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Quantum Infromation and Computing

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Modul No.07

Lecture No.39

EPR and Bell's Inequalities=II

In the last lecture we introduced you to what is known as Einstein Podolsky and Rosen paradox in short the EPR paradox and just to give you take you back to the last lecture.

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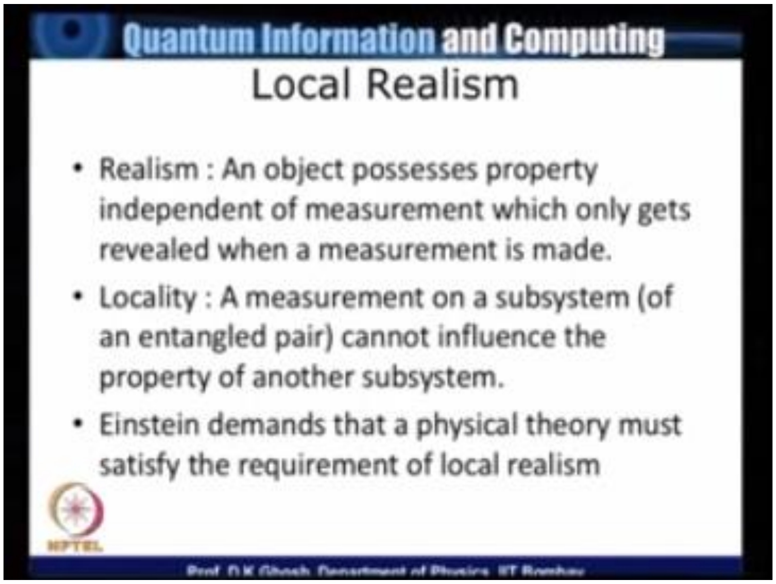

We said the Einstein insisted that an object must possess a property in dependent on the process of measurement while the quantum mechanics is said that it is during a measurement that the particle acquire a property there were two elements 11 realism and the second one is locality so we said that a measurement on a part of a system for instance in our case the entangled pair cannot influence the property of another system another subsystem if they are well separated physical and Einstein demands that any acceptable physical theory.

Must satisfy the requirement of this duel property of locality and reality we then introduced a person due to David bomb which talked about a particle of spin zero disintegrating into to spin a particles and then of course because of conservation of angular momentum the state of the particle must be that of a singlet.

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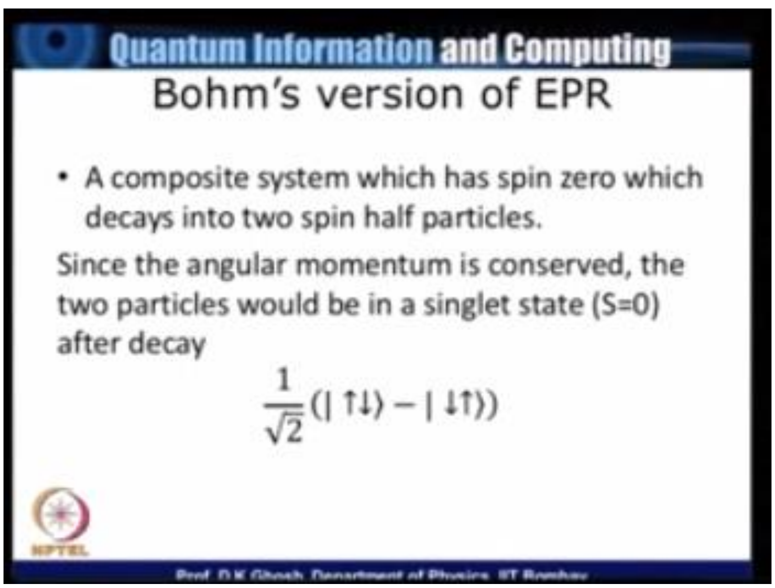

And that is given by 1√2 up down minus down up which is another way of slighting down 01 - 1 0 in our qubit model.

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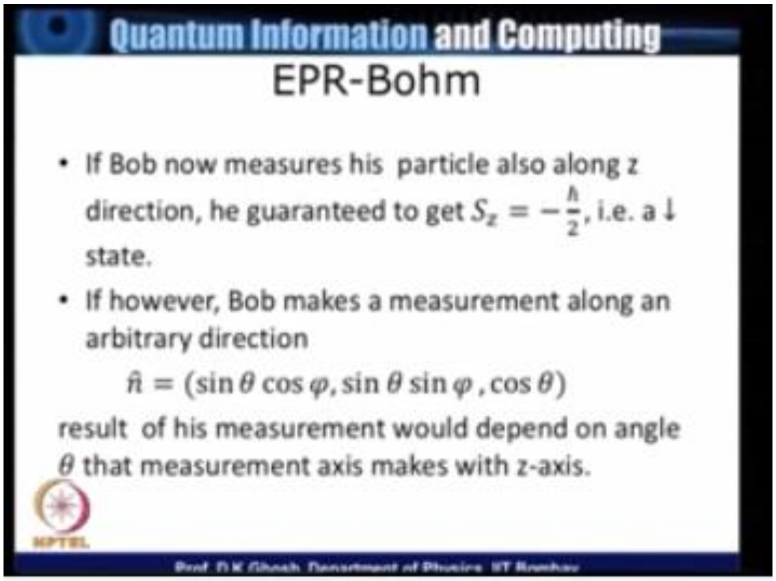

Then we said that if bob is to measure the particle also along z direction if Ali's had measure her particle that is particle number 1 along the direction and got as the difficult to h /2 bob is guaranteed to get is an equal to minus h cross wait that is a down state in other words there is perfect anti correlation between LSS measurement and Bob's measurement even though that to our measuring on two different particular now let us look at what happens in Bob instead of measuring along the z direction decides to make a measurement along and arbitrary angle θ ith the red access the usual θл coordinate in the spherical coordinate system.

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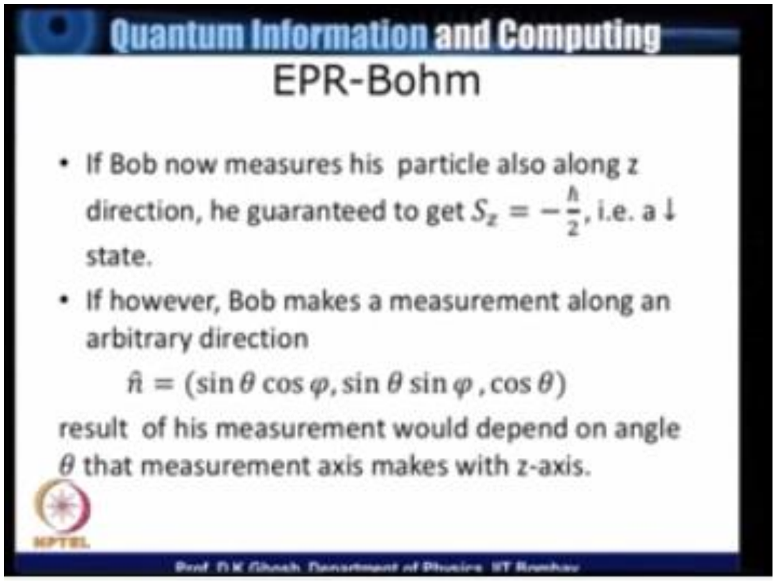

So we have an n arbitrary direction n.

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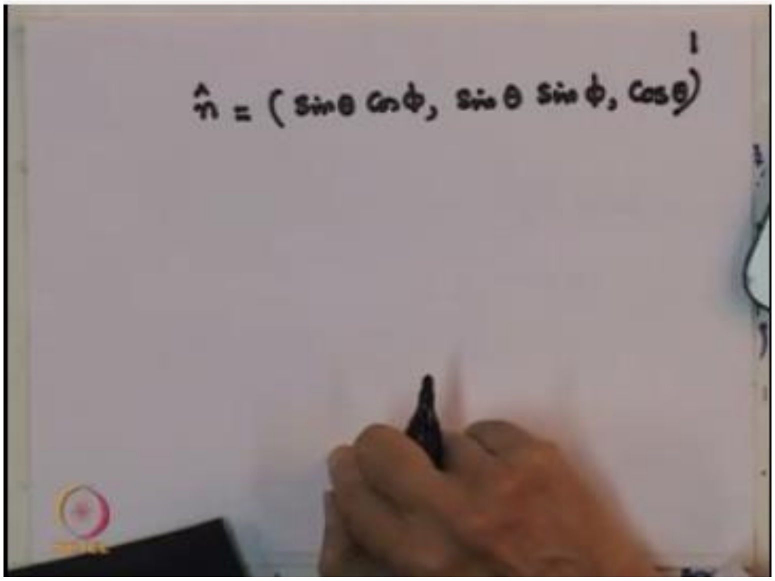

Which is given by the Cartesian component sinθ cosϕ, sinθ sinϕ and cosθ now we will show that though the anti correlation that exists now is not perfect in the sense if l is got +1 Bob would get minus 1 and vice versa there is still some correlation between the two measurements and in order to do that.

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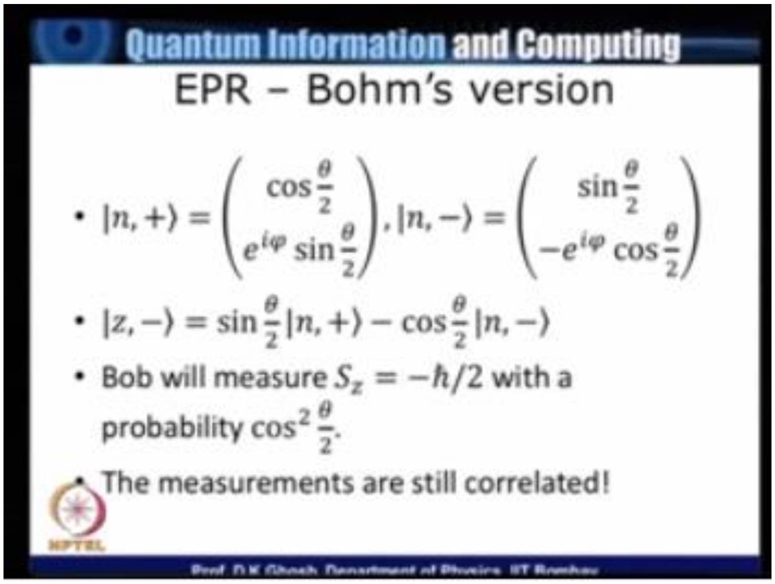

Recall that if Bob's measurement is taken along the arbitrary n direction.

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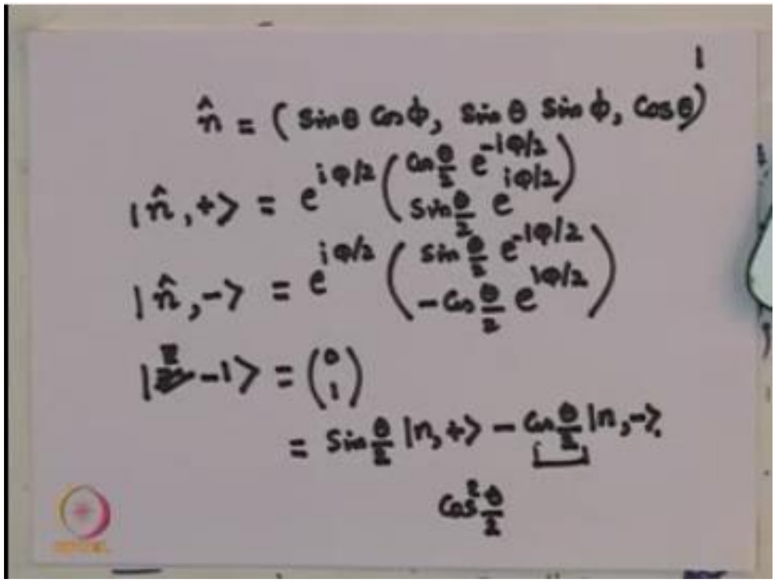

Then the eigen state of sigma n having an eigen value + 1 is given by as we wrote down $e^{i\phi/2}$ $\cos\theta$ /2 x $e\text{-}^{i\phi/2}$ this is a symmetric way of writing and we will always ignore this overall global factor $\sin\theta/2$ $e^{i\phi/2}$ and the eigenvector corresponding to - 1eigen value is given by $e^{i\phi/2}$ $\sin\theta/2$ $e\text{-}^{i\phi2}$ and $\cos\theta/2$ which $e^{i\phi/2}$ now notice my Sigma z if I am looking at the eigen value of Sigma z corresponding to z = - 1 Sigma z the eigen value corresponding to -1 along the z direction let me indicate it by like this.

Now that is we have seen is 0 1 state now since we said that if bob had to measure it is along the z direction he would get - 1 so he would get this state now you can it is a trivial algebra to show that this happens to be $\sin\theta/2$ n + - $\cos\theta/2$ n mine now you can immediately see what it tells me that is Bob will measure the SZ = - h cross /2 with the probability $\cos^2$ then other words the measurements are still coil it.

Though there is no perfect anti correlation as would be the case if Bob were to make his measurement also along the z direction.

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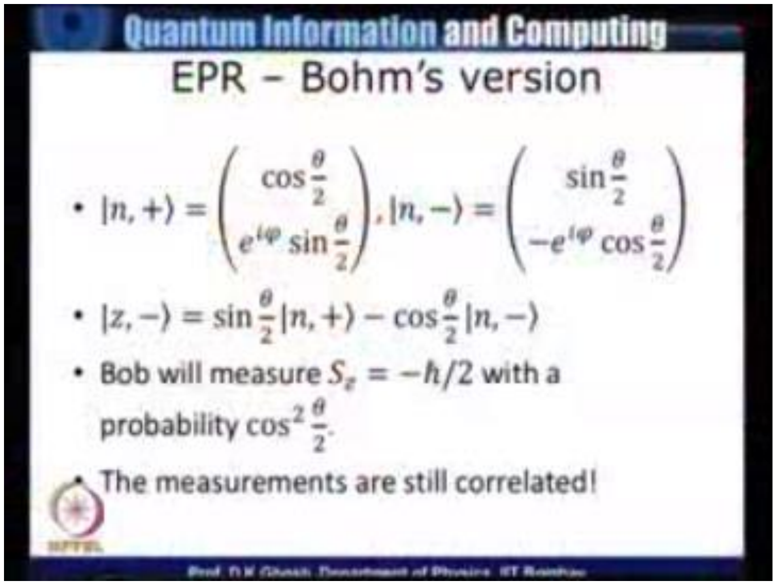

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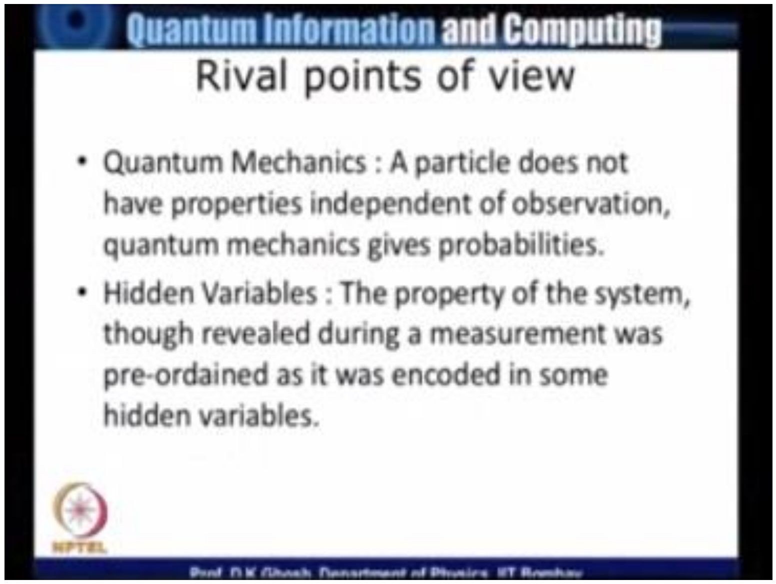

So we had seen the conflicting point of view which is summarized in this slide but since I had discussed it earlier i am not going to go through it.

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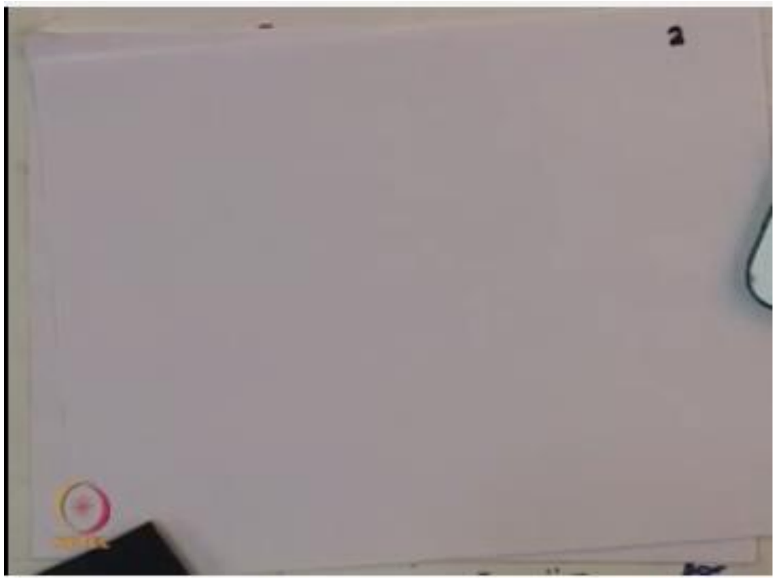

Now we said.

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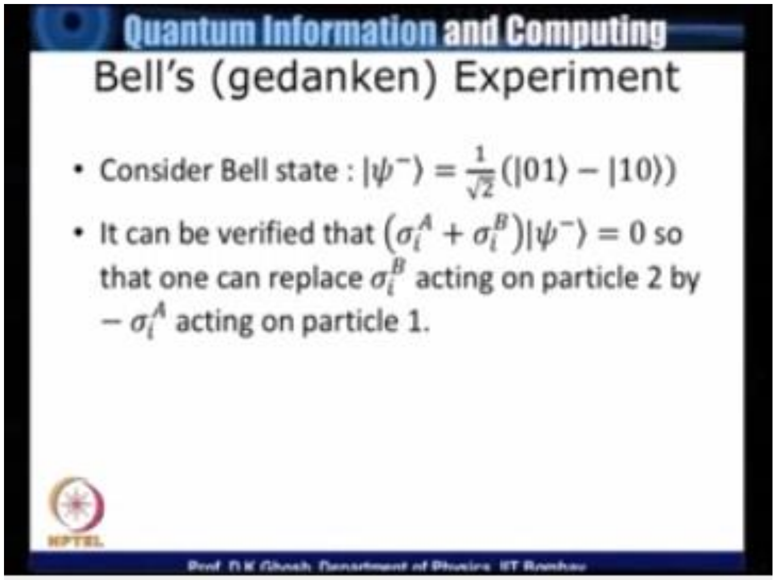

That bells made a gate and connection and for that let us consider the same stage only thing is I will not write as up down-down up but I will write this as.

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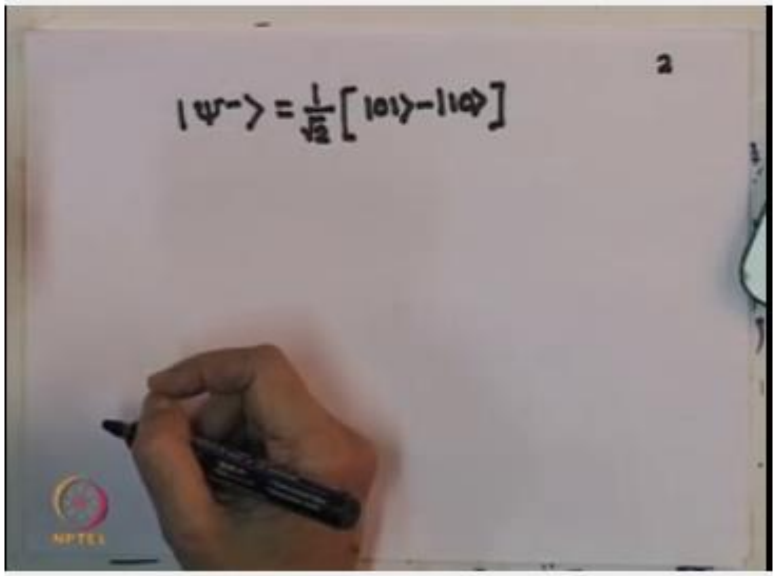

The state Ψ – 1 of the four Bell States which is equal to 1√2 0 1 – 1 0 now I proved it last time that if Alice is to make a measurement on particle number one and bob is to make a measurement on particle number two.

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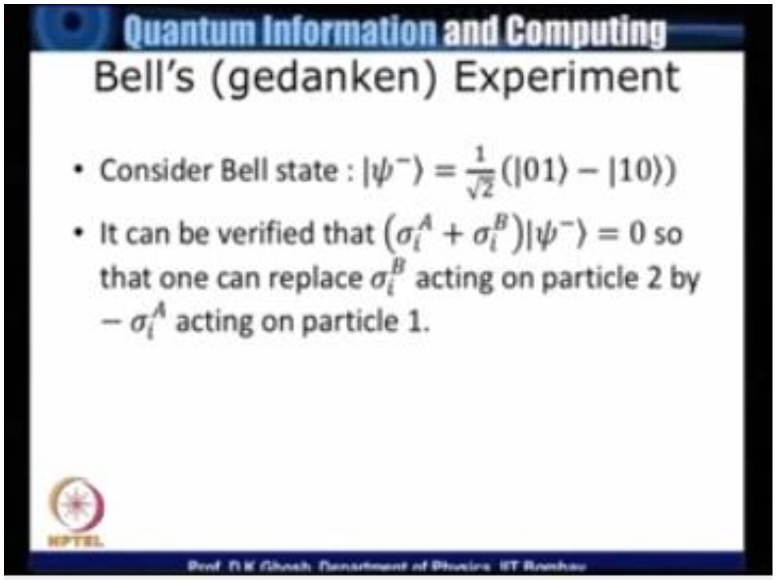

Then as long as they make a measurement along the same direction it does not have to be that direction along any direction then this quantity.

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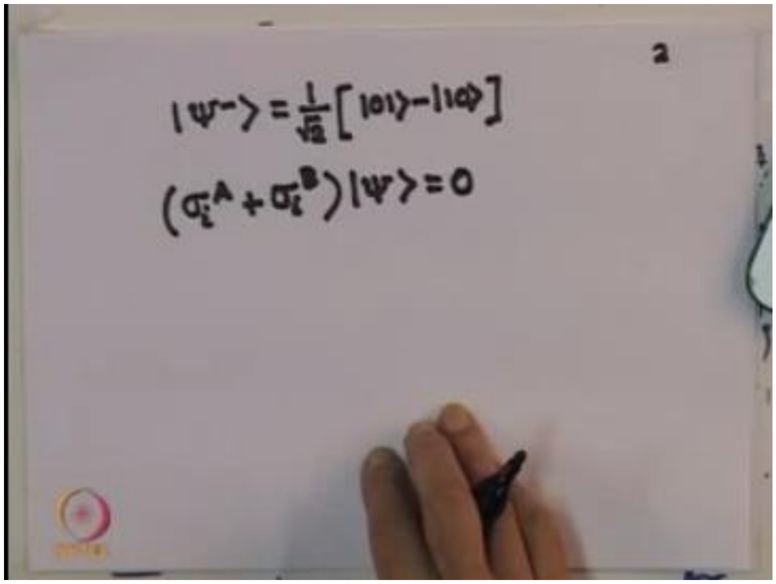

$\sigma_i^a + \sigma_i^B$ acting on the state side gives me you which means that σ_i^a active on Ψ is the same as $-\sigma_i^B$ acting on side now what does it actually mean it means that if I have an operator σ_i^B and which is acting on particle number two I can equivalently replace it with σ_i^a on particle number one and that is what is shown in this slide.

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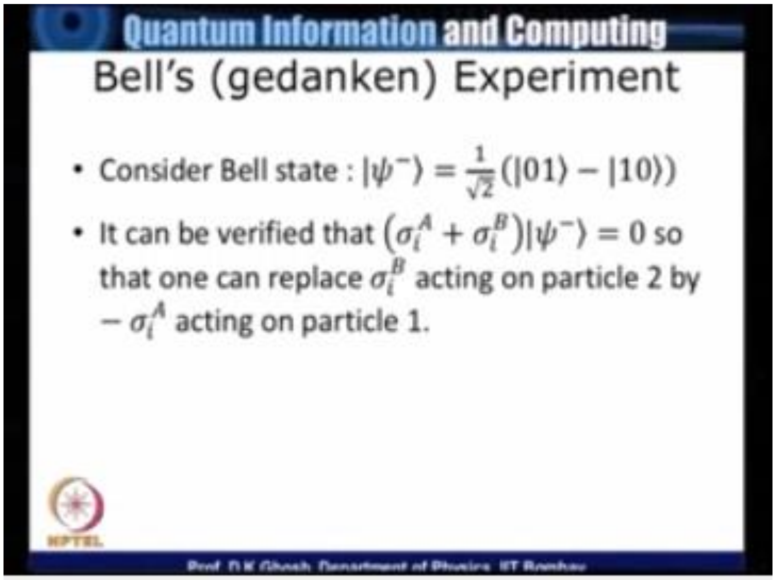

Now let us look at.

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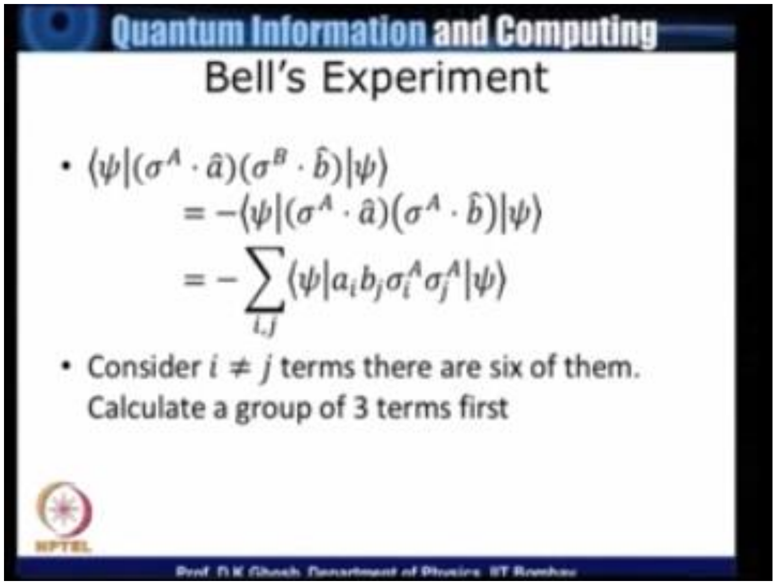

This quantity so we are looking at.

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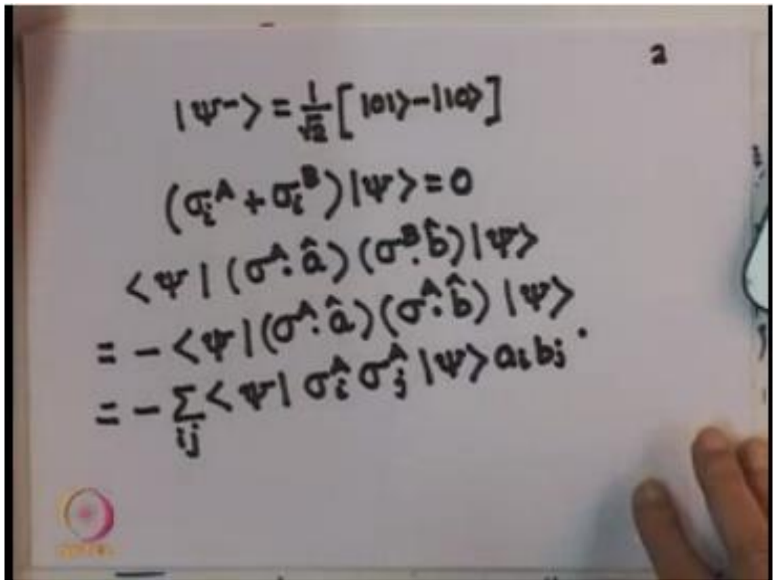

What is the expectation value of Ψ inside in the state Ψ if Alice is to make her measurement along an arbitrary directly let us call it σ^A. a dot a is the unit vector and bob is to make his measurement along and unit vector direction b Ψ which all good what is this quantity now according to what we said just now this is equal to i can replace the σb which is acting on particle number two with a – σ^a but then σ^a always acts on particle number one so this is equal to - of Ψ σ^A a σ^A.b.

So I do not refer to Bob any longer now expressing the dot products I get this as $\sum_{ij}$ Ψ σAi σAj Ψ and of course multiplied with ai bj this is my the quantity that I get now let us actually take a bit of a time in calculating this quantity what is this but first there are several terms.

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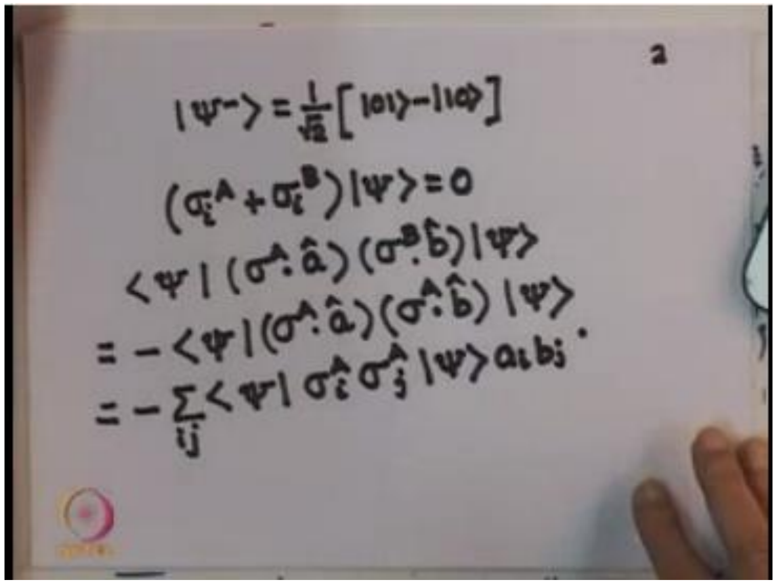

Here but let me try to see to be precise there are nine terms there let me see what are the off diagonal government namely terms for which I is not equal.

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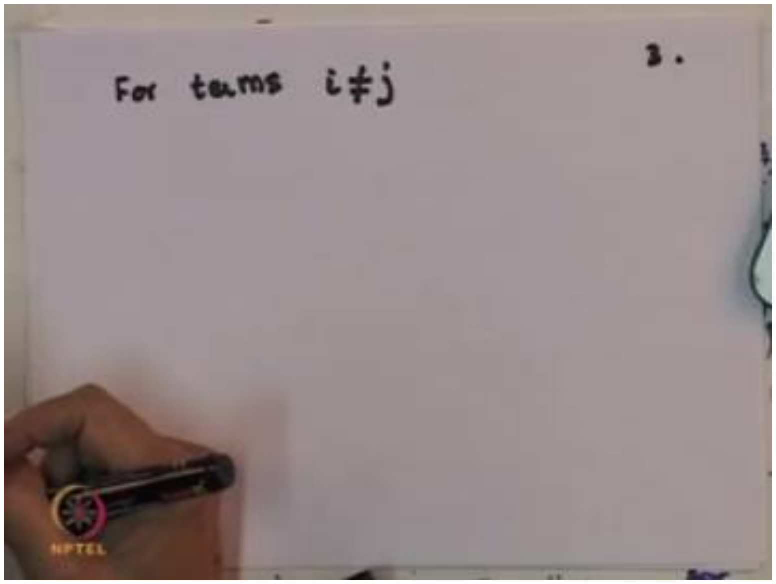

So for terms i not equal to there are six terms now I would like to group them into two pairs of three terms each so let me look at.

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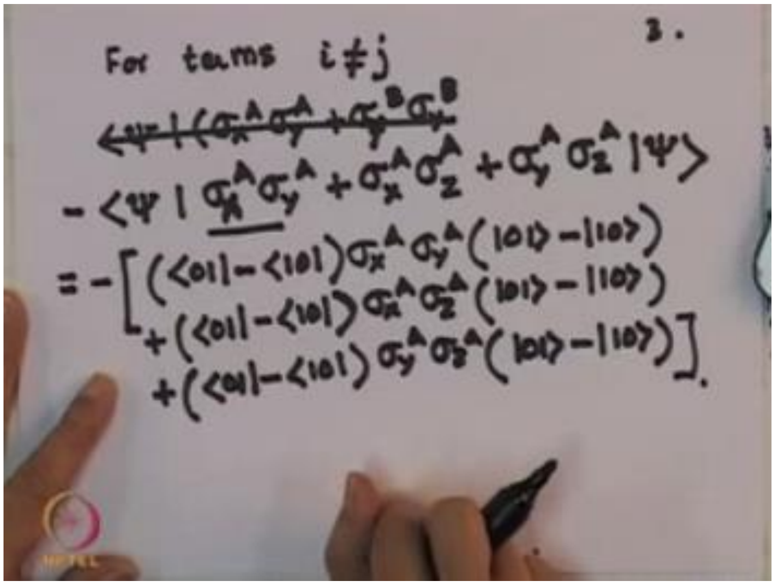

The following quantity σ_x^A σ_y^Aa + σ_x^B σ_y^B sorry this is σy let me cut this out σ_x^A σ_y^A +σxA σ_Z^A+σ_y^A σ_Z all of them refer to a because we have replaced it with the -Ψ now let us look at this calculation not all that time consuming calculation so this is - so I had 0 1 – 1 0 this is the bra and so let us take this term so I get σ_x^A σ_y^A acting on 0 1 – 1 0 + the same bra σ_x^A σ_z yes mid a get me down there is nothing being done other than routine algebra now look at that remember what σ_X does σ_X simply manages to flip 0 to1 and 1 to 0. σ_Y does essentially the same so but σ_Y acting on a 0.

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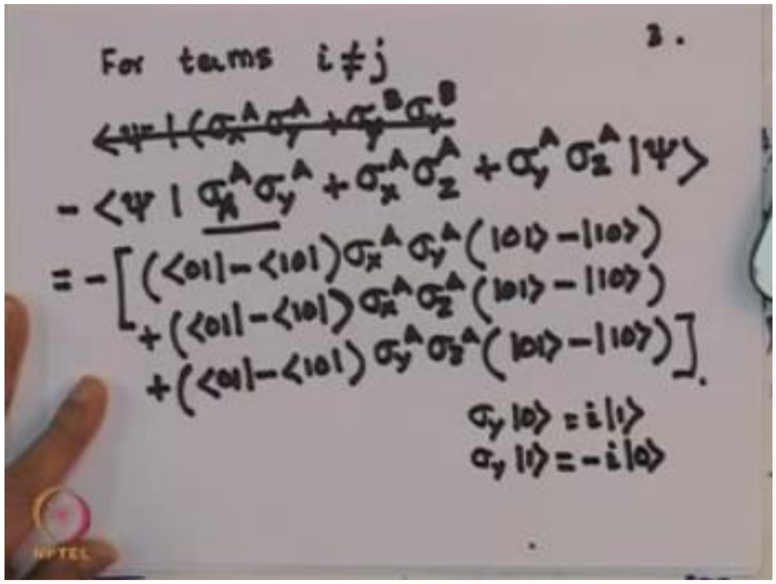

σ_Y acting on a 0 gives you i times 1 there is a phase factor there and σ_Y acting on a 1you have to be careful it is -i times. Now that tells me that this term that is there.

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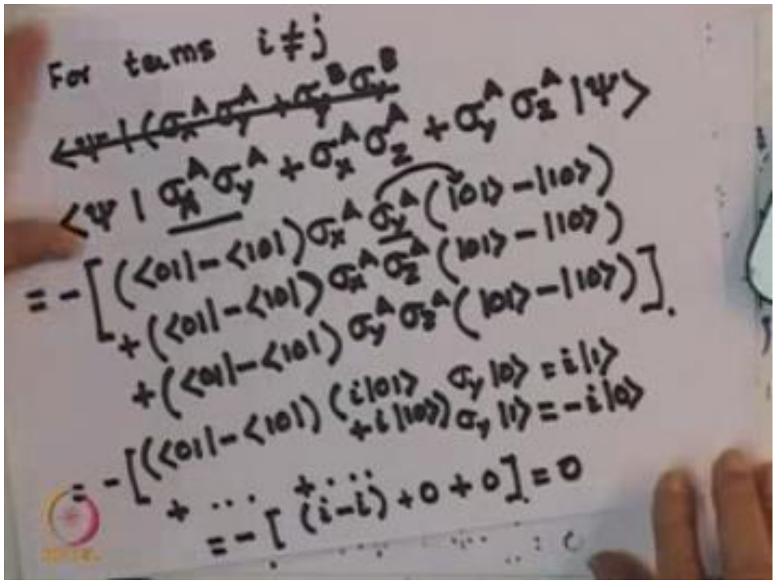

So notice what is happening here σ_Y remember all of them are acting on the first particle and I have to put them in the order in which they appear so in this case the σ_Y^A acting on 0 gives me i times 1, σ_X flips it back so therefore other than for this state flipping from 0 to 1 back to 0 I have simply collected a factor i. Now let us look at this, here also the same thing happen, so look at what am I getting here, so I get again the σ_Y^A acting on this state and so this will give me -sign is already there.

[(<01|-<10|) into as we said (i|01>+ because that was the - i|0> okay, now look at what this actually is this state 01 is of course the same as this state 01, so I get a factor i but 10 is orthogonal to 01 so I get 0. But this, this state is the same as this state but there is a -sign here so i get a –I, so i -i I get 0. Now these states are much more straightforward, in his case so this is I will write down the second and the third term earlier so this term as we have seen is i-i so that is equal to.

Now if you look at this term σ_z acting on 0 does nothing to it, but σ_x acting on 0 after that changes this 0 to 1, so instead of 01 I will have a 11. Now similar is the situation here σ_z acting on 1 gives me -1, so this -sign will become +sign, but then σ_x acting on that 1 will give me 0, so at the state that I will get here will be 11- 00 which is of course orthogonal to the state, so therefore I have my second term is 0.

The third term similarly gives you 0 because it is very similar to this term where I got 00+11 here also I will get very similar thing but because it is not if a Y operator rather than X operator there would be some i's coming in, but never the less the resulting term will be orthogonal to this so therefore, this will also be 0 so the result is 0. Now this is what happens to the non diagonal term there are three motto but then it does not matter because there will be σ_Y^A σ_x^A, σ_Y^A σ_Z^A, σ_Z^A σ_Y^A and identically those three terms will also turn out visit. So what I am I left with I am left with only the diagonal terms, so I have.

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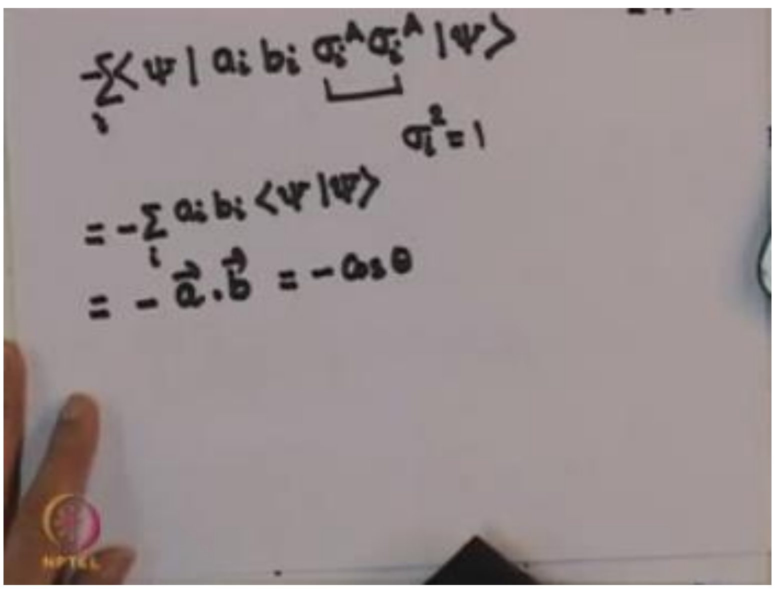

ψ I get $a_i b_i$ because only i=j term I will sum of course over i then σi^A σi^A again $|\psi\rangle$ -sign has taken care of the fact that there was a σi^B there but I now have written to $\sigma i^2=1$ so therefore this quantity is $-\sum i\ a_i b_i \langle\psi|\psi\rangle$ and of course the state ψ is normalized so therefore I have this as –a.b and which we have said is nothing but -cosΘ because Bob is making a measurement along an axis which is making an angle Θ with that of Alice's axis. So you notice again we have a reiterated using explicit calculation that if Alice is to make a measurement along one direction a and bob is to make a measurement along a second direction which is b then the.

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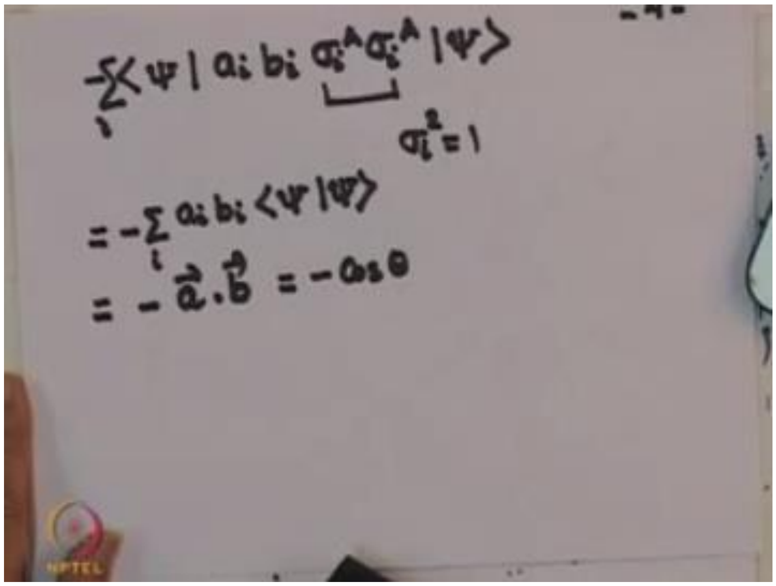

Their results of Bob making that measurement of his spin Alice is making a measurement of her spin is given by what is cosΘ, so there is still a correlation.

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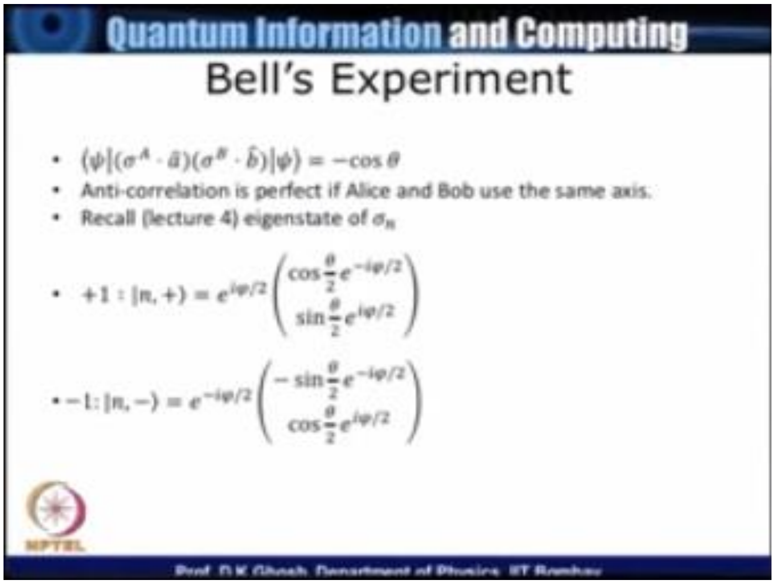

Now it is this correlation that we will consider in the further discussion of what it means in terms of quantum mechanics. Now this slide simply reminds you which I have talked about a few minutes back that the eigen state of Pauli operator along an arbitrary direction n are given by these matrices.

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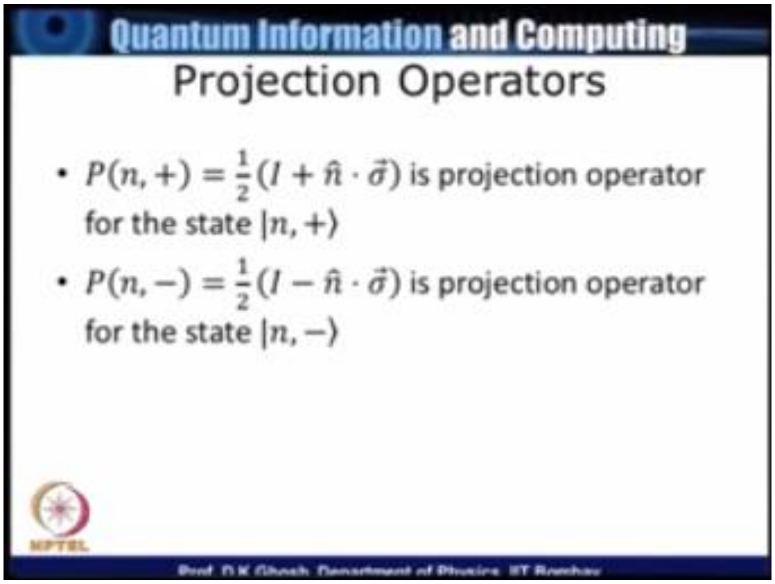

Now suppose Alice is making a measurement along a particular direction and gets a particular value supposing she gets a +, now what I might actually doing, I am projecting out the N equal to if ψ x+1 I am projecting out the eigen state n having an eigen value plus. Now you can check it immediately which I will indicate.

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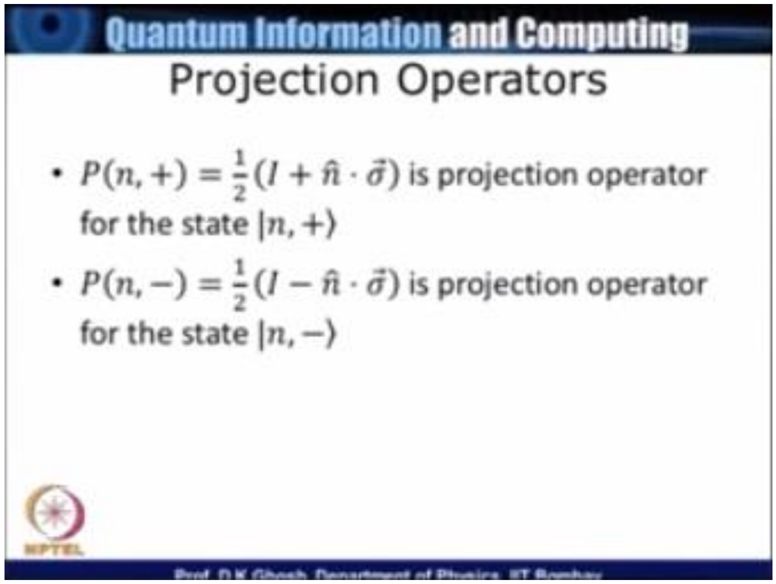

How it is done that this quantity half of I+n.σ is a projection operator for the state n+ and identically half of identity operator –n.σ is the projection operator for the state -.

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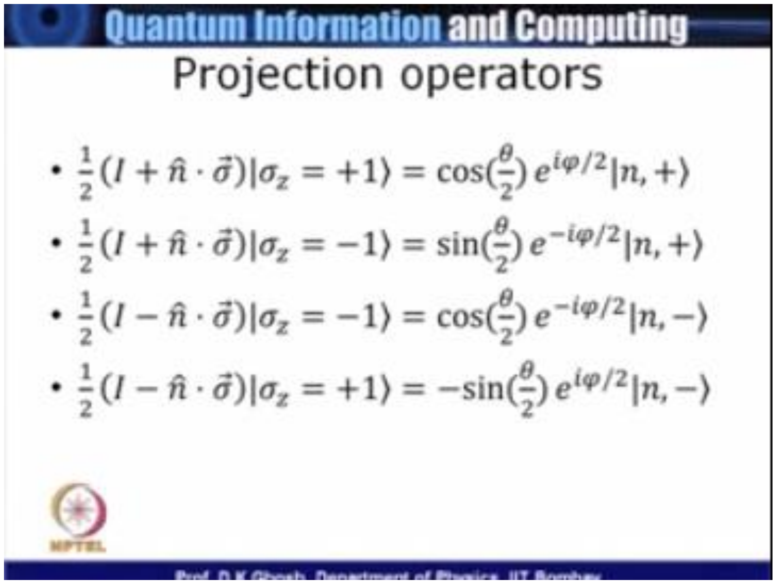

Now this is easy to work out but let us do this, I will briefly do this algebra for one of the relations and then I will sort of urge you to do the calculation for the others and see that that is coming.

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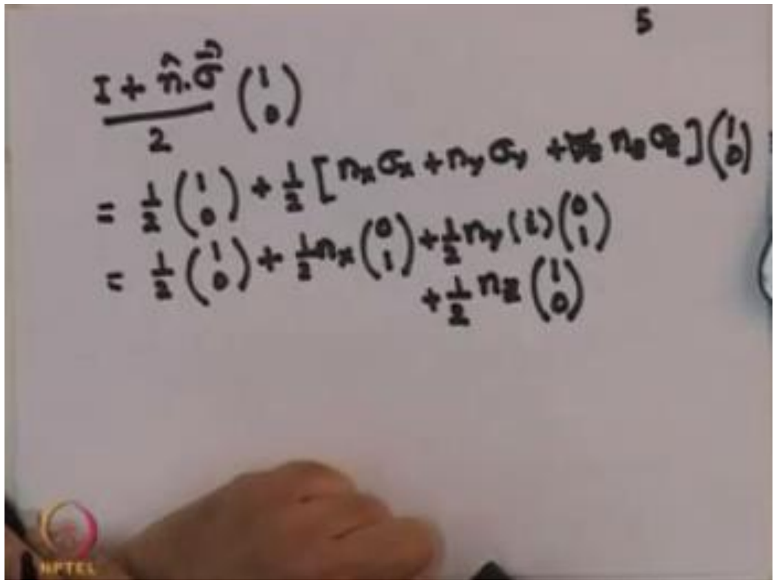

So suppose we are saying I+n.σ acting divided by 2 of course acting on let us say up this is z=+1 σ_Z eigen value +1, so what does it give me. So since this is an I, I get simply ½(10)+ let us expand this out, so I have n_x σ_x + n_Y σ_Y +n_Z σ_Z acting on the same state (10). Now what do I get here, so firstly let me keep this n_X operator as it is σ_X n_X direction as it is the component Cartesian direction.

σ is separating on 10 I know just inverts it that is flips the qubit, then I have ½ n_Y now σ_Y acting on 10 also flips it but there is a factor of i there and as before σ_Z does nothing so therefore it 10. Now remember this term will pick up a- sign if it were to act on the 01 state.

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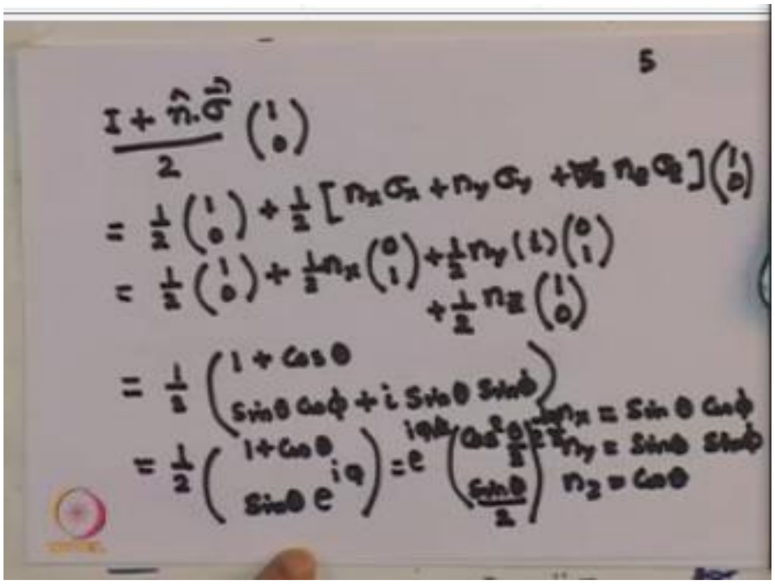

Now let us write it as a single matrix, so look at what am I having I have a half there I have 1, now recall that my n_X is sinΘ cosϕ, n_Y is sinΘ sinϕ and n_Z is cosΘ so I bought 1 and only a term coming from n_Z so it is 1+cosΘ the other element is there is nothing from here there is a n_X here so let me write it as sinΘ cosϕ+ this is i times sinΘ sinϕ. But then this is equal to 1+cosΘ if you take sinΘ common here I get cosϕ+ i sinϕ which is nothing but $e^{i\phi}$ which is other than for a global factor which I can pull out $e^{i\phi/2}$ so which is remember that 1+ cosΘ/2 is $\cos^{\Theta}/2$.

So this is if I pull out $e^{i\phi/2}$ I have $\cos^2\Theta/2$ $e^{-i\phi/2}$ and this is I have got sinΘ/2 so if I write down here sinΘ. Now remember sinΘ there is a half there so let me keep that sin σ/2 remember that sin σ can be written as 2 sin σ/2 x cos σ/2 so that this 2 and that2 will cancel out and I will be pulling out a factor cos σ/2 and I will be left with a cos σ / 2 this is shown in the slide.

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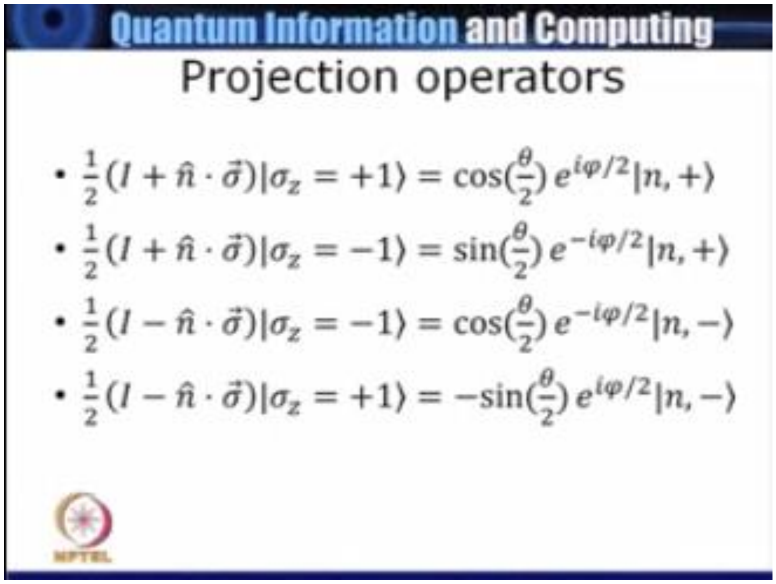

$\cos \sigma/2$ $e^{i\pi/2}$ which I have taken out and the remaining thing which is there is nothing but the Eigen state along n direction with an Eigen value the remaining relationship are also as shown there, so what we have shown is that.

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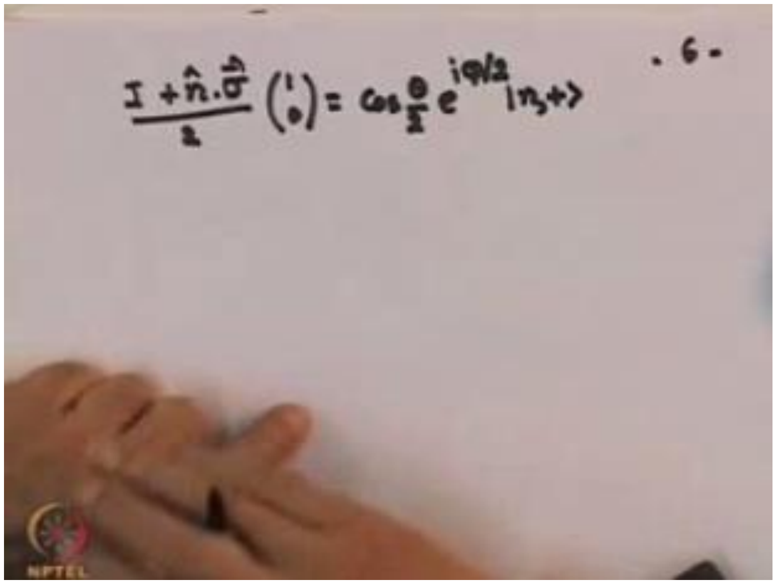

i + $n.\hat{}$ / 2 is a projection operator for the state n, with an Eigen value plus and this acting on for example the state 10 we have shown he gives me 0 sin σ/2 times $e^{i\psi/2}$ an immaterial phase factor and envious the, the slide shows the other.

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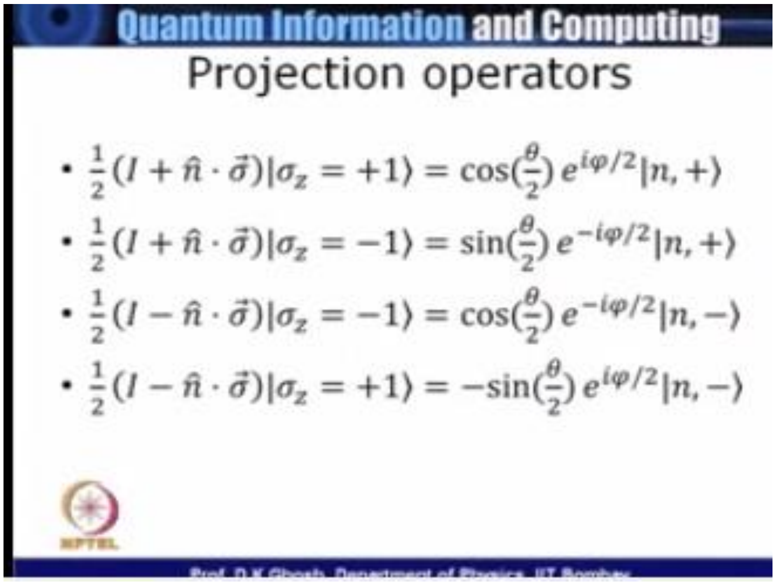

Possibilities that what happens when the same operator acts on the state σ_Z = -1 you find that it gives you essentially the same state but with a factor of sinΘ/2 and 1 minus n dot Θ/2 is a projection operator for the Eigen state n- and acting on state -1 gives you this and that is as written in the slide. But then let us look at what this quantity gives.

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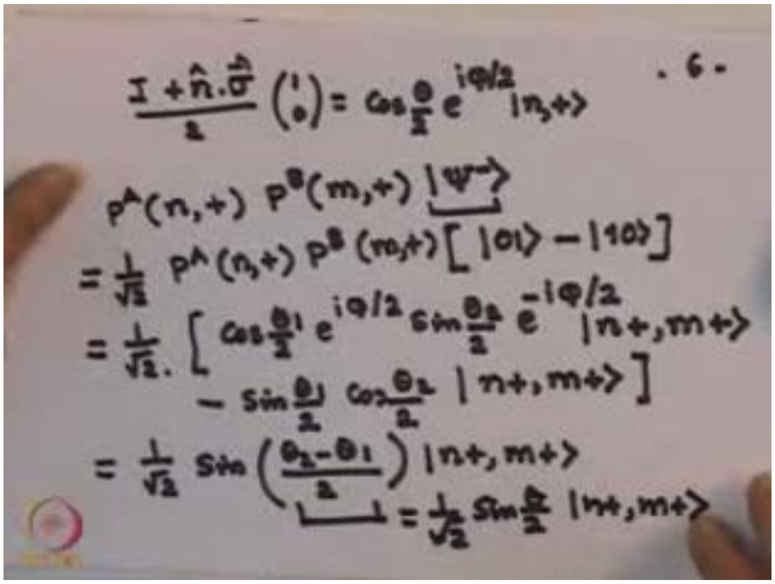

Alice makes a measurement along the axis a using this operator Bob along some direction m also plus ψ, now look at this since this side is 1/√2 which we written as it is and so let me rewrite this n + p^B M + acting on 0 1 - 10 use the 4 relations that I have shown in the slide so this is giving me 1/√2 now let us suppose the direction n next an angle σ1with the z-axis and the direction m makes an angle theta 2 with the z-axis in that case I get the following.

Since this is acting on 0 I get cosσ1/2 there is of course $e^{i\psi/2}$ there and this one because this is b in fact even one so you get sinσ2/2 because this is direction m into $e^{i\psi/2}$ and what this is projecting is n +, and m+ - identical way this is because it is acting on one so I get a sin σ1/2 the exponential factors will cancel out as in the first term and I get cosσ2/2 and this is again giving me the same n + and m + because the projection operator use in both cases is the same.

So that is equal to 1/√2sin of σ2 – σ1/2 n + M plus and if the angle between the two axis is σ this is simply equal to 1/√2 sin σ/2 n + m now I can, I can do the same calculation for the other three projection operators and get results.

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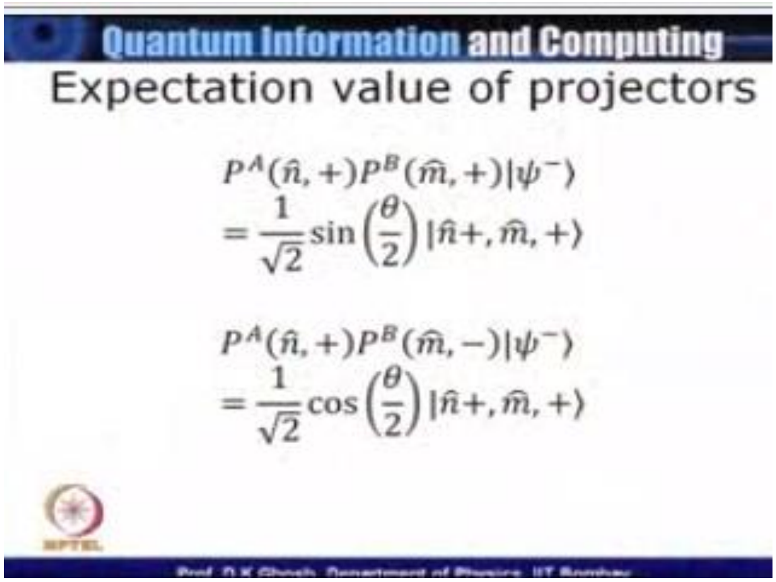

Which are tabulated in these slides, so therefore if we now decide that Alice and Bob decide to make their measurement along direction n and m respectively then.

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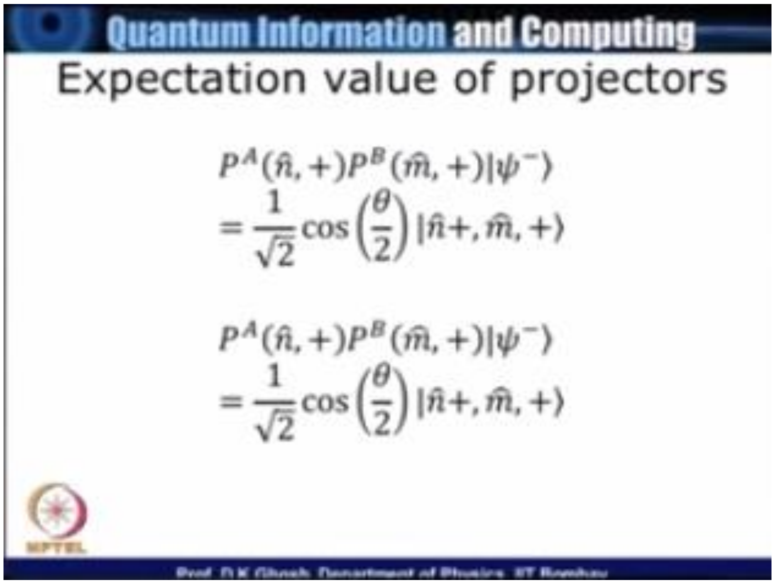

We can calculate.

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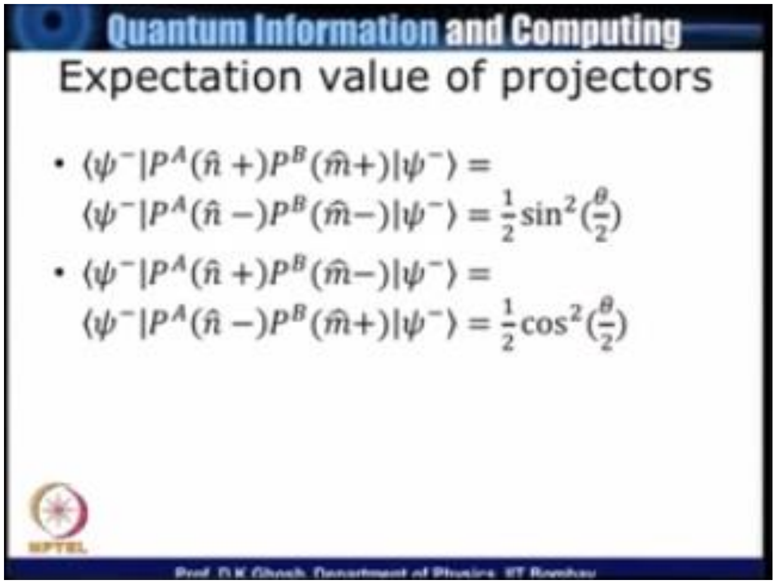

What is the expectation value of let us say T a n + P b m+ remember that they all that I need to calculate this is to realize the projection operator squares our projection operators themselves so I can write this P^A as P^A square P^B as P^B square and I can make one pair act on the ket 1 pair act on the brother and get these relations as have been written up. So you notice that when they get P^A plus, plus that is P^A plus P^B also gives me plus they or minus, minus my probability is 1/2 $\sin^2$ σ/2 and when they are dissimilar it happens to be cross particularly.

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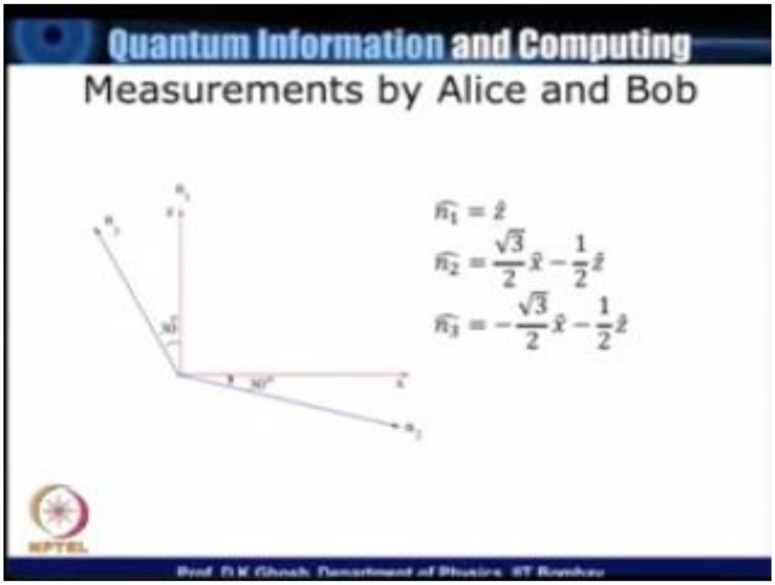

Now what is suggested now is we have not said what these directions let me take choose direction n 1 to be the z-axis n2 to be a direction which is in the fourth quadrant making an angle of 30degrees with the positive axis x axis and m3 in the second quadrant making an angle of 30 degrees with the z-axis the respective unit vectors are written down there.

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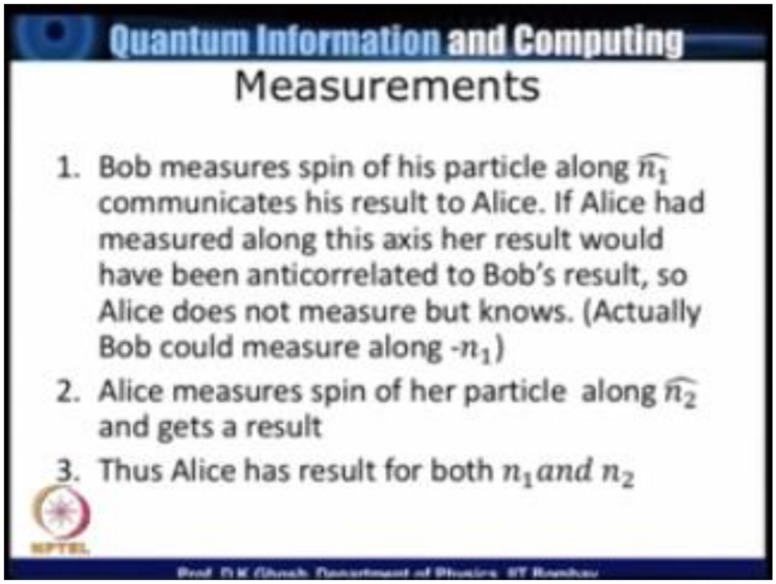

Now what happens is this Bob measures the spin of his particle along in one direction actually what Bob would do is to measure it really a long- and one direction so that Alice does not have to worry about changing that minus one Bob will make the directive measurement along minus one direction communicated to ask Alice, now you see when Alice makes a measurement along on her particle along and two directly see is no longer able to find out what the her result would have been hand she made a measurement along then one direction.

But Bob's information lets us know that so therefore Alice can have the information about the result that you would get on a pair of direction at the same time n_1 n_2 n^1 and 3 and n_2 so what she does is the same.

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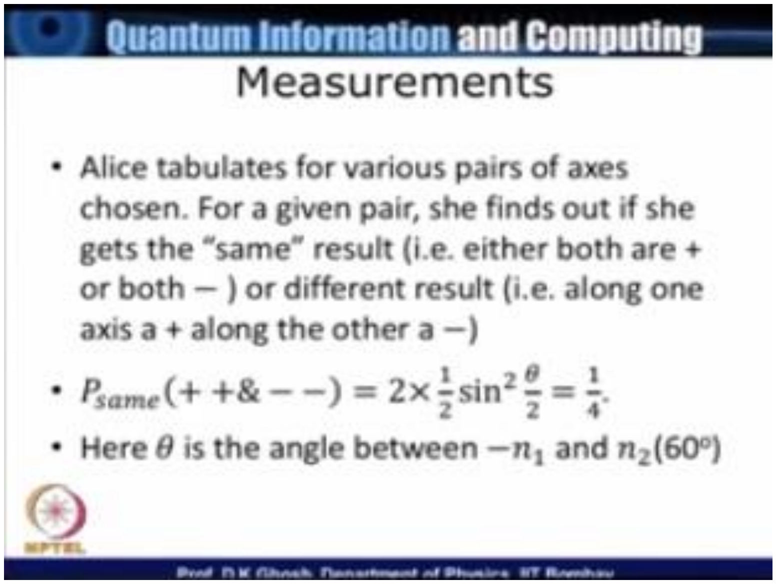

That looks at that for a given pair of axis does he get the same result by same I mean that is he get + + or – so concentrating on only the same results the probabilities would be because there are two possible is also 2 into half $\sin^2 \Theta/2$ and that works out to be equal to 14 because of the choice of dangling that we have taken.

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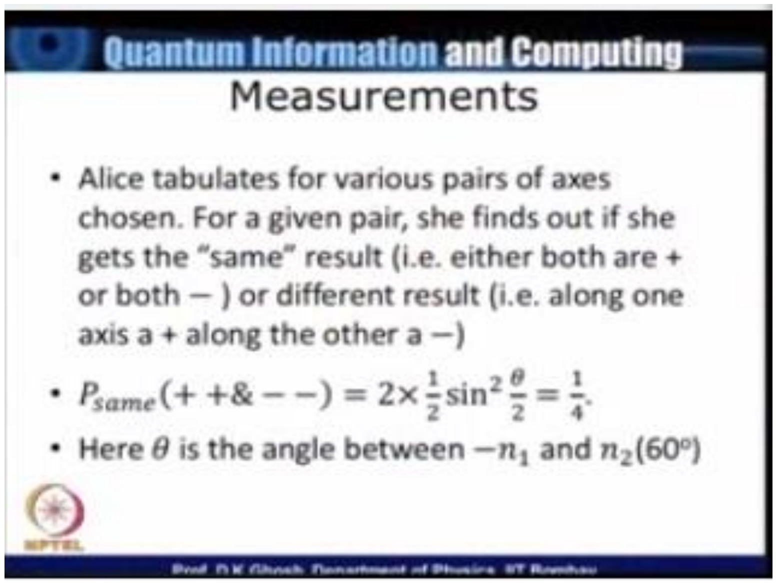

Because sin σ is 1/ 2 for 30 degrees now with this we conclude that if see only concentrates on get the probabilities of having the same level when she does measurements along n_1 $n_{2,}n_{2,}$ n_3 or n_1 & n_3 & sums of the probability for the same result the result must be equal to 3 / 4 which is less than.

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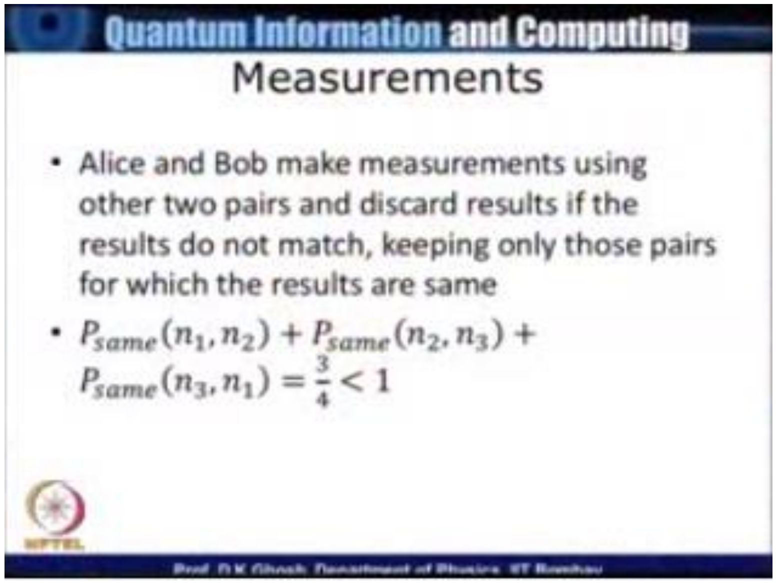

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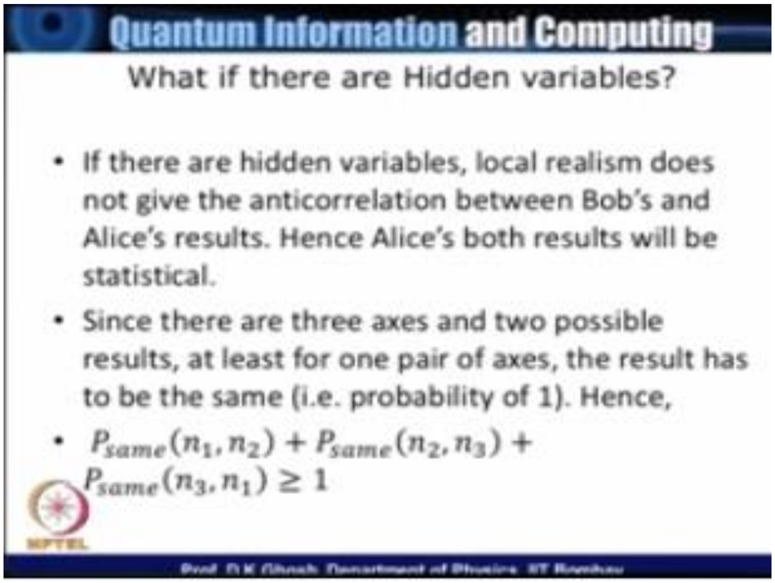

Now what happens if there were hidden variables in the problem now local realism does not give any anti correlation between Bob's and a Alice brother so therefore Bob scene forming Alice that this is the result I got it is totally made up, but then Alex can make the measurement along the plexus because there is no nothing which is there is no collapse of the way form now so therefore we recalculate P is same for n_1 n_2 n_2 n_3 and n_3 m but remember I have only three possible result + I can have the ++, -- or +- and -+ whatever happens one of those results is guaranteed to give me the same.

Because it could be ++ or – I do not care we could have ++, +- ,-+ or --, +-, -+ this is s this version but then one of those events has the probability of one and then this tells me that the probability of the same result.

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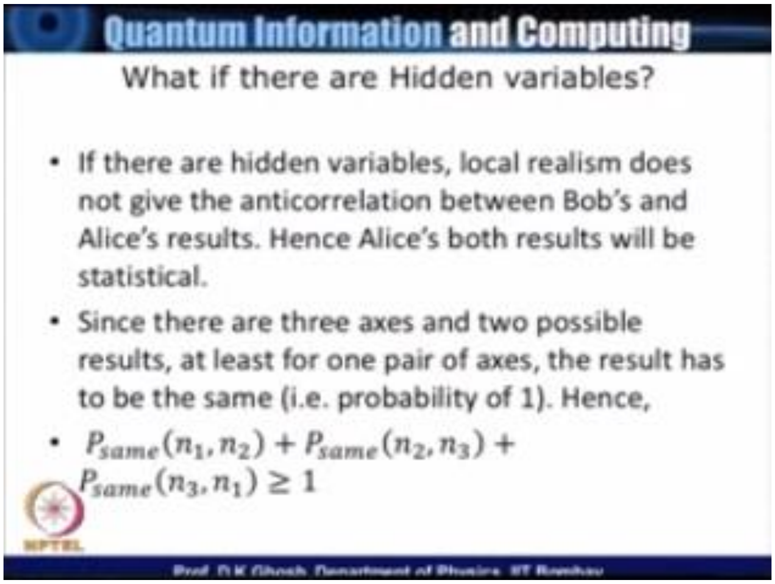

It greater than or equal to 1because at least one of them terms is 1 so this provides a test for which one is right one of them says it is less than one the other one says it is greater than one and if invariable theory of slides you should get greater than or equal to one if quantum mechanics is correct you should get less than one and that is what will determine if one can do this experimentally that you want to determine by the quantum mechanics is correct or the invariable theories.

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