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Quantum Infromation and Computing

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Modul No.06

Lecture No.33

Quantum Error Correction-IVShor's 9 Qubit Code-II

In the last lecture we introduce Shor's 9 qubit code which attempts to correct both bit flip and phase flip or a combination error in a single qubit or a 0 + b1 that is being sent over a qubit. We had seen the coding circuit consulted 6^{th} situation and we said that our purpose is to encode.

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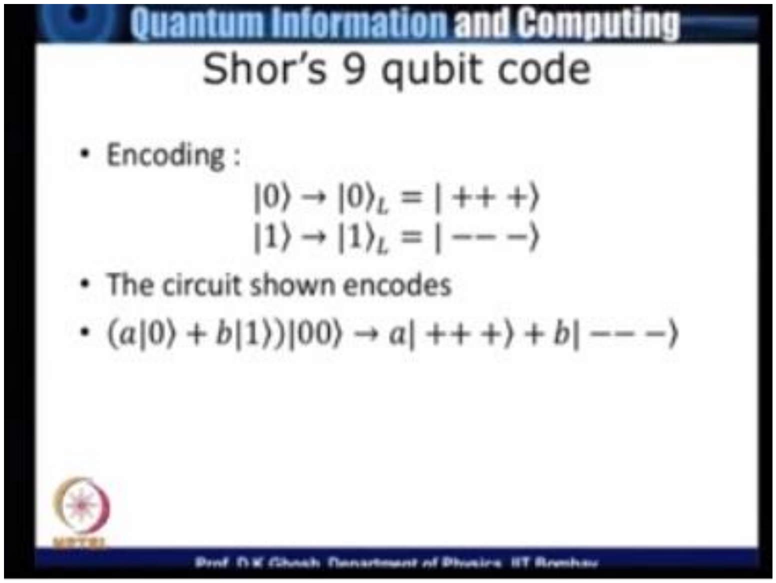

The state 0 as 0 logical which is return in a compact form as + + + + and 1 as 1 logical it is written as - - - there the + + + stands for 000 + 111/√2 written so let us look at this circuit again.

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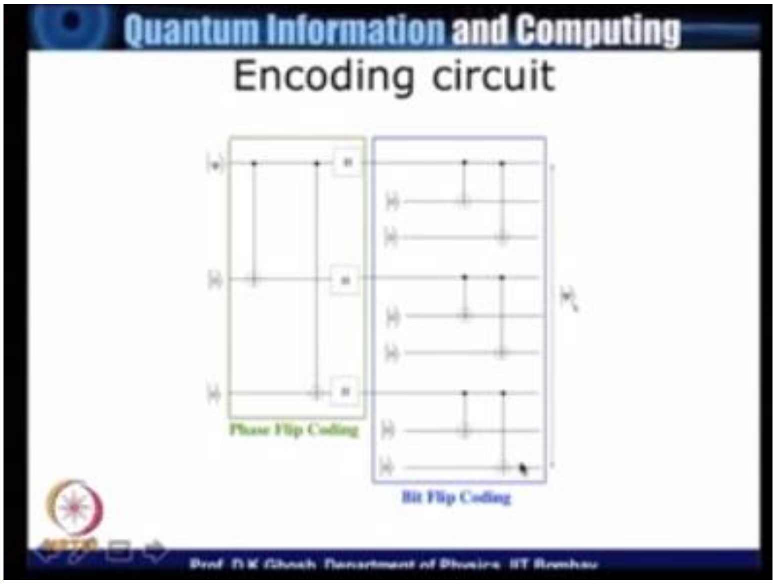

So this is the circuit which we talked about last file where Ψ is the qubit which I am interested in sending which are the structure ab + b1 and instead of sending the three quibit code we have now generated s9 qubit code in which we have done two phase one is transmit coding and other one bit flip code then so and that end of it what we got it was a state which is.

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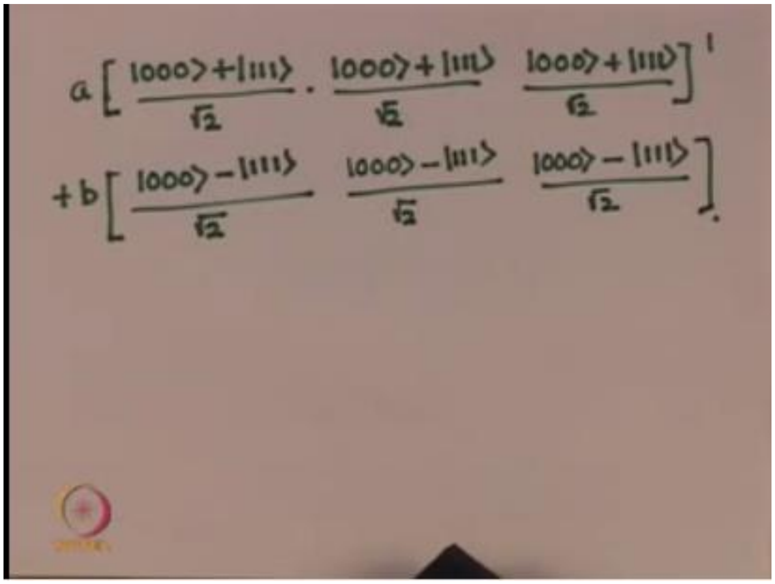

A 000 + 111/√2 let me two times that is how we have 9 qubits and this my coding actually plus b times 000 – 111/√2 that is the, I am writing down minus111/√2. Now we need to correct for that instead of being that general theory now let me sort of schedule that we will assume this helps the analysis but on the other hand the discussion will be generally show that any of the qubit that could take it out. Now let us suppose that in the 9 qubits that you will receive I have a single qubit error and that error is happening and let us say first qubit which is our both bit flip as well as phase flip.

Now you have to realize that I am treating the 9 qubits as 3 blocks of a qubit and the special thing about the phase flip is the following that even though I said with that the phase flip is occurred in qubit number one, but remember I would not do the difference if the phase flip has occurred in qubit number 2 or 3 of it, because it will simply give you a relative minus sign qubit so let us look at what happens.

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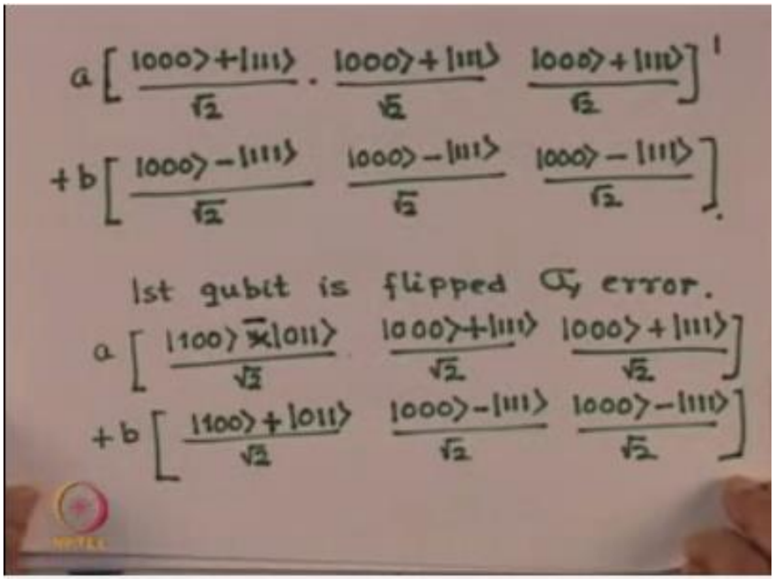

So we will assume first qubit is both as σy error. Now in that case what do I get, what I get as a result is this since the first qubit is flip instead of 000 I get 100 + 011/√2 and in two 000 + 111 this now change occurs here. I will change the first qubit to take care of the fact that there is a phase flip also, but let me first write down what happens when I got just a bit flip. So this 0 has become 1, 1 has become 0 and the other two I will retain exactly the instructive that they had.

Now this is what would happen if I had just a σx there, but as I have said that I also have a σy error so that would made this + or – and this – or + this is the completely decoding circuit so let me go on gradually analyzing the decoding.

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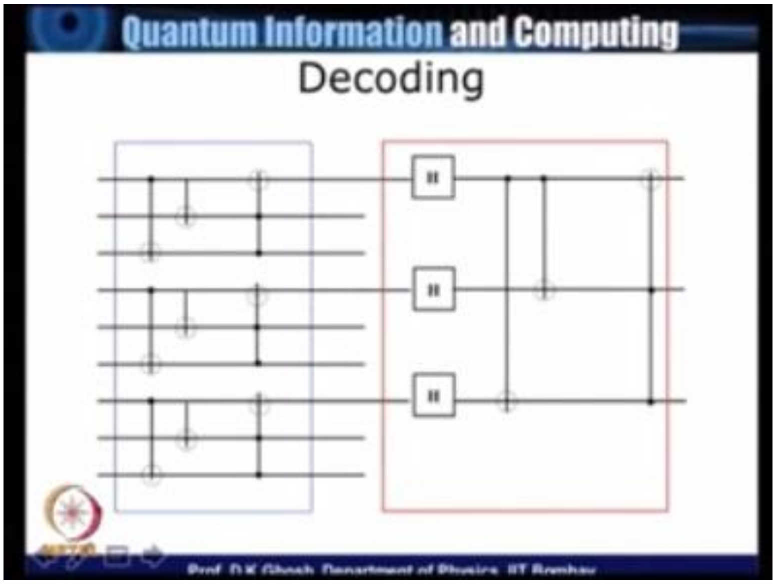

So this is what we have received, this is what I wrote down.

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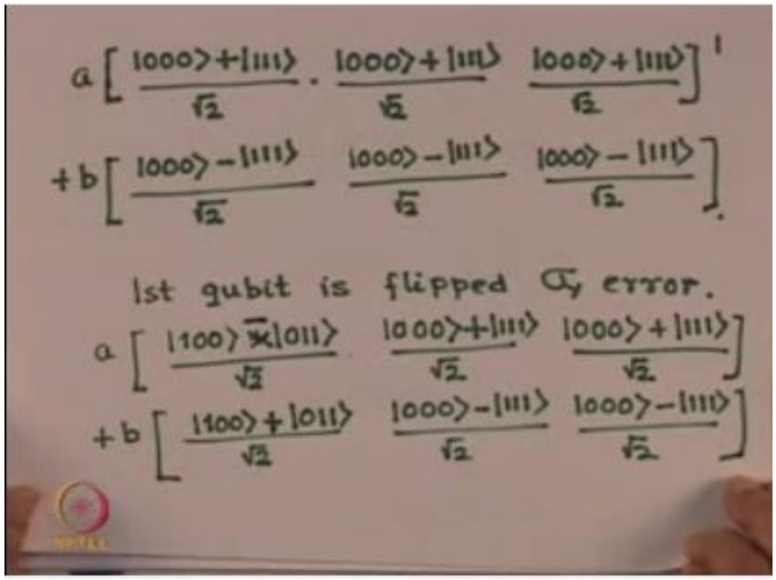

Because of the noisy channel this, what I show.

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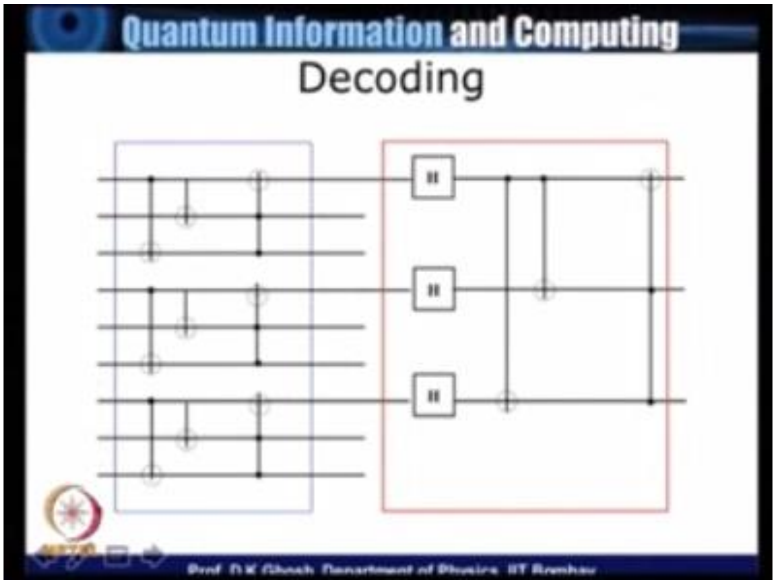

There that is what I received. Now it would help if we treat each of the three as 1 block. So let us look at what I am doing here, so here what I am doing take a qubit number 1 as the control, I am applying a CNOT gate on qubit number 2 and 3 and again taking qubit 4 of the control and apply a CNOT on 5 and 6 and taking 7 as control I am applying 1, 8 in that. So what does it given me, so what I get is this.

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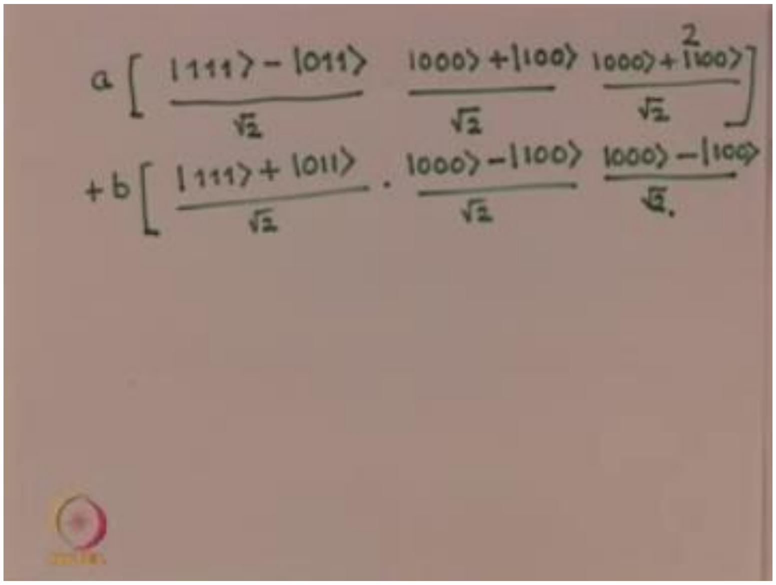

That A now remember I have said that take the control the first qubit so therefore I get 111 – now nothing happens here, because the qubit number 1 had 0, so therefore they remain the way it was this divided by √2, this into |000> + remember qubit 4 is the control now and that is already 1, so therefore these will become 00/√2 and likewise this is the same thing |000> +|100>/√2 not much is happening to the second and the third block because that is the way I am dealing it.

So this will now be |111>+|011>/√2 and this will be the same as before but it is a – sign, at that stage what we do is.

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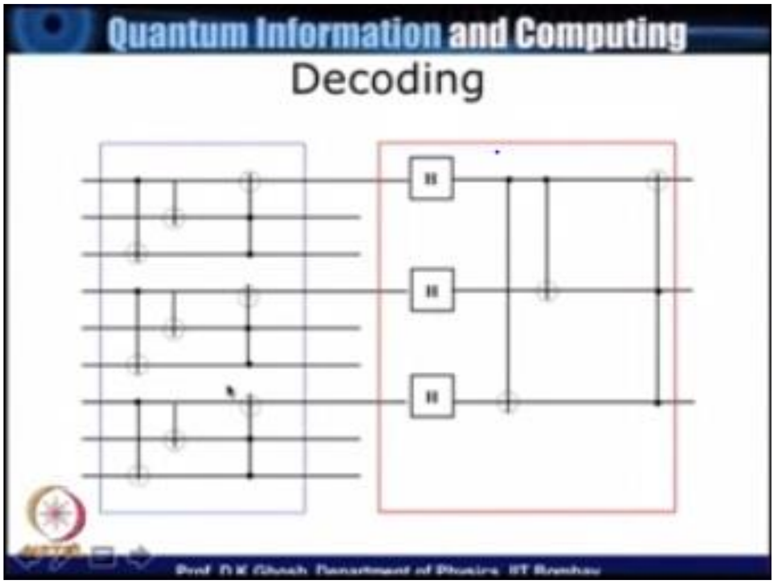

Go back to slide again that I apply a CCNOT gate taking the qubit number 2 and 3 will be controlled and apply a CCNOT on qubit 1 likewise taking Eigen 6 has control on qubit 4 and 8 and 9 has controlled on qubit 7. So what I get now is.

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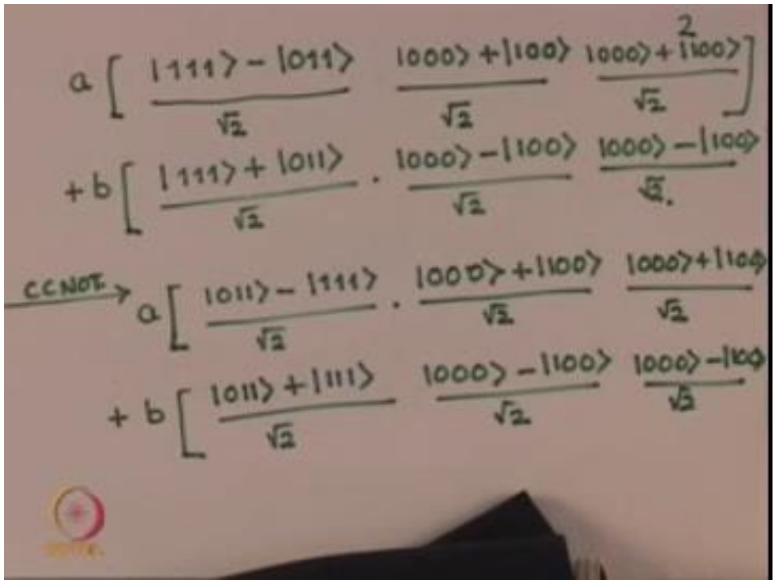

So this is the pre CCNOT that I apply, so I get a times, now you remember how CCNOT works, the CCNOT will work if both the controls are 1 and this is exactly what is happened here so therefore I will get |011>-|111>/√2, in this case I will get |000>+|100>/√2 |000>+|100>/√2 and the second term is b times the same thing sort of here |011> but with the +sign |111>/√2 and |000>-|100>/√2 |000>-|100>/√2. So this is if we look at the slide.

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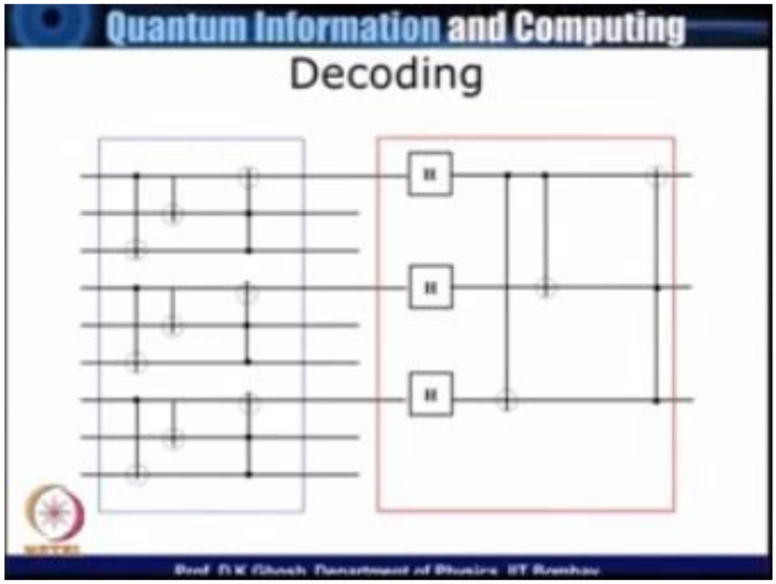

This is what comply, now what I do now is that I do not worry about that is happened to this second set of ancilla that I have written and concentrate only on the first qubit it was send and the two sets of ancilla that I will repeat. So in this case what will happen in this I could carry on writing all the 9 qubits but it is actually not particularly important so I will now because we are whatever was their status they remain as it is, so what I will now do is to simply look at what the state was.

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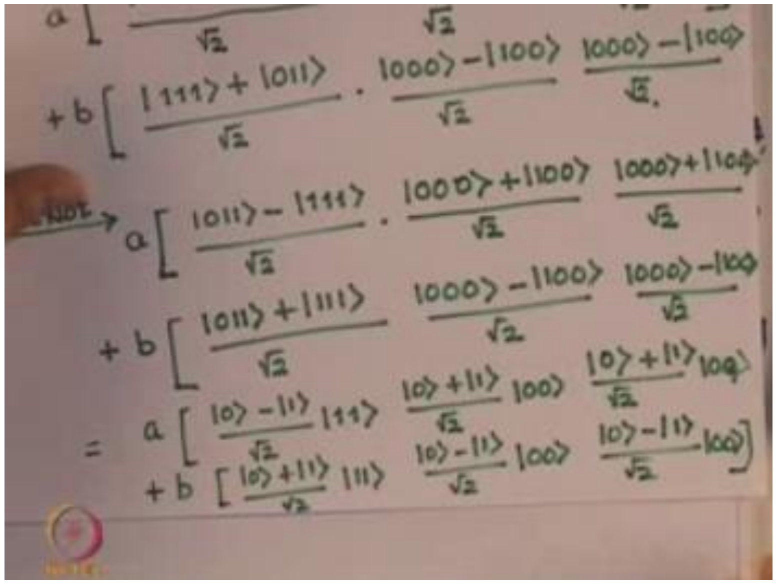

Look at the state you can actually factorize you notice this, this state is equal to a times |0>-|1>/√2 times |11> this is qubit1, this is that we have qubit 2 and 3 and |0>+|1>/√2 |00> |0>+|1>/√2 |00> and likewise b times |0>+|1>/√2 |11> |0>-|1>/√2 |00> and |0>-|1>/√2 |00> as I said I ignore this second set of qubits that I have introduced and I will be now be looking at only the first.

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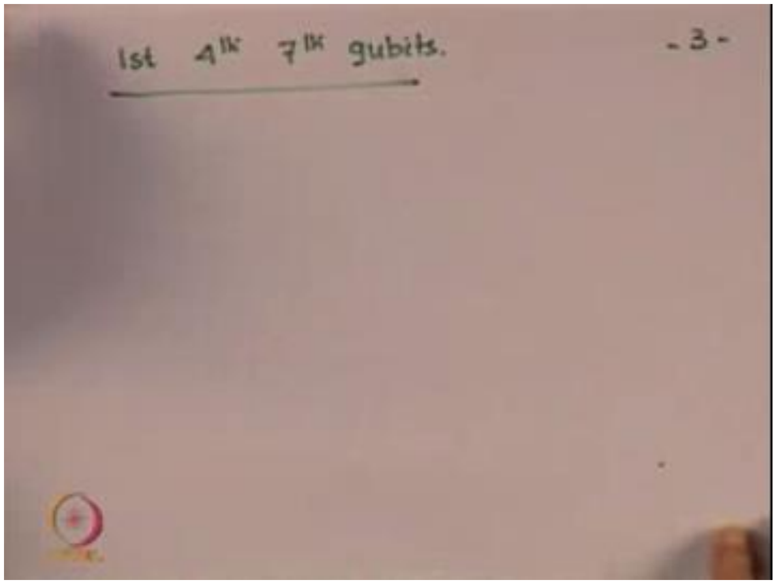

The 4th and the 7th qubits, if you look at.

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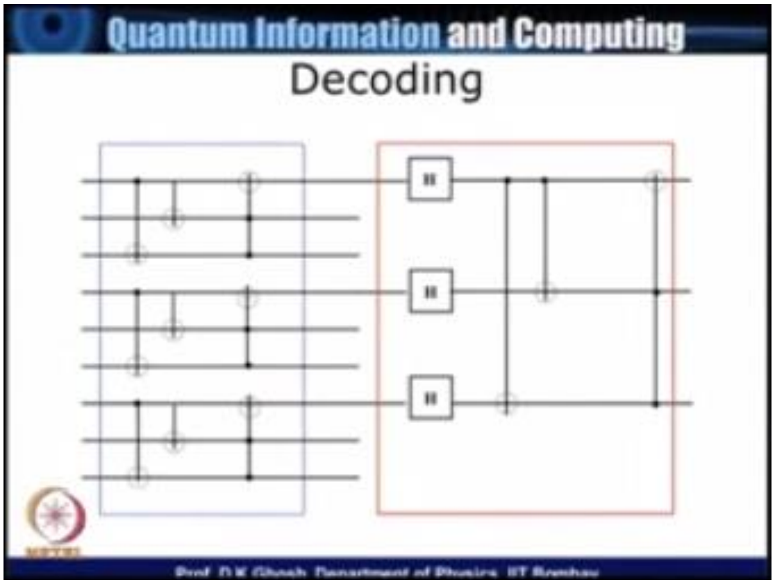

The slide again you notice that on each one of the, I have introduce a Hadamard gate you look at what the Hadamard is going to do.

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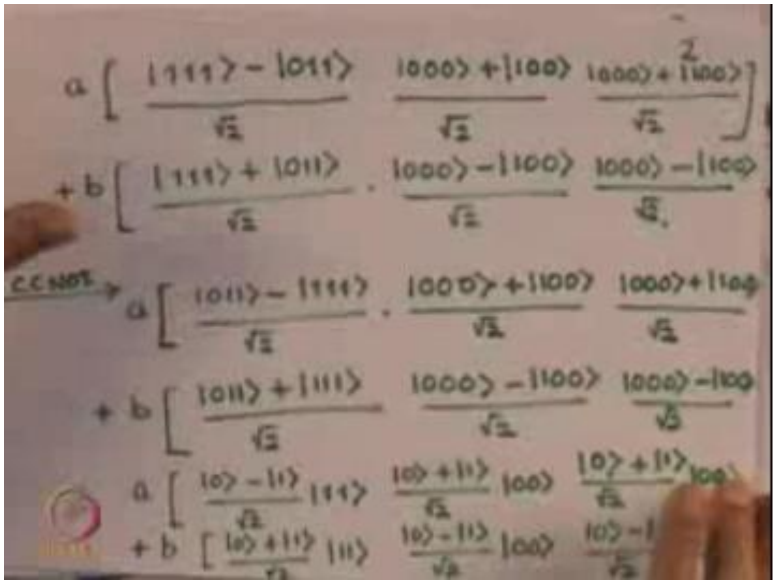

A Hadamard applied on qubit number 1 is going to make it a 0, because this $|0\rangle-|1\rangle/\sqrt{2}$ and on this one, on this term it going to make it 1, and these things I will ignore they will remain what it is because nothing has been done to this.

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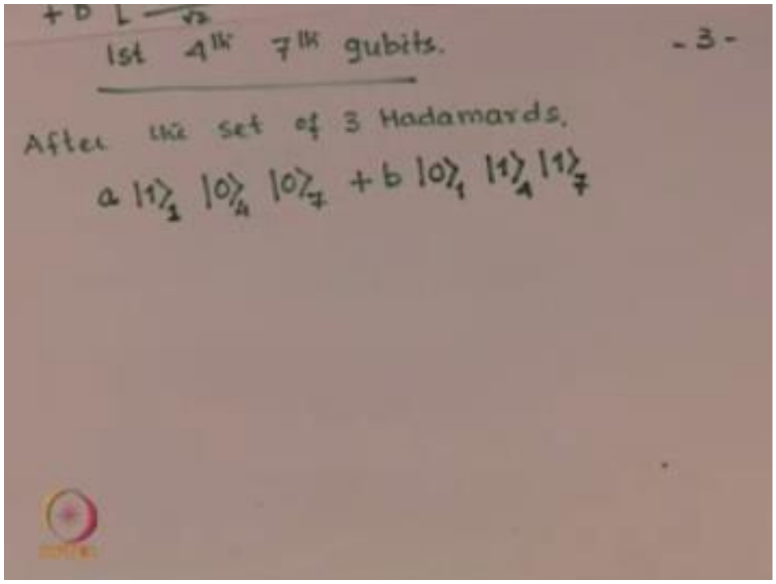

So therefore, after the set of 3 Hadamards I get the following, I get a times remember this was |0>-|1>/√2 so therefore let me write it as 1 on a Hadamard and just to make sure that I am talking about line 1 I indicated that then the fourth one is 0, and the seventh one is also 0, plus b times |0> |1> and |1> and of course in this element. This is what happen if you look at the slide.

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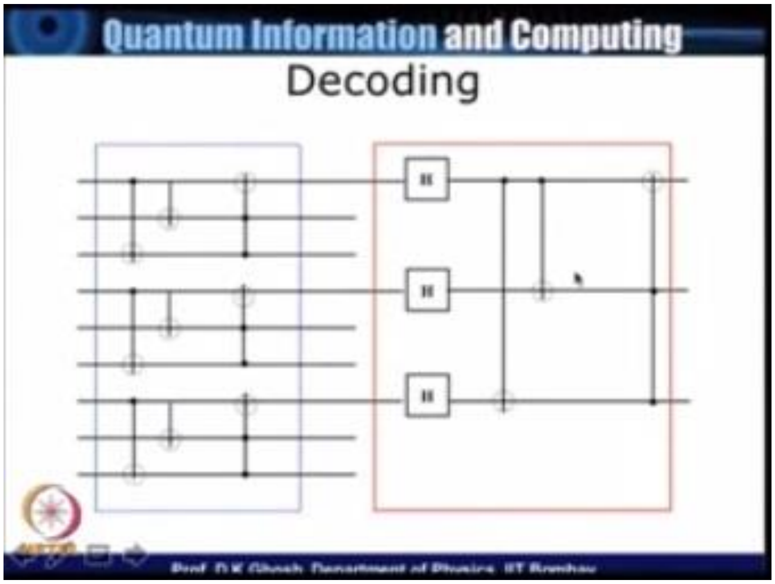

Which has 3 Hadamard, now what I am going to do at this state, I am going to take you remember that my interest is actually in qubit more than. So I am going to take qubit number 1 as the control and apply a CNOT on the qubit number 4 and qubit number 5, so let us do that, so CNOT

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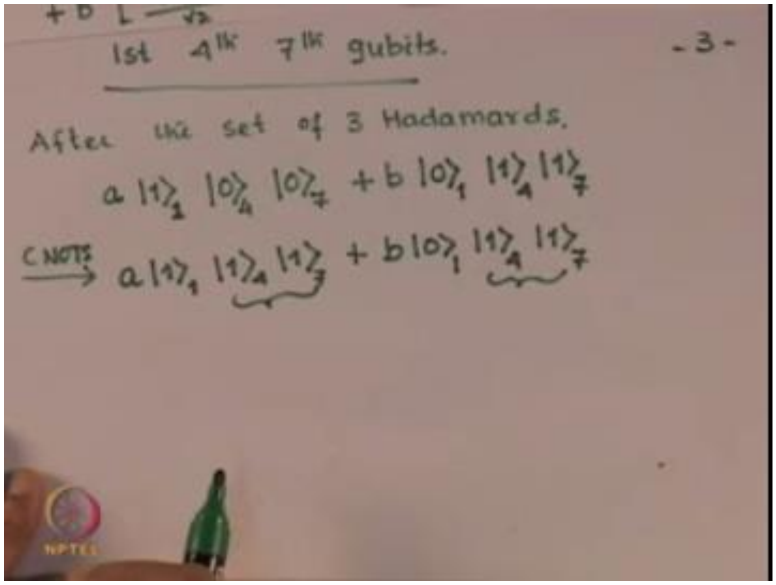

And that gives you a 1 since the control is 1 the 0 becomes of 1 and this 0 also become to 1 + b times this is the 0 and this also remains 11. So this is qubit 1 and this qubit 7. 4 and this is 7. The last bit of an action that I need to do now is obvious, because you notice that what I have bought here is 4 and 7 having 11 that is actually telling you that I could use this there to apply a CCNOT on the qubit 1 which I have interested in to begin with.

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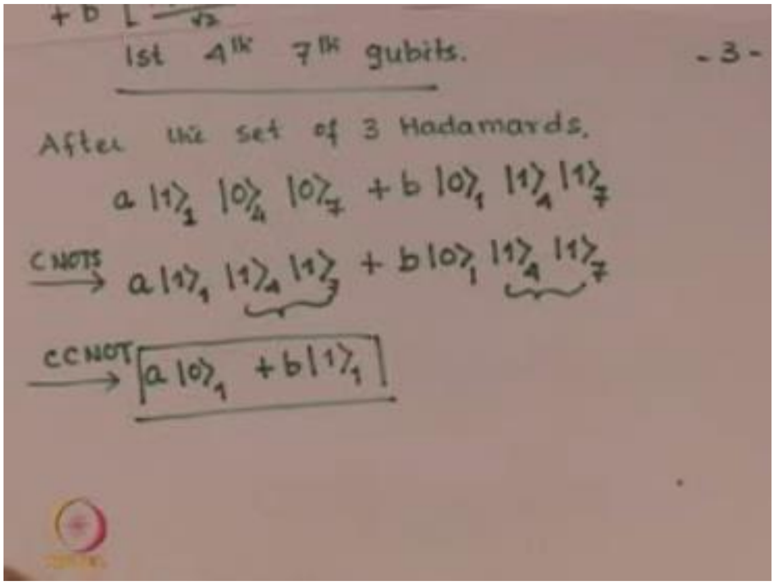

So therefore, we apply CCNOT and that will make it a 01+b1 and this if you recall is the original qubit that was sent before I did coding and then decoding. And this is roughly what the whole structure is about, but let us conclude this with the general policy because have illustrated it for only qubit number 1, but as you could see that what I am thinking is that input apply do it for anything. So let us see what does want do so if you remember that what I had was.

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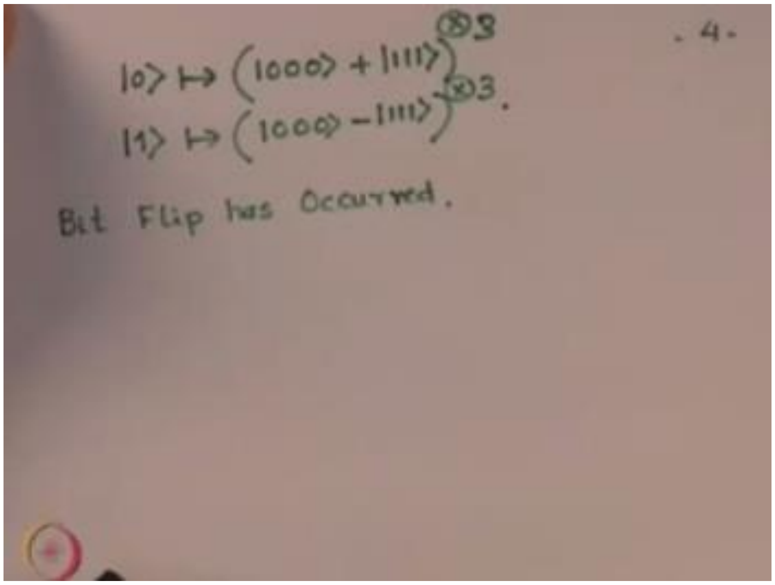

A 0 encoded as 000 +111 instead of writing three times let me just put our usual notation of three and the one was encoded as 000 -111 supposing a bit flip has occurred. How do you take care of a bit flip how do you check this on which bit the error is occurred? Remember I work it out for qubit number 1 but other than how did I know to begin with that the error actually had happened there and the way to do is this that I want to check parity.

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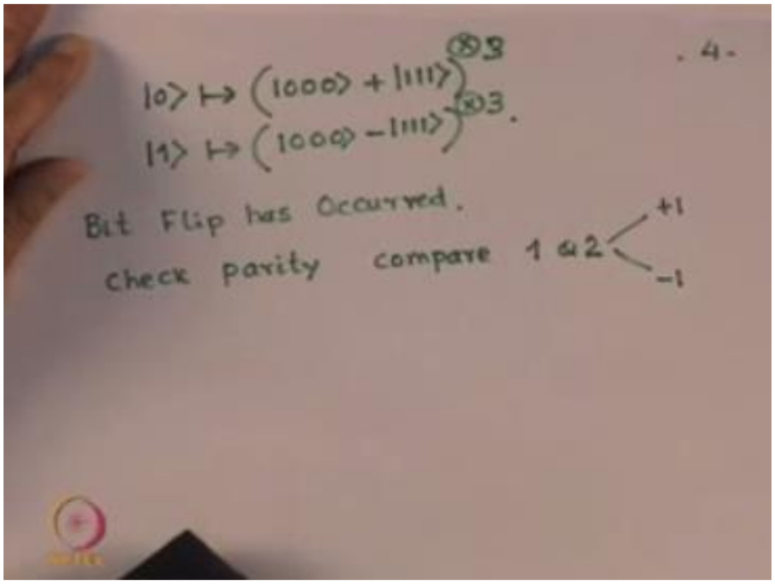

I want to check parity in the following way I compare 1 and 2 no, when I compare the parity of 1 and 2 I get either a +1 or I get a -1. Now since I have already made a statement that at best I have one error only when I get a -1 then I know that the error has occurred in one of the qubits. And that could be either 1 or 2.

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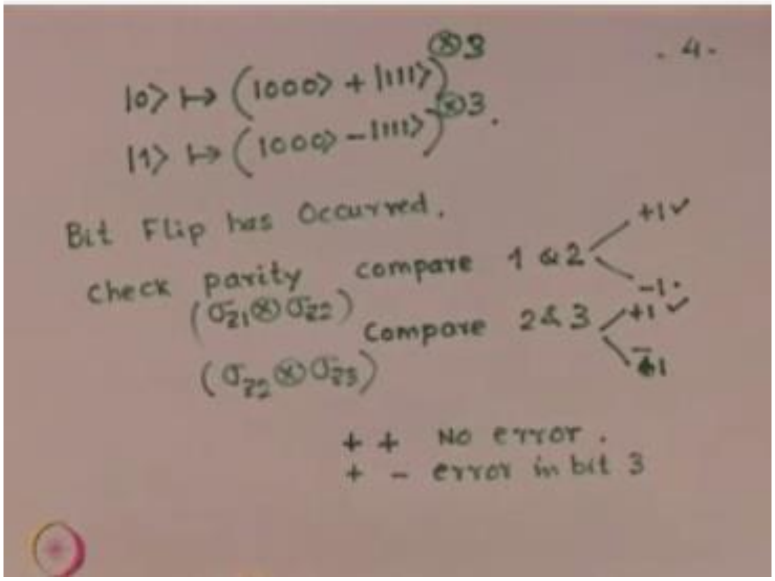

Then what I do is compare 2 and 3, now this is very trivially done comparing 1 and 2 by finding out what is σ_{z1}, σ_{z2}. And similarly if I want to measure 2 and 3 I just do σ_{z2}, σ_{z3} measurement. Now here again I can get a +1 or I get a -1 now notice what are my possibilities. So the possibilities are I get +1 here and the +1 there which means no error had occurred I get a+1 there but a -1 here.

Now remember +1 here meant 1 and 2 did not have any other but if I got a -1 here it means that I have an error in bit three, if I have a -1 there then either bit 1 or bit 2 have an error, but then if I have a +1 there bit 2 does not have an error and hence 1 must underneath. But if I get -1 there then it means wither 2 or 3 must have an error and since when first argument we have given either 1 or 2 have an error so therefore 2 has an error And likewise I can find out in the error occurred in bit 3 or not. And suppose I am talking about not a phase flip error.

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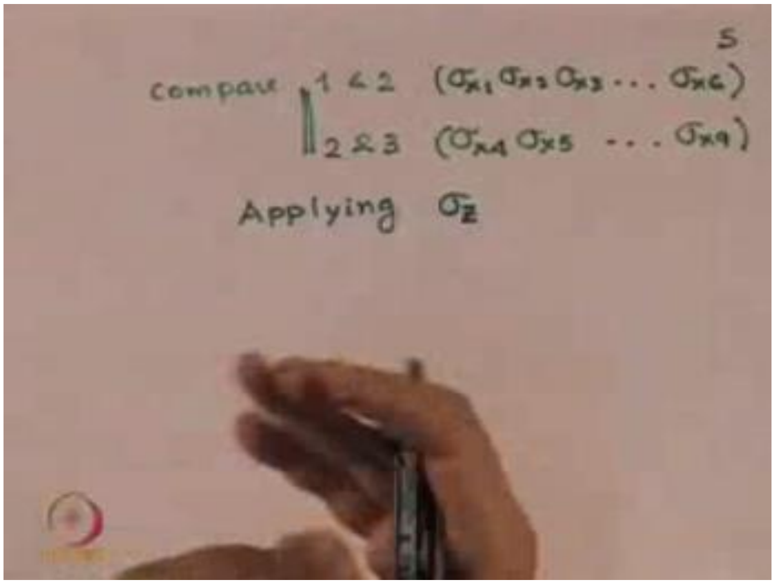

Then what I do is to compare 1 and 2 and this I do by σ_{x1} σ_{x2} σ_{X3} write up to σ_{x6} and also do 2 and 3 which can be done by doing σ_{x4} σ_{x5} up to σ_{x9}. Now I am not going work out as I did for the previous case but I leave it for you to work out that this will enable you to find out where the face flip is occurred and if a phase flip has occurred it can be corrected by applying a σ_x .I should have said in the first group then after I found out which qubit has had an error I could have corrected it by applying $\sigma_{x.}$

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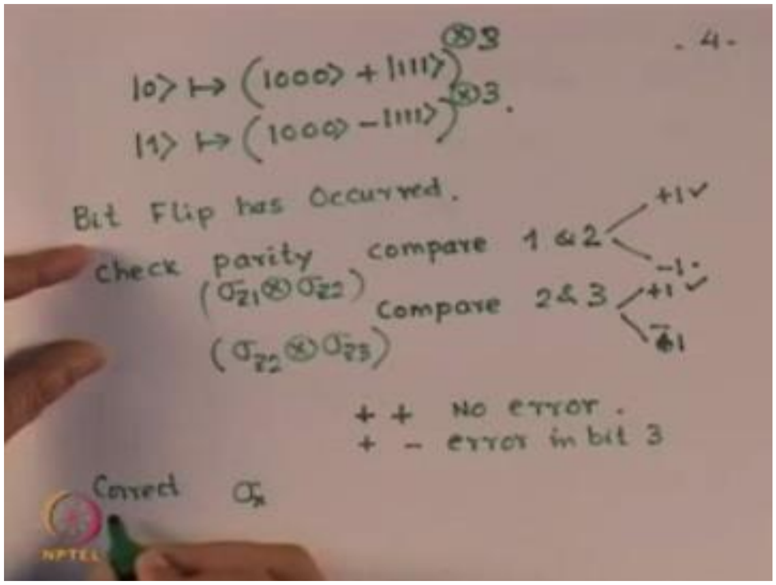

And here by σ_z okay.

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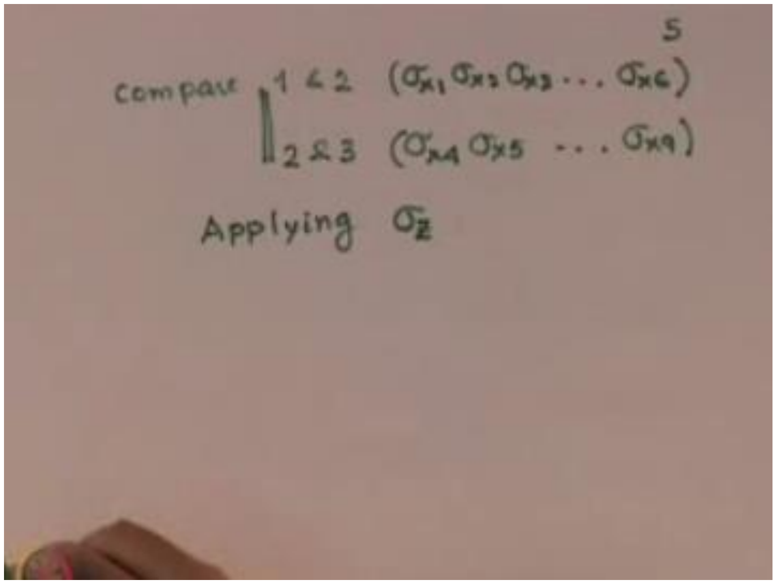

Now suppose this analysis leads me to conclude that there have been both a phase flip and a bit flip then each one of the correction must be sequence theory of that, because as you consider not interfering with each other. So this is essentially a brief summary of quantum error correction course which has very important con sequence in communication because keeping the purity of a quantum state is of almost important in dealing with any quantum algorithm.

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