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Quantum Information and Computing

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Modyl No.01

Lecture No. 03

Postulates of Quantum Mechanics – II

In the last lecture we had talked about the first two postulates of quantum mechanics the first postulate was that every physical system is represented by a ray in an abstract inner product space known as the Hilbert space. The second thing that we did is to point out that there is really no difference between a state which we represent by let us say $\psi\phi$ and another state which is represented by a scalar multiple of the same state ψ where the scalar multiple is in general a complex number.

And because of that the states are actually represented not by a given vector in the Hilbert space but by a ray in the Hilbert space. Having done that we had defined operators which are linear operators in the vector space and towards the end of the lecture we were telling you that there are matrix representation possible both for the state that is vectors and the operators which act on this space.

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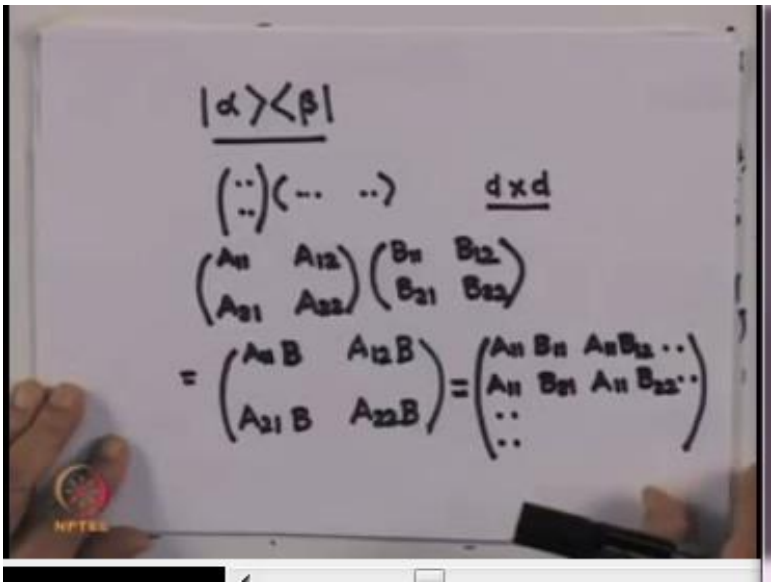

We had seen that a general operator has a structure like a ket α followed by a bra vector so that it can act on an arbitrary vector by giving me another vector. Now supposing these two have the same dimension, if these two have the same dimension D then this is a product of a column vector with a bra vector having the same dimension. So if this is a column vector of D dimension that is there are D entries in the column and if this is the bra vector then I get a d x d matrix.

Now you have to realize that this is not an ordinary matrix multiplication, but is what is known as direct multiplication of matrices are also known as Kronecker product of the matrices. Now the way it works something like this, supposing I have a matrix which is I write as A_{11}, A_{12}, A_{21}, A_{22} and try to multiply this with a matrix which is let us say B_{11}, B_{12}, B_{21}, B_{22} then the way this product works is, you take the element A11 and multiply with the matrix B itself.

And similarly A_{12} with B, A_{21} with B and finally A_{22} with B, now this of course you have to now expand it out and I will not write down the entire form for a matrix, but the way it goes like this. So I have A11 multiplied these two by two matrices so I will have B_{11}, A_{11} with B_{12}, A_{11} with

B_{21}, A_{11} with B_{22} and like this I have, so it will become A/4 matrix. So this is what is meant by a matrix product or Kronecker product.

Now we talked about that physical observables such as energy, angular momentum, momentum etc are represented by self ad joint or Hermitian operator both the languages are the same. Now according to this postulate of quantum mechanics which I still in system calling it second postulate a state ψ, a state ψ has a definite value λ. I will explain what it means by that.

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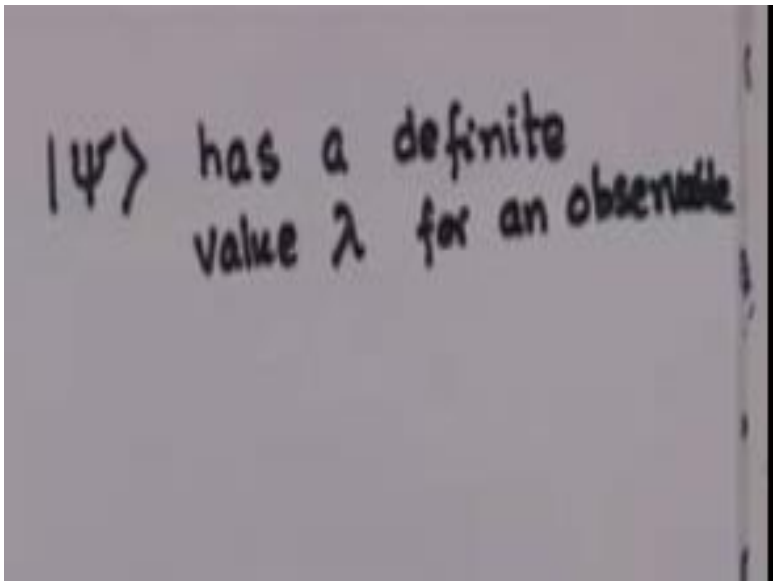

Definite value λ for an observable what it means is that supposing I am interested in finding out what is the energy of the state ψ so my observable is energy. Now then if I measure the energy of ψ then I can make a statement that the energy has a particular value, only if this state ψ happens to be an Eigen state of the operator corresponding to energy with an Eigen value λ.

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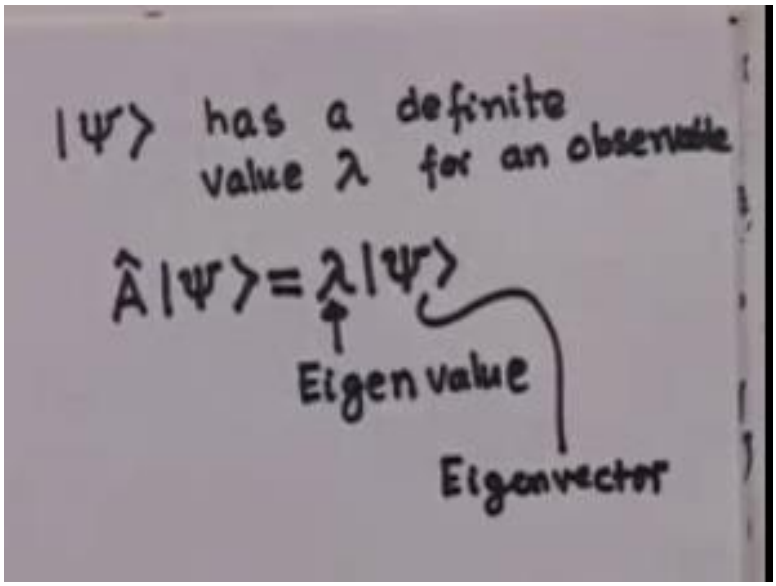

So they postulate simply says that supposing you have any observable A or this is operator corresponding to the observable A. Now if this acts on an arbitrary state ψ and gives me a number of times the same ψ, then this λ is called the Eigen value of the operator A and the corresponding ψ is called an Eigen vector corresponding to this Eigen value. Now an arbitrary state is not necessarily an Eigen state of an operator, but the if you take the collection of Eigen states corresponding to a given operator in the [indiscernible][00:06:20] space then this set of Eigen states they form a complete basis.

What is meant by a complete basis, it simply means that an arbitrary vector in that state can be expanded in terms of the Eigen states of any operator corresponding to whatever observable you have. In other words I can expand an arbitrary state in terms of let us say a complete set of Eigen states of the energy of it, or I can do the same thing in terms of the complete set of Eigen states of let us say angular momentum operator.

Now so suppose I am identified one particular observable in which I am interested in, no particular reason but let me for specific purpose call it an energy. So I am interested in finding out what are the Eigen states of the energy of it. Now and I have determined the complete set of

the Eigen states just to give you a familiar example to you that you have solved the problem of hydrogen atom and found out that there are possible energy levels which are given by the bourse energy condition, the ground state be having an energy -13.6 electron volt, the next higher states having energy -13.6/4, the next one -13.6/9 etc., etc.,

Now each one of these the state corresponding to each one of these values these values are called Eigen values. Is an Eigen state corresponding to that Eigen value, so what we are saying in this context is an arbitrary state of the hydrogen atom can be expanded in terms of these Eigen states corresponding to definite Eigen values. And what this of the, that the postulate said is that when I make a measurement of the energy, then the best I can do is to predict the various probabilities with which they will occur. And I cannot definitely say if you do a measurement you will get -13.6.

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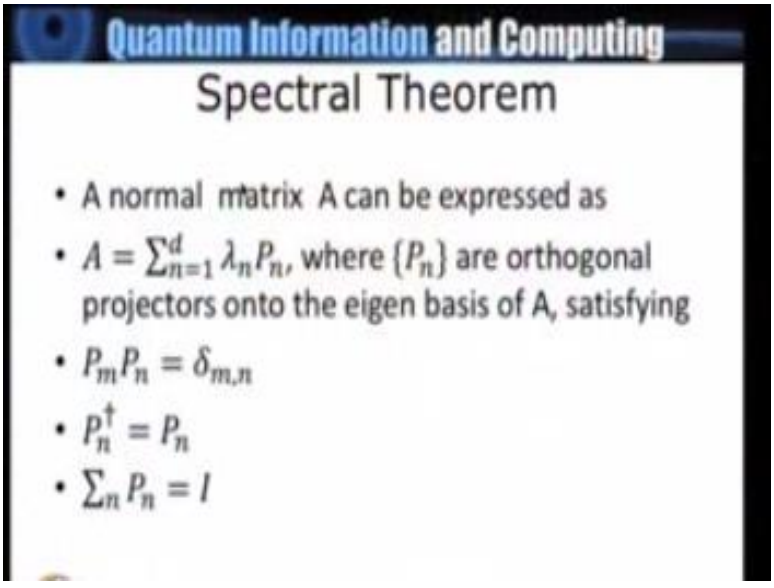

Now one can show that if we consider a normal matrix they, actually the theorem is I am interested in that theorem for only the Hermitian matrices, but in practice the theorem is actually true for much more general matrices called the normal matrices, normal matrices of this property that $A^+ A$ is equal to A^+.

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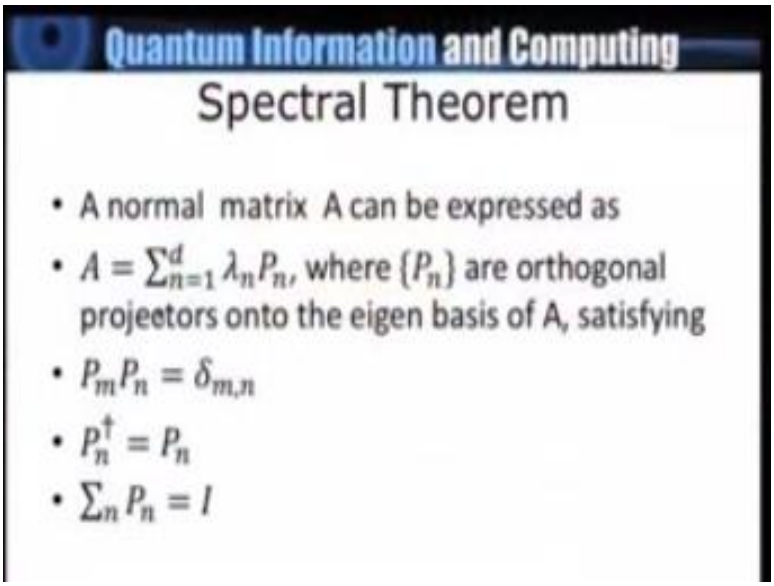

Now if you look at that type of matrix and if you look at any normal matrix there is a theorem which says which says, which is called spectral theorem and the spectral theorem simply says that this matrix or the operator, the operator and the matrix they will be used interchangeably can be expanded in terms of like this, the Eigen value corresponding to the operator times another operator called P_n and this P_n is a projector the P_n is a projector corresponding to the Eigen state λ.

Now if you think about it is not all that difficult to understand what it means see basically we are saying that if you have an operator. Now you can resolve that operator into various operators which project the state along a particular Eigen state and spectral theorem simply says that. Now so obviously there are some properties to be satisfied by these operators this projection operators.

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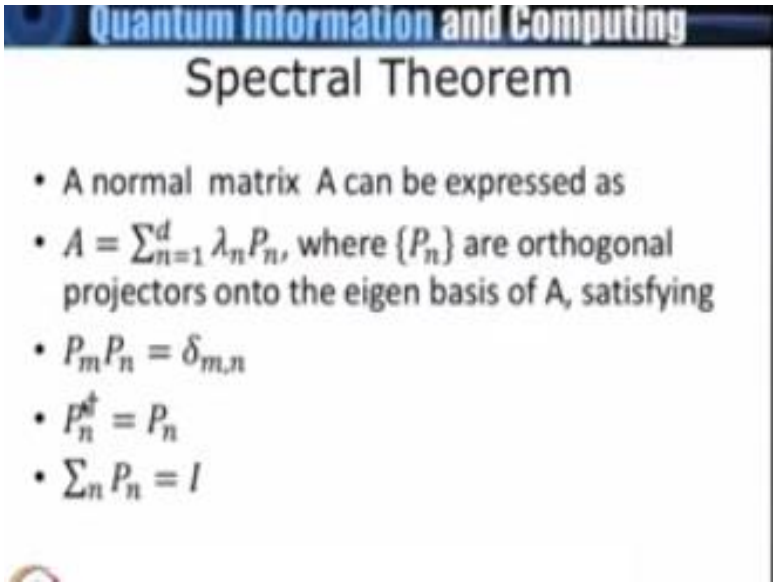

So one of them is P_m x P_n is $\delta_{m,n}$ means if you multiply actually P_m x P_n is δ_{mn} x P_m there is a mistake in that slide.

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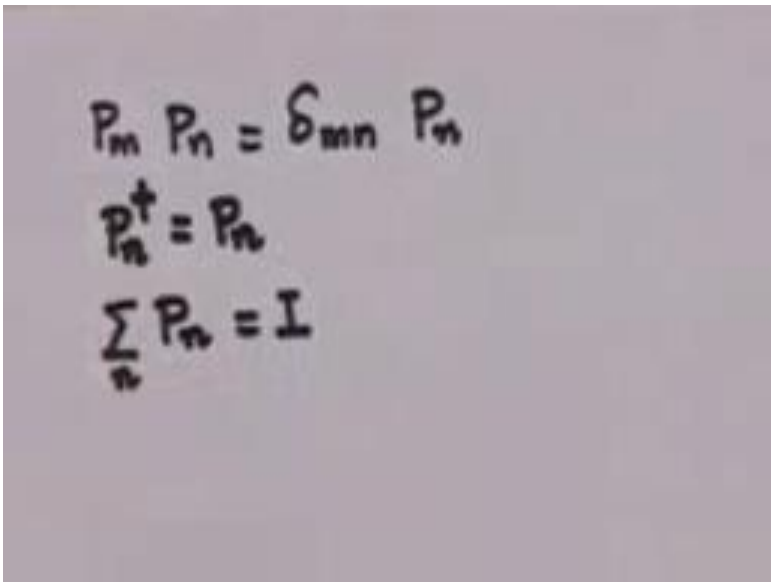

So P_m. x P_n is equal to δ_{mn} times P_m. Now this is actually obvious because once you are projected in that direction these are orthogonal projections once you are projected in let us say direction N, now if you now try to project something in the direction M obviously it is going to be 0 unless it is in the same direction. And if it is in the same direction then P_m square must be equal to P_n. The other thing is that these operators are hermitian, matrices are also our hermitian and secondly I must have a completeness that is sum over P_m must be equal to i because I am resolving an arbitrary operator in terms of its orthogonal projections. So obviously when I add up all of them they have to give me the same identity value just to illustrate the spectral theorem let me look at the basis in C_2 again.

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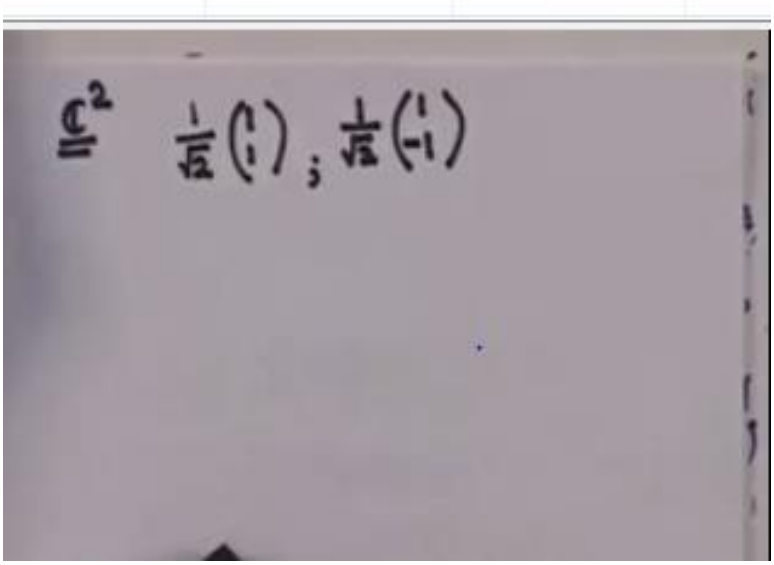

The C_2 is one where we keep on coming back with various types of bases I have told you that C_2 has a basis which is 1001 but let us look at another basis in C_2.

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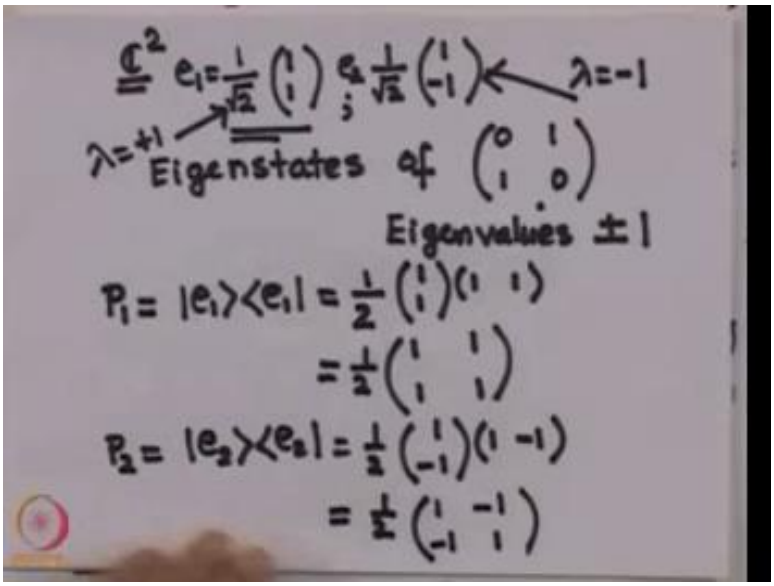

This is normalized basis 1 over square root of 2 11, and 1 over square root of two 1-1. In other words in terms of these two bases I can expand any vector in that two dimensional space. Now these vectors are Eigen states of a matrix which looks like 0, 1, 1, 0 we will later on see this matrix has a very important place in our physics that will be doing. Now so this is an Eigen state of this operator with an Eigen value let me write it as λ=+1 and this one corresponds to λ=-1.

You can easily see the Eigen states of this is plus or minus 1 so this as the Eigen Values plus or minus 1. Now let us look at what does this spectral theorem say, so first one is supposing I call it e1 and supposing I call this e2 then my operator P 1 corresponding to this Eigen state is e1, e1 this is the general structure and you can immediately multiply. So I have 1 over square root of 2 into 1 over square root of 2 which is equal to 1 by 2, I multiply 11 with 11, this is a direct product so therefore I get 1 by 2 you can see 1 into 1 1,1,1,1, this is p 1.

And the corresponding thing for the other Eigen value is e2, e 2 and that is equal to 1 over 2, 1-1, 1-1 you can multiply these easily. So I get 1-1, -1 and 1. So these are my two projection operators. And you can immediately see what I am trying to tell you in the spectral theorem is.

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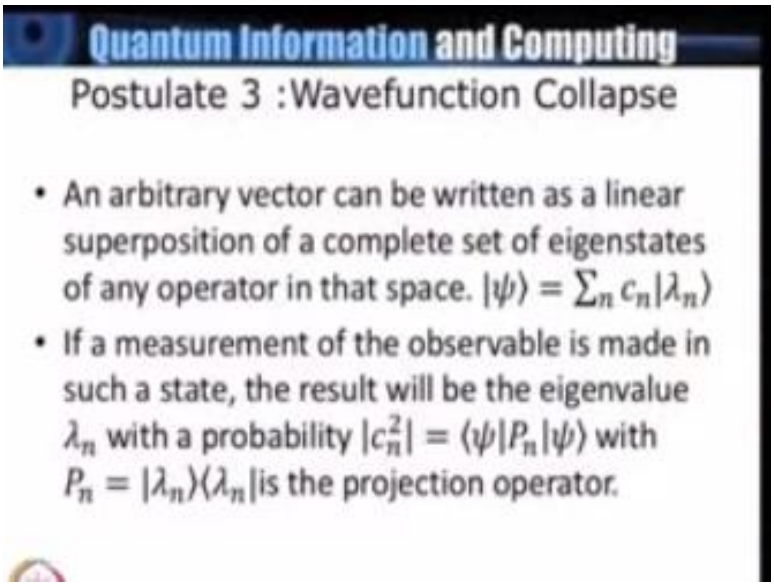

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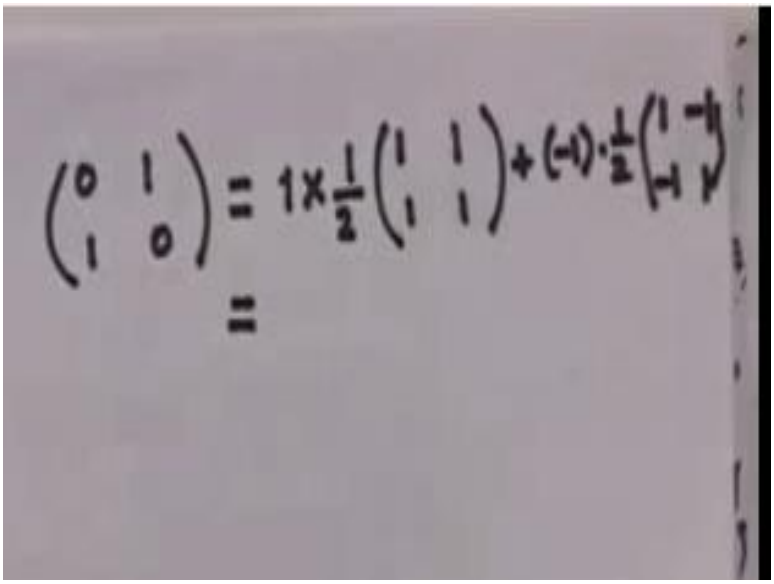

That my original matrix which is 0, 1, 1, 0 is given by λ1 which is equal to 1 into the first projection operator which is 1/2 1, 1, 1, 1 + -1 times 1/2 1-1, -1 1. You can see that this does it did give this one. So this was my second postulate and we have done some linear algorithm. Now I come to a rather important philosophical process which is the component of Copenhagen interpretation which was the most contested component of the Copenhagen interpretation by as I have told you in the beginning [indiscernible][00:15:33]. And this is what is known as the wave function collapse.

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Now the wave function collapse is a postulate which has certain amount of philosophical significance, let us try to understand what it means. So what we said in the beginning is that if I look at and arbitrary ket ψ.

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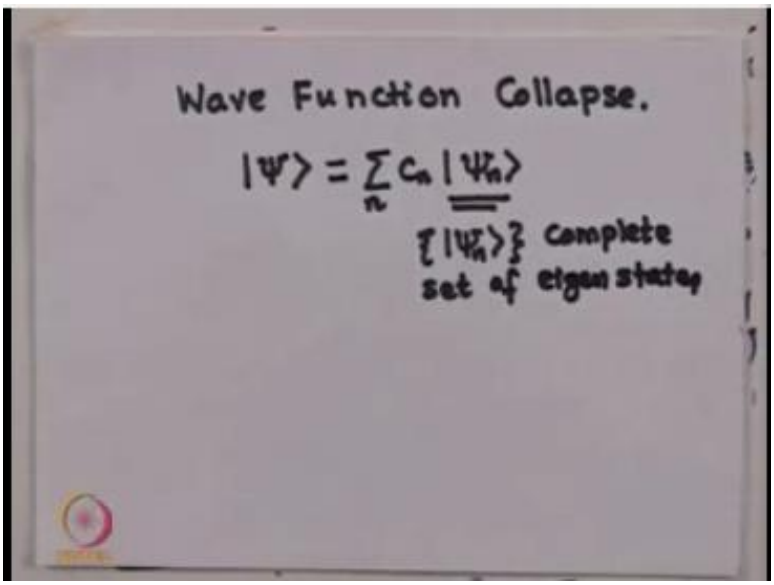

Then I can write it as a linear combination of C_n ψ_n where this ψ_n from a complete set of Eigen states of corresponding to any operator okay, complete set of Eigen states of any operator. Now supposing I have a system in this state and I have expanded this in terms of complete set of Eigen states of an operator let us again for specific idea let us say its energy operator. And I now using this state I want to measure the energy of the system, what the Copenhagen interpretation says is the following, that the result that you would get when you make a measurement of the energy will be one of the possible Eigen Values of this set.

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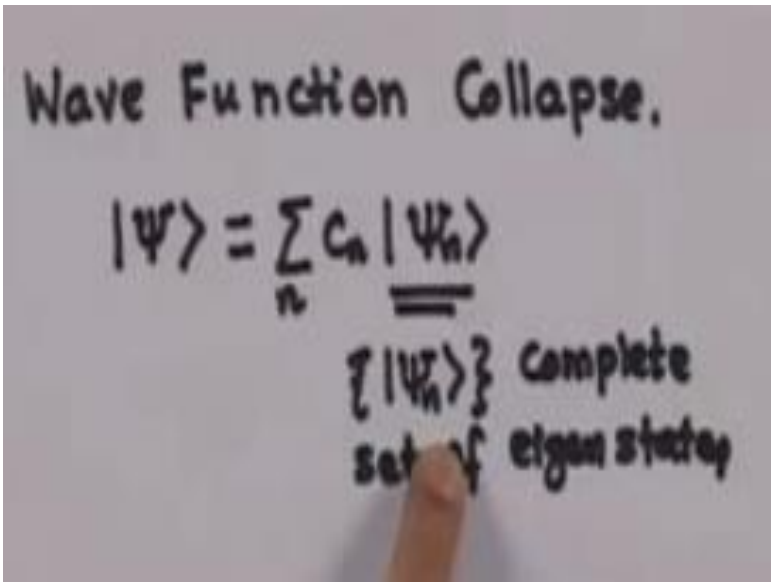

Remember we said ψ_n are complete set of Eigen states means the $\psi 1$ corresponds to $\lambda 1$, $\psi 2$ corresponds to λ towards etc. So what we are saying is that if you take a measurement of the state I cannot tell you what will be the result I will get.

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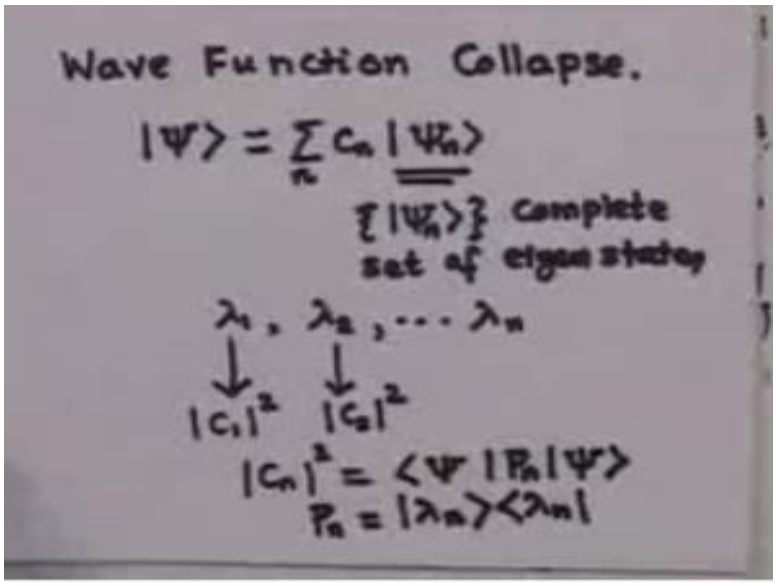

But I can tell you that you will get either λ1 or λ2 or up to λm. And I can say to something more, I can say that the probability with which the value λ1 will occur in a measurement will be given by $C1^2$. Likewise probability with which λ2 will appear will be given by $C2^2$ itself. And in general this Cn^2 you can easily see by definition of my projection operator is nothing but $\psi P_n \psi$. And P_n then is nothing but supposing I represent λ_m, λ_n I had written it as ψ_n does not matter as the projection operator corresponding to the Eigen value λ_n then this is what the probabilities are.

So on making a measurement the wave function of the system would suddenly collapse to the Eigen state corresponding to the Eigen value that you have measured this is called collapse of the wave function. Now what it means is something deeply philosophical, it tells you that quantum mechanics has two parts. One who can know observation is being made, what happens there, how does the wave function change which I am going to talk in a moment.

But a process of measurement is a totally discontinuous process. During that process of measurement, the state of the system would collapse to one of the Eigen states of the system. And there is no way for anybody can predict which Eigen state if you collapse to, the best that one can do is to tell you that this will be the probability with which a particular state will appear.

Now the question is how does not calculate such a problem that is another problem. The way to calculate such a probability would be to make identical copies of the system large number of them.

And make the measurement over and over again on different systems but identical, and find out how many times for instance λ1 is appearing, how many times λ2 is appearing etc, etc. We will see their later that this is itself a tolerant, because there is a theorem which says you cannot make identical copies of the system, but that is a differential, that is a philosophical question. Now what happens to the state of the system, once it had collapsed.

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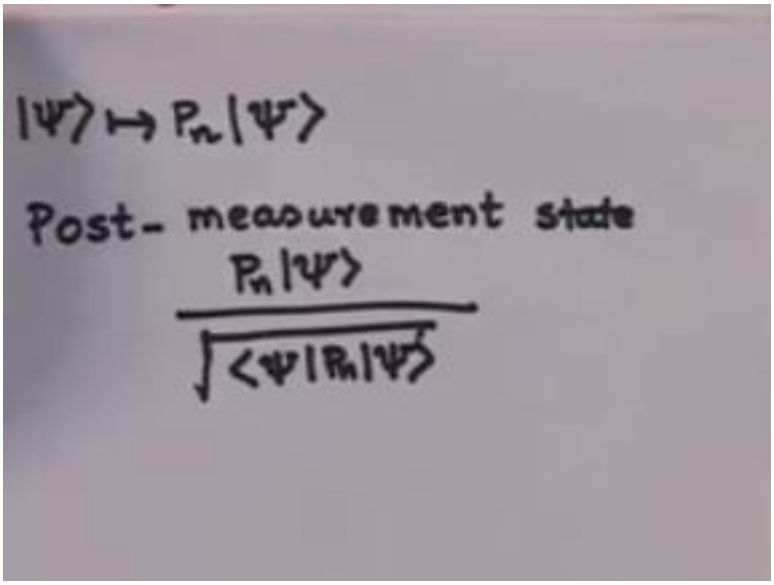

So we have agreed that what has happened is that the state has gone from state ψ to a projection of the state on to the Eigen state corresponding to λm. Now if I now normalize this state, I can talk about what is the post measurement state of the system. And this is given by $P_n\psi$ and I need to normalize it simply which means I will divide it by its length, so this will be $\psi P_n\psi$ and you can see why, because this should be this quantity multiplied with the corresponding bra, but then I will get a Pn^2, but Pn^2 is Pn so therefore this is the structure.

So this is the post measurement state of the system let us look at a matrix example to make it very clear what is happening suppose I have a state.

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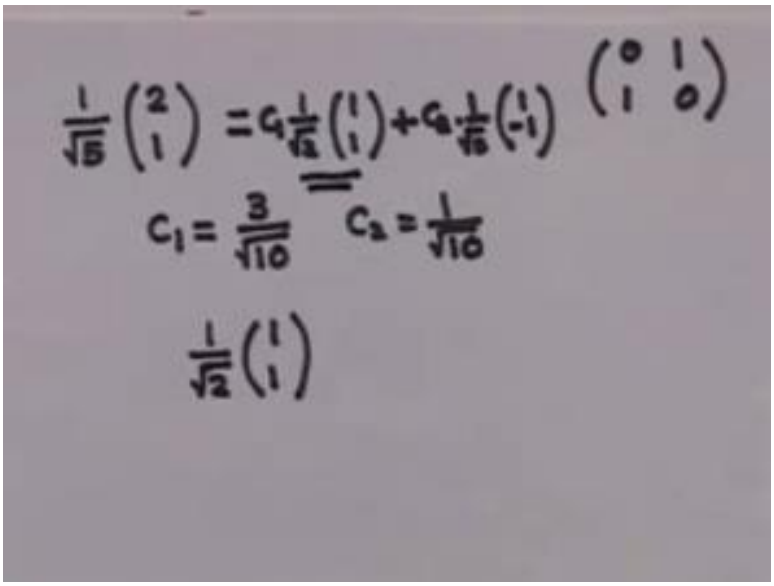

Which I have normalized it one over square root of five 2, 1 remember normalization simply means square of this square of this one divided by square root of that sum. So supposing I have this state I want to expand this in terms of the Eigen states of that matrix which I talked about, the Eigen states of 0, 1, 1, 0 which was I had taken this to be 1 by square root of 2 1,1 + so I write it as C1 + C2 into 1 over square root of two 1-1.

Now this is easy to calculate what is the 1 and 1 C2, because this is a matrix identity. So I just put this whole thing and you can do a trivial algebra and so C1 is equal to 3 by square root of 10 and C2 is equal to 1 by square root of 10. Now what it tells me is this, that if I make a measurement and I get this Eigen value to be $\lambda=1$ then the post measurement state of the system it is normalized value will be 1 over square root of 2, 1, 1.

So this is the third postulate, I will complete the quantum postulates with the fourth postulate which is I will not be using it much accepting philosophically that when I am not observing a system what happens, when I am not measuring a system what happens the time evolution.

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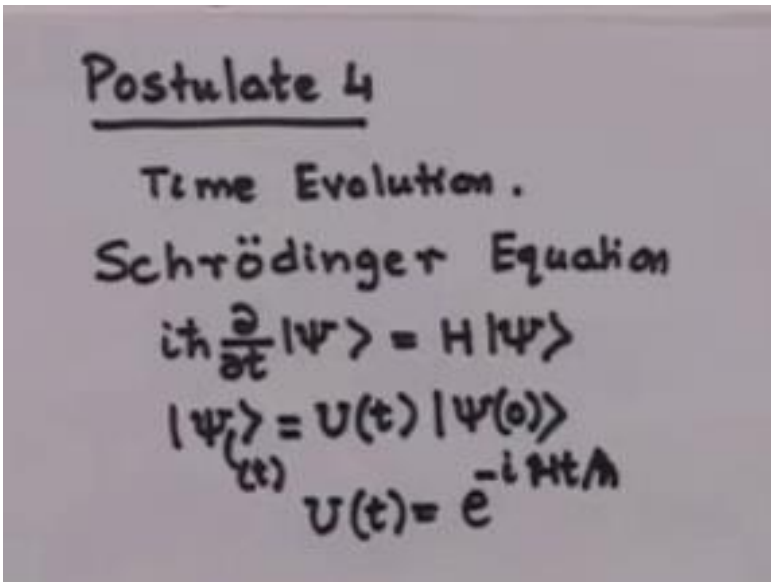

So I will call it postulate for convenience remember postulates are hypothesis on which theory is based. So therefore you do not question where did it come from. So time evolution how does a state ψ develop with time, what it says it follows an equation remember classical physics we say that the evolution or the, with time the dynamical quantity follows Newton's law. In this case the corresponding equation is known as a schrodinger equation which is given like IH dψ/dt is equal to hψ where H is the hamiltonian of the system, hamiltonian operator.

Now this is, you can do a formal solution of this and find that ψ develops with time following and unitary operator I should write ψt and a formal solution for u(t) is $e^{-iHt/h}$. Now this is an important one that the unitaru operators, the operators which determine which way a state will develop are operators which an unitary operator. Now what is the unitary operator and unitary operator is an operator which preserves the norm of a state or a vector so just here is the definition.

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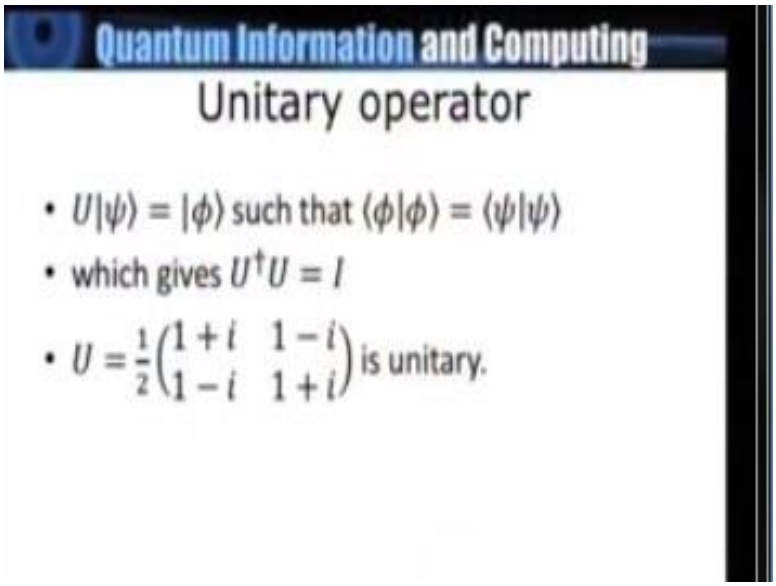

That uψ is equal to ϕ if this is what happens, the operator u is unitary if the state ϕ and state ψ have the same norm that is ϕϕ is equal to ψψ. Now this is automatically implies that $U^+U=i$, i is an identity, you can check for instance hat this is an unitary operator, this is the unitary operators. Now what is important take in the second last one is that all quantum processes occur develop unit area. And as a result the operators which we deal with in quantum computing are all unitary operators.

So these are the operators which decide how a state develops with time when you are not making another one. The process of measurement makes it a discontinuous process, makes the state collapse to one of the Eigen states of the system, and so therefore, the quantum computing deals with two aspects of this computation process, one is just to give you a classical analogy it means that when it is passing through transistors and gets I am not trying to find out intermediate results I am perfectly alright.

The system simply follows a unitary operation, but the moment I decide I want to do a read they find out what is the result I must make a measurement and when I make a measurement I can at best give you a probabilistic answer. And the result that I will get will be one of the permitted results, but I have no way of telling you which result I will get. And this causes a big difficulty in

extracting information out of the measurements because we are interested in only relevant in moment we will see as we go along how one gets it down.

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