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Quantum Information and Computing

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Modul No.01

Lecture No.04

Qubit- the smallest unit

In the last lecture we had talked about the basic quantum postulates and I had essentially brought in what are the quantum postulates necessary for you are being used in quantum computation. The basic idea was that a quantum state is represented as a ray in an abstract linear vector space know as the Hilbert's spaces and we said is that it is not a particular vector which represents a physical state but a ray.

Meaning thereby that there is no differences physically between a particular vector and a vector which a constant multiple of the same vector. So this one amount of subspace of the Hilbert space essentially respects any physical state having done that we talked a little bit about the type of operators which would be there in such a Hilbert space the job of the operator would be to act one such vector and take it to another vector in the same Hilbert space. And we had seen that the algebra which with we are concerned with our linear algebra.

Now what we want to do today since we are interested in computation we are going to be talking about what the smallest unit of the information that one can talk about while talk dealing with quantum mechanical region for instance in classical computation you are aware that we talk about bits which take the value 0 or 1 and the these bits which in order distinguish from there quantum counterpart I will be using the word Cbit for it.

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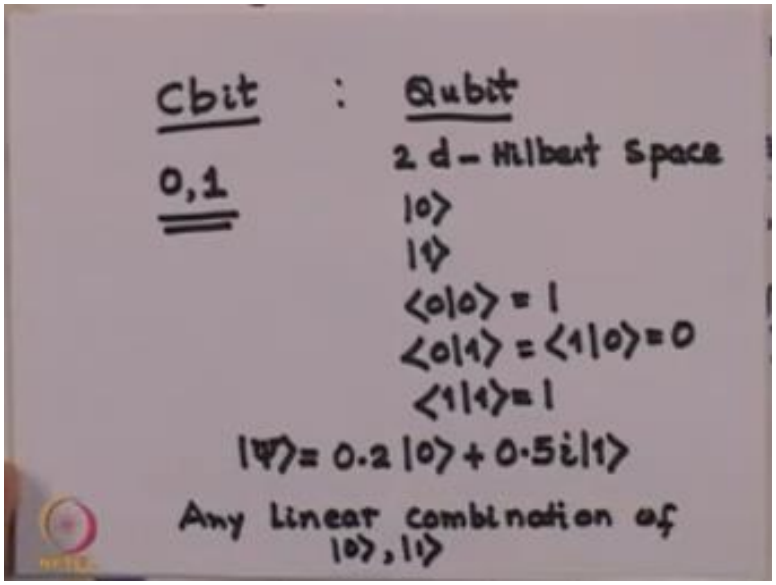

Cbit meaning a classical bit and the corresponding quantum weight will be calling it a Qubit, so if you recall my classical bits can take value 0 and 1 and all the classical computation is essentially based on the manipulation that you can do using these bits or a collection of bits. Now the corresponding quantity in quantum computation is known as a Qubit and there is a very big difference between them. So what I will say is this that these, I am working now in the smallest space that is possible.

This is two dimensional Hilbert space in which I will respect these states by a 0 or a 1 okay. The, since these are quantum states I choose them as an orthogonal bases in that state such that my 00 scalar product is equal to 1 and a 01 equal to 10 is equal to 0 and of course 11 is also equal to 1 ,this properly normalized bit. But there is a very big difference between them the difference is that while a classical bit or Cbit can take the value either 0 or 1 at a time in quantum mechanics is the smallest bit of information is can be a linear communication of 0 and 1.

For instance, in this context I will say some statements like this 0.200 + let us say 0.5i. i is the square root of -1 times 1 this makes perfect sense. So any linear combination of the basic Qubits namely 0 and 1 with complex coefficients that makes perfect sense. Now I told you last time that since we are talking about a vector in the Hilbert's space in general when we talk about and Hilbert's space of n dimensions I can represent it by a collection of n complex numbers and meaning thereby I can represent a state equivalently by means of a column vector. In this case also I also talk about the matrix representation.

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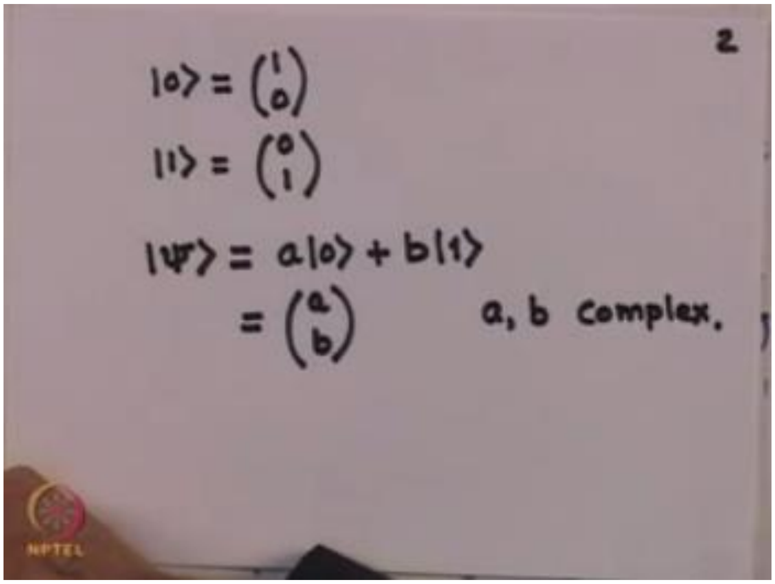

For example the state 0 we can use the matrix representation 0,1 the notice it is already normalized and similarly a state 1 is given by the matrix representation 0 1. So therefore when I say I talk about a linear combination of this 0 as 1. For instance, we talk about a times 0 + b times 1 where a at b in general are complex it is matrix representation will be a, b, so a, b complex. Now what is the difference between this and the corresponding classical other than the fact that I am talking about that a quantum state can be simultaneously in a linear combination of the basics states which are 0 and 1.

The point is that when you talk about a linear combination the amount of information that it has is of course lot more than there would be if it was either just this case 0 or the state 1. But nevertheless this information is hidden, this information is there in the Qubit but you are enable to get. Now in order that we can extract information from such a Qubit we will have to make a measurement. Now can I make a measurement you remember last time I told you that there is a Copenhagen interpretation of the state, the quantum state and the Copenhagen interpretation says there is a probability of its occurrence.

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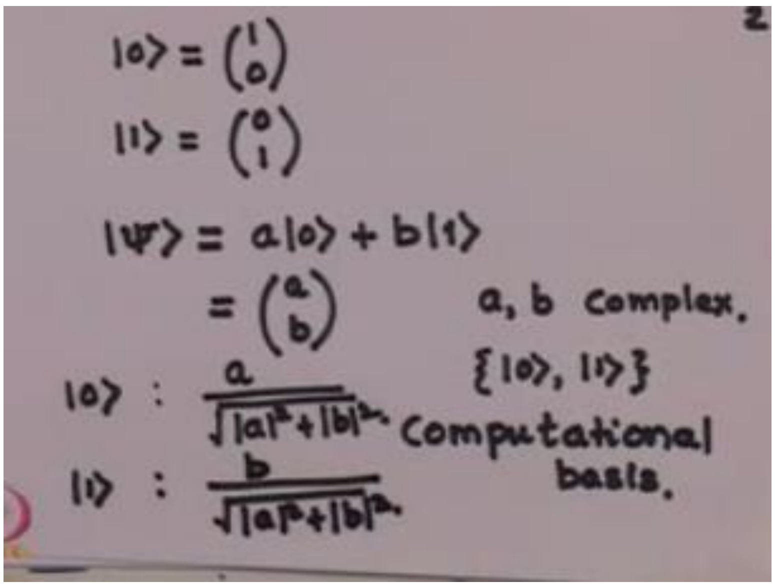

Now what happens is this that if you are trying to measure such a state in what I am at this movement calling it computational, this is you know illustrated little more as I go along. The computation of this simply means you are measuring equipment is also using the state 0 and state 1 as the bases for measuring the states. So this is what is known as computation of bases. So if you have a state like this and you are making a measurement in the computational basis, then you are not going to get a linear combination as result.

You are going to get the state 0 or the state 1 which is what I call as the collapse of the way function postulate of the Copenhagen interpretation and you will get state 0 with the probability given by a by square root of a^2+b^2 and this is the probability amplitude of this state being measured as 0, now square of that of course is the probability. And likewise the state 1 has a probability amplitude of being measured as b by square root of a^2+b^2. So probabilities are just squares of this number namely aa*/ $|a|^2+|b|^2$. So therefore the representation is measured, the quantum state is measured with the probability.

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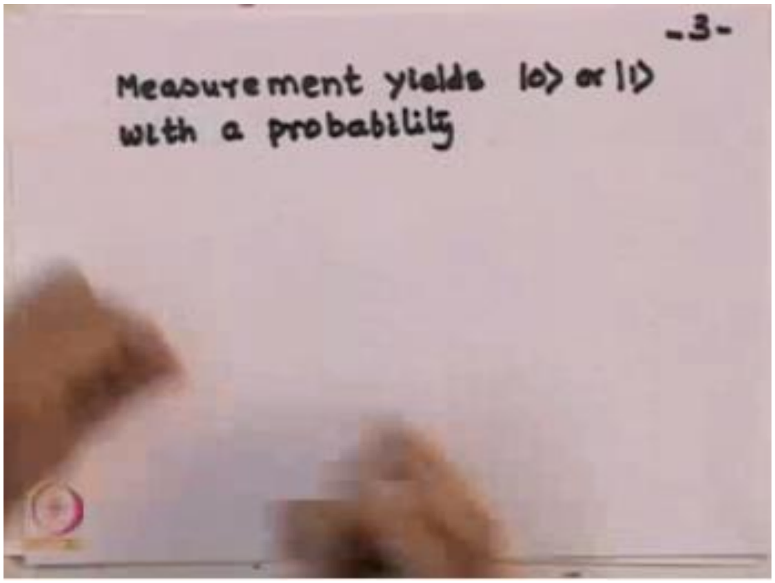

So measurement yields either 0 or 1 with a probability. So let us now see what is this Qubit, how do it get it physically? I have said already that a Qubit is the smallest unit of quantum information just as the smallest bits of classical information where 0 and 1. Now remember that even can we made this statement that the smallest unit of classical bit is 0 or 1, we always know that there is a method of getting the 0 or 1.

For instance one of the way they fetch one gets the state 0 or the state 1 as we use two electrical levels which are separated by let us say five electron five volts also. So the ground state that is the 0 voltage state could be taken as 0 and a five volt state could be taken at 1, this numbers may differ but that is roughly the idea. Now the part of this what do I mean by a Qubit? What type of physical system it represents. Now there are many way of looking at it. But basically what it means is that if you have any bi-stable system meaning thereby a quantum system which can exist in one of the two states at a time.

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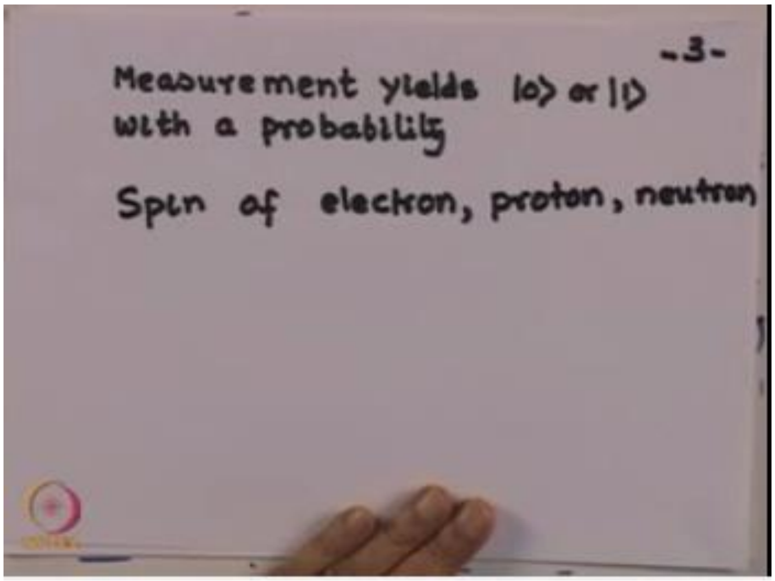

So for instance, the most common way of understanding it is to realize that to talk about spin of it could be spin of an electron, it could be anything which is been our for instance. For example, it could be spin of a proton, a neutron and I do not have to necessarily restrict my result to the elementary particles like this, I could even be talking about the silver atoms spin and things like that.

Now when such a system an electron, proton, neutron or a silver atom or whatever you have, is put in a magnetic field, which we will take it as arbitrary z direction, then the, there are two values of the spin that comes with it. This that component of the spin can be either a plus half so this is what is written.

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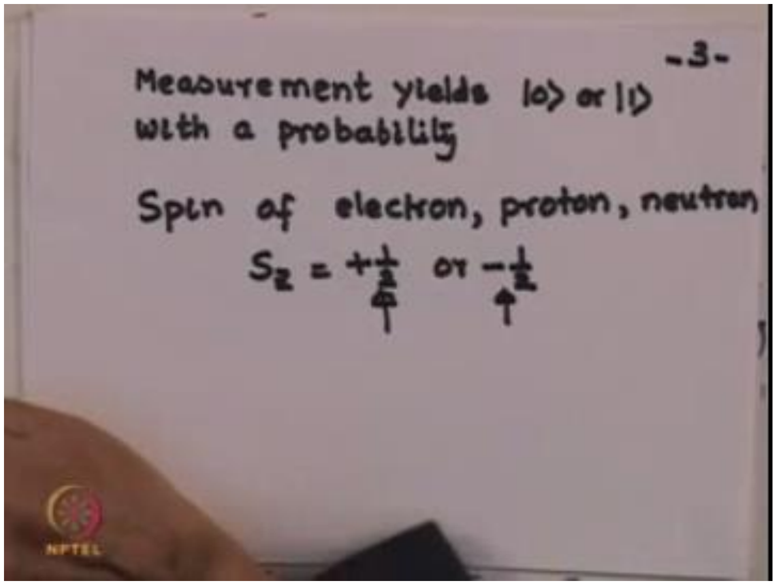

An S_z is equal to either a plus ½ or a – ½, so this could stand for one of the states let us say 0 and this could stand for the other states let us say 1. And the equipment with which you controlled is what is known as the [indiscernible][00:12:31]. But a much easier way of understanding or realizing this, is to talk about.

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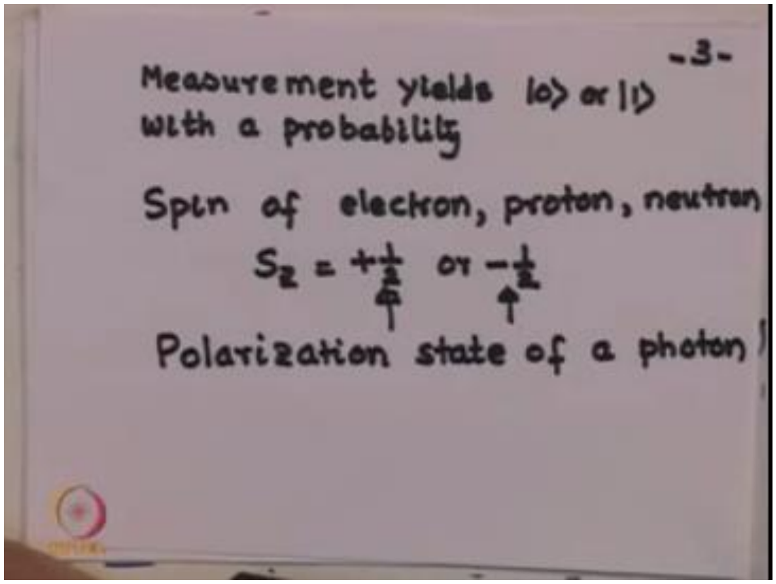

The polarization state of a photon as, you know photon is the essentially the particle nature of life, so if you have a single photon which let us say is propagating along the z. Now such a photon has its direction of polarize either along the x direction basically the direction of polarize and is in the x, y plane which is perpendicular to the direction of the propagation. So therefore, since as you know that the electric field and the magnetic field associated with such a photon will be mutually perpendicular if I am talking about a photon which is polarized.

So I can take the electric field associated with it, along one of the directions let us say the x direction and so therefore, again I am talking now about two mutually perpendicular direction perpendicular to the direction of propagation. So if the direction of propagation is the z direction then I can take x direction as one of the direction of polarization which I will call as the horizontal direction.

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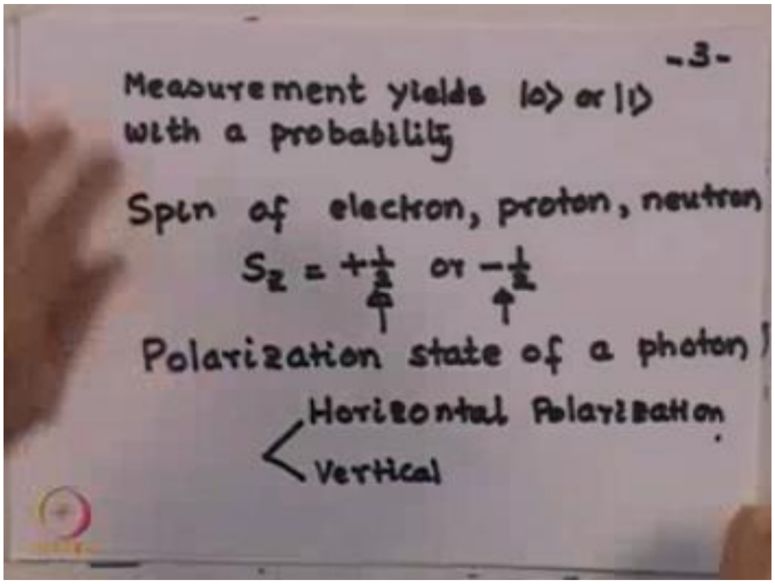

And the other perpendicular direction and the y direction I can take at the vertical polarize, these are just nomenclatures. Now this is just illustrated, this is just illustrated because what one could do is instead of taking along the x and the y direction I could also take for instance if the photon is propagating along the z direction I can take all right.

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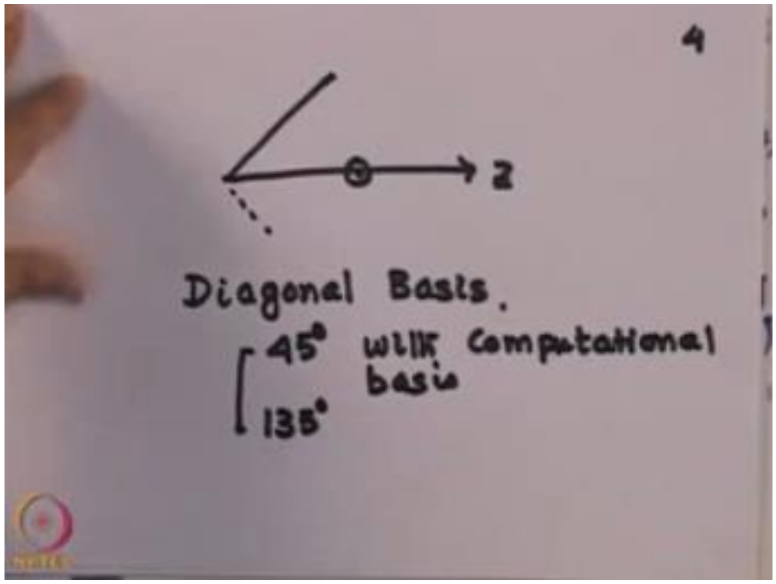

So this is my z direction of propagation and I can take one of the directions as making an angle of 45 degrees with the z direction. The other one making a 135 degree to the z direction, so this is does not in the same plane the ideally above I should put them and it slightly different notation. So this type of a basis will be called as a diagonal basis. So in a diagonal basis one of the directions is taken as making 45 degree we let us say x axis or 45 degrees with computational basis.

And the other one is at 135 degrees with z direction, so this is at there. Now if I represent, if I represent the state which is making 45 degrees angle with the horizontal direction I can write that state.

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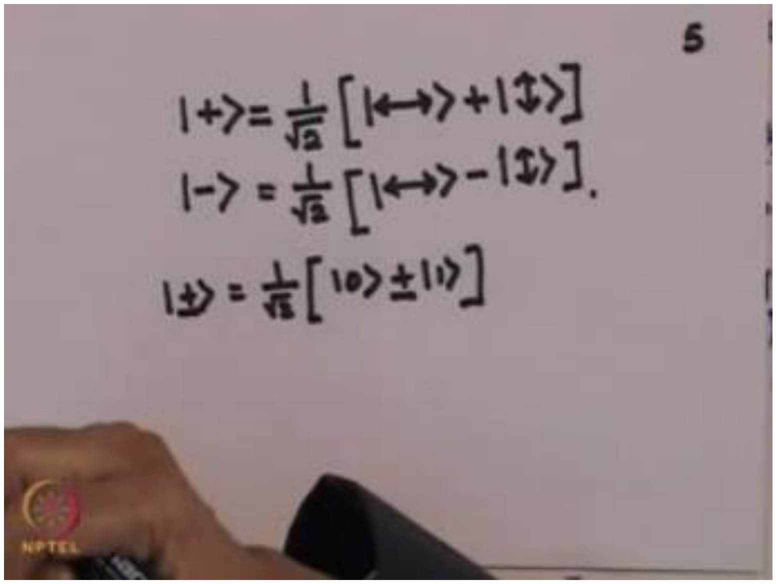

As a plus which is a notation I use and normalize it one over square root of 2, okay this is pictorial I could write it as 0+1, but let me write it pictorially as horizontal plus vertical, which is just another way of writing, one over square root of 2 to 0+1 and the minus state which is one of over square root of 2, -1. This is not a very convenient notation to used in writing so therefore, we will be using that plus is equal to one over square root of 2 of 0+1 and minus is one over square root of 2, 0-1.

So these are two possibilities, we could also consider a situation where I take an atomic system when the electron is in the ground state of the atoms, that is one of the state and when the electron goes to the excited state that is another state. So for instance, just to make it clear supposing you take hydrogen atom, I know that in the ground state and the electron is in the ground state the energy is -13.6 electron volt.

Now in the electron is excited to the next excited states and is equal to two states, then I know that the energy is -13.6/4 and I compose. Now you know it is there is a substantial difference between the two steps. So therefore, I could not use when the electron is in the ground state or the one of the states stable state when it is in the excited state it is another state. So are the spend a half system or buy stable system or there the only possibilities.

Though we will be talking only about such situations in principle for quantum information process it is possible to think of a situation where we talk not about two states but about let us say three states, a state of triplet, the triplets of an atom atomic states. Such states instead of being talked about as they Qubit will be talked about as a Qutrit other than talking about this names I will really have anything to do with this.

So this is basically three state system and in general if you are talking about a D-dimensional system the corresponding unit of information is called a Qudit.

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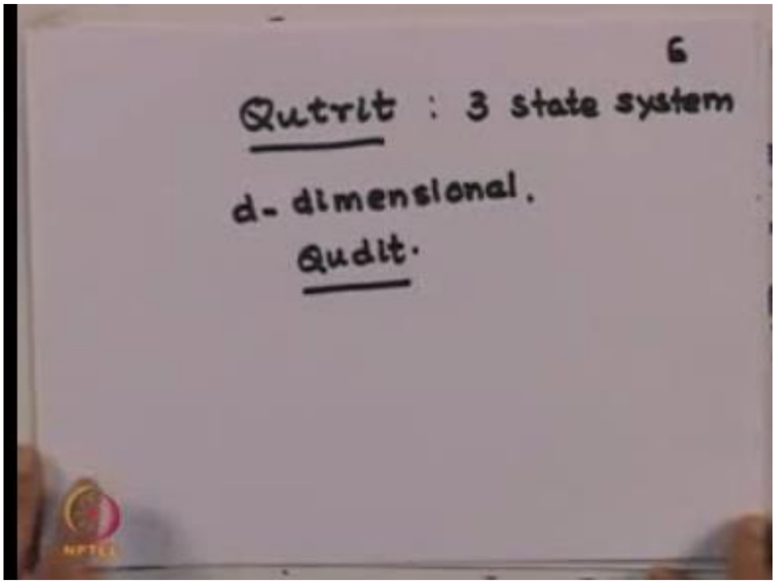

But these are just nomenclatures and will have the relative to do with it. So I will continue with my discussion of the Qubit.

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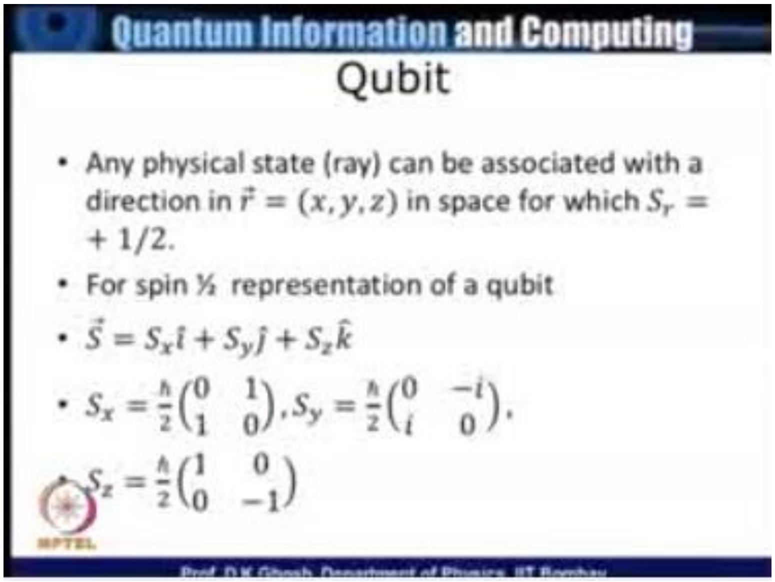

So basically I am talking about a situation where I have a y stable state the as I said a physical state will be associated with a ray.

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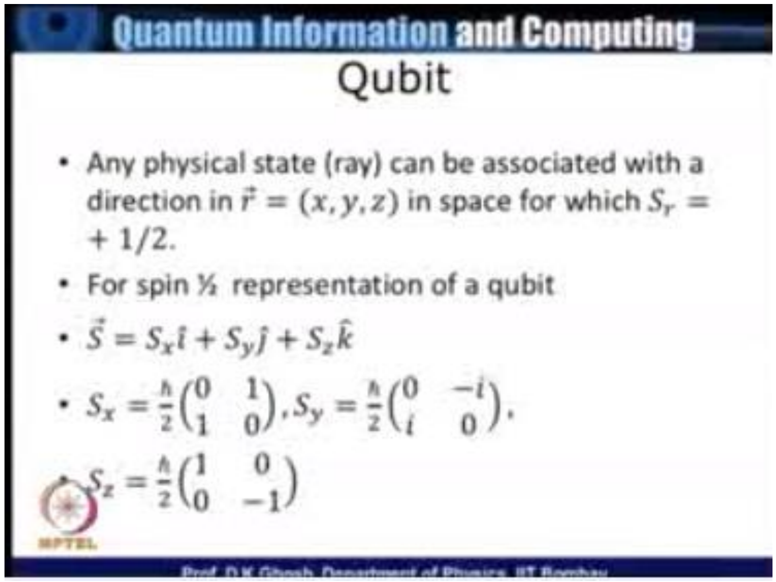

Which is the direction in space, now there is a beautiful lecture I have seen that works if you associated with the ray with the space direction I have written it that direction as r which is the x, y, z, componential states and the direction is such that the r^{th} component of the spin is equal to ½. So let to us look at the algebra associated in such a thing, now we are aware that for spin ½ the Qubit representation is given by vector as is equal to S_xi + S_yj + S_zk this is an important point which is used frequently so I would spend a bit of a time on it. The S_x is the x component of the spin.

And as we know that the spin projections along any direction has a value either +ħ/2 or –ħ/2. Now if you look at the spin operators then the x component of the spin operators is represented as ħ/2 a matrix (0, 1,1, 0) so let we come back on the, so S_x is h/2 (0, 1,1, 0). Now remember I am still in the 2 dimensional Helbert space so that my operators are too big, because they have to act on states like αβ or a, b and giving me a state of this type.

So therefore, the operators which act on such a state they are of this structure. Now S_y is equal to ħ/2 (0 –i, i 0). S_z is ħ/2 (1, 0, 0, -1), now you do wonder why am I writing like this that these mattress which are written as σ_x ħ/2 σ_y and this is ħ/2 σ_z these are known as Pauli matrices.

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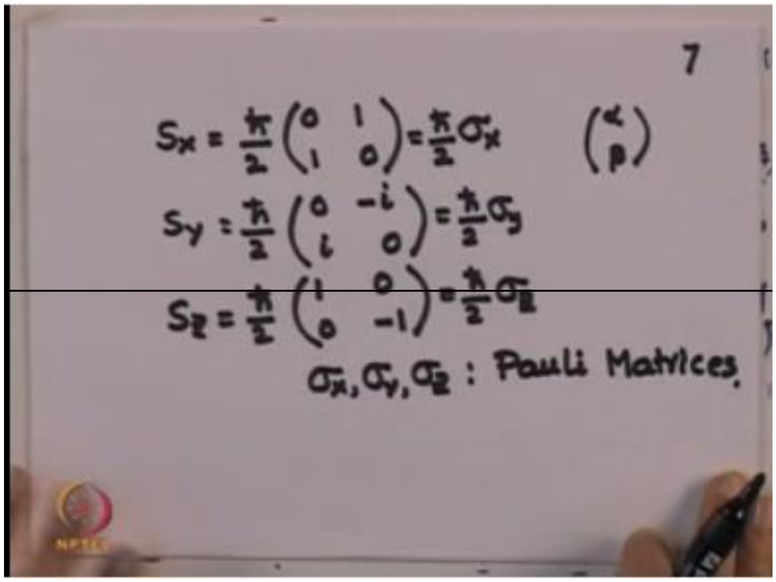

The Pauli matrices have a very important role for play in deciding what, how spin vectors there states comes from and so therefore, we would be dealing with it quite a bit and other thing is that it can be shown that along with identity matrix, now identity matrix is obviously (1, 0, 0, 1), if you take this collection of Pauli matrices any 2 by 2 matrix can be represented as a linear combination of these four matrices.

This Pauli matrices, they do not commute along themselves we will be talking about that a little bit. Now I talked about a beautiful geometrical representation.

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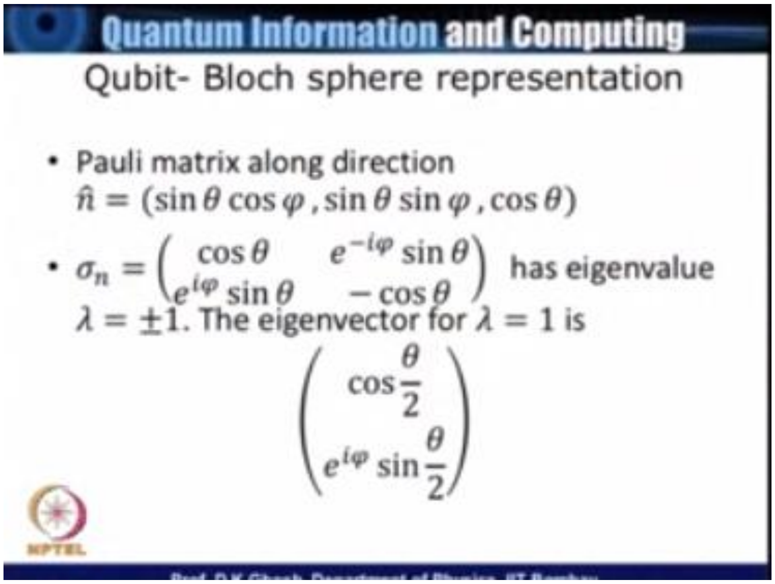

So I said that what we do is choose a direction along h the r[th] component I am choosing a direction r or if you like a unit vector n along which the component of the spin has a value ħ/2. Sometimes the ħ is you are having not explicitly mentioned but it is always understood.

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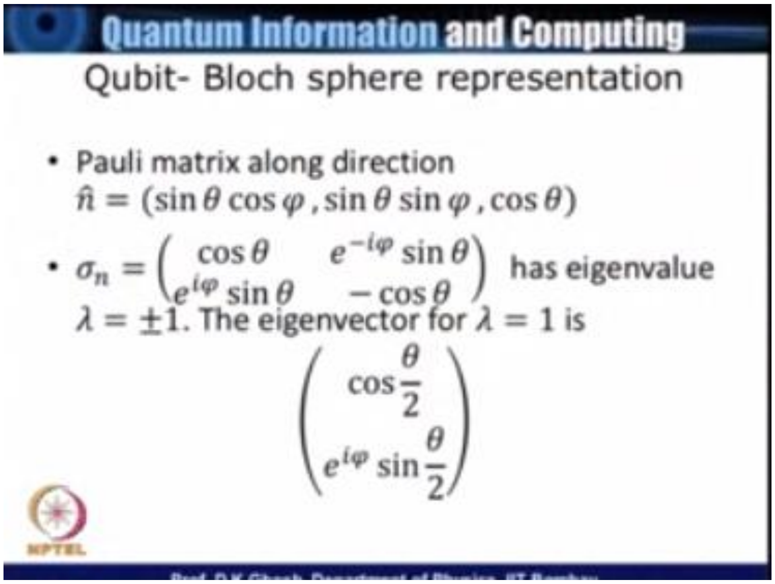

Now let us look at a Pauli matrix along an arbitrary direction. So I have given out the Pauli matrices along the x, y and the z axis. Now what we will do in the next lecture is to talk about a representation of the Pauli matrix okay in an arbitrary direction. And so far we have been talking about the spin projection, the vector along the z direction. Now what we will do is this we will find out what are the Eigen values and the Eigen vectors of a Pauli matrix long an arbitrary section.

And then out of that we will choose that direction in which the Eigen value of the Pauli matrix happens to be +1, remember that I said that the spin has a projection $+\hbar/2$. So I am remembering that $\hbar/2$ because the spin vectors and the Pauli vectors have a factor $\hbar/2$. So therefore, I will be interested in that direction in which the Pauli matrix vector has an Eigen value +1.

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