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Quantum Infromation and Computing

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Modul No.06

Lecture No.31

Quantum Error Correction-II Three Qubit Code

In the last lecture we started talking about the errors that might arise in quantum communication. We have found that like in classical communication one could of course have a situation where the amplitude of the qubits are being changed related amplitudes of various basis states will be changed. But in addition to that there is also a question of the relative phase between the various numbers have been changed. Now what we what to do today is to talk about steps that could be taken in order to correct these things.

I had already explained last time that the option of sending multiple qubits and deciding which is the correct one by a majority effort as is done in case of classical bits is not really an acceptable alternative because of the quantum no cloning theorem which precludes multiplication or repetition of states. We will talk today about an error code and the possible steps that one could take and I must alert that this simple minded code that I will be talking today is not a complete code in the sense that while it takes care of the correction of amplitudes.

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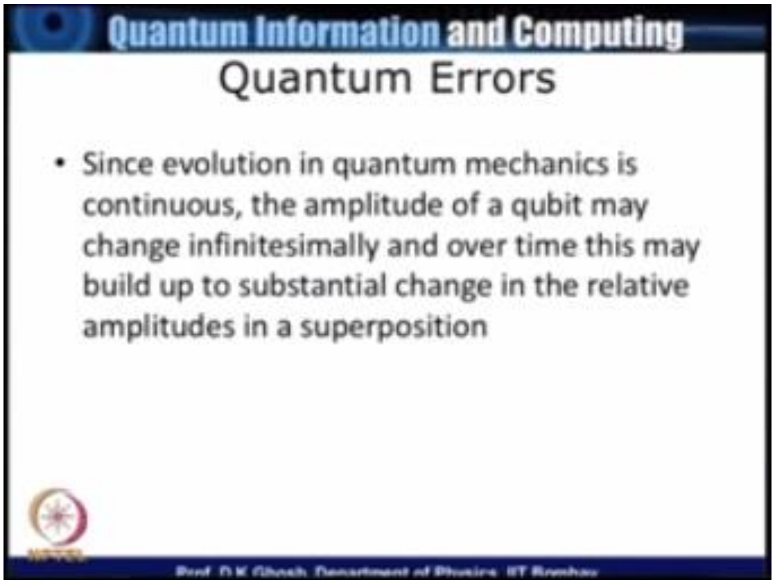

That is correction of bit flips, the does not quite take care of changes in relative changes of in case of a superposition. So let us see I again use this phrase that suppose I have a sender whose name is Alice and I have a receiver whose name is Bob and let us see how does this communication works. Now Alice wants to send a state |ψ>.

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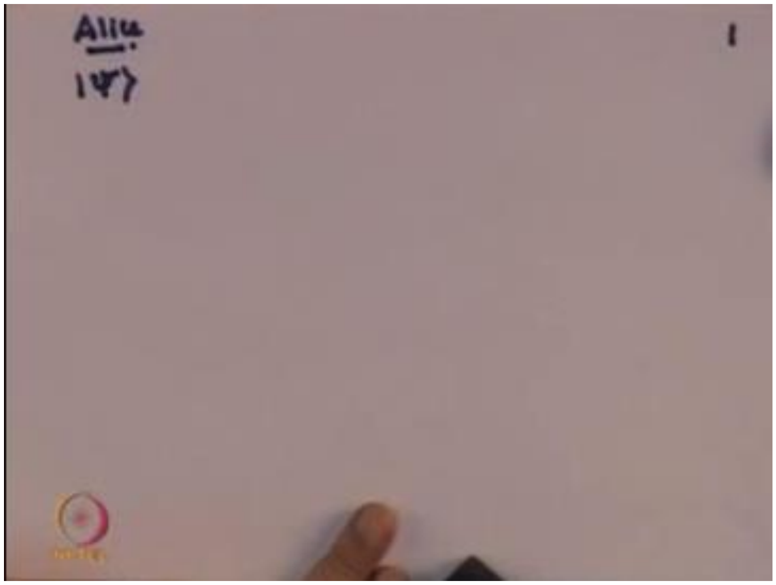

So this is Alice’s protocol we have seen that Alice cannot duplicate this state, but let us see what Alice can do, so this is some state.

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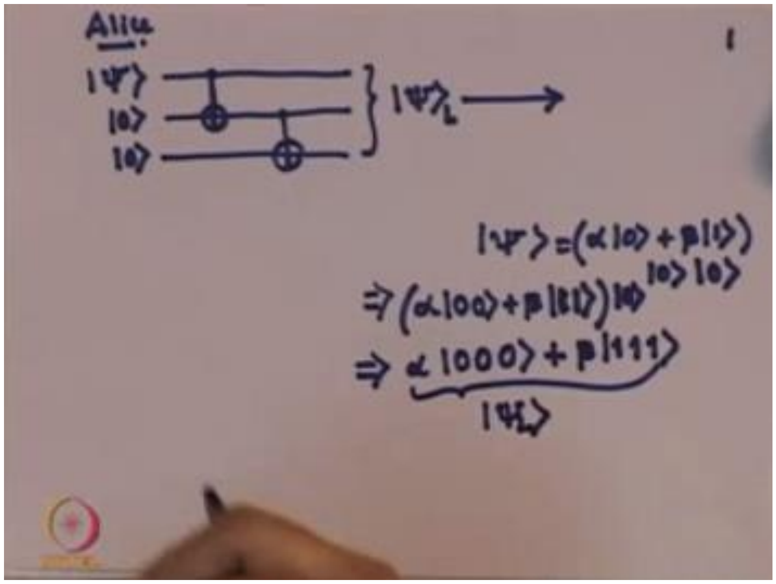

|ψ> which is in my example I will use it as sum α|0>+β|1>, so let me write it here, the state |ψ> is α|0>+β|1> properly normalized so that mod α^2+ mod β^2=1. What Alice does is to introduce two ancilla these ancillas are initially set to |0> then she executes a quantum circuit of this type, and whatever comes out here is what we will call as ψ logical this is what will be sent to your communication. Now look at what is this situation, since this state ψ is α|0>+β|1> and these two states where 0 of each what I get here, so I initially start with this is the second bit, this is the third bit.

In this case the control is 0, so therefore at this stage when I do this operation I first get α|00>+β| now the control is 1 for this bit so therefore this will make it 1 so I will get 11> and of course the third bit has not yet been changed so it remains it. In the third case the second is used as a control so I get α|0 since the second bit is 0 third bit remains 0. But when I come to this term my second bit is 1 so I get β|11 and this 0 will be flip to 1.

So this is my ψ logical, this is very similar to what we had in classical bit excepting it is not then repetition of ψ,ψ,ψ.

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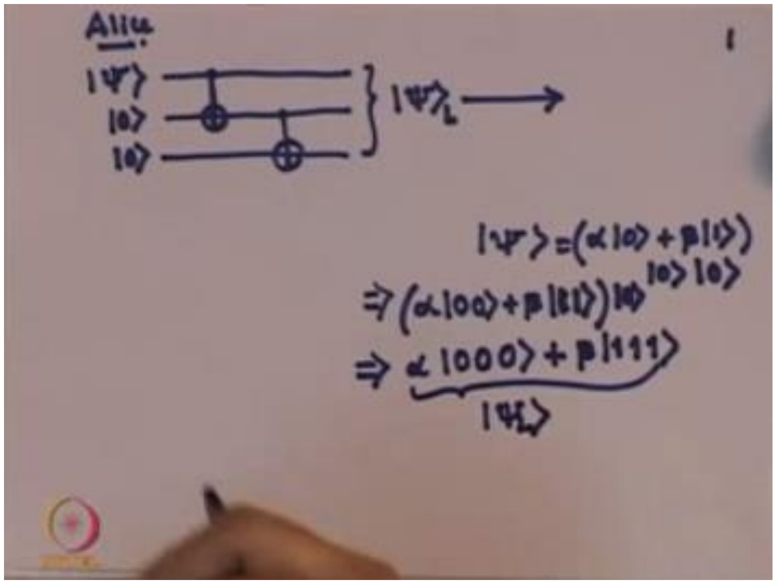

But ψ has this structure and it has sent to this. Now I have a noisy channel which flips a bit and let me say it flips with certain probability P.

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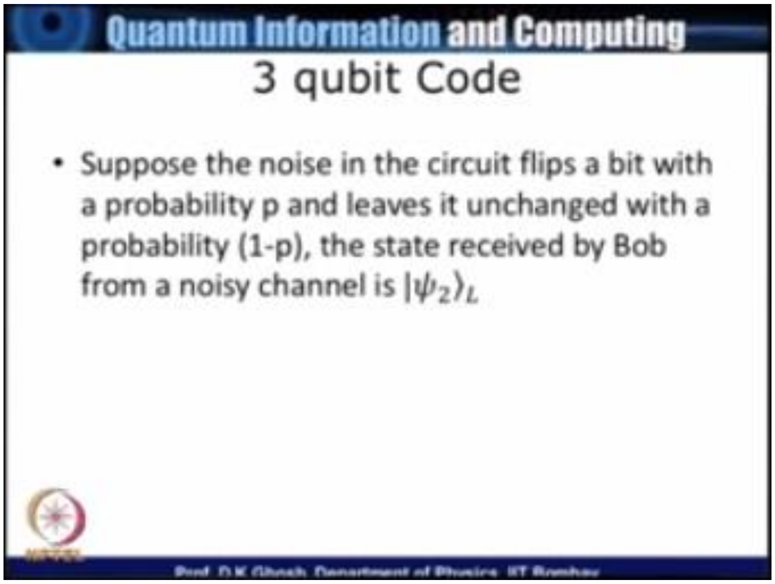

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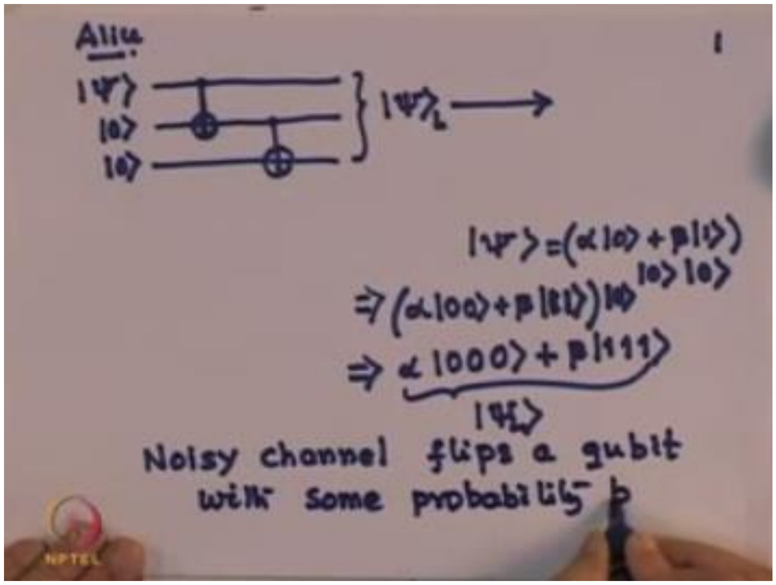

So, noisy channel flips a qubit with some probability P.

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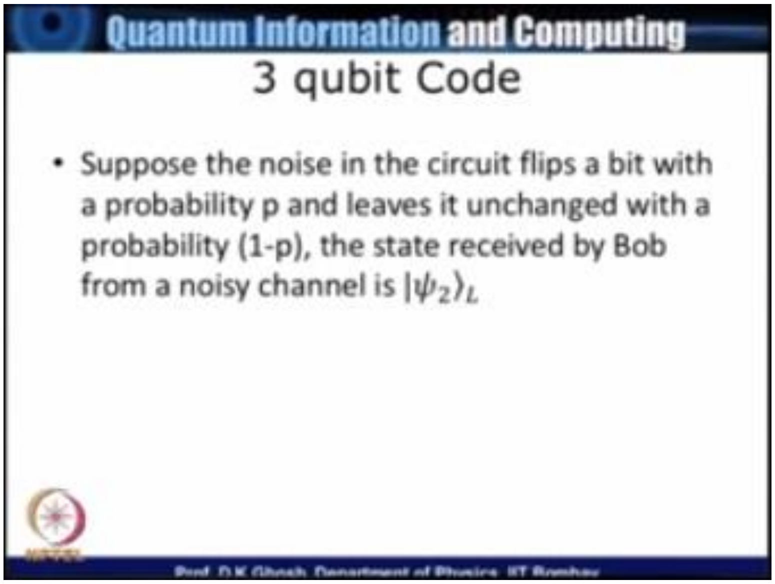

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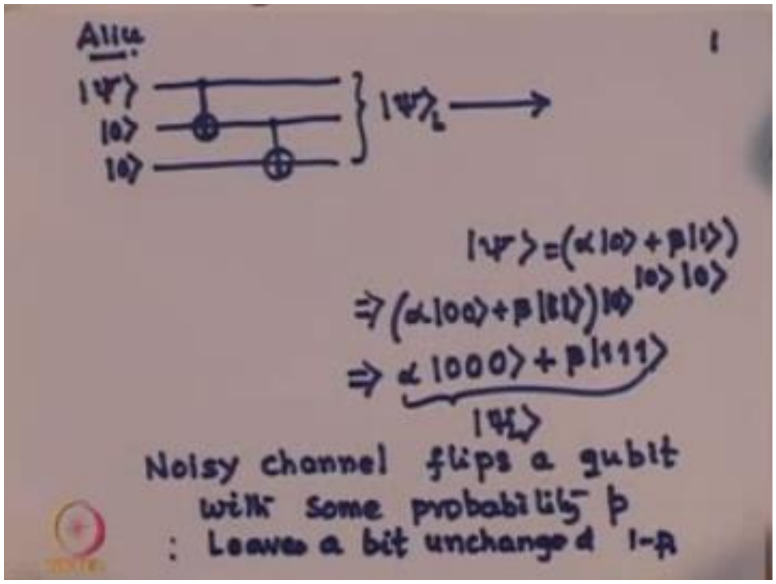

And leaves a bit unchanged with a probability obviously which is 1-P, this will give me this situation as we shown in the table.

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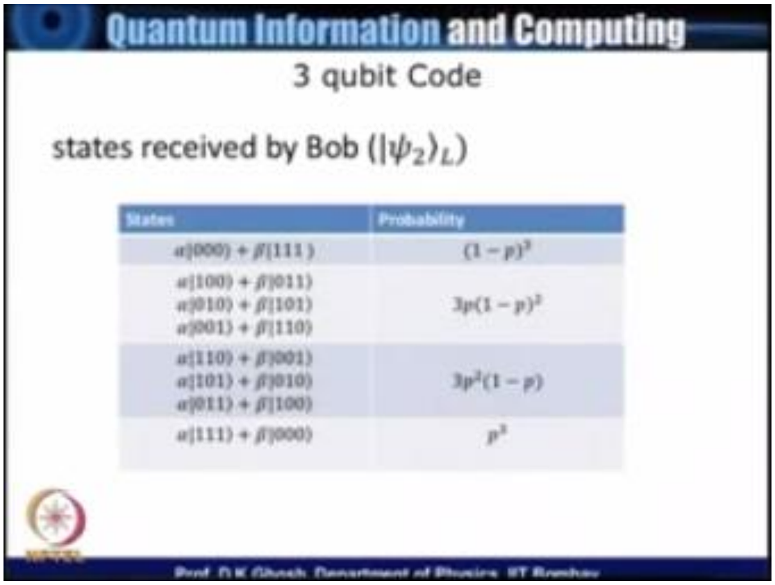

But I will a bit of a time in explaining some terms there. So first is the possibility that.

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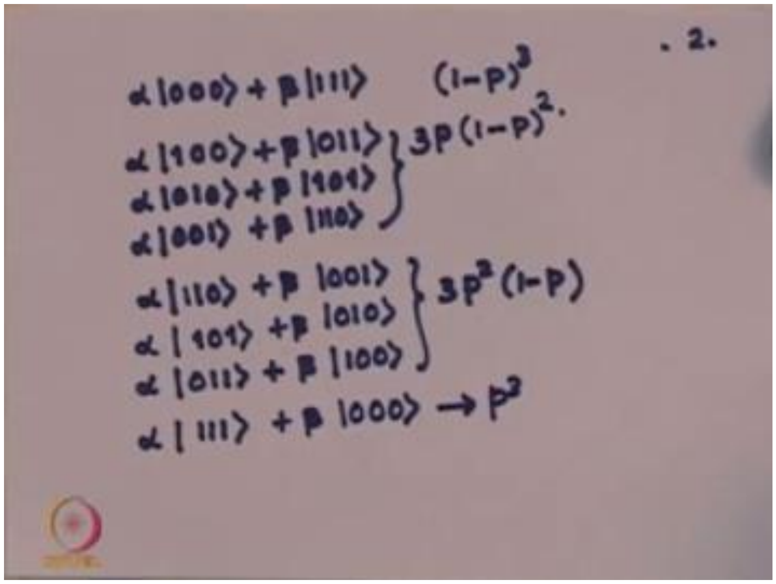

I had α|000>+β|111> and Bob receives it without any change, which means each of the 3 qubits have been left untouched. So the probability of that happening is obviously $(1\text{-}P)^3$. Now let us look at the single qubit flip situation, now suppose my first qubit is flipped then my state will be 100>+β|011> and you flip the first qubit. And the probability of this happening will be clearly P times because that is the probability of a flip into $(1\text{-}P)^2$ and not only this, this will be prove of for example second qubit flip 010>β|101> or third qubit flip which is 001+β|110>.

And in each case the probability is this, so the total probability of a single bit being flipped is 3 that is there. The situation with respect to 2 qubit flips, now if 2 bits are being flipped then it could be 1,2, 1,3 or 2,3 so I can get if 1,2 gets flipped 110>+β|001> if 1,3 get flipped I get 101>+β|010> and if 2 and 3 get flipped I get α|011>+β|100> and the probability of such a thing happening in each case if P^2 (1-P) so therefore I get 3 times P^2(1-P).

And finally the probability that all the three bits get flipped gives me the state α|111>+β|000> and that would happen obviously with a probability of P^3 and this is what the slide here indicates.

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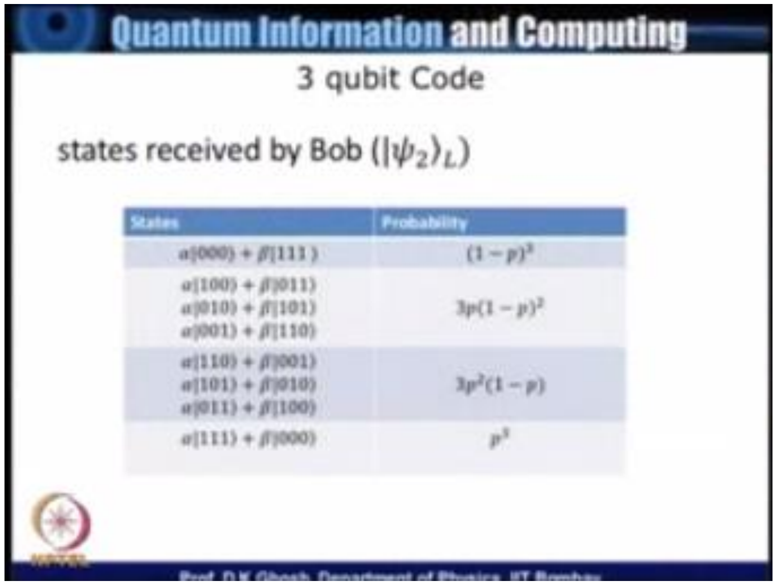

Okay, having down that, let us look at what can Bob do, now Bob knows that there is a probability with which the logic channel is working and so what he has received is what is, what I showed in the table. Now when Bob receives this Bob does certain other operation, Bob at his end adds.

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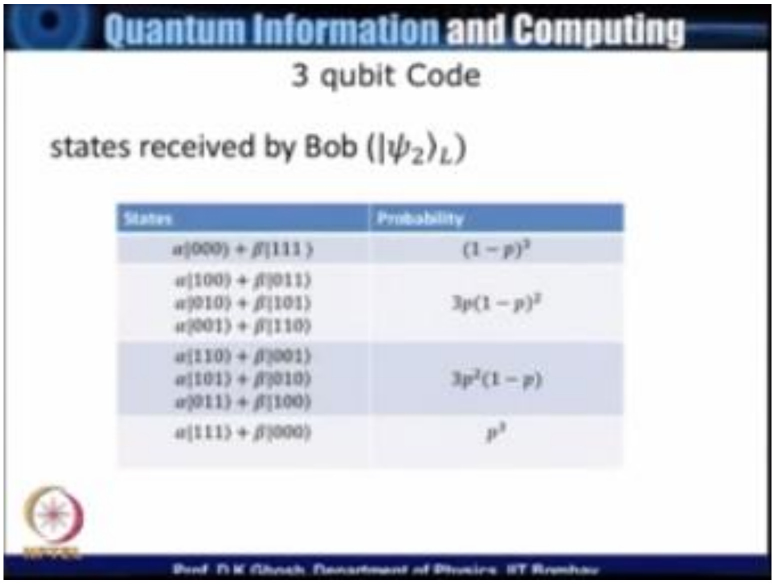

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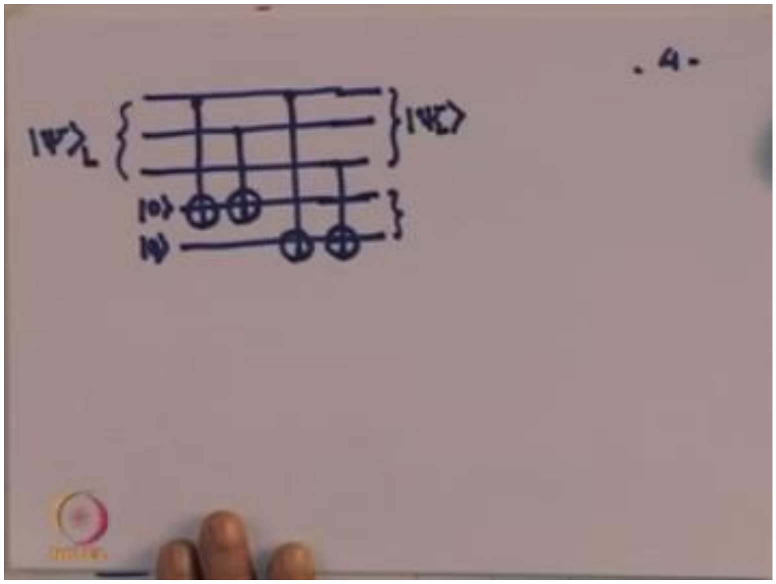

Let me first draw lines corresponding to the bits received, so we have been calling this $|\psi>$ logical both I adds two ancilla qubits which are initially set to 0 and Bob executes the following operations. The first thing that Bob does is to use the first qubit has the control and apply a CNOT gate on the first ancilla bit, then he uses the second line as the control and again does the same thing to the first ancilla bit.

For the second ancilla bit his operations are slightly different the reason we will see here little later and this is he uses the first as the control and the third as the control to execute CNOT gates. Now so at this stage this situation here is I still have the $|\psi_L>$ because you have not disturbed then but in this case the ancillas has been disturbed, so look at the slide for the, the picture of the circuit.

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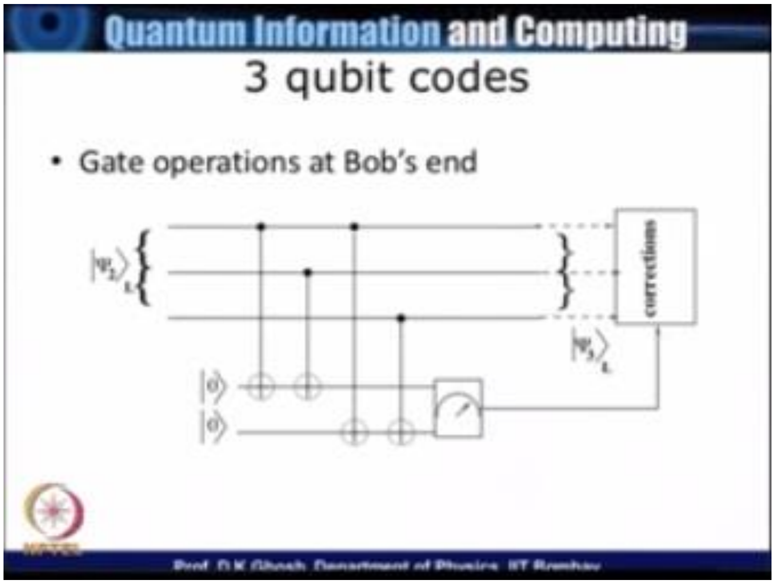

Now so what is happening in this slide is this that the ancilla bits have been changed by whatever I illustrated just now and the situation comes here. I will work out how does the ancilla change, but at this stage Bob will do a measurement and depending up on the state of the measurement he will do some corrections on the bits that he have received. So let us look at what Bob does and what are the possible results of Bob's operation. So Bob's operation will now generate what will be calling as ψ_3.

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So ψ_2 which was the state of the situation which Bob received they were as we have given 8 possibilities we have given.

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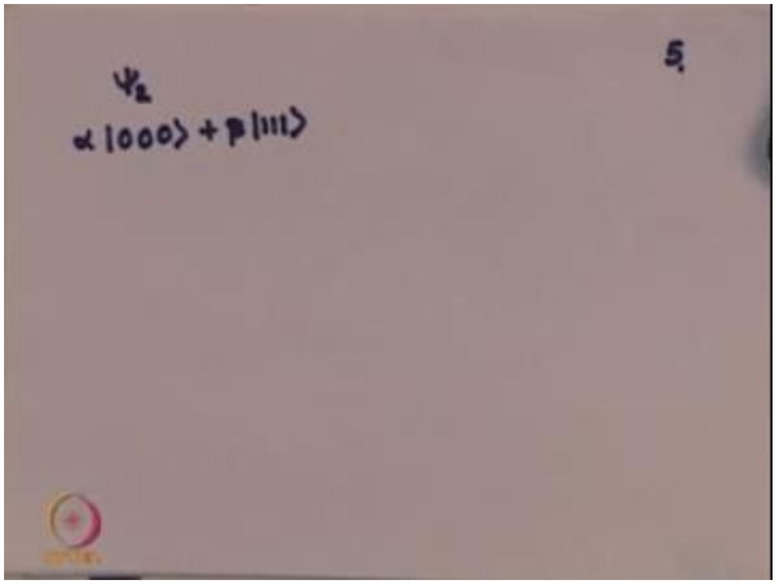

So this was $\alpha|000\rangle+\beta|111\rangle$ I had given some probability with which it occurred, and let us look at.

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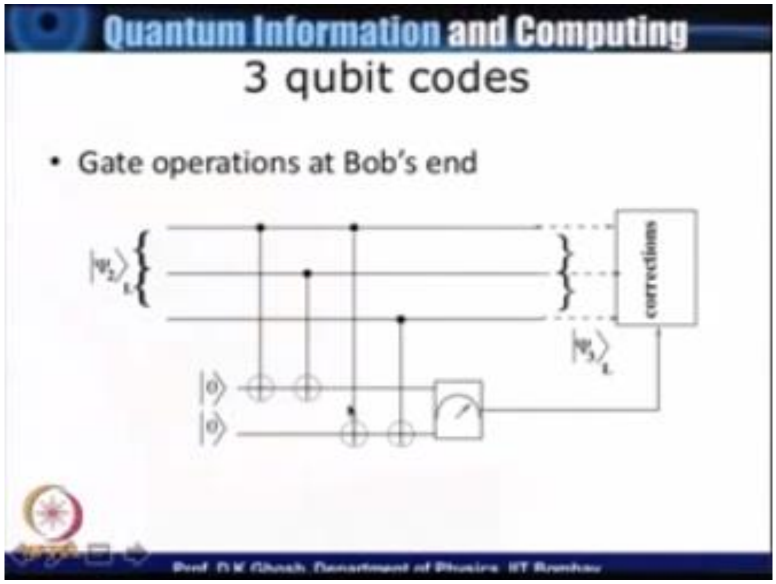

The situation here now in this situation when Bob executed this, what happen.

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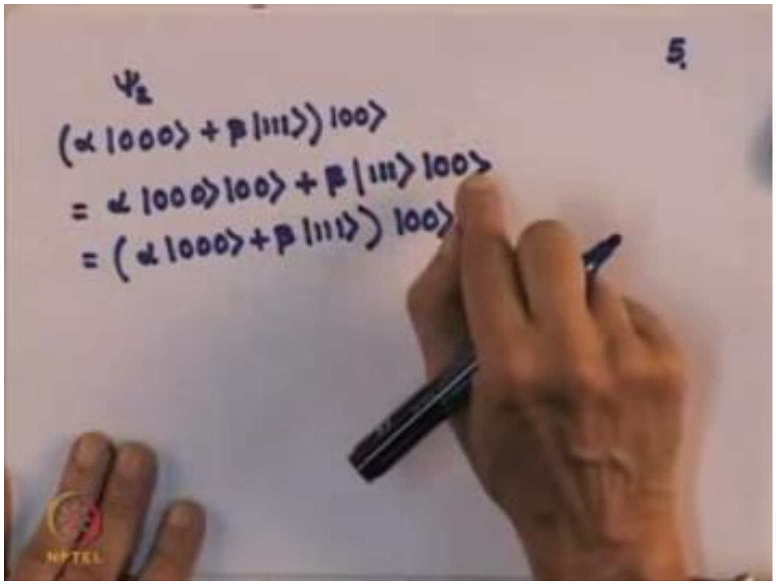

Come back here then they will, so my ancilla states were 00, since Bob decided to use the first and the second as the control to change the first ancilla bit, in this case the first control was 0 so that kept this 0 as 0, second control was also the 0, so therefore nothing happen to this. So therefore, I would write this as this happen α|000> and |000>, but let us look at what happens to second term.

So this will be, now remember first and the second qubits are the controls for that so far as the first qubit states here. So because this control bit is 1 it will flip this 0 to 1, but the second control bit which also one will flipping back to 0, so therefore what I will get is β111 which is the control which is always left alone times 000. So in such a situation I will have α000 + β111 times 00 that the ancilla bits are not affected at all.

And you can check that the situation with respective the second qubit is also the same because any time the two control bits are 1 then there is the double flip.

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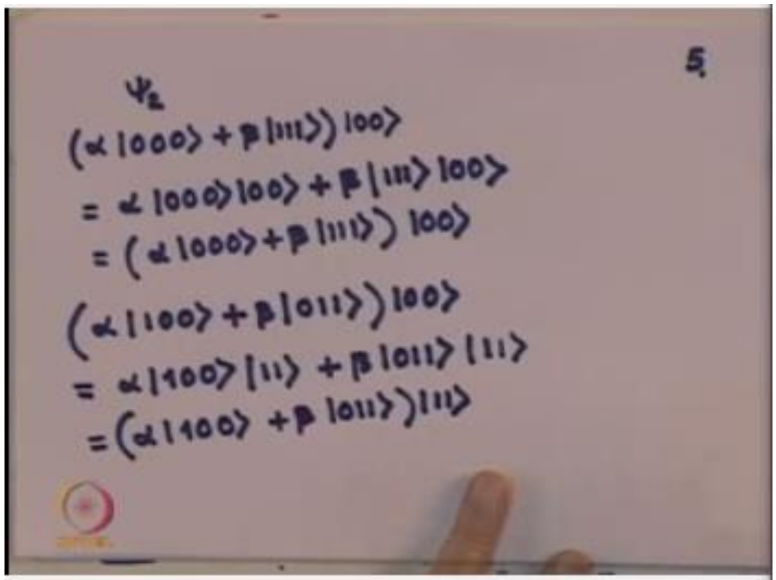

So original bit flip that took place after the first seen how to get is canceled because of the second flipping also. But this of course is a very special case will let us look at one of the other cases so let us say that in the second case we have said with the probability of p x1-p^2 and likely to get a state like this, the three of them but let me illustrate it with only one of them first.

So I have this situation here, now what I will get so my control x not is the first in the second now you notice that I have a 1 and a 0 so this will give me α100 so far this qubit is concern 0 that will be flip so I will get 1 there now remember that the second ancilla bit is being control by the first and the third and here I will have 1 and 0 there, so the second one also gets flipped.

+ β011 the CNOT control was the first and the second qubit for the first qubit so there is a 0 and the is a 1 so that must be flipped so therefore, I get it 1 and the control for the second qubit was the first and the third so which is also going to make it 11 again notice that what I got was 100 + β011 which is what I received times 11 the ancilla bits got change naturally because I am doing operations on ancilla and the complete list of what we received. What the states are after Bob has done this operation is given in this table, is a look at this table which little clumsy.

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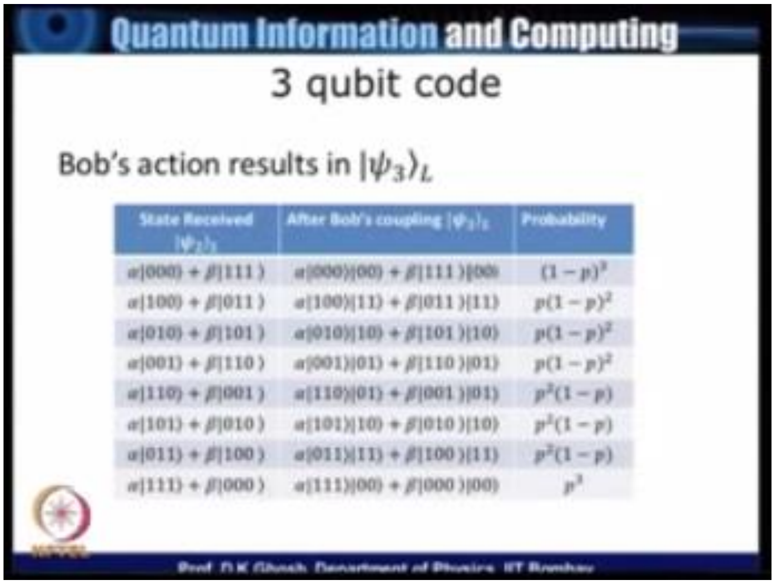

Because there are too many items there we convince you the following that as result of the operation is that we did from two ancilla bit the first ancilla bit with the first and the second line as the control and the second ancilla bit with the first and the third line as the control will change the ancilla bit in such a way that the ancilla bits state in both this terms to be the same. So look at these and remember that these had different probabilities as we see in that.

So in the first three cases here I had $p(1-p)^2$ of the probability in this case it was this and final was p^3 but at this stage if I measure an ancilla the ancilla bit.

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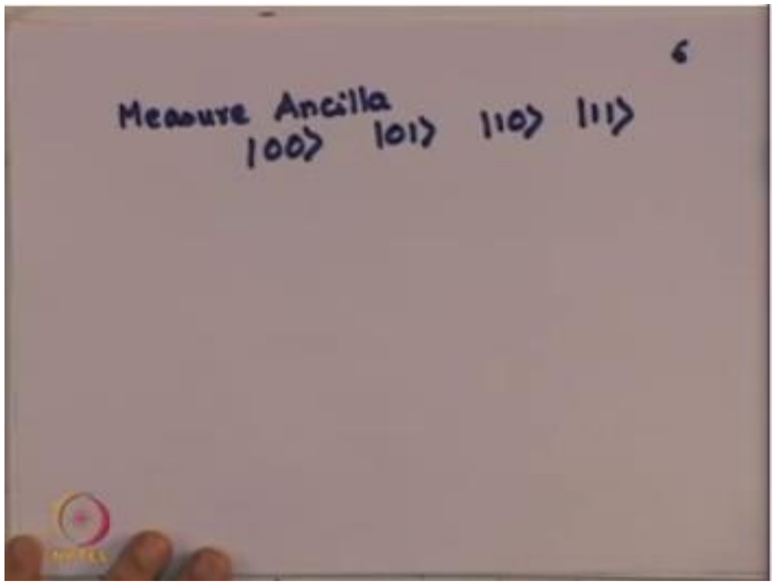

I am likely to get either 00 or01 or 10 or 11 each of these terms come in two terms.

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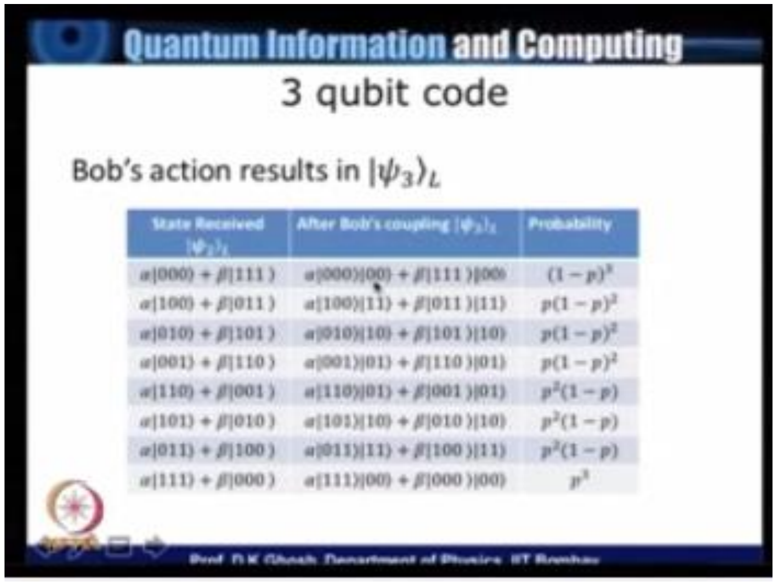

So let us look at what happens to the case where my ancilla bit is 00 if you look at it this line the first line has 00 11 and the last line also has the ancilla bit is 0 but there is a difference. In the first case the first three lines have been unchanged this probability was $(1 – p)^3$ but in the last line that this is not what Alice sent but this is what Bob received with the probability p^3.

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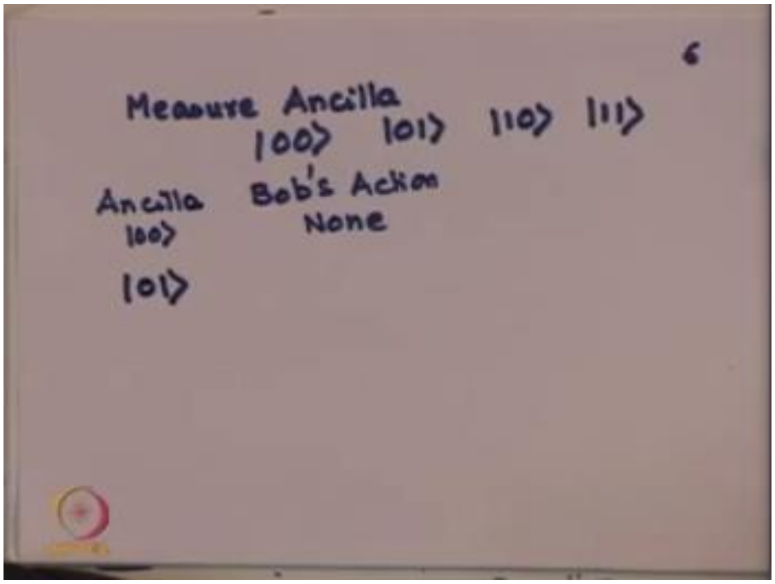

And so therefore, in this case if ancilla state is 00 Bob takes no action but then his states are all the received without any error with the probability $(1 - p)^3$ but received with error in all of three bits with the probability p^3. Now let us look at supposing the ancilla bit was 01, I will not do all of them but I am minus illustration.

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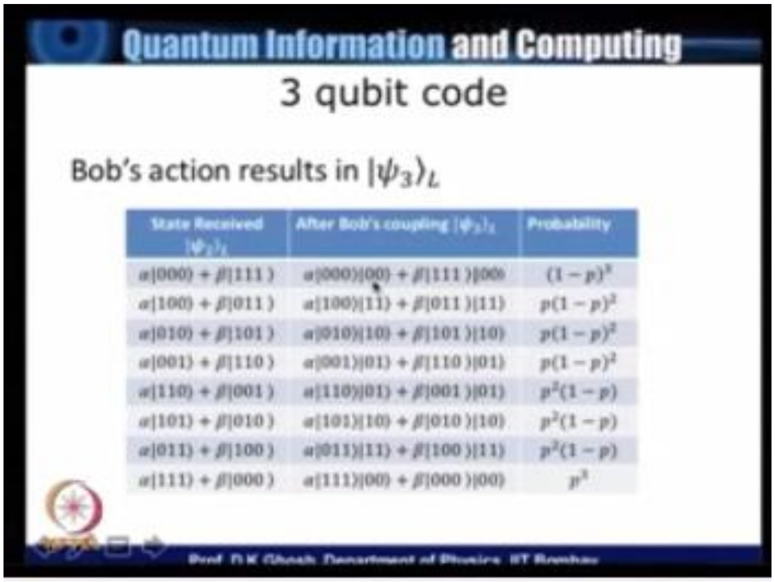

Look at where is 01 for the ancilla bit so my ancilla bit is 0 1 in this case.

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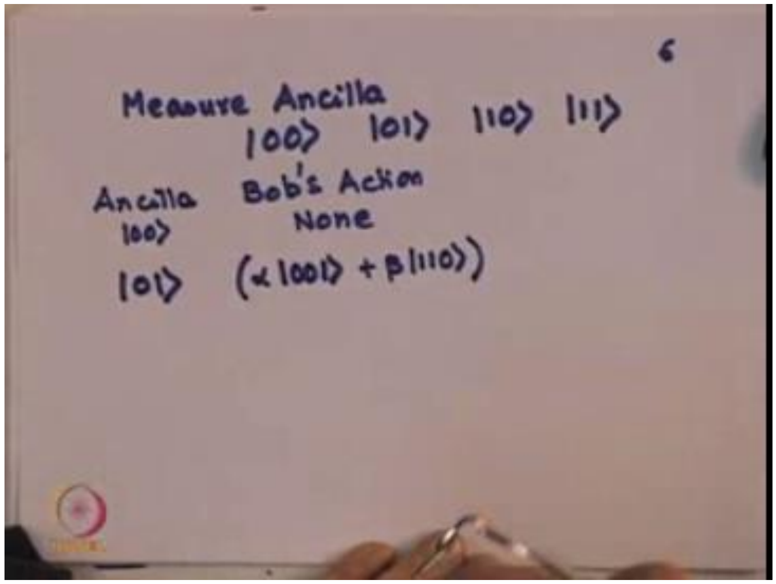

Which is α001 + β100 this gives me 01.

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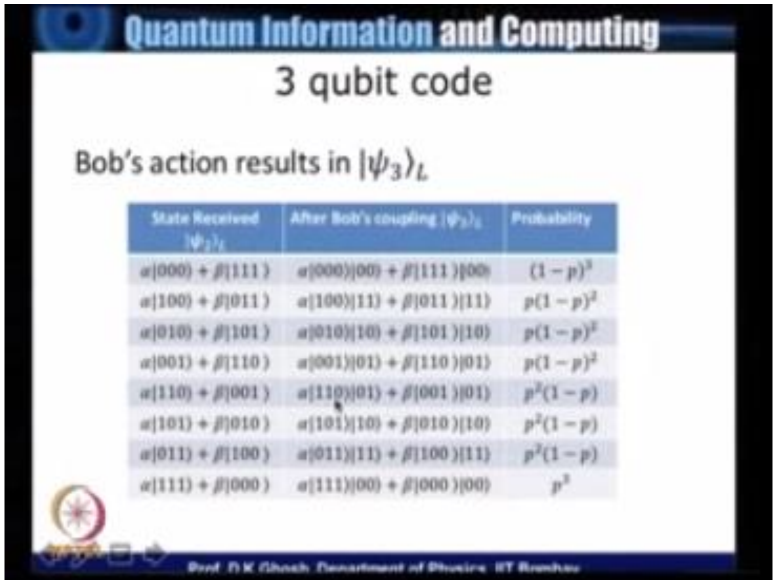

And there is also this other case which is α 100.

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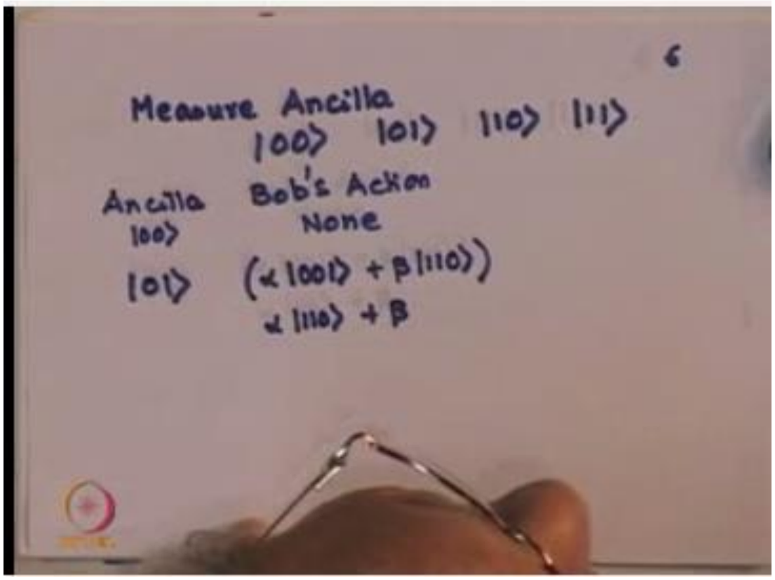

+β 001.

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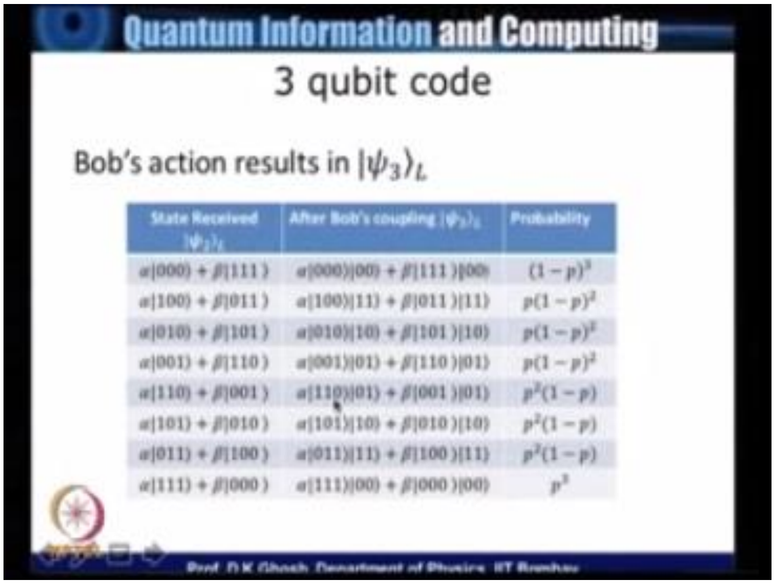

Both these term had 01 as the ancilla bit.

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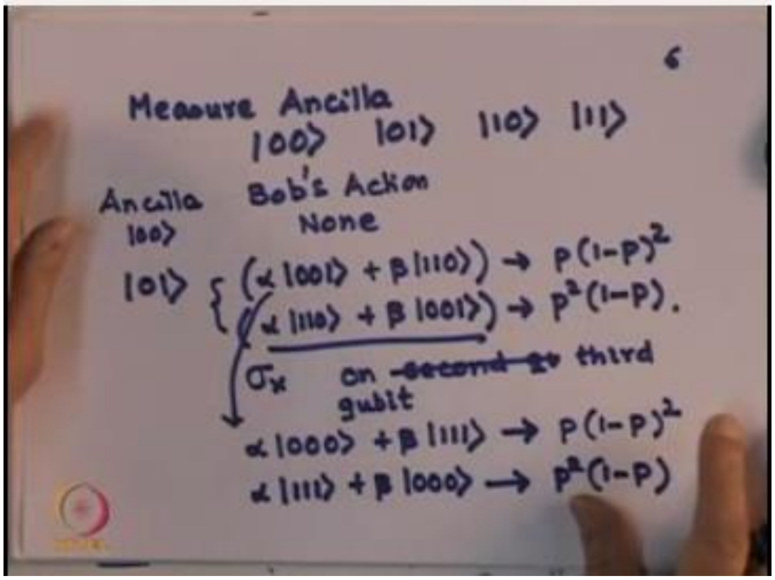

But this one came with the probably p up p x 1- p^2 but this one came with the probability p^2 x 1- p. Now this means that Bob received with one error this situation and with two errors the situation. So what want does of he receives either of them which we apply and x gate that is σ_x operator on the third qubit. Now Bob applied σ_x of the x gate on the third qubit this state will become α 000 + β 11 as he is expected.

So these states which you are received with the errors $p(1 - p)^2$ will be corrected but this will make this state α 111 + β 000 which has been received with the probability p^2 x 1- p this is of course wrong look at the slide again.

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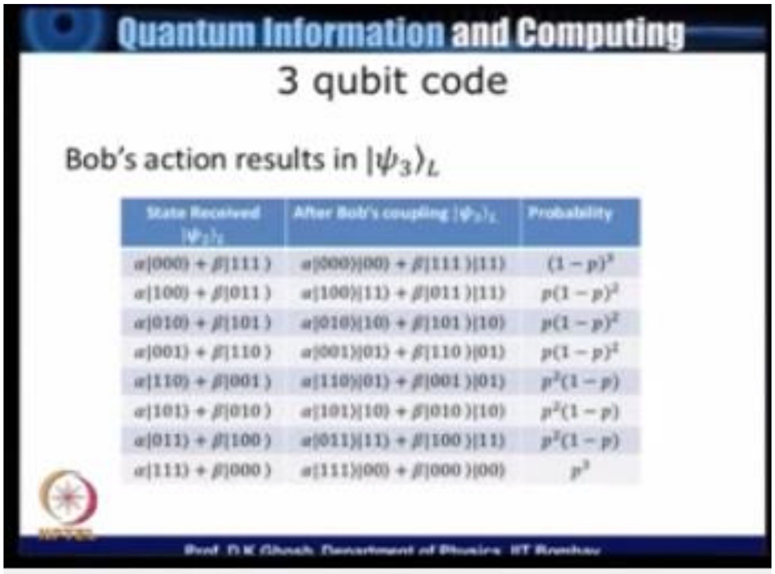

If the ancilla bit is measured to be 10 Ancilla state is 10.

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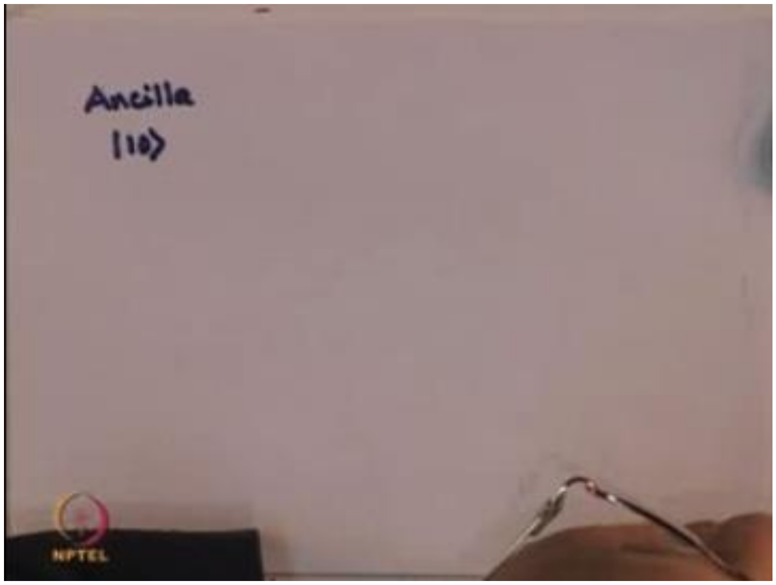

Again there are two places where such a thing happens.

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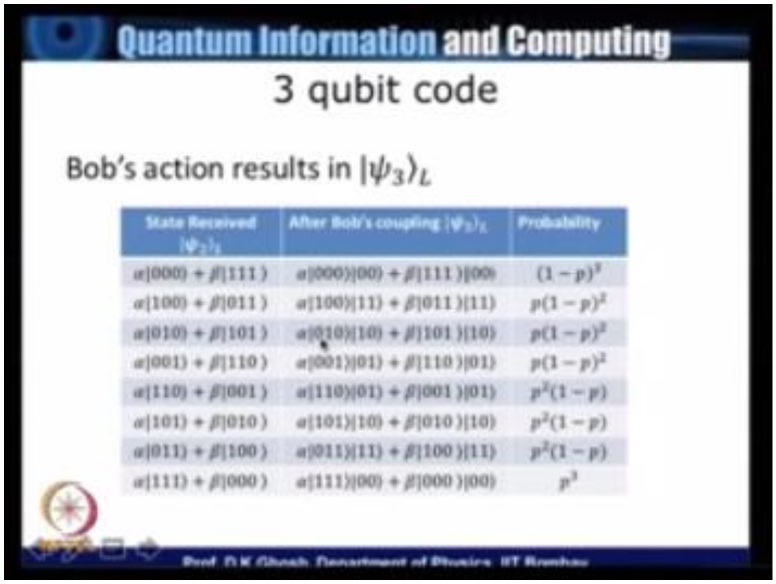

State Received $|\psi_2\rangle_L$	After Bob's coupling $|\psi_3\rangle_L$	Probability
$\alpha|000\rangle + \beta|111\rangle$	$\alpha|000\rangle|00\rangle + \beta|111\rangle|00\rangle$	$(1-p)^3$
$\alpha|100\rangle + \beta|011\rangle$	$\alpha|100\rangle|11\rangle + \beta|011\rangle|11\rangle$	$p(1-p)^2$
$\alpha|010\rangle + \beta|101\rangle$	$\alpha|010\rangle|10\rangle + \beta|101\rangle|10\rangle$	$p(1-p)^2$
$\alpha|001\rangle + \beta|110\rangle$	$\alpha|001\rangle|01\rangle + \beta|110\rangle|01\rangle$	$p(1-p)^2$
$\alpha|110\rangle + \beta|001\rangle$	$\alpha|110\rangle|01\rangle + \beta|001\rangle|01\rangle$	$p^2(1-p)$
$\alpha|101\rangle + \beta|010\rangle$	$\alpha|101\rangle|10\rangle + \beta|010\rangle|10\rangle$	$p^2(1-p)$
$\alpha|011\rangle + \beta|100\rangle$	$\alpha|011\rangle|11\rangle + \beta|100\rangle|11\rangle$	$p^2(1-p)$
$\alpha|111\rangle + \beta|000\rangle$	$\alpha|111\rangle|00\rangle + \beta|000\rangle|00\rangle$	p^3

This is α 010.

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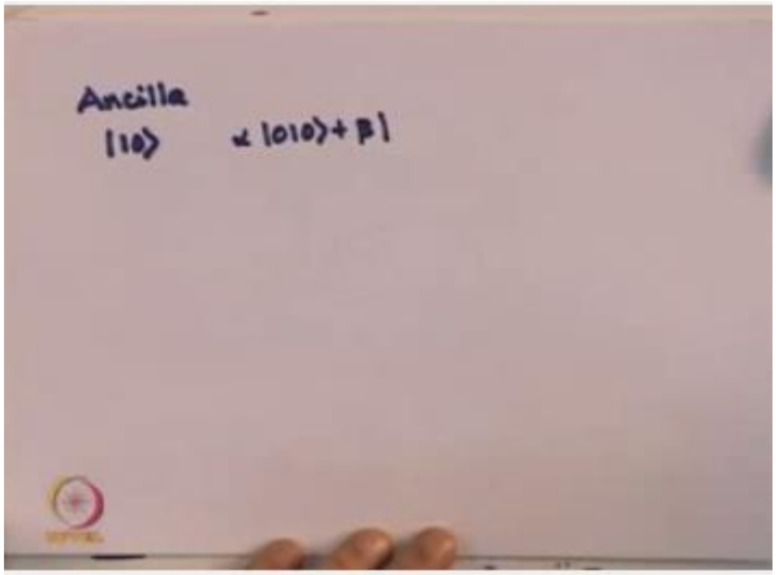

+ β101.

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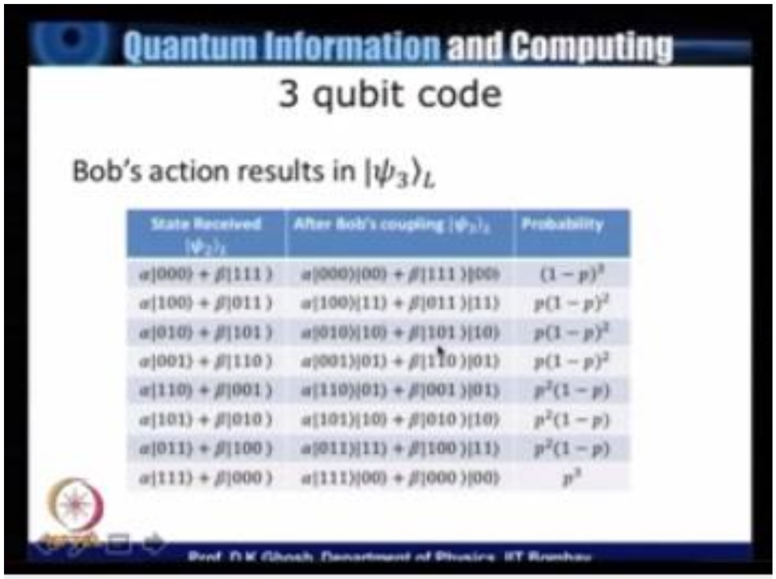

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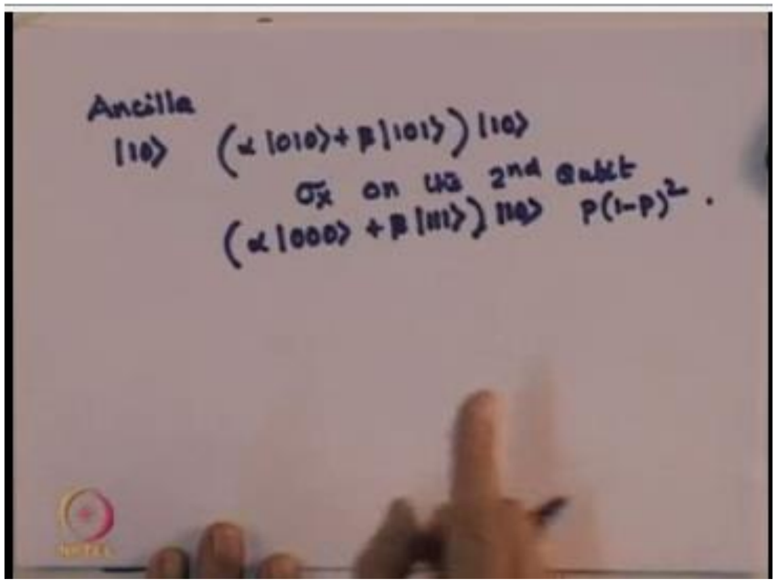

With 10 now what Bob does in such a case is we applied σ_x or the x gate on the second qubit that will make it α000 + β111 as was sent and that here seen was again something which had a problem probability of 1- p x 1- p^2 but the second alternative that was there.

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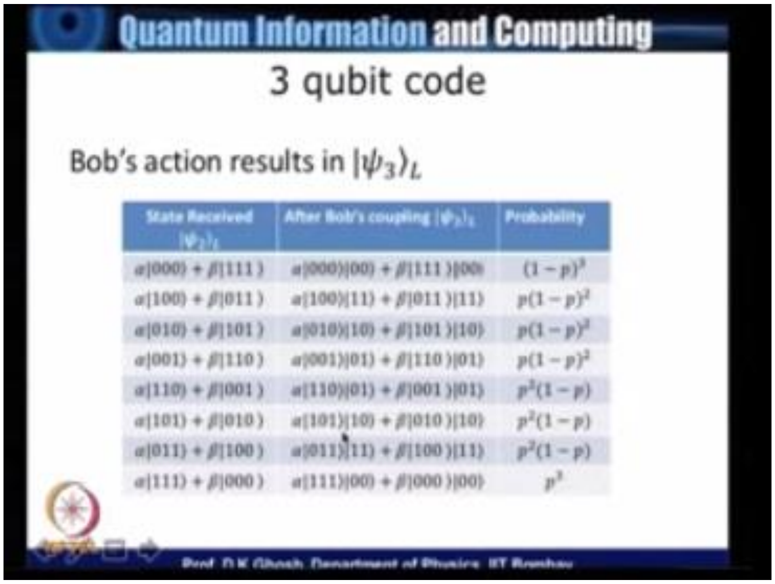

Where the second qubit was 10 was α 101.

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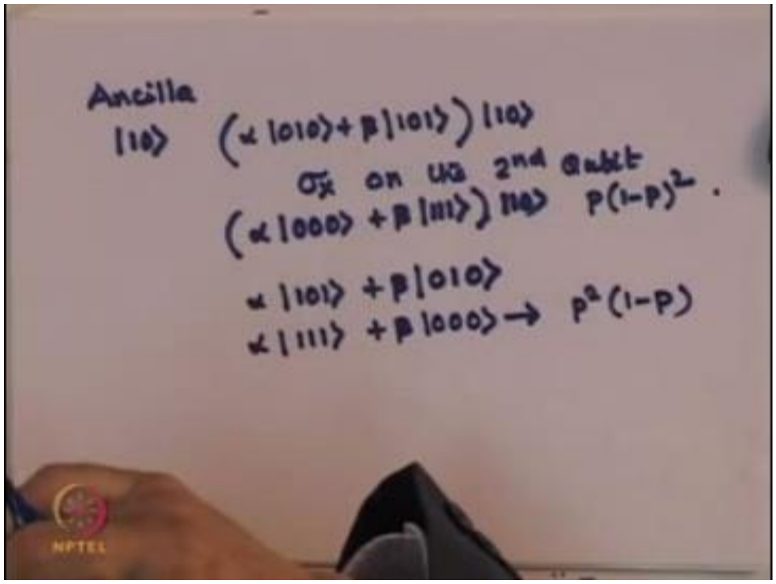

+ β010 if you apply second x gate on the second qubit it will get α 111 + β 000 and this state as we know appears with p2 1 – p. So in other words one second Bob's action has being with correct the error that occurred in the second qubit.

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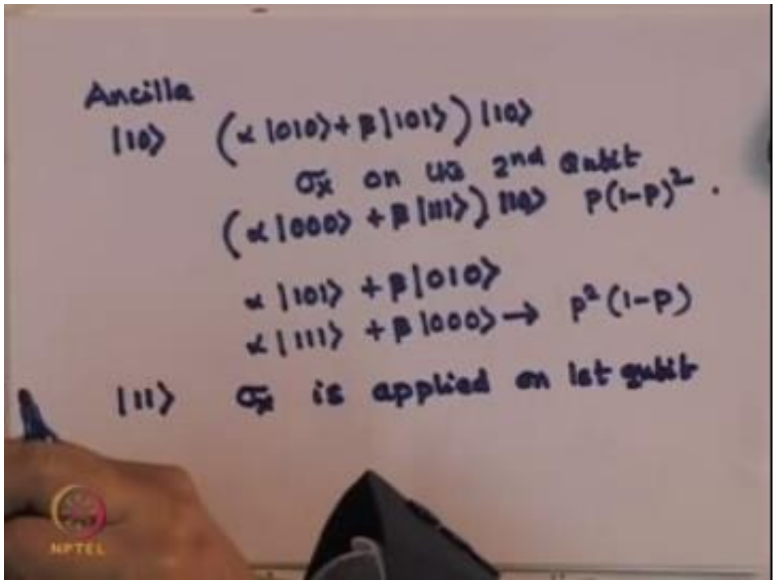

Or he was able to correct it with the probability p x 1 – p^2 and the, he still has an error which occur with the probability p^2 x 1- p. Finally if Ancilla was measured to be equal to 11 then this x gate, x is applied on the first qubit and you can work it out easily to check that this application would once again correct the third qubit position which was received with the probability of p x 1- p^2, but then the other state which had the same value of ancilla which has the probability of p2 x 1- p that does not bit corrected as before.

So in out of the 8 cases in 4 cases it will be corrected and the other 4 cases will continue to have errors. So let us look at how many of these states continue to have errors.

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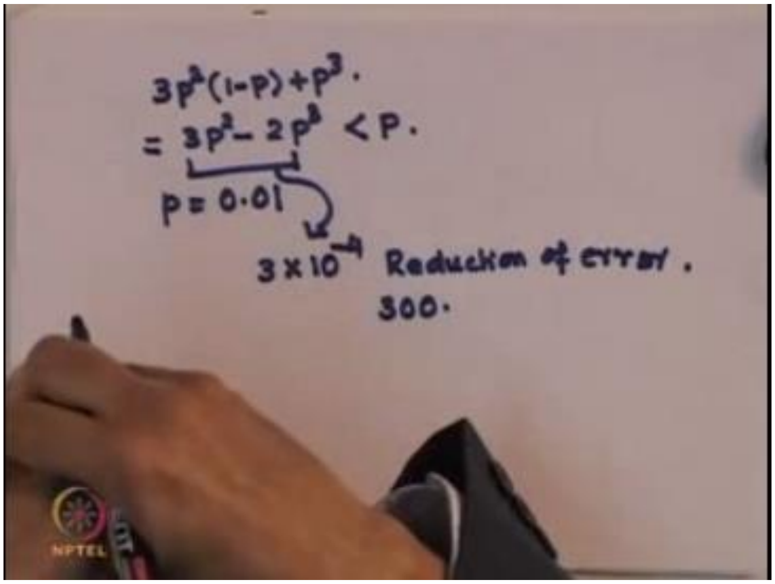

So firstly the probability of that happening is $3p^2$ x 1- p + p^3 these states are the, this is the total correction so if you add it up it becomes $3p^2 - 2p^3$ which is less than p the p is less than half supposing p = 0.01 you can calculate it you will find that the, those cases which is now where is not corrected is with the probability of 3×10^{-4} which is great implement because what we have found is that we have been able to reduce error by a factor of 300. So this is the way a three qubit code is able to re caller error to a great etc..

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