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Quantum Infromation and Computing

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Modul No.06

Lecture No.30

Quantum Error Correction

Till now we have been talking about various aspects of Quantum computing. However as we all know that even though computers where originally designed to do computing operation as the name suggests over the years the use of computers have being primarily in the field of communication. Of course the computing still remains the prime interest to scientist and engineers but people generally use computers today to have access to the communication medium, to social media, the E-mail and so many other things.

So we will now be spending a bit of a time in talking about use of quantum computers as applicable for the purpose of communication. To realize of course that the principle of quantum computing that we have found out like the classical computing they are going to be common to both communication protocols and the computation etc. So therefore all the required things are in place. But let us look the communication as we understood in classical review of course the simplest communication that we talk about is our talking across directly and for people who unfortunately cannot speak.

They can even use sign languages, but in any case that is a communication, the communication the idea is there is a person whom in our language you call usually a sender and there is a receiver of the communication. Then we jump to a traditional telephone where there is actually a wired connection from one point another and something on the telephone at the sender send or the transmitting and as a more appropriate for this case, and the person on the other hand he can pick it up and receive whatever communication we are sending that will be called a receiver.

Of course, over the years this simple model that we talked about has undergone several changes. For example, an intermediate connecting wire is not necessary if you look at the way the mobile communication works there is a connectivity through micro wave from tower to tower and there were really. Now what we want to talk about today is the process of communication as is likely to be seen in the quantum computer regime, but before we do that let us look at the following, that for instance we are writing something.

Now if I make a mistake in the spelling there is usually no problem in understanding what I meant in spite of the spelling, because we do not really understand the meaning of every word there but human being has this ability of assimilating the sense that is complete by a whole sentence. Now this later on in the computing language came to be known as fault tolerate computing, what fault tolerant computing or communication would mean is this, that your communication protocol or even a computation should be able to with stand certain amount of errors.

Now telephone is the biggest example of that, humans speech has a wide range from 20 Hz to 20,000 Hz, but telephone equipment essentially transmits things maybe up to 5000 tones. But we have absolutely no problem in understanding what the person at the other end is talking about. And that is because the human hearing it does not really distinguish unless you are a very keen listener of classical music particularly the western variety it does not really distinguish between different ranges of frequencies all that easily.

And this is basically the principle of fault tolerant communication that is the communication should be able to withstand certain amount of errors. Now what we are going today because the subject of errors in communication is a course by itself, but such a course cannot be complete unless I give you at least some sense of quarter errors and how does one actually handle it. So this lecture and the following one I would spend on talking about errors that may arise in quantum communication in particular of course I will introduce the corresponding classical thing as a reference, because of the fact that in going from sender to a receiver the communication passes through what we will be calling as a noisy channel.

But this noisy channel introduce as certain amount of errors in whatever are being sent in our case the bits that will be sent are subject to errors. So we first thing that we will be talking about today is how to detect errors or in principle what are the errors how does one detect it, and having detected it how does one correct it. The situation in quantum computing would have its own problem and let me begin by first talking about the way the classical errors occur and the way they are detected.

So first thing that we talk about are the following that in a classical communication a bit might just get with it. So for example, if you are sending a 0 it might become 1, or a 1 might become 0 this is because in classical communication these are the only two things that are being send bit by bit. So therefore, the only type of error that can arise in classical communication is a particular bit or several bits together will undergo change. But if you want to classify them there are basically three types of error that we talk about.

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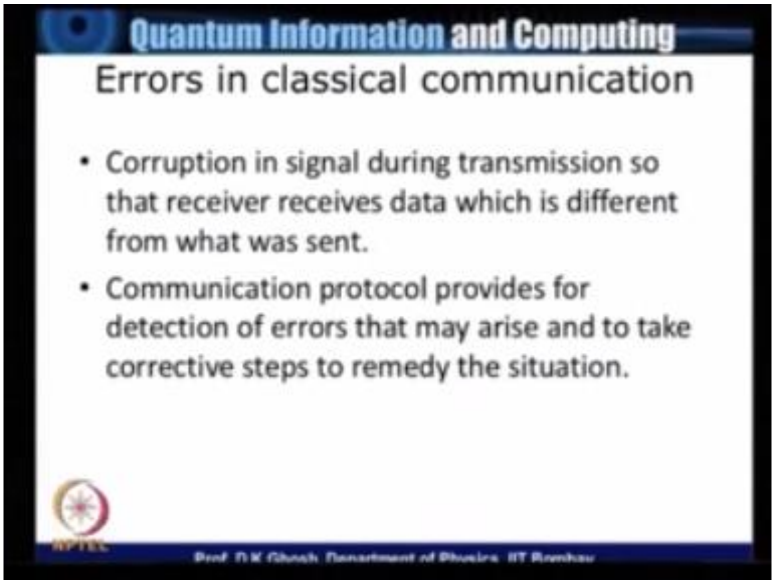

So we use the word corruption in signal to mean that receiver receives data which is different from what was sent, because of noise during the transmission. And our communication protocol provides for detection of error that may arise and to take corrective steps to remain the situation.

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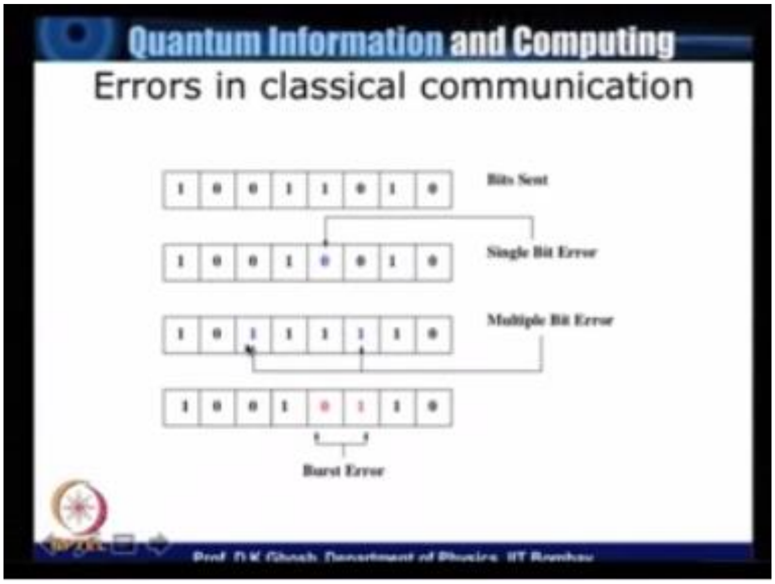

So let us look at what are the various types of classical errors. So notice in this example that I am showing on the slide I am the sender sends 1001010 the bits themselves have no significance accepting that I am giving an illustration. Now if you compare the second line with the first one this is not very clearly visible, but there is a different color it is a blue color there you notice that all other bits have remain the same, but this 1 has become 0 all other things have remain the same.

This is what I would call as a single bit error. In the third line that is there we have notice that in the third place the 0 has become 1 and again in the fifth place 6^{th} place the 0 has become 1. So in another words there are two errors the only thing that I emphasize is these two errors are in different places they are not adjacent, and I would call such a thing as a multiple bit error. The third type of error that occurs in classical communication is the error where consecutive bits are corrupted. So for instance, in this case the fifth and sixth, the 1 has become 0 and 0 has become 1, and this is called as the burst error.

So how do we detect errors, the, classically the simplest would be if the sender could tell me what did he or she said, then I would simple be able to find out what I actually got, and you know try to see whether they are the same and take a corrective as it.

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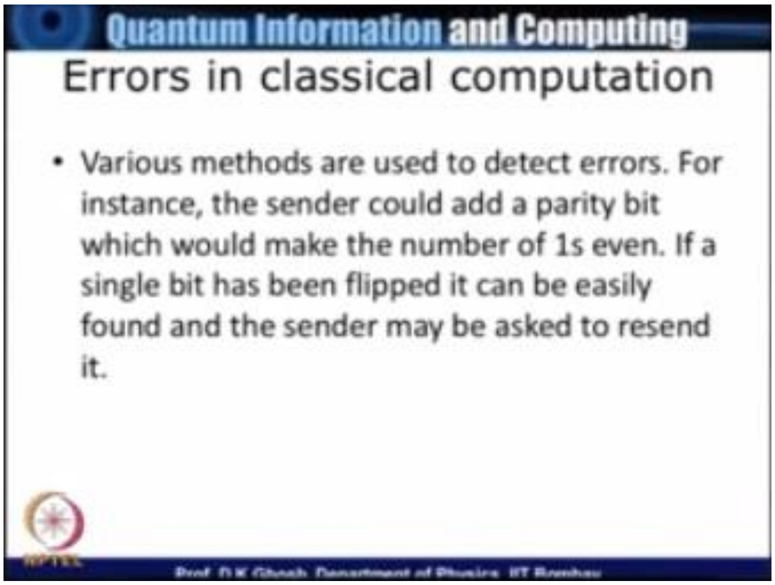

But there are various methods which are used to detect errors. Now one of the simplest would be suppose the sender and say what we would call as a parity bit. The parity bit is like this, that if you have a communication of a word which let us say has 8 qubits, you 8 bits then I add another one to it, the length is not important, you could have 7 and you want to 1 bit. But we agree on a protocol that whatever the sender sends it has let say even number of 1s. Now this additional bit which needs to be added in order to make this protocol work is called a parity bit.

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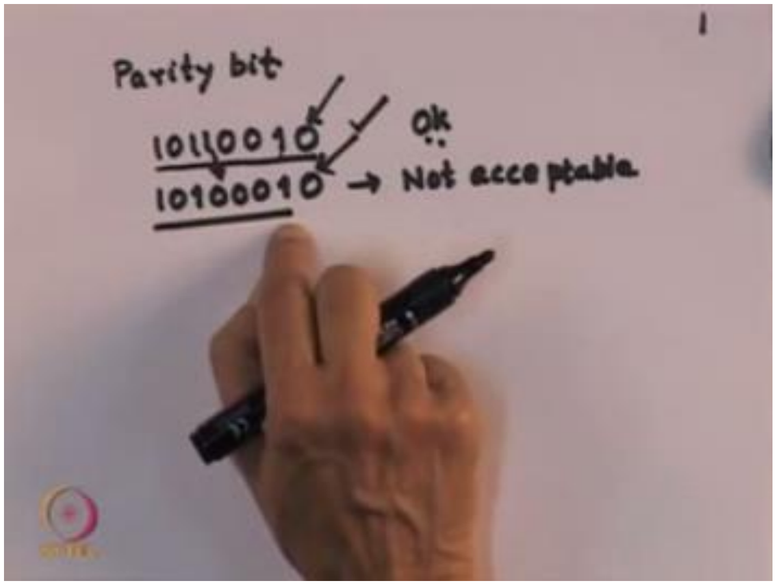

So notice this that suppose I have a situation like this 10110010 this is what was sent let us suppose this has four 1s here, here, here and here. So since the number of 1s is even that the protocol which we agreed this is, okay this is correct situation. On the other hand if the receiver receives a situation like this, you notice there are only three 1s in it, so this is not acceptable. Now in this example that I have given you the parity bit which is let us suppose added at the end happens to be 0.

Because this was intended to be sent up to this, and there are already four 1s in it, so the parity bit could not be a 1. But what was the received was something where this place, the fourth place developed an error. So the receiver can easily recognize that this could not have been the word that was sent because it happens to have odd number of 1s. Now even and odd is not important one could agree for example that the number of 1s could be odd.

But it depends up on a protocol that we have talk about. Now so what does the receiver do on realizing that he has received something which is not acceptable. Now this is very simple all that he does not classical communication is to ask the sender that can you resend it, and it is very unlikely because remember that these are changes that occurred in the bits that are being sent because where noisy channel and that by definition is random.

So it is very unlikely unless you have such noisy channel that the probability of corruption is very high and once should not use such channel in communication. When the sender resends the message nothing really it is unlikely that it would be repeated, so just to make this point clear.

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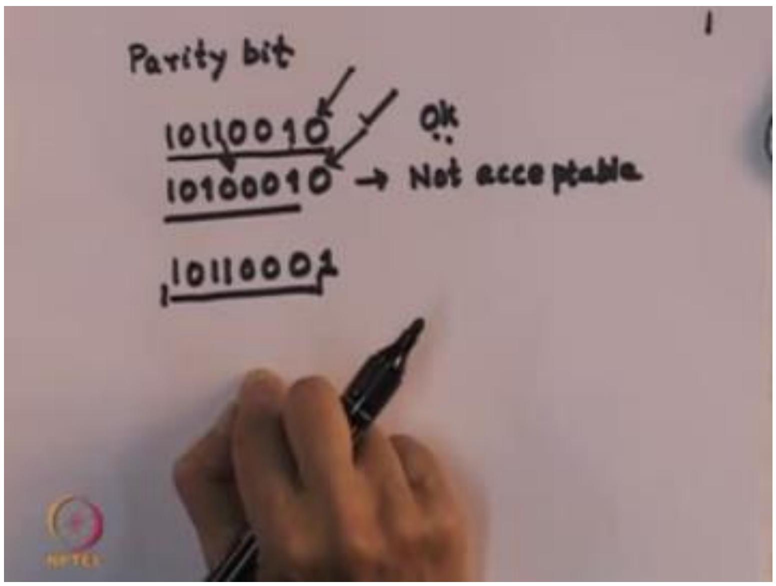

Supposing the original intended message I will take a 7 qubit situation, supposing it was like this 1011000, now notice at this case this is what was intended to be sent. But it has only three 1s in it now that is not an even number so therefore the parity bit 1 is added. Here we did not need it we just till added a 0 because all the communication that are being sent according to this protocol would be 8.

Again I emphasize that the lengths are not important I am giving it as an illustration. Now you will say that it is looks like a rather clumsy procedure that we measure detect in error tell the sender I did not receive what you send. So what was done or what is done in classical communication is to built in what is called a redundancy.

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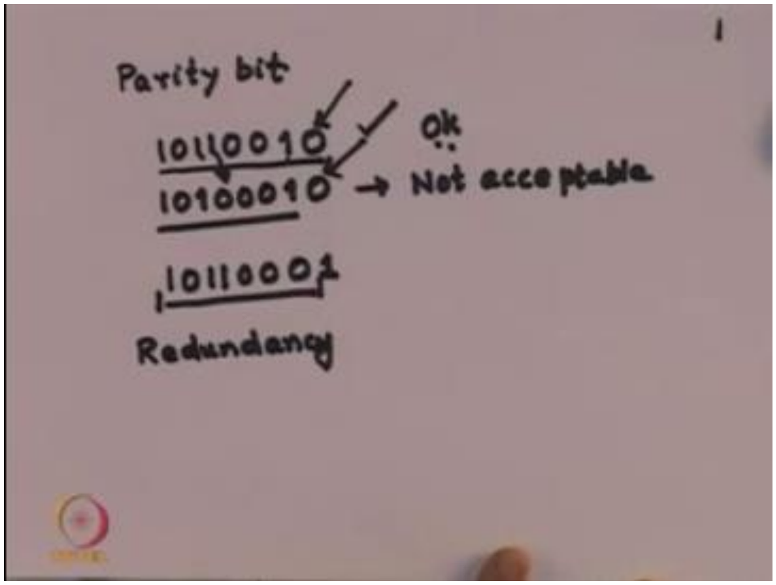

Now let us try to understand what is meant by redundancy. So redundancy is the following that supposing you want to send a particular bit it could be 0, it could be 1 it does not matter. But instead of sending a single bit in its place I send multiple number of bits just to illustrate I will assume that I am sending three bits all that is required is it should be odd number. Now in that case I will be sending.

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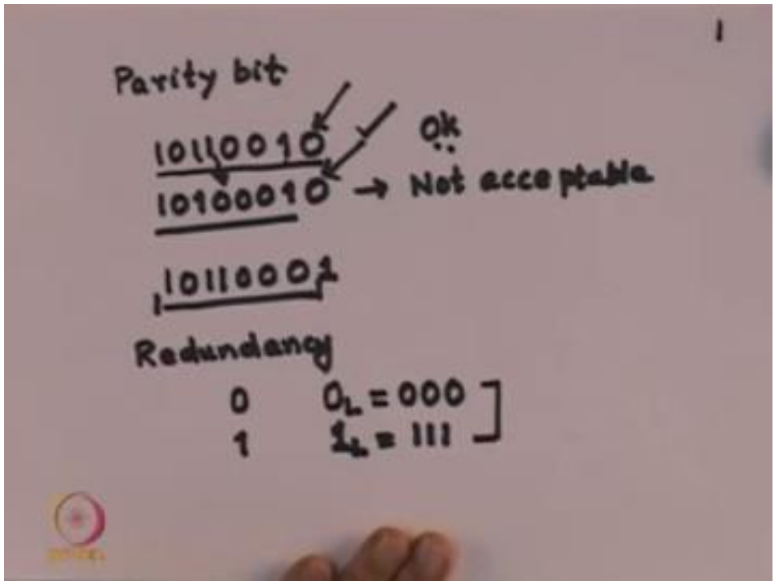

Instead of 0, I will send what I will call as a logical 0 which is defined to be equal to 000, three bits will be sent in place of 1. And likewise in place of 1 I will send the logical 1 bit which will be then 111. Now in such a situation when the communication is received the system can automatically check by monitoring every group of three bits whether they happen to be identical if they are not then an automatic correcting mechanism can be worked out.

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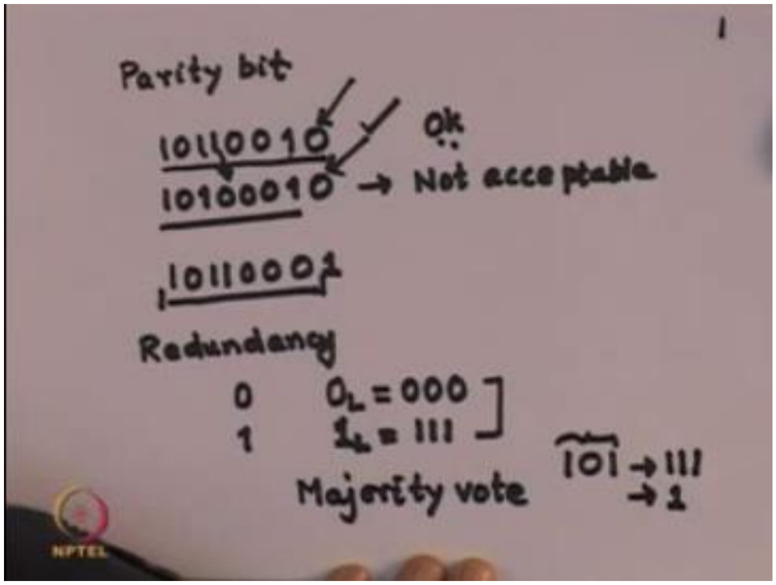

By what we call as a majority vote, this is a very simple minded procedure which says supposing you received 011 or 101, then since this group did not have equal number of 1s or 0s I look at what is the maximum number of, what is the identity of the maximum number of qubit bits. And in this case it happens to be 1 so I know this actually is should have been 111 logical which corresponds to the bit 1 which was originally intended but very simple procedure and it is a workable situation.

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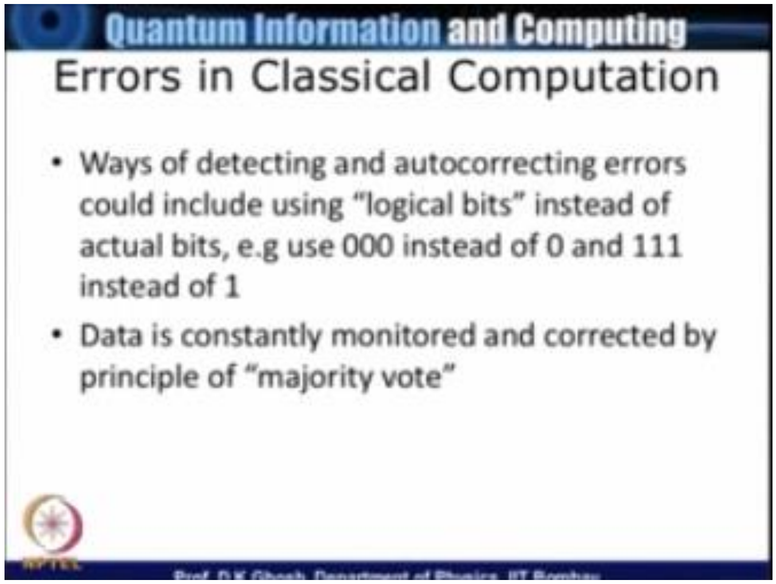

And of course, naturally you would think that y3 there is nothing special you could use 5, you could use 7 whatever it is. But the point in classical communication is that the data is constantly monitored and you have certain procedure for correcting them automatically.

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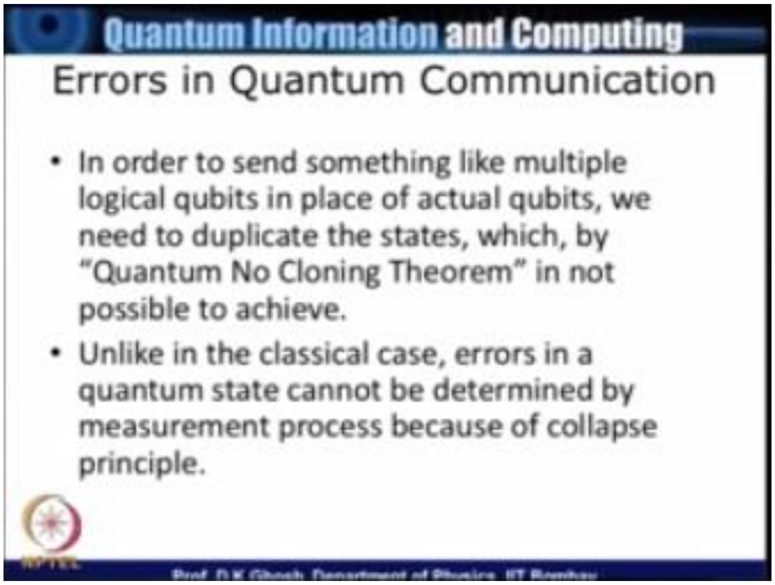

Now let us look at the corresponding situation that arises in quantum communication. So we assume that we have some quantum channel, so instead of sending bits 1 and 0 I am actually communicating qubits which could be, now remember in this case which could be in state 0, state 1 or a linear combination. Now this part that the single qubit that I am sending could be in a linear super position of the basis state is something which is special to quantum computers as we have seen in the last several lectures.

So in other word there are two issues there supposing I want to send multiple copies of the qubit that I want following the method that I talk about. So first thing is could I am not necessarily sending 0 and 1 but I am sending a linear combination there. Only thing that can I send three of them instead of one, the answer obviously no because we have gone through a theorem known as the quantum no cloning theorem which says that quantum states cannot be duplicate there is no unitary transformation which will be able to do it. So that is the first obstacle to replication of bits as it is common in classical computing to in case of quantum computing.

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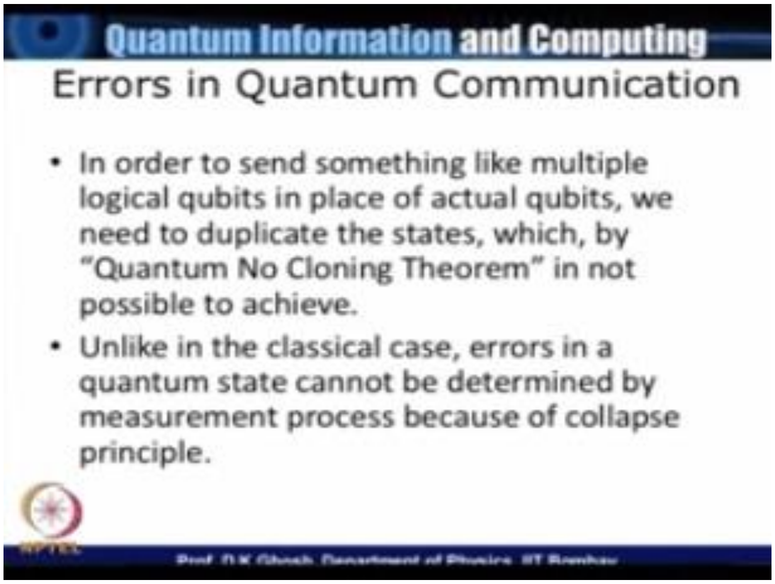

The other thing which makes the problem lock more serious is unlike in the classical case where the receiver actually does a measurement looks at the bits and says that I have not received what I wanted whether receiver physically does that or in a scene does it an important. But there is something or somebody who will be looking at the bits that was sent examine them and try to find out the correction. We have seen in quantum computing this is not feasible.

Because monitoring essentially means making a measurement and the process of measurement would make the state collapse to one of the ideal states. And so therefore this question of majority vote etc it does not quit one.

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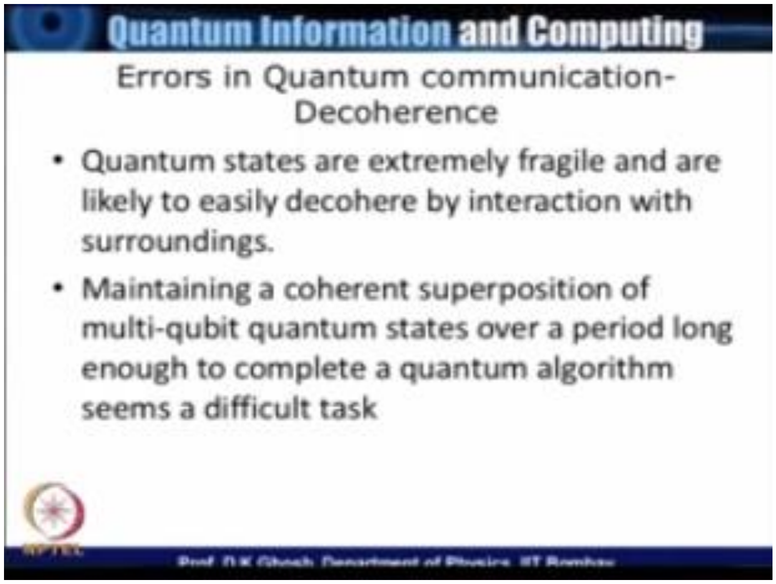

So we realize by making this statement what we are saying is quantum states are, you know I mean there they are own problems one of things is quantum states are extremely fragile and they are likely to, I will be introducing this right now decoherence, decoherence means losing face relationship, we have seen that a linear superposition of states have a certain face relationship.

For instance, simplest example is a qubit which is let us say 0 + 1/ √2 or there could be a relative complex phase between them. But supposing the 0 + 1/ √2 because of noise and interaction with the surrounding become 0 + 1/ √2 then we approved it.

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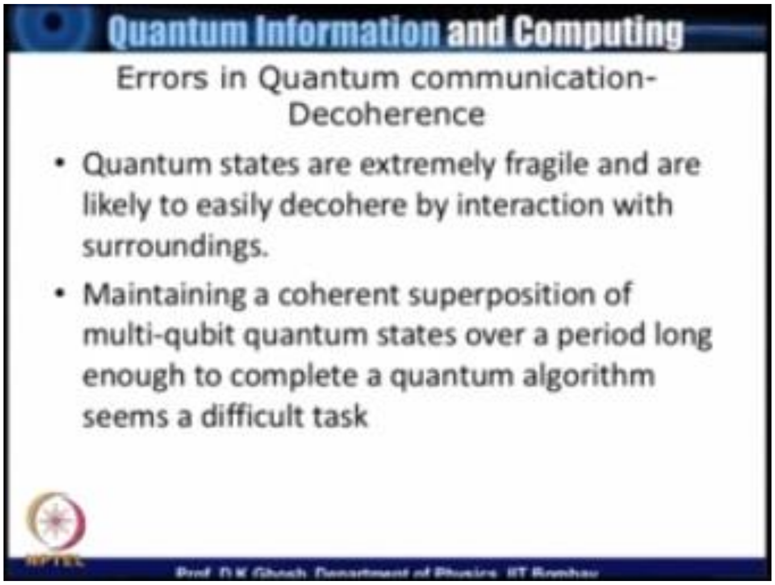

Now maintain coherence superposition of multiple qubits over a long period long enough which is to complete a quantum algorithm happens to be a very difficult task.

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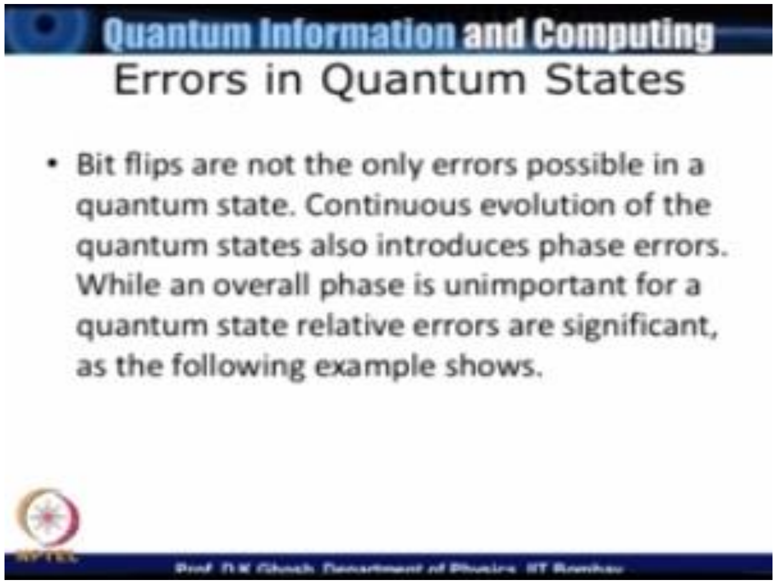

The other problem is while in classical computing the only possible error was that due to a flipping of a bit or flipping of a bits in case of quantum states there are different types of errors possible, remember that quantum states evolve continuously. So it can introduce errors in both its amplitude and its face and even though such errors that are introduced during evolution could be small in magnitude over a period of time they could build up.

And that is primarily because we are not in a position to do intermediate read operation because they all mean measurements, while having at overall face between.

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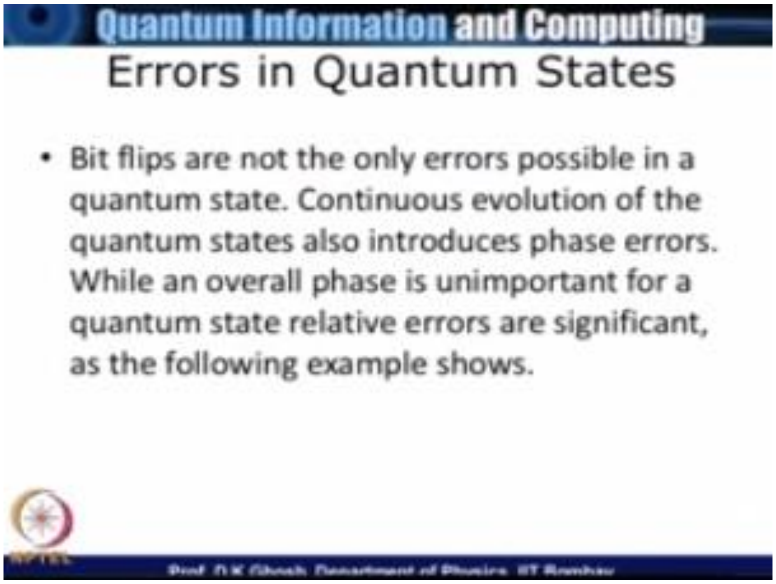

Different components of the qubit then important we have seen overall the face do not make any significant difference to whatever we are doing, but having a relative face between the components in terms of basis states is important. Now suppose I started with an initial state of let us say 0 + 1 /√2, and let us suppose because of interaction with outside world environment as we have been calling it a relative face was introduce between this two bits making this, so this I will call a ψ and making this ψ′ which is let us say 0 + $e^{i\alpha}$1/√2. Now what happens if we make a measurement now.

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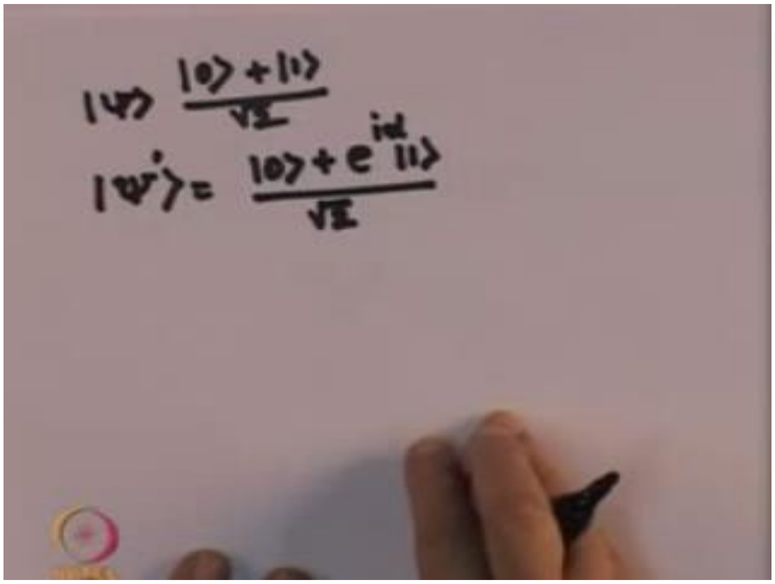

Now if you are making the measurement of the z component let suppose that these represent the up and the down states of a spin the 0 and 1. If you are making a measurement of the z component of the spin the distribution of the Eigen value which are + or -1 remains exactly the same. Here also the probability of getting a 0 is 1/2 and 1 is 1 / 2 and here it is also the same in spite of this $e^{i\alpha}$ that you have got. But suppose instead I am going to measure the x component of the spin.

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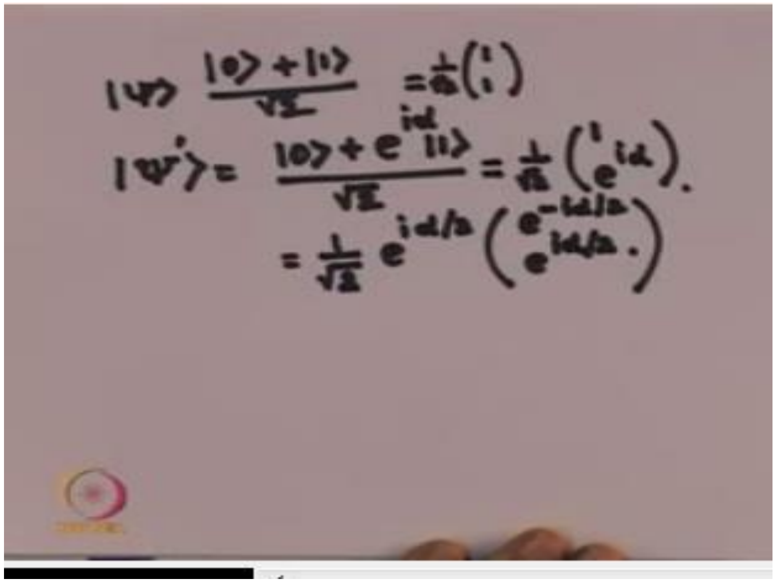

So what actually happened is let me write it this down in terms of my matrix notation remember that this is 1/√2 11. But this 1 is 1/√2 1 and $e^{i\alpha}$ which by taking out a face $e^{i\alpha /2}$ I can write as $e^{-\alpha/2}$ and $e^{i\alpha /2}$, now if you measure the x component of the spin. We know that this is given by its projection operation corresponding to either the Eigen value +1 or -1 so let us look at what is the probability with which the Eigen +1.

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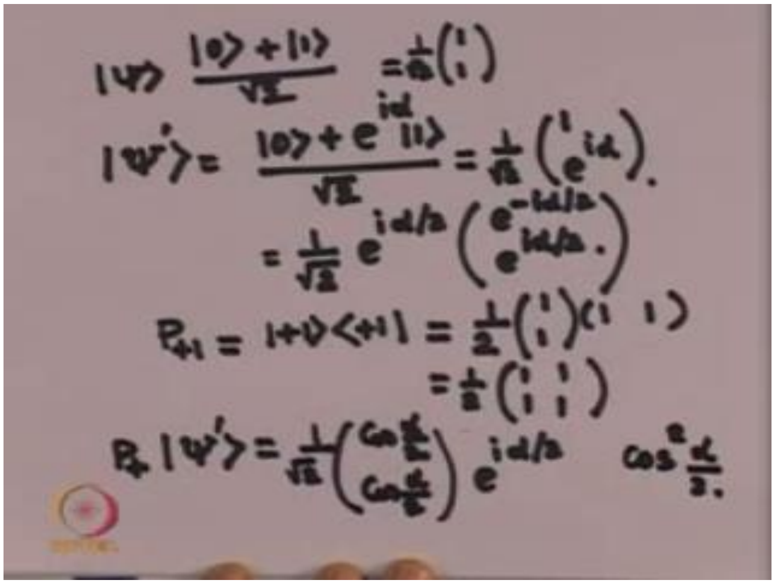

Now for that I need the projection operator corresponding to Eigen value +1 and that we know is nothing but this state and a quick calculation shows that this is 1/2 11,11 this came from 2 $1/\sqrt{2}$ which is 1/2 11,1 1 so if you apply the projection operator for this on ψ it is a trivial matrix calculation to show that this is going to be $1/\sqrt{2}$ Cos $\alpha/2$ and also Cos $\alpha/2$ this is just do this matrix multiplication. So therefore the probability with which the Eigen value +1 will appear for σx measurement is $\cos^2 \alpha/2$.

So what we will seen in this that a face perturbation or a change in face also is a an issue that we have to worry about. The bit flip continues to be here, but loss of coherence or as we use the face decoherence of a quantum state due to external perturbation is a new thing that we have to introduced. In the next lecture we will be talking about a simple way of taking care of such errors and we will see that well completely fault tolerant quantum computation is still probably distant in realization there are course available which can do error correction.

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