase damping and phase damping basically involves one parameter gamma and the operator A_K is the Sigma 3 operator of written down the poly matrices is just to remind us and the equation can be solved easily, it can first be simplified are in terms of the rotating frame what the rotating frame means is to simply go into a frame of reference that is rotating with this Hamiltonian H(t).

And then we get a density matrix ρ tilde of T and in terms of ρ tilde of T we can write the equation of motion very easily as dρ tilde of T by DT is $\gamma\ \sigma_3 \rho\ (\sigma_3 - \rho)$, right? This equation can be solved by simply noticing what happens to a density matrix ρ in terms of the three block parameters r1 r2 and r3. So what happens to this to this density matrix is that the R_3 parameters are left unchanged but the r1 and r2 parameters change signs, right?

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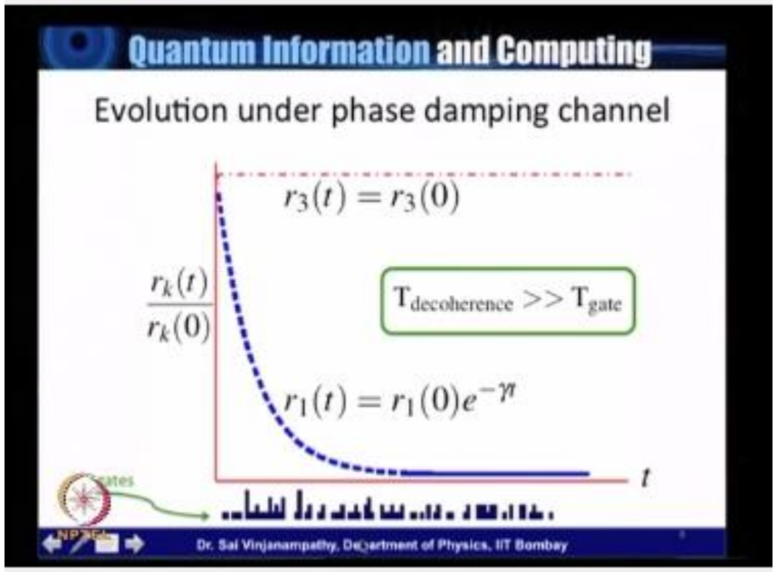

And hence what we what we can see, if we solve the differential equation is the following, the r3(t) simply remains at its initial value and what is represents physically, is it represents the fact that the populations do not change, right.

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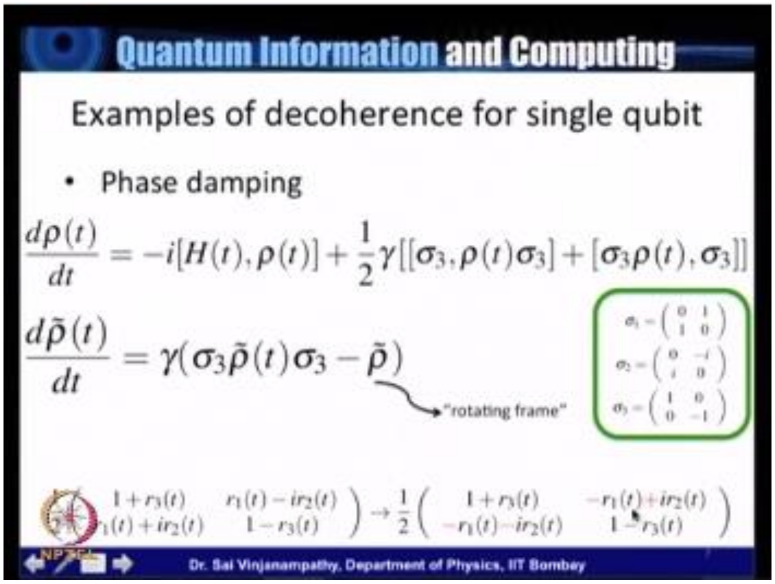

So let me go back and point out that the populations of the density matrix are along the diagonal $1+r_2$ $1-r_2$ whereas r_1 and r_2 which represent the coherence of this qubit are along the off diagonals and the off diagonals basically.

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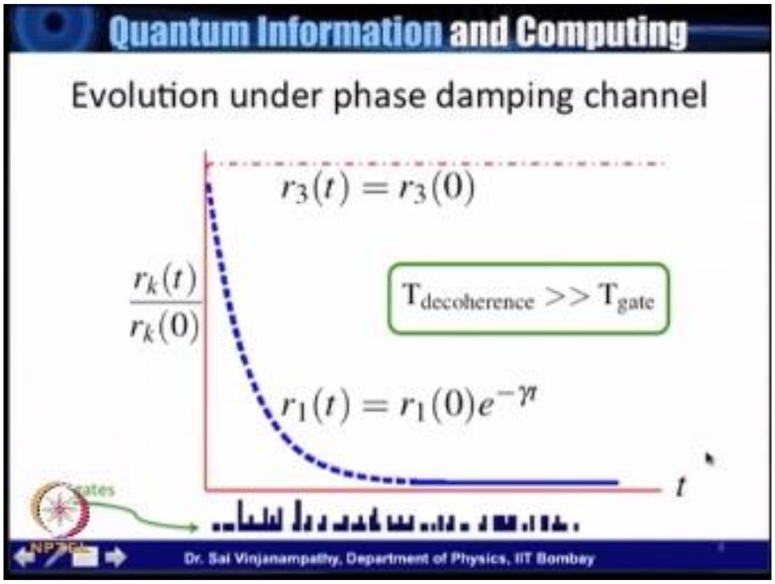

Start decaying very, very quickly and rather exponentially with this rate γ, right. So now we can understand what this criterion T decoherence much larger than T gate actually means, what it means is the following, so let us imagine that we have a quantum computation that proceed thus the, so there are a bunch of gates that we would like to apply and in this cartoon that I have drawn below imagine that the single qubit gates are given by the small bump whereas several two qubit gates are given by the longer bumps, right the taller bumps.

Then this quantum computation which is you know, which we would like to act on the system if, if this represents the real time that it takes its plotted on the same axis we have t, if this represents the real time then it takes to execute each of these gates. Imagine what happens, so at the beginning this gate is, is acting on a fully coherent quantum system and so the quantum computation proceeds accordingly.

But by the time you come somewhere here, by the time this much time elapses this computer is no longer effective because it is suffering from the decoherence that that is induced by the environment, right. And by the time become basically half way through the quantum computation that we would like to perform. The coherence in the off diagonal sense the coherence of this quantum computer is completely gone.

Now what that means is that this computation cannot proceed at all and the answers that we get are at best unreliable and more often than they not they are just not available at all, right. So this is in fact the, the problem that, that DiVincenzo is trying to solve by demanding the T decoherence be much larger than T gate T decoherence in this case is simply the inverse of γ 1/γ because our 1 decays as our1 e^{-T}/ T decoherence if I write γ as 1/ T decoherence right. So that is the entire point of criterion 3 in the in the list, so let me move on by showing you one more example of T decoherence.

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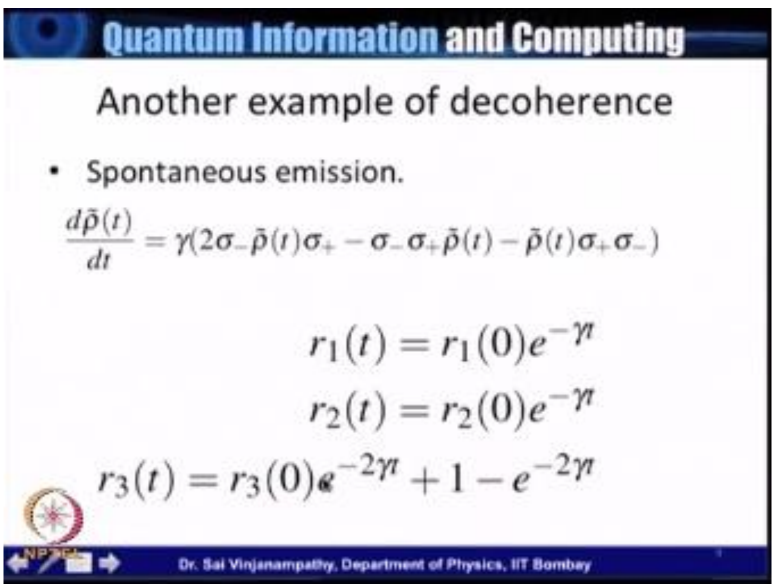

So this is spontaneous emission, where instead of σ3 ρσ 3 we have two operate σ- and σ+ and, and this differential equation can also be solved and what one notices is that the again in terms of the r_1 r_2 r_3 parameters of the density matrix both the off diagonal terms decay exponentially. Whereas the, the diagonal terms basically settle down in the ground state like this, so they settle down in a way where this where the quantum system settles down on the ground state which is exactly what spontaneous emission does.

So if you start a bunch of qubits with a lot of coherence day or the system basically loses all its coherence and the qubits all end up in their ground state. Again rendering this quantum system useless to do quantum computing, right so this is the entire point of decoherence and, and the point that is made in, in a demand 3 the of the DiVincenzo list DiVincenzo who is arguing that, that we must fight this decoherence to be able to do quantum computing effectively. So let me just take us quickly to a take-home message.

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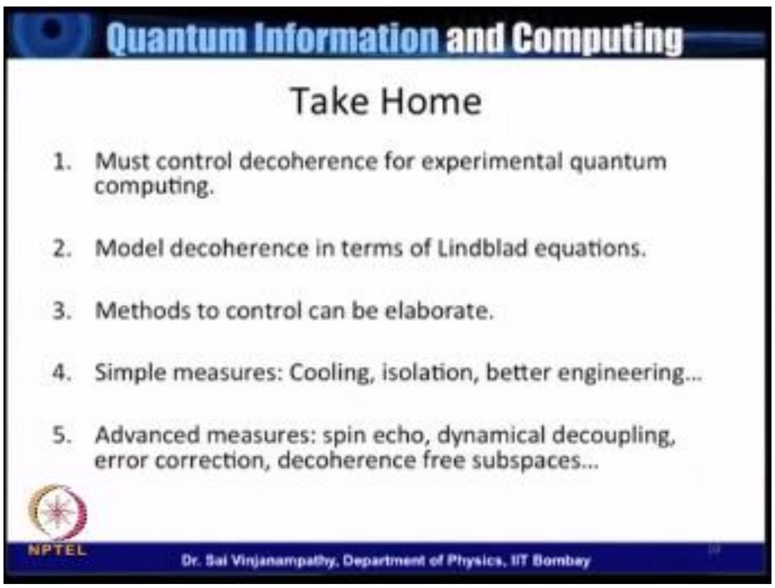

Which is the following, which is that we must control T decoherence for experimental quantum computing, right. What we saw quickly was one model of decoherence in terms of what are called Lindblad equation, right. The methods to control can be rather elaborate, right they can start from very simple methods like simply engineering a better qubit to isolating this qubit from all it is environment, right to cooling the qubit.

These are all rather simpler measures there are more advanced measures such as the spin echo technique, dynamical decoupling, error correction and decoherence free subspaces, right. And I will introduce some of these qubit. To introduce them what I must now tell you is what the popular implementations of quantum computing are.

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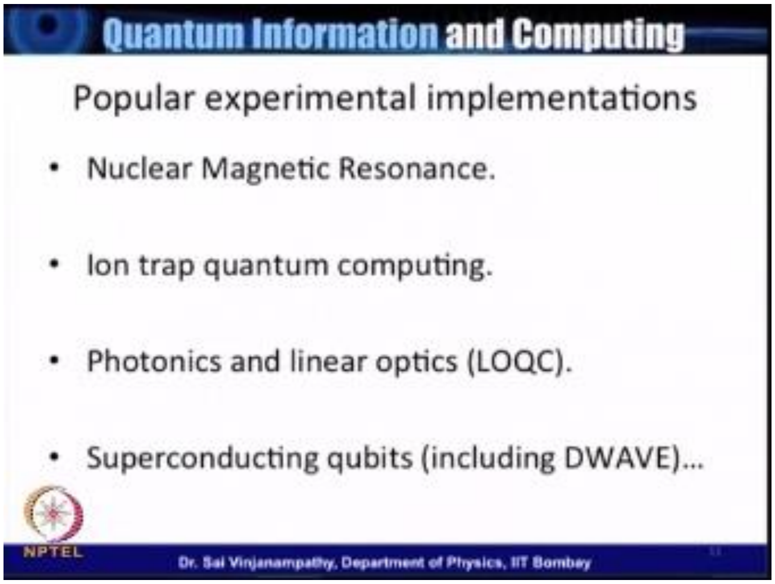

And so here are some experimental implementations of quantum computing the, the framework or the architecture over which quantum computing is built. The first of these is nuclear magnetic resonance which will consider rather elaborate. The next experimental implementation is Ion trap quantum computing which is going to be the topic of the next lecture. The third is photonics or linear optical quantum computing these are slightly separate, but, but this is just rather popular quantum computing architecture as well.

The last is superconducting qubits I would like to point out for anyone who knows what the DWAVE machine is that the DWAVE machine is in fact built on superconducting qubits. So, so let us look at NMR quantum computing.

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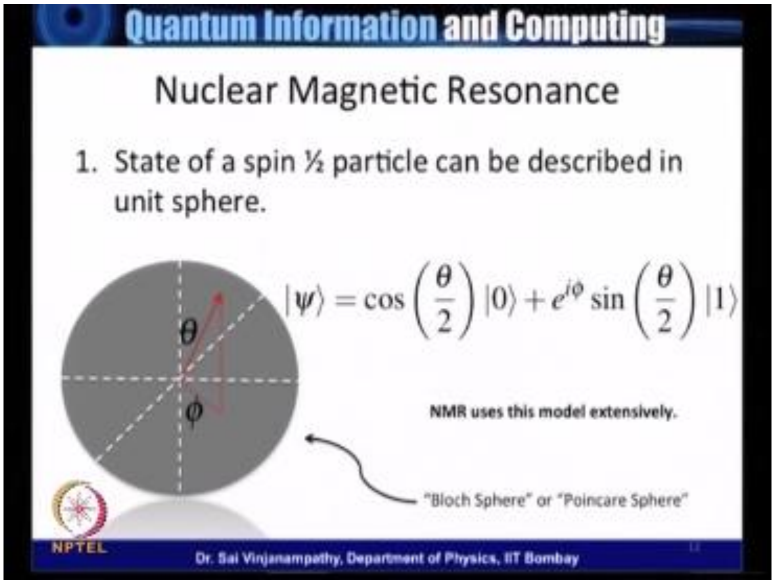

So NMR quantum computing is built on nuclear spins which are qubits to discuss them let me describe to you the blocks fair representation right the Block sphere representation is simply a representation of a pure state in terms of two variables θ and Ψ right and these θ and Ψ describe a points on a unit on a unit sphere so this blocks sphere or punker is here as it is sometimes called an optics so the Block sphere representation is used extensively in nuclear magnetic resonance and we too will make use of it in our discussion.

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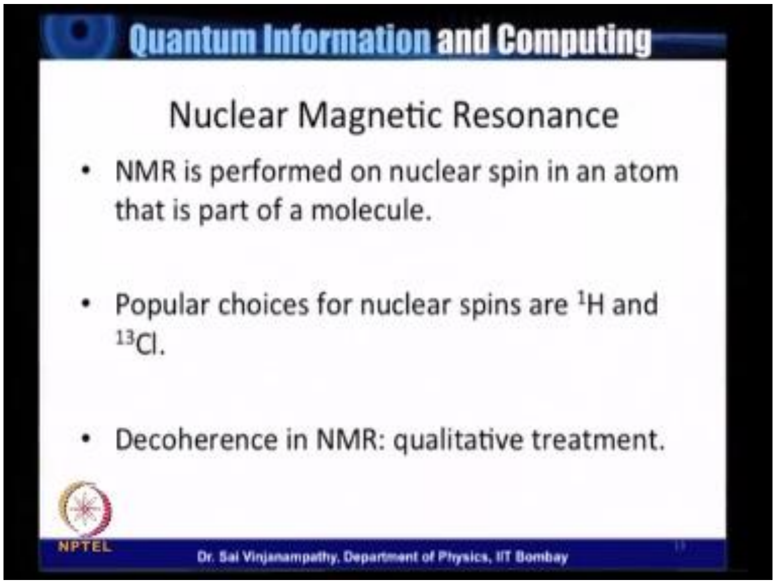

Today let me point out that nuclear magnetic resonance is performed on several nuclear spins but some of the most popular choices are the hydrogen atom and chlorine right to discuss decoherence in nuclear magnetic resonance we will again appeal to this block sway representation that I have this discussed.

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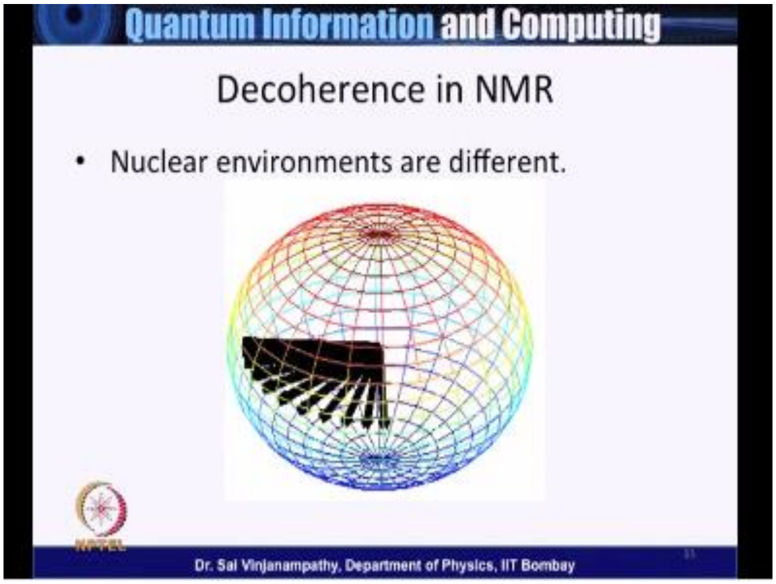

So the decoherence model that nuclear spins suffer from can be described rather simply so what happens in nuclear magnetic resonance is that typically it is either the nuclear spins are either in solid state or in liquid state so let us consider the case where the nuclear spins are part of molecules and these molecules are part of a are part of a liquid they are suspended in a liquid ach nucleus did not suffers from a slightly different environment as a consequence when the nuclear spin processes each nucleus in processes by a slightly different amount.

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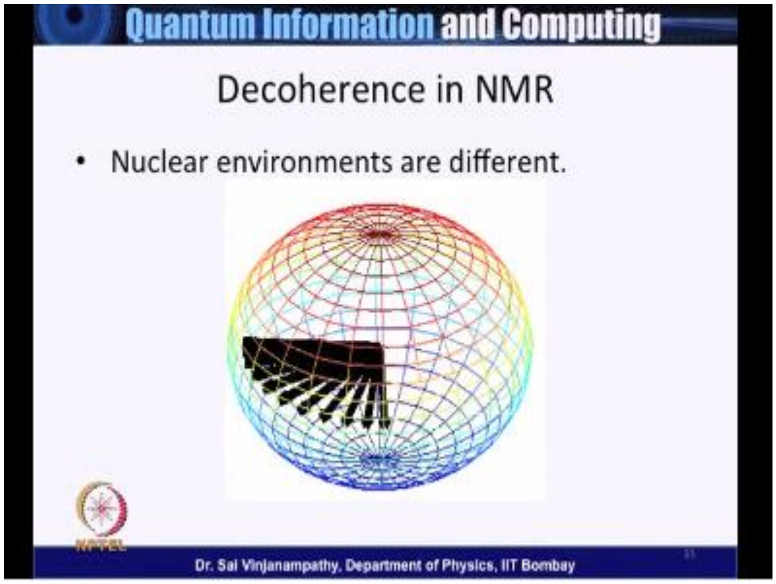

So what this produces is this figure that I have Illustrated so when imagining that the procession is happening in some axis what happens is that if the nuclear spins are all initially lined up with this central spin because their environments are different these nuclear spins basically fan out like this so when we averaged over several nuclear spins which is the signal that is received in, in NMR we see that the signal is d cohered because, because of the averaging process right so how does.

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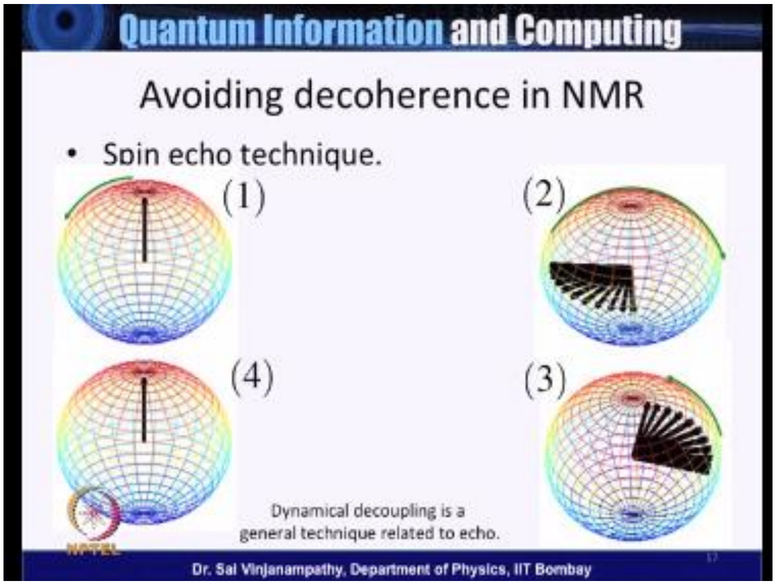

One fight this one fights this again by looking at the Bloch sphere for, for inspiration and this technique is actually called the spin echo technique so imagine that a nuclear spin is pointed along the North Pole the spinnaker technique starts by actually starts by rotating this nuclear spin along the - y axis right and then what happens is that as a consequence the nuclear spins as a consequence of their local environmental differences the nucleus in start fanning out once they found out what one does this applies a pie pulse.

Which rotates the orientation of all of these nucleus things from the - y to the + Y axis in this diagram now because the nuclear NY environments of the nucleus spins are still rotating them in the same direction what happens is that the is that all the nuclear spins start accumulating and echo here and once the spins of echo here there is another л over two pulse that is applied to bring it back to the to the multiple so such a technique actually preserves coherence over the time scale that it takes for us to do this these four operations and hence the spin echo technique is able to keep coherence over a much longer time by actually.

Performing these four steps and Rico hearing the spins at a at a later time dynamical decoupling which I mentioned briefly is a general technique related to spin echo.

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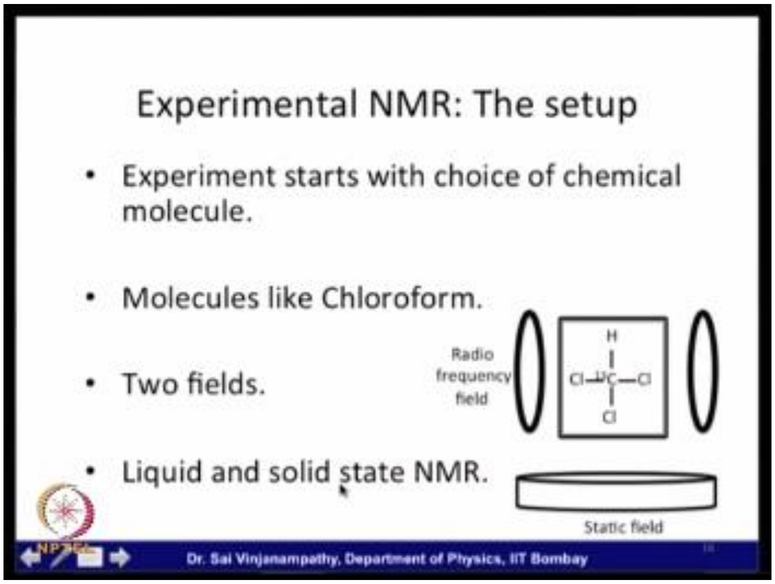

So let me describe briefly the experimental techniques that are involved in NMR the experimental setup of NMR involves a compound such as chloroform suspended in a liquid which have diagrammatically represented here is this box between to freeze one of them is a static magnetic field and the other which is a radio frequency field there are two common types of NMR liquid and solid-state NMR.

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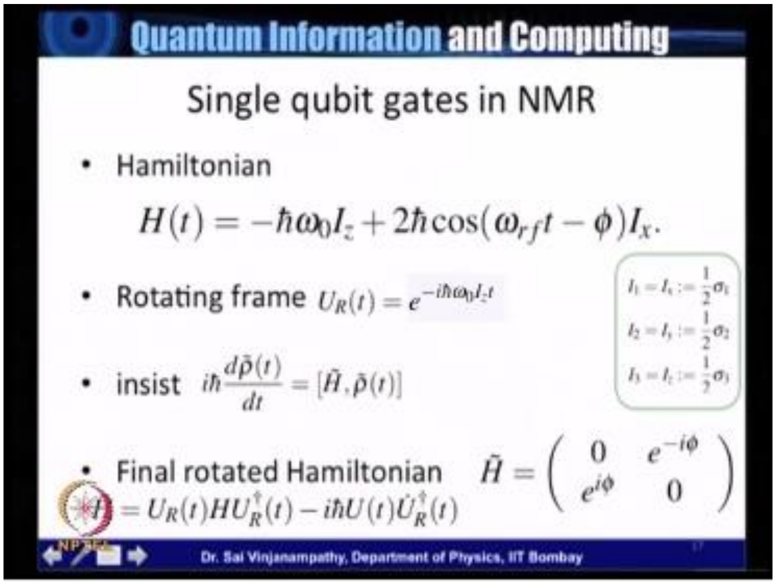

Let me describe how single qubit gates are implemented in nuclear magnetic resonance the Hamiltonian is composed of two terms and this is a direct consequence of our choice of radio frequency and static fields so the first term relates to the static field and it produces a Hamiltonian which is – h - bar $\omega_0 I_z$ here the I matrixes are simply half of the σ matrices and the second term basically is a radiofrequency times I_X right like before we can go to a rotating frame with respect to the unity e - IH bar ω_0 I_Z times T and if we insist that the density matrix in the rotating frame have the same form as the label equation which is ih -bar rho. dot = H0 then what we come up with is a final Hamiltonian in the rotating frame which basically looks like an off diagonal $e^{-i\pi}$ Phi and $e^{+i\pi}$ right.

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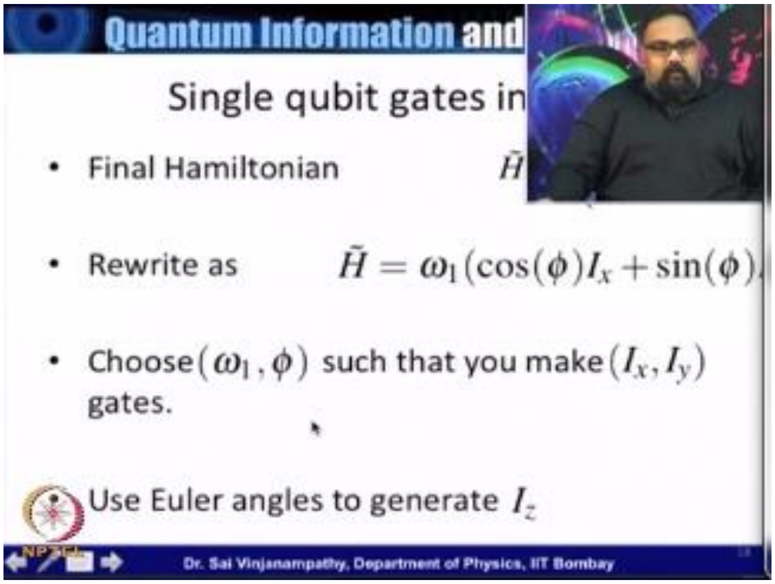

So this Hamiltonian can be written as a linear combination of the I_X and I_y matrices right which are again just the σ_X and σ_y matrices up to a half right. Now we understand how any single qubit gate is possible because if you choose different values of ω 1 and 5 then what that dons is it changes the Hamiltonian and you are able to implement either the I_X part or the I_y part or some linear combination of I_X and $I_{y.}$

As we fully know by using the Euler angle rotation we can implement any series of gates so long as we have σ_X and σ_y matrices right. So this is the way in which all single qubit gates are performed by simply choosing various values of ω 1 and 5 right, so let us now describe briefly how.

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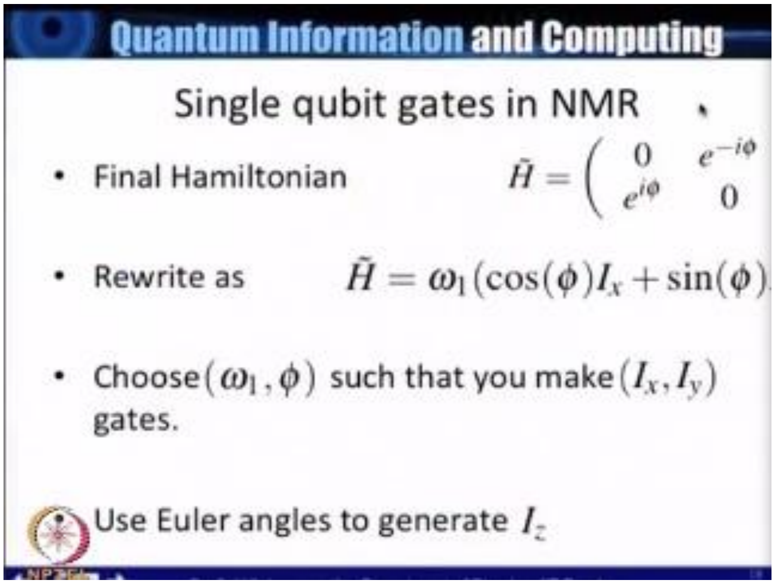

Two qubit gates function in this.

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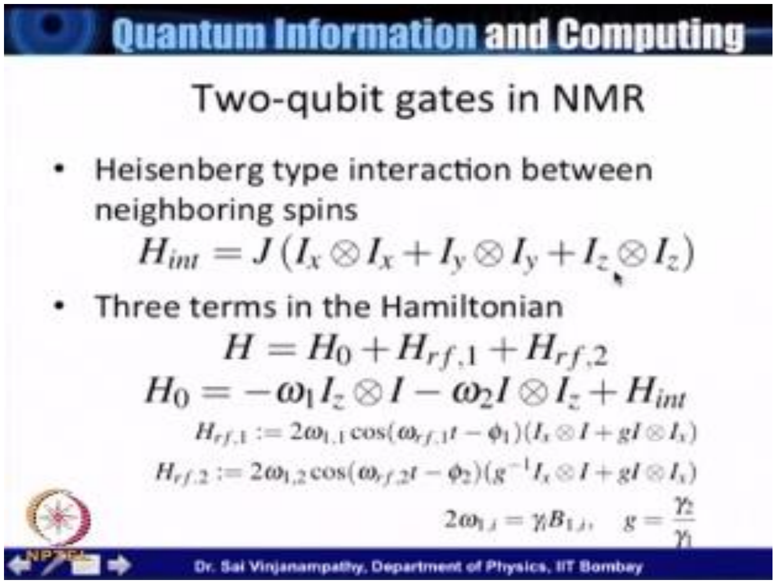

In nuclear magnetic resonance for two qubit gates to be implementable what one needs is some sort of interaction between nuclear spins. So for example, in NMR the Heisenberg type interaction is quite common between neighboring spins, so this Hamiltonian is given by I_X times the product I_X + I_y times the product I_y and I_z times of product I_z right, and the coupling ratios is the quantity J right this is the coupling J, there are three terms in the two qubit Hamiltonian and they refer to H_0.

Which basically is the is the bare σ_Z is just a static field σ_Z H_{Rf1}which refer which represents an RF field on the on the first qubit and H_{Rf2}which represents an R_F field on the second pivot right and I have written what $H_{R\ f1}$ + $H_{\ RF2}$ are here right note that H naught also has this interaction Hamiltonian which is the Heisenberg type coupling between right, so we have several operators here of perform I_x tensor I_x I_y tensor I_y I_z tensor I_Z.

We also have I_z tensor identity and the radio frequency fields bring with them I_x tensor identity and identity tensor I_x right, so by using suitable commutator between these operators what we can do is implement all 16 operators of the two qubit algebra right there are four operators relevant to the first qubit which are the identity matrix the poly σ_X σ_Y σ_Z and there are four operators that are relevant to the second qubit which are again the identity operator poly σ_X σ_Y σ_Z right.

So there are 16 operators in total and again by simply choosing different values of the radio frequencies and the static fields we can manipulate an engineer an effective Hamiltonian that basically is any one of these operators or any linear combination of these operators right, so this is exactly how two qubit operations.

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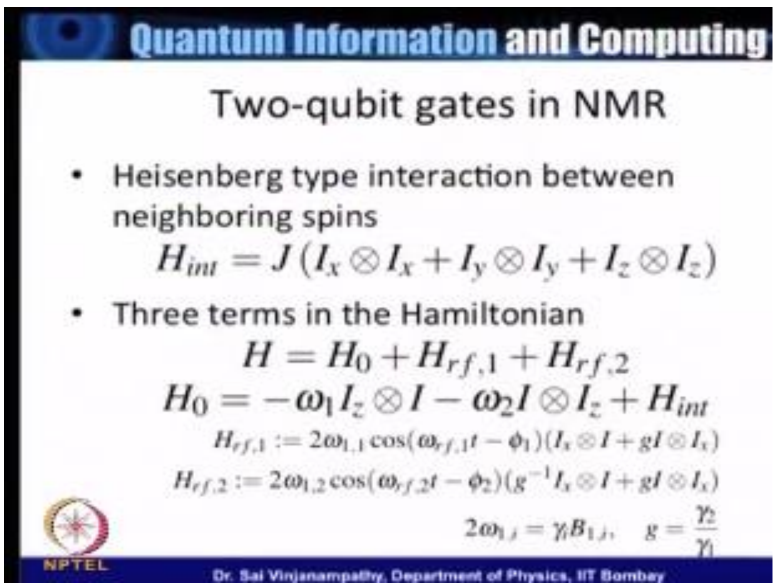

Follow in NMR.

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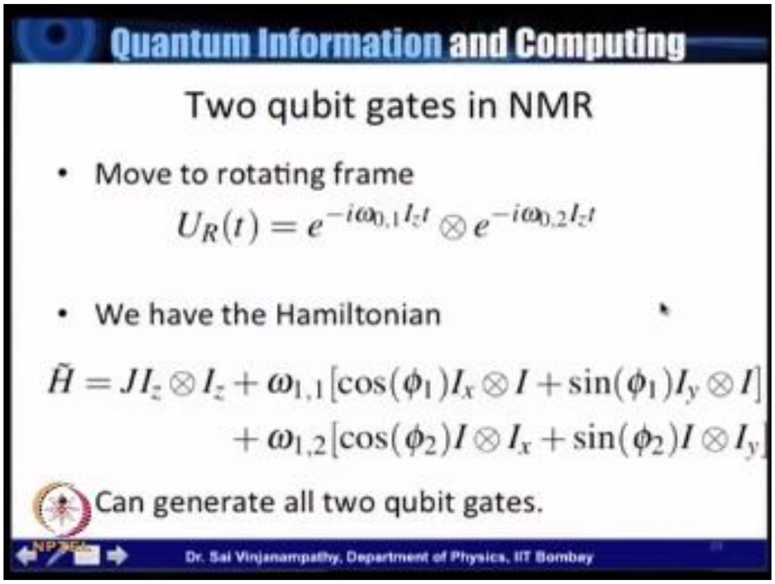

So again technically the way to do it is to move to a rotating frame with respect to the bare Hamiltonian and then we have the Hamiltonian in the rotating frame it has this ~ which is given by I_z tensor I_Z and then there are several terms which now have I_x I_y, I_X and $I_{y,}$ so we can generate all 2 qubit gates by taking H ~ and simply applying it over and over again but by choosing different values of J ω1and ω2 ϕ_1 and ϕ_2 right.

So this is the way in which two qubit gates actually proceed in nuclear magnetic resonance so to summarize.

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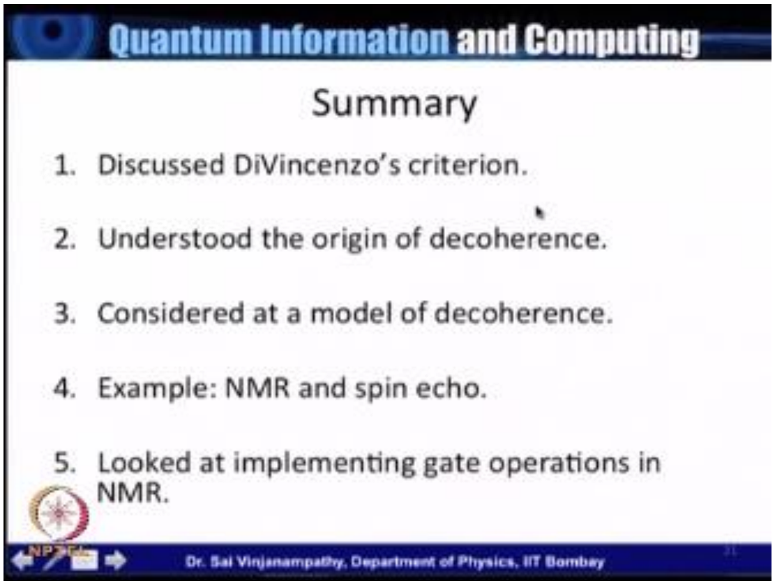

What we discussed today were the DiVincenzo criteria which are the crucial criterion for experimental quantum computing. We also understood the origin of decoherence we actually considered a model of decoherence and studied what decoherence does to qubits and how it is detrimental to the quantum computing process. As an example we considered nuclear magnetic resonance and we discussed spin echo as a way to mitigate the decoherence process.

Finally we looked at implementing single qubit and two qubit gates in nuclear magnetic resonance. In next lecture we will discuss why the NMR qubit is not a particularly well suited qubit and we will discuss an alternative qubit which is the high on trap unit we discuss how to trap ions and how to cool them and how to perform various gate operations on them and this will be a much more viable architecture for implementing quantum computing, thank you.

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Teaching Assistants

Pratik Sathe
Bhargav Sri Venkatesh M.

Sr. Web Designer

Bharati Sakpal

Research Assistant

Riya Surange

Sr. Web Designer

Bharati M. Sarang

Web Designer

Nisha Thakur

Project Attendant

Ravi Paswan
Vinayak Raut

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