NPTEL

NATIONAL PROGRAMME ON
TECHNOLOGY ENHANCED LEARNING

IIT BOMBAY

CDEEP
IIT BOMBAY

Quantum Infromation and Computing

Prof. D.K.Ghosh
Department of Physics IIT Bombay

Modul No.07

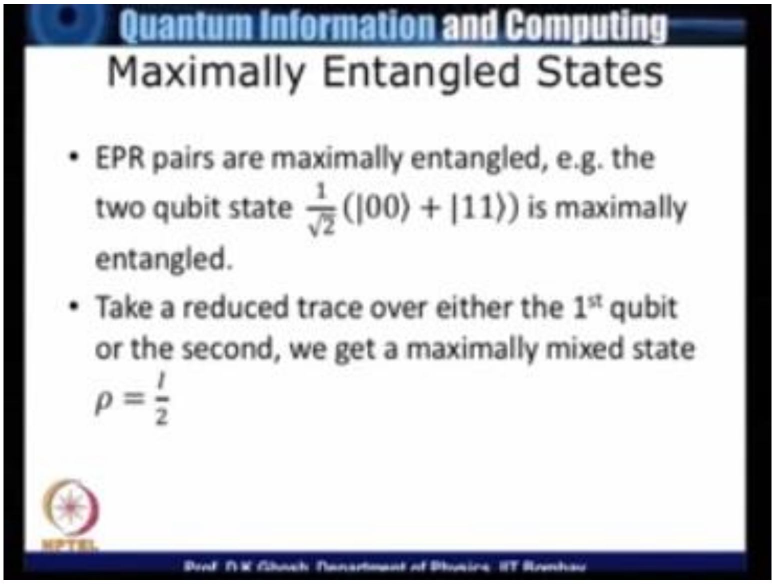
Lecture No.38

EPR and Bell's Inequalities

In the last few lectures we have introduced to the area of quantum information and in particular we talked about concepts like Shannon entropy and it is quantum extension namely one on an interval we wish to end this series of lectures with a discussion on quantum cryptography but before we do that a question naturally arises how reliable is quantum mechanics, how robust is the information sent using the processes of quantum mechanics over a quantum channel or over a public channel in order to do that we would need to establish the supremacy of quantum mechanics.

Over it is classical counterpart I have pointed out several times that right from the beginning even distinguished people such as Einstein did not believe what is known as the Copenhagen interpretation of quantum mechanics. Today we will discuss what is this objection that Einstein and his co-workers had which goes by the name of Einstein Podolsky Rosen paradox or in short EPR paradox. Firstly let us talk about what we have been saying as the entangled state.

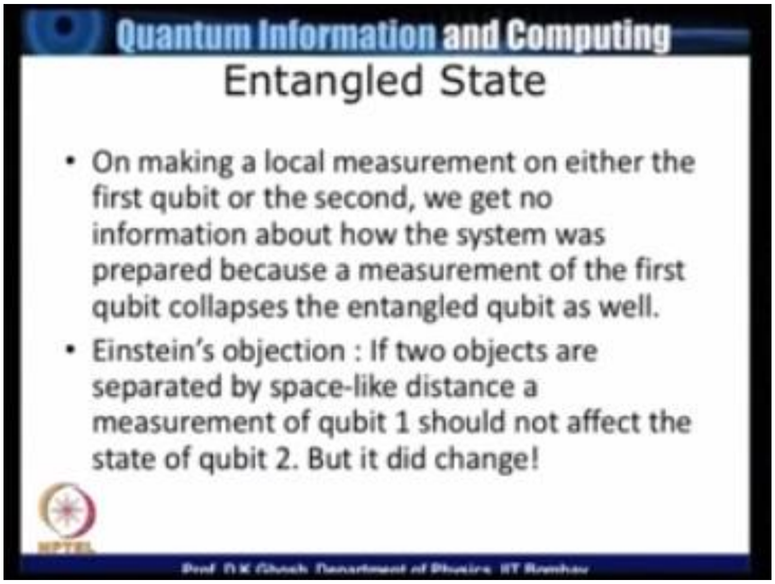
(Refer Slide Time: 01:52)



Now what is meant actually by Entangled state I had given you several examples take the example of one of the EPR pairs these bell states these are two qubit states given by for instance one of them is 1√2 that is a normalization factor into 00 + 11 state and this is the maximally entangled state what I mean by a maximally entangled state is this.

Supposing I am to take a reduced rate over either of the qubits then the result that I would get would be not a pure state but a mixed state with maximum mixed that means the density matrix row turns out to be equal to half. The beautiful thing about entangled state beautiful or curious thing about the entangled states is that if you made a measurement on one of the qubits the second qubit automatically gets fixed and the information that could be there in the entangled state otherwise gets lost.

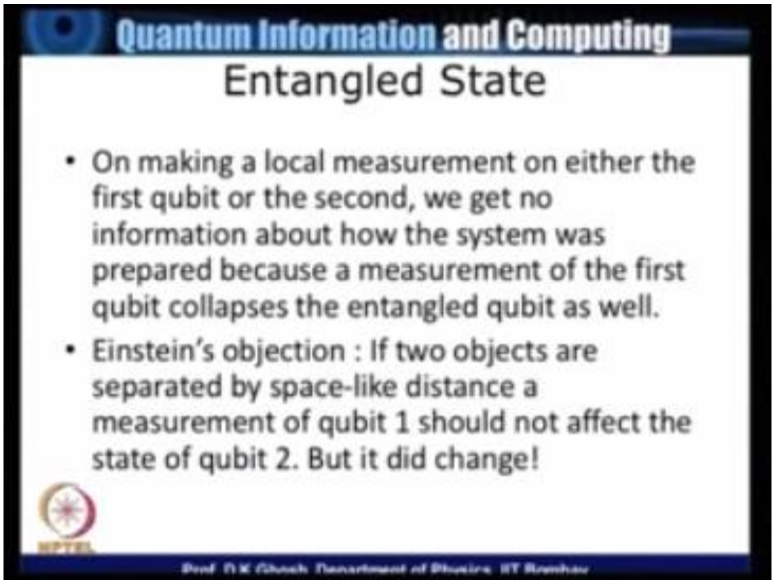
(Refer Slide Time: 03:08)



So the idea is this that a measurement of the first qubit would collapse the second qubit as well now what was Einstein objection to this whole thing what Einstein said is this that supposing there are two objects in this case my qubit number one and the qubit number two of the Belfair they are separated by a space like distance what is meant by a space like distance it means a distance which cannot be even travelled by time. Well in practice what it means is that supposing you are making a measurement on one and you are making a measurement on the second one.

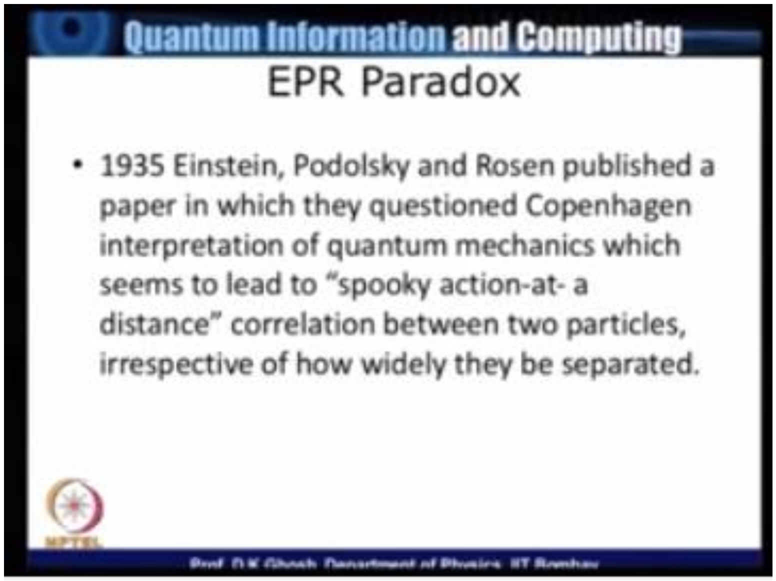
During the time that it takes for the light to travel for one qubit to together any measurement made within this time interval cannot have influence of one measurement over the other.

(Refer Slide Time: 04:04)



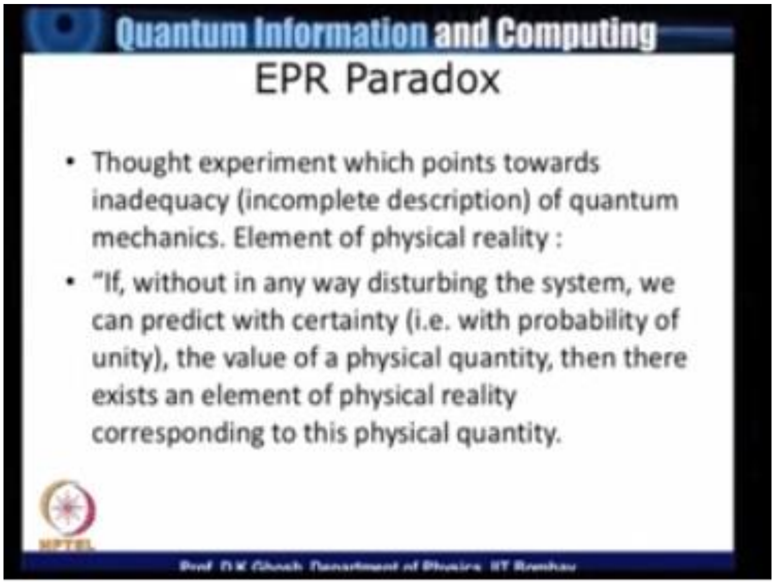
But however according to the dictates of quantum mechanics the moment you make a measurement of qubit number one the qubit number two gets fixed so in other words the despite Einstein's objection these second qubit did collapse as a result of the first qubit measurement.

(Refer Slide Time: 04:26)



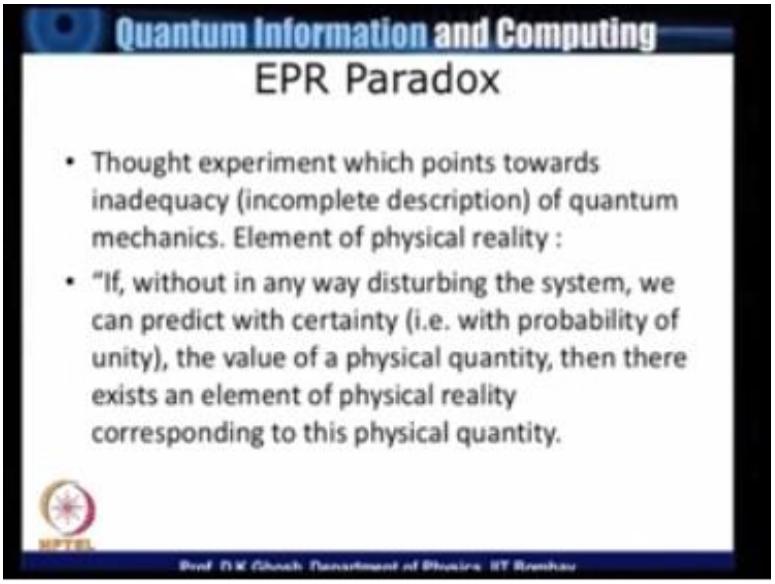
So in 1935 Einstein Podolsky and Rosen this paper is a very well celebrated of talking about people this is known as EPR paradox they questioned the Copenhagen interpretation of quantum mechanics and they it led to a phrase called "spooky-action-at-a distance" this column correlation between the two particles, irrespective of how widely they be separated.

(Refer Slide Time: 04:56)



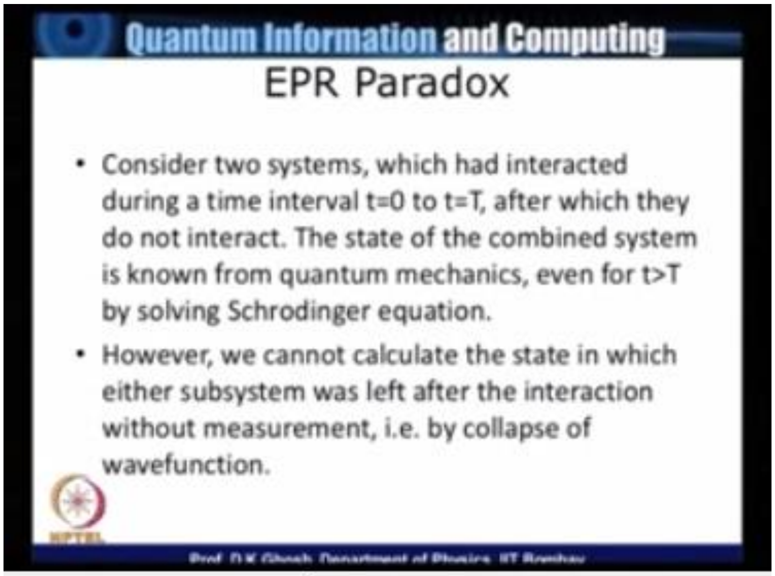
Einstein thought of a thought experiment and which points towards what Einstein called as the incomplete description provided by quantum mechanics, because according to Einstein a system must have what he called as an element of physical reality so let me explain in Einstein's words what physical reality actually means to quote.

(Refer Slide Time: 05:25)



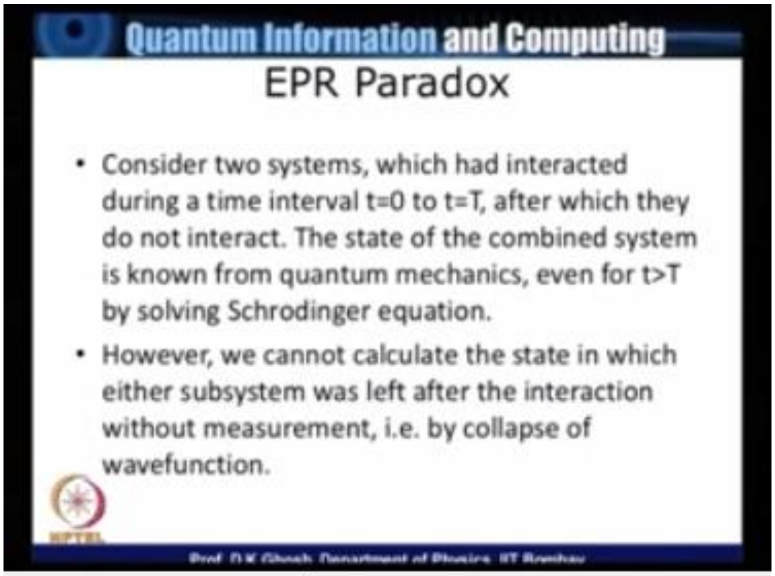
Einstein if without in any way disturbing the system we can predict with certainty by certainty he meant with probability of 1 the value of any physical quantity then there exists an element of physical reality corresponding to this physical one.

(Refer Slide Time: 05:46)



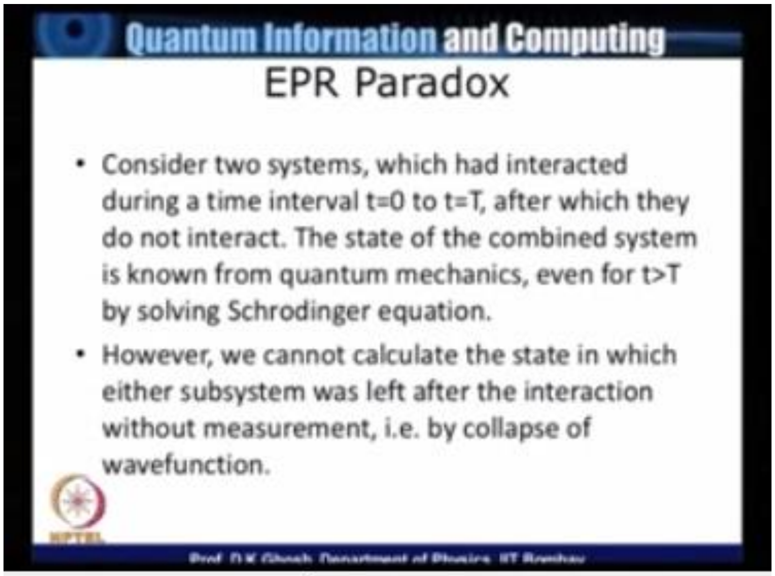
Now how is it different from what the quantum mechanics says, remember quantum mechanics says the value of a physical of the pebble has no meaning independent of it is measurement. So in other words the, its value gets fixed the instant we have made another but Einstein thinks that the you got that result because that was the value that was some or other encoded in the system.

(Refer Slide Time: 06:19)



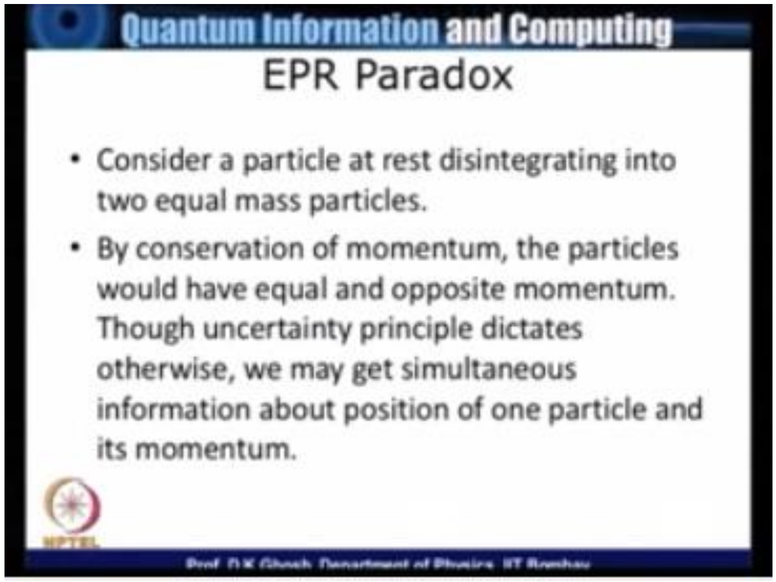
So let us look at a little more about Einstein's objection, supposing we have a combined system which consists of two systems and they had interacted during a time interval from t = 0 to p = T and after that they got separated and they did not interact. Now this combined state of the system this is known from quantum mechanics and their development is decided by quantum mechanical laws namely the Schrodinger equation even for time t greater than T when they are no longer interested now the problem is this.

(Refer Slide Time: 07:03)



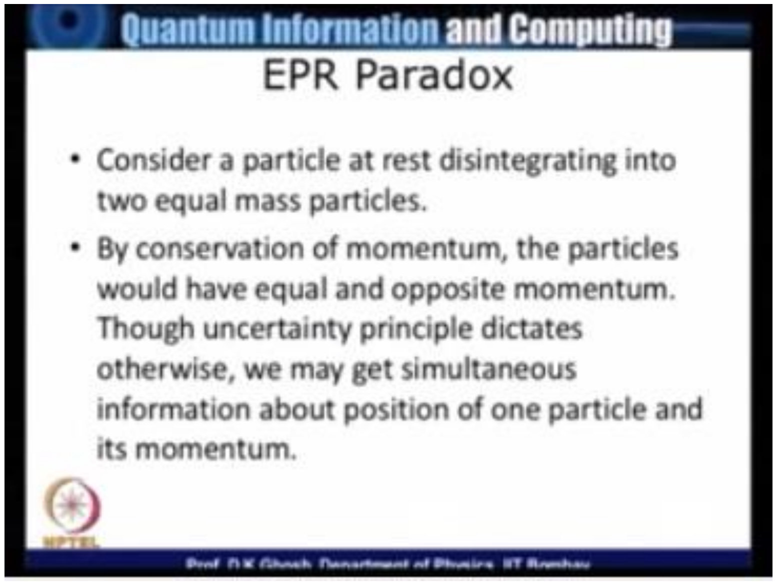
That we cannot calculate the state in which either subsystem was left after the interaction without the middle that is by collapse of the wave function.

(Refer Slide Time: 07:18)



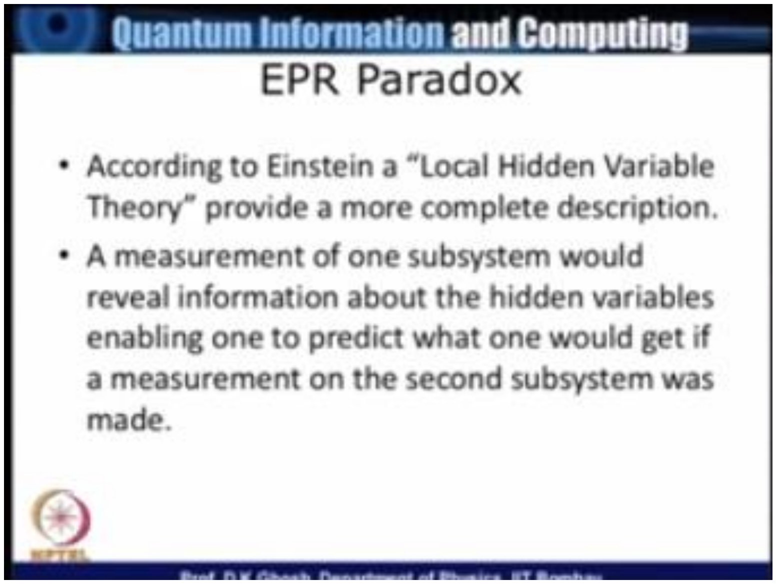
Now let us look at a simple example, I could give some more but this example is consider a particle at rest which disintegrates into two equal mass particles we have learnt that if the particle was at rest it momentum was equal to zero. So therefore since the disintegration process is an internal process this will conserve momentum. In other words after the system of one particle has disintegrated into two the momentum of the sphere will continue to be zero now let us look at what happens.

(Refer Slide Time: 08:03)



Now Einstein objection will prove I can fix simultaneous the position and the momentum of one of these particles either of these particles in spite of the objections that is posed in quantum mechanics due to what is known as Heisenberg's uncertainty principle. In this particular case let us consider suppose the two particles disintegrated and moved away from each other along the x direction. Now if you are looking at one of the particles let us call a particle number one then according to Heisenberg's uncertainty principle I cannot fix the position of the particle as well as its momentum that is I cannot talk about simultaneously about it is position and momentum at a given time now let us see whether that would be correct or not.

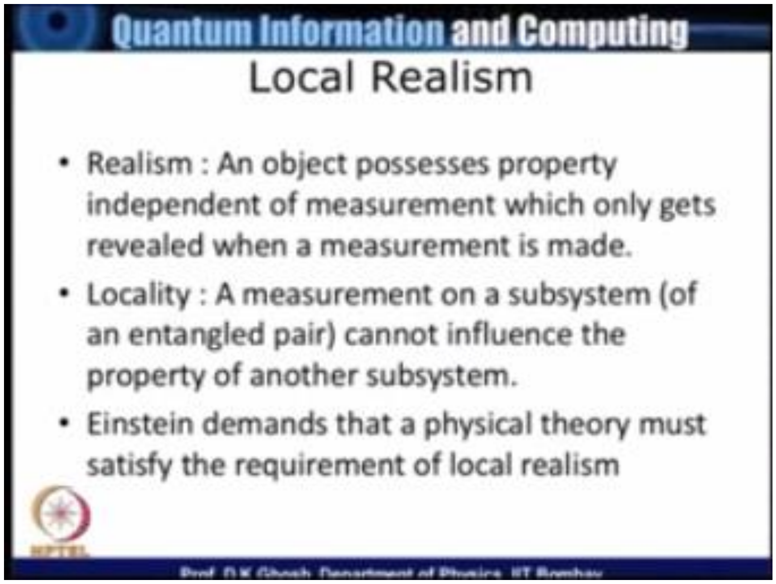
(Refer Slide Time: 09:05)



Einstein's objection would be the following that suppose you made a measurement of the momentum of particle number 1 and position of the particle number 2 now this is possible because Heisenberg certainty principle does not raise any restriction on different measurements being made on the same particle at the same time. But however, since the momentum is conserved a measurement of the momentum of particle number 2 will fix the momentum of particle number 1 as well.

So if the momentum of particle number 2 was P2 the moment of particle number 1 will be –P2 because the net momentum is equal to 0. Similarly, if at the same instant I measured the position of particle number 1 because of the fact that the center of mass must remain fixed I would be able to calculate the position of the particle number 2 as well. So in other words at the same instant I have information.

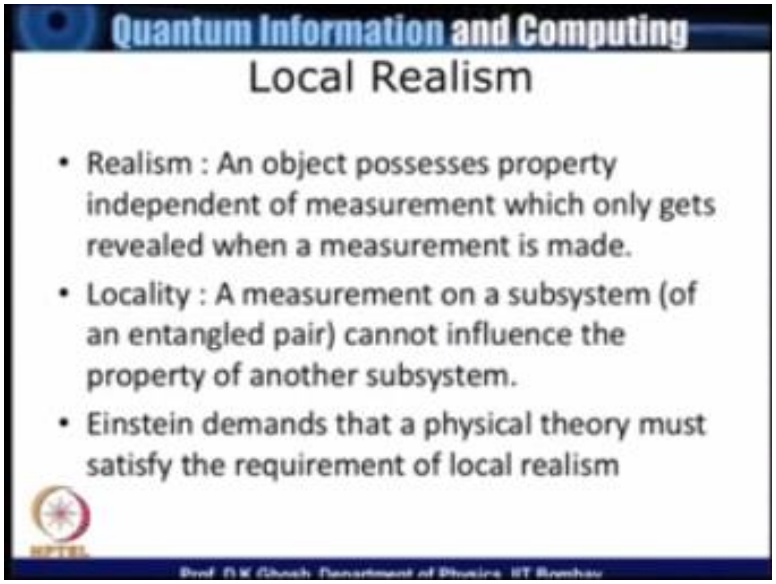
(Refer Slide Time: 10:14)



Regarding both the position and the momentum of the particle and that is something which was Einstein's primary objection to the Copenhagen interpretation of quantum mechanics I could give other examples, but let us look at the following thing. So what is meant by local realism, so realism according to Einstein's interpretation means an object possesses property independent of the measurement and this property gets revealed only when a measurement is made.

In other words measurement is not the process which gave the attribute to the matrix the measurement simply is a revealing process. Secondly the Einstein another phase is locality, now what is meant by locality.

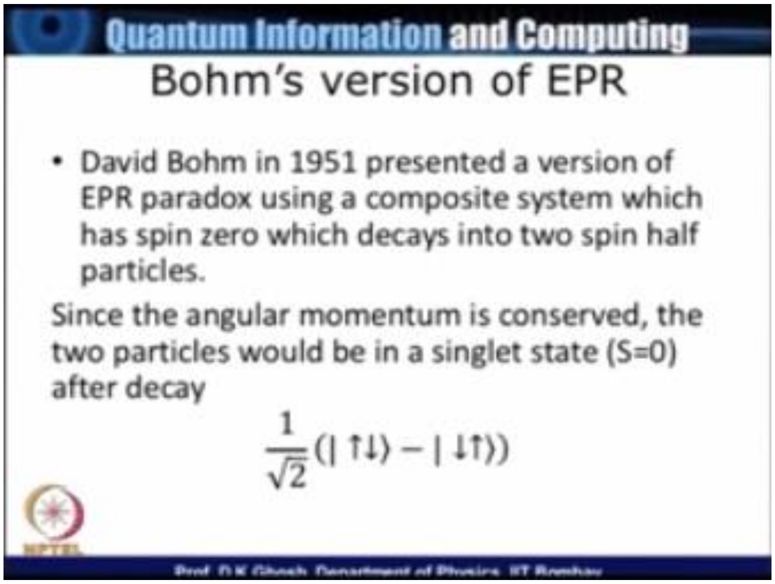
(Refer Slide Time: 11:15)



Locality says a measurement on one of the members of the combined system, in this case one of the members the entangled pair cannot influence the property on the other member, that is a locality because a measurement that you make in one place, cannot influence the property of another particle in another place well in particular if they are separated by a distance such that the property measurement that I am making on the second particle is shorter than the time that the light takes to travel from the instant the particle number 1 measurement was made to the vertical number 2.

So this is locality, so Einstein demands that a physical theory in order that it is understandable must have physics local reality this has two components one is locality that is independent of what happens to particle number 1, the particle number 2 processes certain property, and realism that is, it is not the process of measurement which gives the property to a particle but measurement simply reveals what was already coded in the particle.

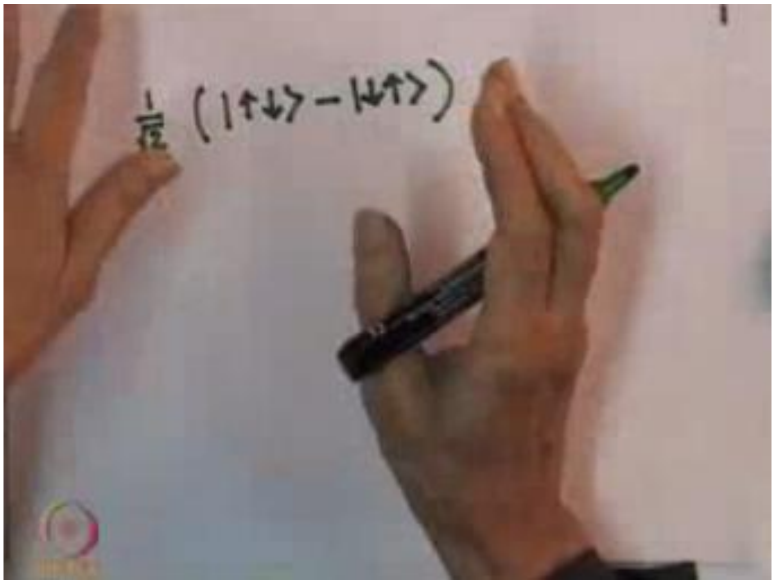
(Refer Slide Time: 12:36)



This was known as probably Einstein did not use that word but this is what is indicated on probably Einstein thought that there are some hidden variables in the problem which encodes the property of the system. To understand it in the language of quantum mechanics let us look at a slightly different version of the same EPR paradox and that is due to David Bohm in 1951, he presented a version of EPR paradox using a composite system which has a spin zero and this system decays into two spin half particles.

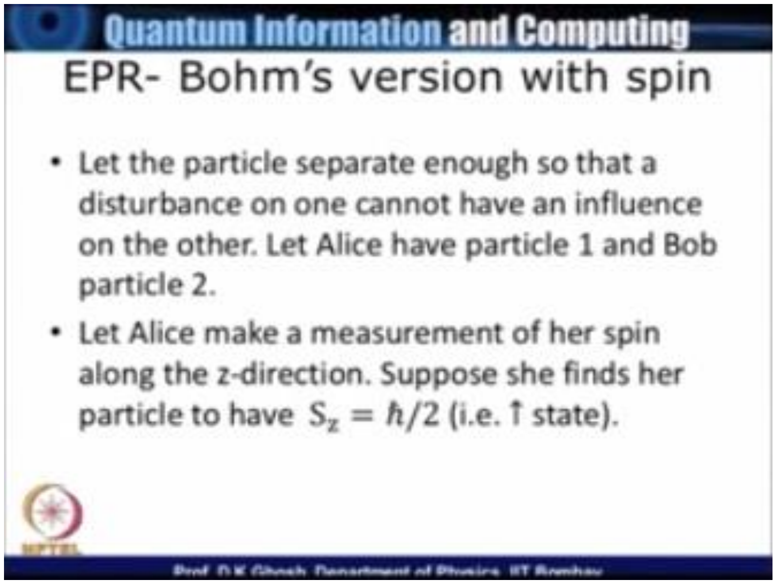
Now this is routine because you will find enough examples in nuclear physics and particle physics where a spin zero particle decays into two spin half particles. So we know again because the angular momentum is conserved and I had started with a spin zero particle the net moment, angular momentum of the two particles into which the combined particle has disintegrated should also be in s=0 state, which means that.

(Refer Slide Time: 13:56)



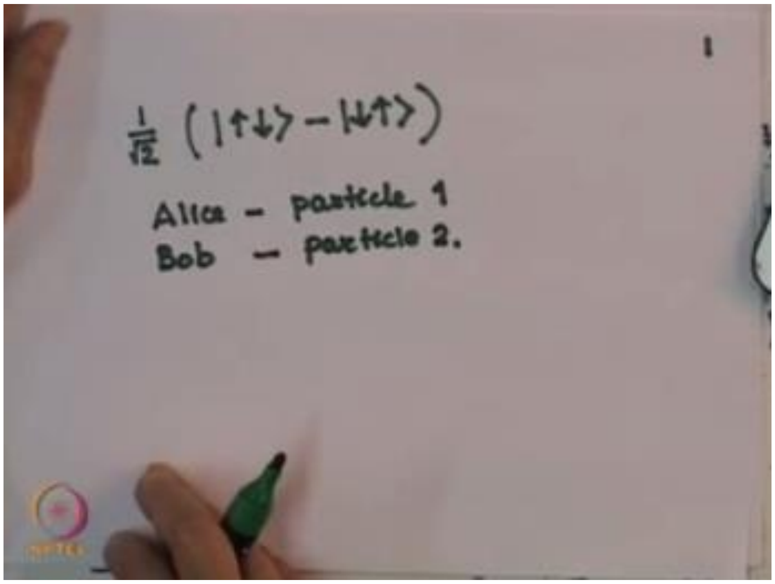
The particle will be in this state, so let me write down (|↑↓>-|↓↑> in the language in which we are familiar in quantum mechanics. So what it means is that the S_Z value of particle number 1 in half spin you could call it as Z=ħ/2 the S_Z value of the particle number 2 is a down spin so (|↑↓>-|↓↑>). Now supposing this qubit, two qubit system get separated by a distance such that disturbance of 1 cannot have an influence on the second one.

(Refer Slide Time: 14:47)



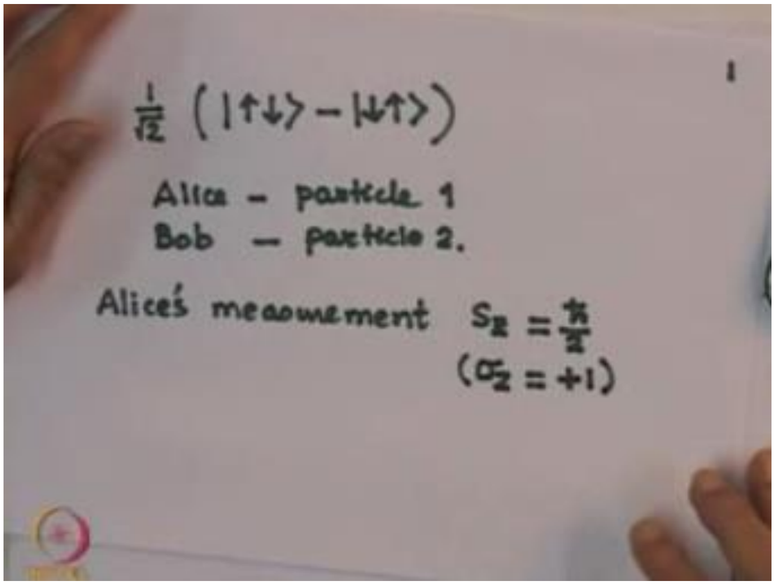
Let us assume that the particle number 1.

(Refer Slide Time: 14:51)



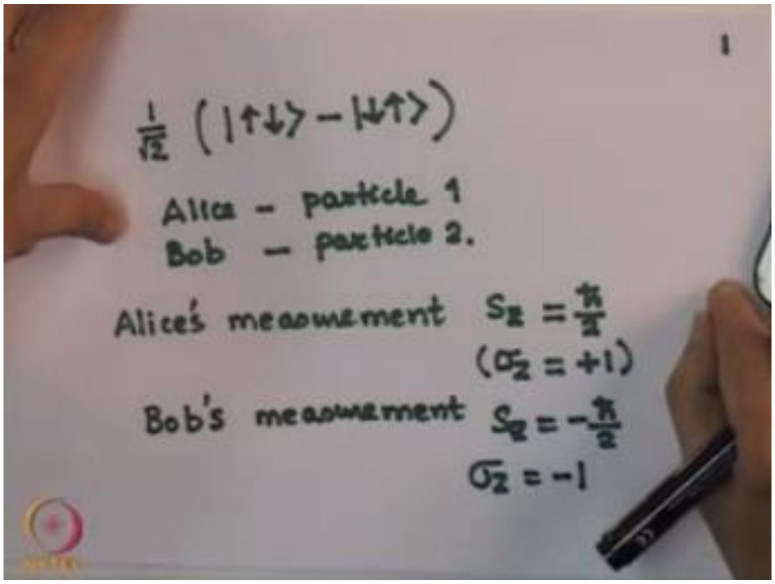
Is held by Alice and Bob holds the particle number 2, now suppose Alice now makes a measurement of her spin her qubit what is the S_Z value and suppose Alice, Alice is measurement says.

(Refer Slide Time: 15:18)



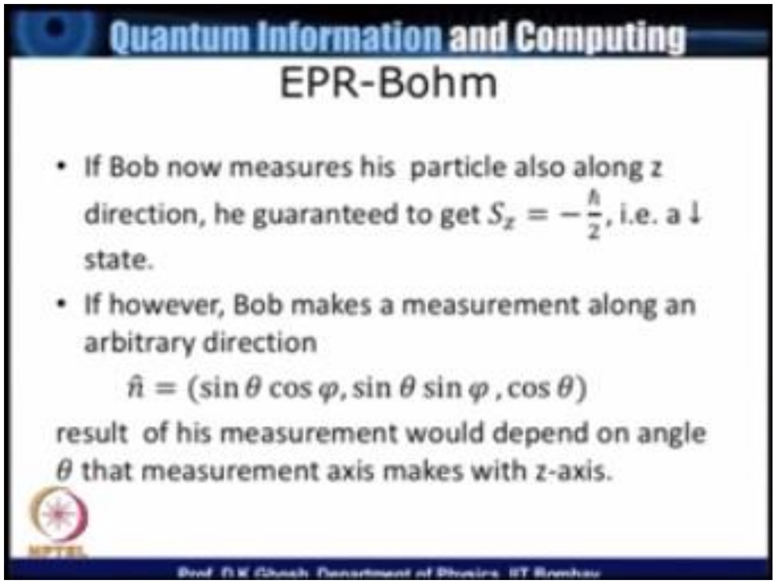
That the S_Z value of particle number 1happens to be in ħ/2 that is we will say σ_z that the value of the Pauli matrix Eigen value is equal to +1. Now what we are saying here is this, that according to Bob.

(Refer Slide Time: 15:49)



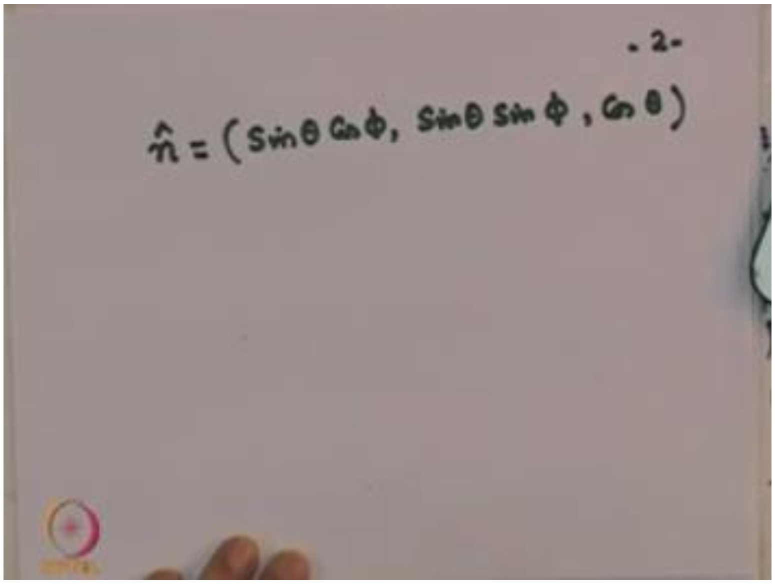
If he now measures the particle that he has he is guaranteed to get so Bob's measurement will definitely give him $S_z = -\hbar/2$ meaning they are by $\sigma_z=-1$. So you notice that what we are talking about here is this that Alice's measurement and Bob's measurement have a perfect anti correlation. If Alice’s get up Bob gets down if Alice gets down Bob gets up. Now suppose Alice is still made the measurement and got a spin up but Bob decides to make a measurement in an arbitrary direction.

(Refer Slide Time: 16:41)



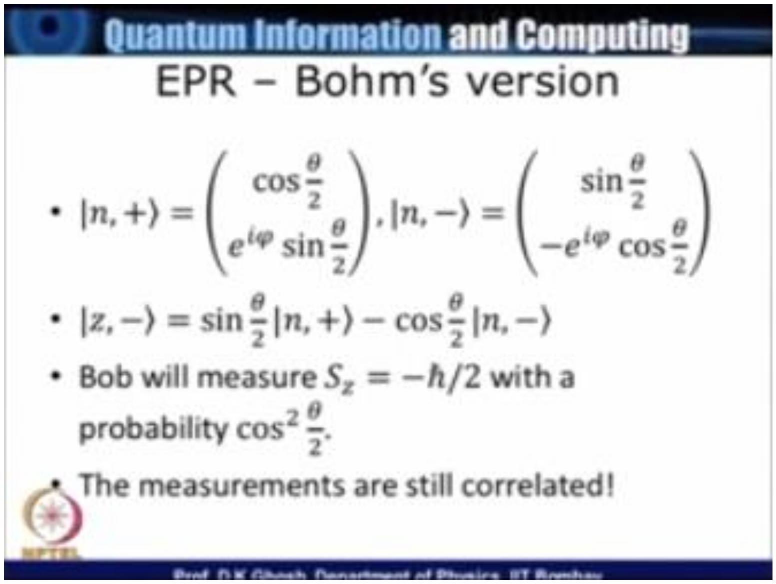
So suppose Bob's measurement direction is.

(Refer Slide Time: 16:46)



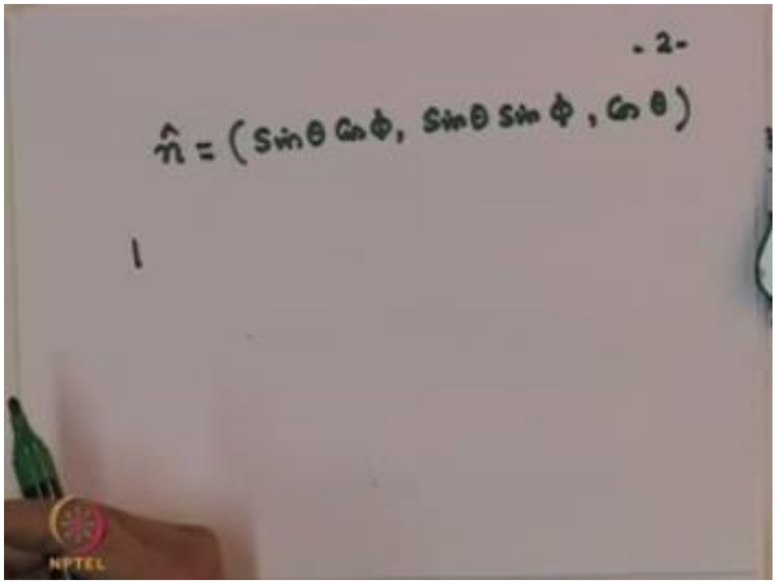
n which is equal to X component $\sin\Theta \cos\phi$, Y component $\sin\Theta \sin\phi$ and of course $\cos\Theta$. Now what happens to Bob's measurement in this case. Remember Alice has got S_z=+1 and Bob is now not measuring along the z-axis but in measuring along the arbitrary axis which makes an angle Θ with the z-axis.

(Refer Slide Time: 17:25)



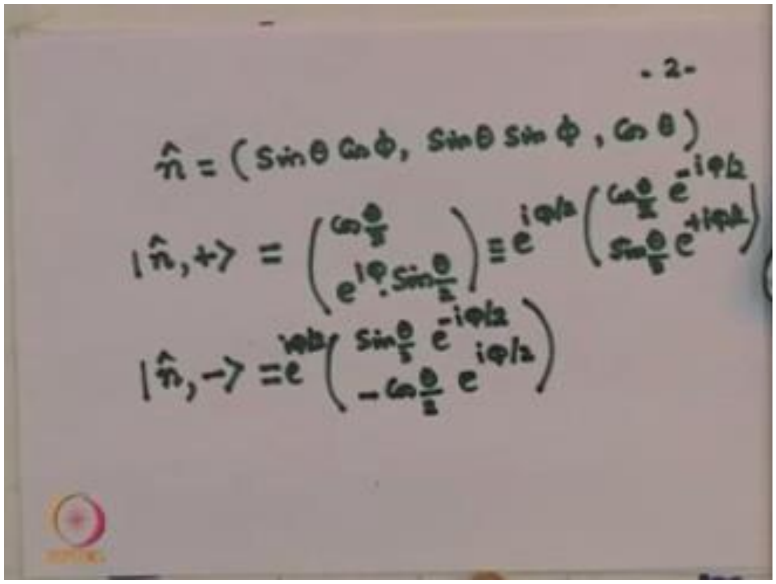
Now let us recall.

(Refer Slide Time: 17:29)



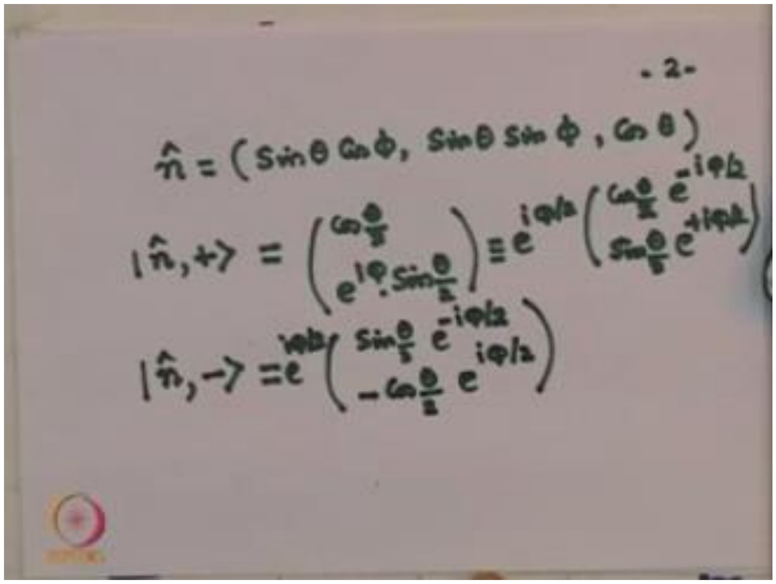
That along an arbitrary direction we have talked about it long back when we had just started introducing the qubit and its Bloch sphere representation, we said that if you look at σn that is the component of the Pauli matrix along an arbitrary direction then the eigen state of the Pauli matrix.

(Refer Slide Time: 17:55)



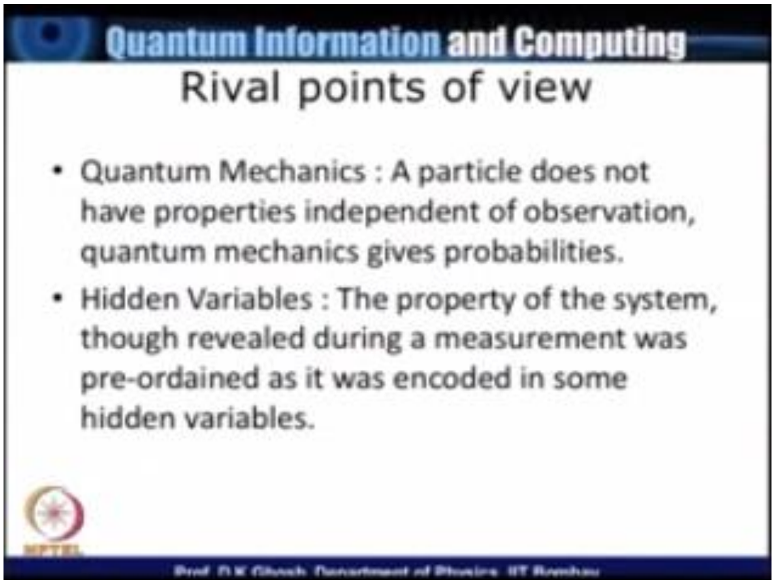
Corresponding to the Eigen value +1 is given by cosΘ/2 $e^{i\phi}$. sinΘ/2 in fact you could write it more symmetrically by pulling out a factor of $e^{i\alpha/2}$ and write it as cos $^{\Theta/2}$ $e^{-i\psi/2}$ sin $^{\Theta/2}$ $e^{i\psi}$ and correspondingly the state Eigen state corresponding to s n = -1 σn = -1 is given by same $e^{i\psi/2}$ this normalization factorize we knowing subsequent discussion I get sine $^{\Theta/2}$ $e^{-i\psi/2}$ –cos $e^{i\psi/}$2 remember that Eigen states belonging to different Eigen values must be orthogonal to each other and hence one of them has a - i and these.

(Refer Slide Time: 19:18)



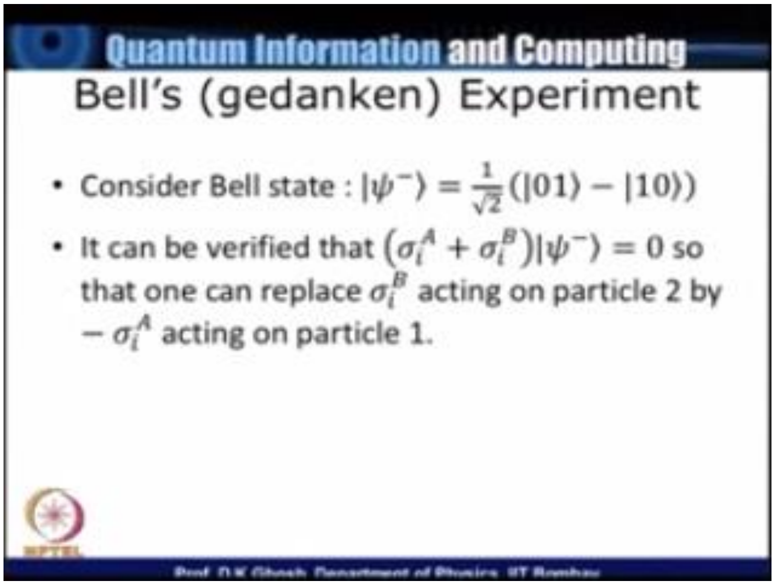
Eigen states are properly normalized.

(Refer Slide Time: 19:21)



So now let us look at what are we rival points of view that is there. So according to quantum mechanics a particle does not have property independent of the observation while and the process of measurement in quantum mechanics at best reveals the result based on certain probabilities. Hidden variable says the property of the system is simply revealed at the time of measurement but was pre-ordained maybe it was encoded in something that we cannot look at it is hidden from our both understanding and our eyes. And hence certain variables which are there in the system they are known as hidden variables.

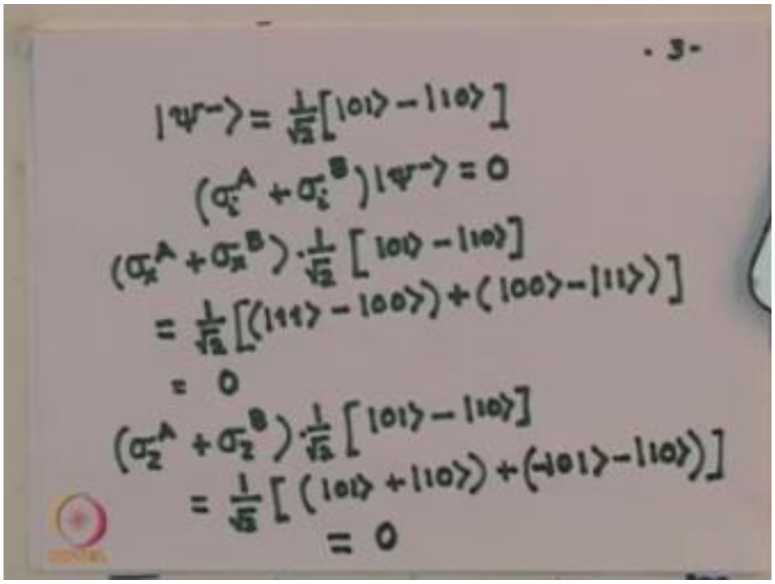
(Refer Slide Time: 20:20)



Now enter John Bell, John Bell suggested a gedanken experiment the experiment gedanken experiment things these are thought experience, that supposing one could do such an expert then what do you want, so what Bell did we are going to be discussing a set of inequalities generally known as the Bells inequalities we will not be discussing Bells original form but there are many subsequent simplified versions which are essentially equivalent and we will be talking about some of them.

But will be clubbing all of them together under a common name called Bells inequalities. So let us look at this remember we had four Bells I could start with any one of them so let me start with the bell state.

(Refer Slide Time: 21:16)



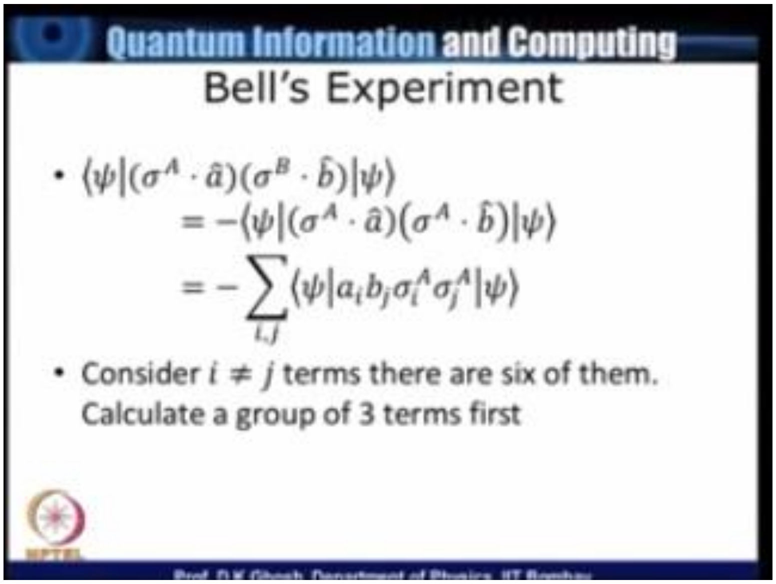
Which is ψ- which is equal to $1/\sqrt{2}$,0 1-1, now one of the things which I can verify with this very trivially is that if you take any component of the spin for the particle number one I will indicate for convenience instead of numbers because these numbers are also there I will always refer to particle number one by A because it is a qubit belonging to Alice and the second one I will call it as B because that is a particle which belongs to Bob so that second number.

Now I claim that if you take i f component of σ, σ first particles and second particle then when it acts from the states ψ - that gives you this is the important relationship I will simply illustrate this with by calculating the let us say i = x if i is equal to x I have σ + a+ σ_x^B acting on this state there is a $1/\sqrt{2}$ it remains as it is 0 1 - 10 remember σ_x what it does is to make a 0 x 1 and 1 x 0.

So first when σ_x operates on these two states I get 11 because σ_x only acts on particle number 1 minus 00 and then when $\sigma_{x \text{ of}}$ B which acts only on particle number 2 I get 00 - 11 because this 0 becomes 1 and you can see immediately it is 0, just to get convince you that this is not a fluke let us look at σ_z^a plus σ_z^B $1/\sqrt{2}$ is there and 01- 10 remember what σ_z does σ_z acting on the state 0 does not do anything.

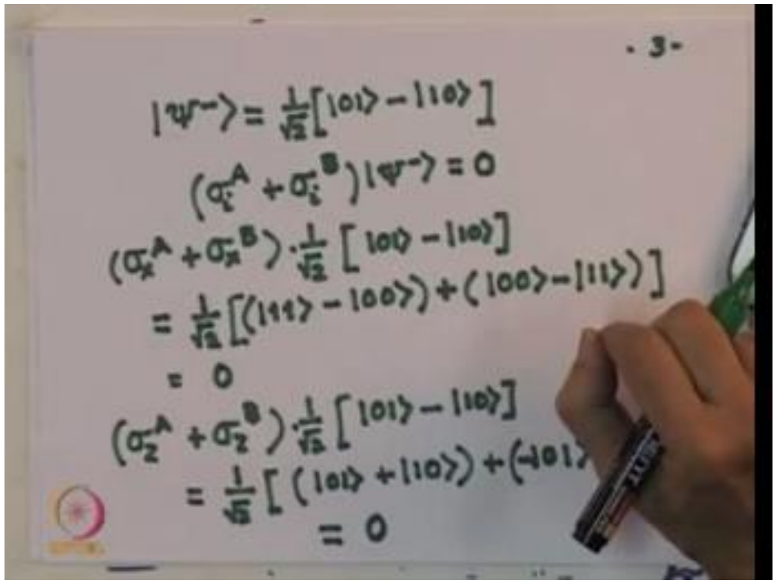
So therefore σ_z^a acting on the first state here will keep it a 0 1 but however acting on the second state this 1 would become a -1 so therefore I would get plus 10 simply changes the phase if the acts on qubit 1, then I have a +σ that is acting on be so 0 is undisturbed this one becomes a minus 1 and hence this minus -1 and 0 nothing happens by z and once again you can see that the number comes cancel out and I am left with this. Now this is an important result to get because it tells me that.

(Refer Slide Time: 24:52)



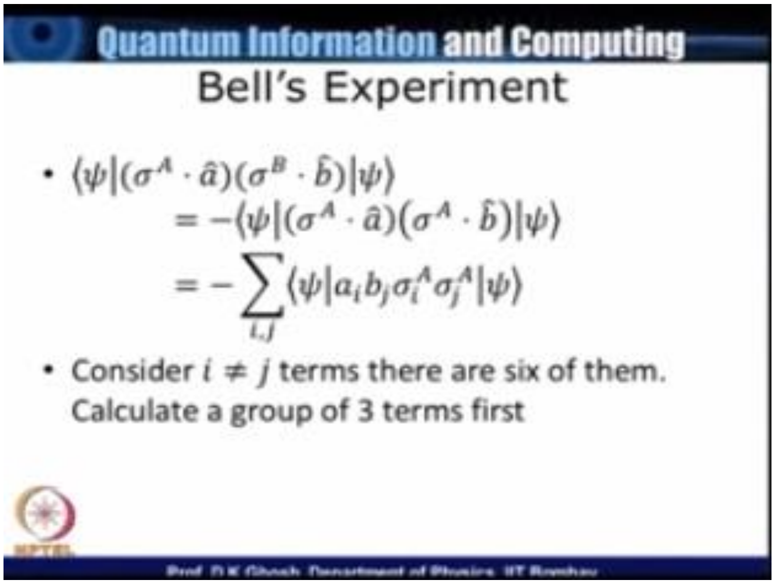
There is a quantity which is.

(Refer Slide Time: 24:56)



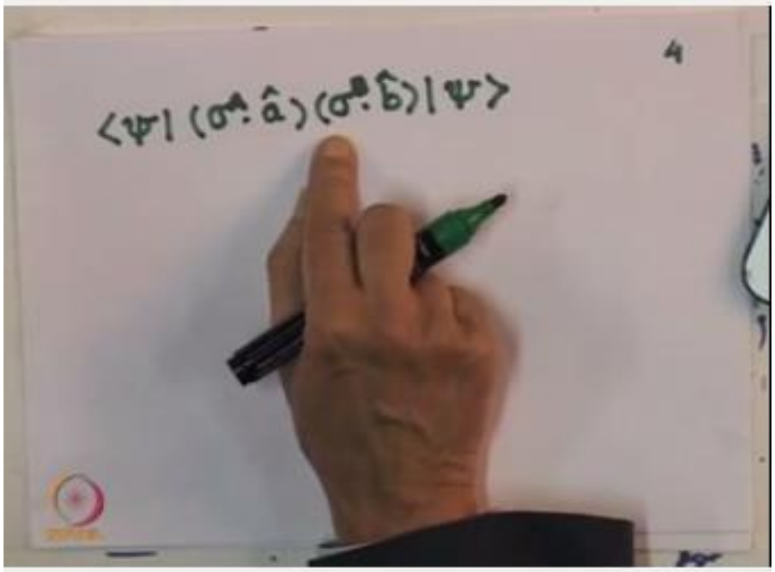
$\sigma^{ia} + \sigma^{ib}$ which gives me 0, so let us look at what does it actually mean.

(Refer Slide Time: 25:09)



This gedanken experiment that I am talking about we will be calculating.

(Refer Slide Time: 25:18)



The expectation value of this quantity σ^A supposing Alice makes a measurement of her spin along some direction a so σ^A .â unit vector a and Bob measures along b so we can easily calculate how much does it give me and one thing we have just now said is that Bob's measurement of σ^B of the particle number 2 because of the fact we have proved that $\sigma^A + \sigma^{iB} = 0$, so if Bob is to reveal the measurement to Ellis then Ellis can conclude what would have happened if she had made the measurement in spite of or not having made them we will continue with this point in the following lecture.

NATIONAL PROGRAMME ON TECHNOLOGY
ENHANCED LEARNING
(NPTEL)

NPTEL
Principal Investigator
IIT Bombay

Prof. R.K. Shevgaonkar

Head CDEEP

Prof. V.M. Gadre

Producer

Arun kalwankar

Online Editor
& Digital Video Editor

Tushar Deshpande

Digital Video Cameraman
& Graphic Designer

Amin B Shaikh

Jr. Technical Assistant

Vijay Kedare

Teaching Assistants

Pratik Sathe
Bhargav Sri Venkatesh M.

Sr. Web Designer

Bharati Sakpal

Research Assistant

Riya Surange

Sr. Web Designer

Bharati M. Sarang

Web Designer

Nisha Thakur

Project Attendant

Ravi Paswan
Vinayak Raut

NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING (NPTEL)