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Quantum Infromation and Computing

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Modul No.05

Lecture No.26

Implementing OFT-3 qubits (and more)

In the last lecture we had started talking about implementation of quantum Fourier transform and we had seen that for a one qubit case this is nothing but an ordinary Hadamard transform and incase of two qubits we analyzed the situation and found that the implementation is done by having a Hadamard transformation on one of the qubits and a controlled phase rotation in on another qubit along with a Hadamard transformer.

The one should in principle one could go and find out what happens to n qubit but the mathematics becomes a little clumsy though straightforward, so what I will do is instead of going to n qubits I will try to tell you how to extend it to three qubits and try to see whether there is a pattern which emerges out of this. But before that let us review what we did in the last lecture for n =1 and n = 2.

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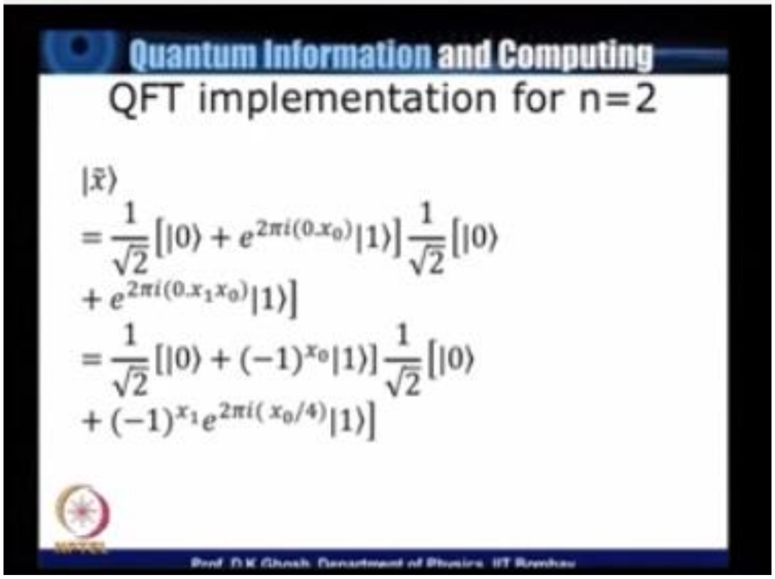

The slide here shows for n =1 this is this slide is for n=2 and you can see that what it requires is a Hadamard transform on the second qubit because I had 0 + $(-1)^{x0}$ |1> and a Hadamard transform on the first qubit $(-1)^{x1}$ along with a controlled phase rotation, I say it is a controlled phase rotation because the rotation is there only if X0 = 1. Now that requires as I said.

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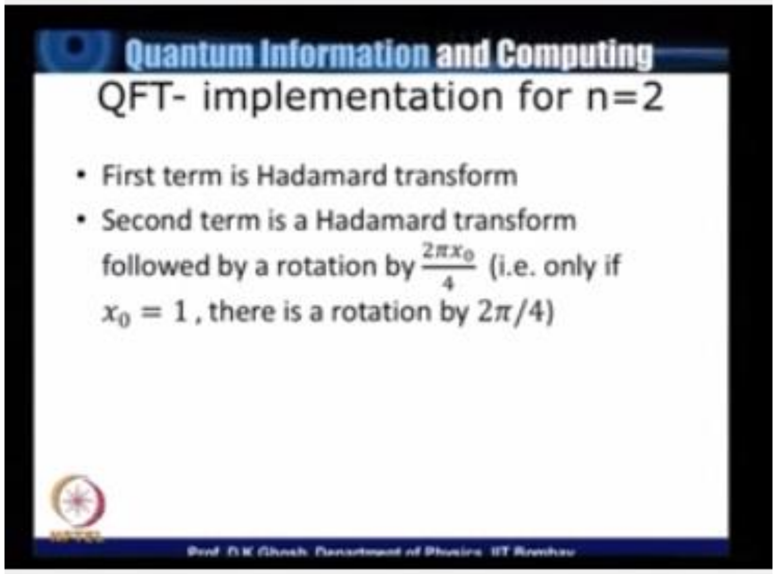

Last time that we cannot change the value of the second qubit before we have taken its value for application on the first bit.

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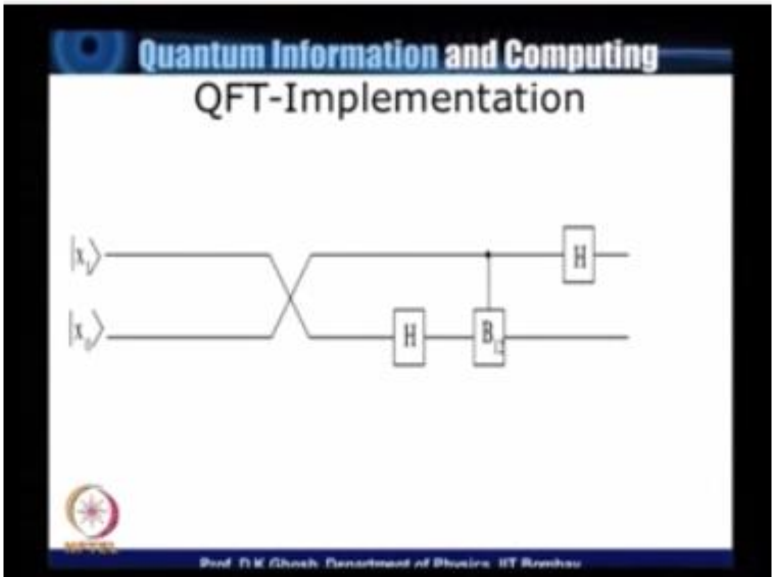

And that was done by realizing that what we need to do is to interchange X1 and X 0 that is the MSB and the LSB as one says in bit language and then apply the Hadamard gate we defined B1 to get last time which I will again have occasion to talk about today and this is the way one implemented QFT for n = 2.

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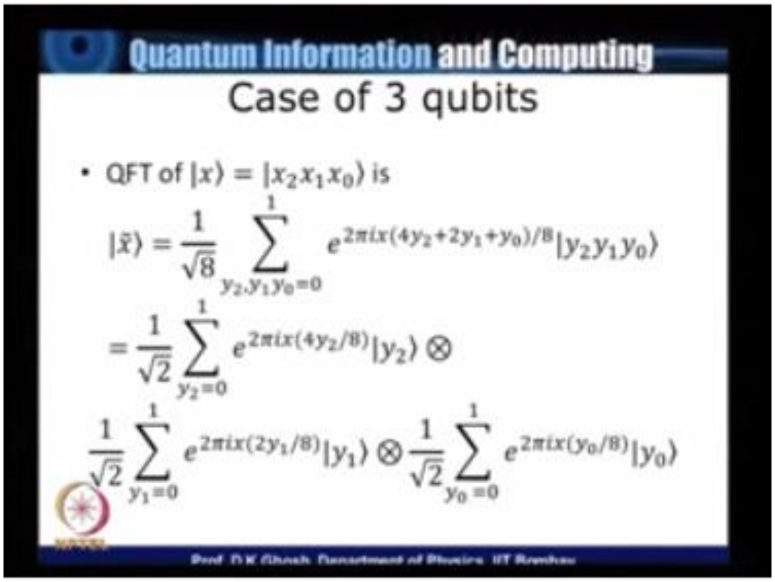

Now let us look at what is the situation with respect to n=3, now if I have n = 3 so 3 qubit case.

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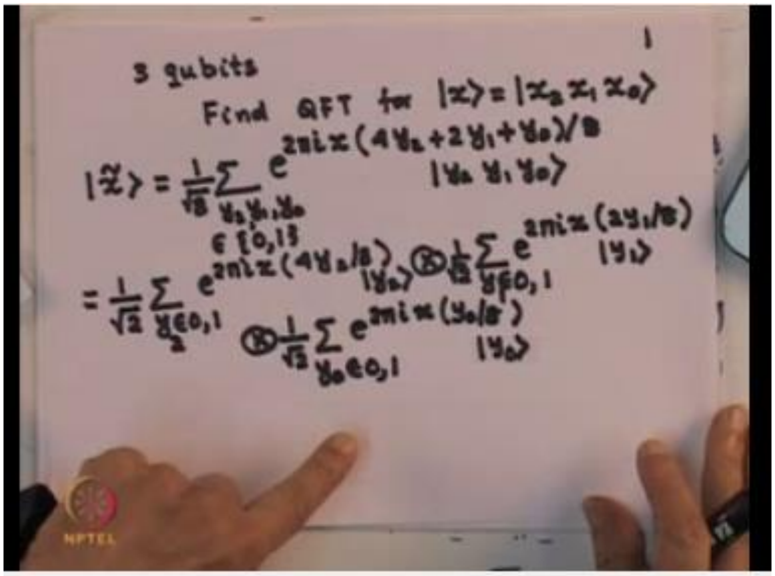

So we need to find out for X which is a shorthand notation for writing X2, X1, X0 by definition of the Fourier transform my X tilde then is 1/ √8 because I have three qubits sum over using the same notation as we have been using last night y2, y1 and y0 each taking the values 0 and 1. So I got $e^{2\pi ix}$ at the moment I do not expand X like I did last time and now I need to write down the y but then y is $4y_2 + 2y_1 + y_0$ / n = 8 in the this case.

And I have the state y2, y1, y3 now as before I will split this into three terms, so and I will also distribute this 1/ √8 as I did before, so I have 1/√2 sum over y2 = 01 $e^{2\pi ix}$ so this is $4y_2/8$, so therefore (4y $_2$/ 8) |y 2> then I have 1/ √2 again sum over y belonging to 0 and 1 $e^{2\pi ix}$ this is 2y1/8, so let us keep them like that because it is much easier to handle that way and of course y1 and finally for the last qubit I have 1/ √2.

So this is y2 this is y1and I have y0 belonging to 0 and 1 $e^{2\pi ix}$ and y0$_{8.}$ Now let me look at each term in turn.

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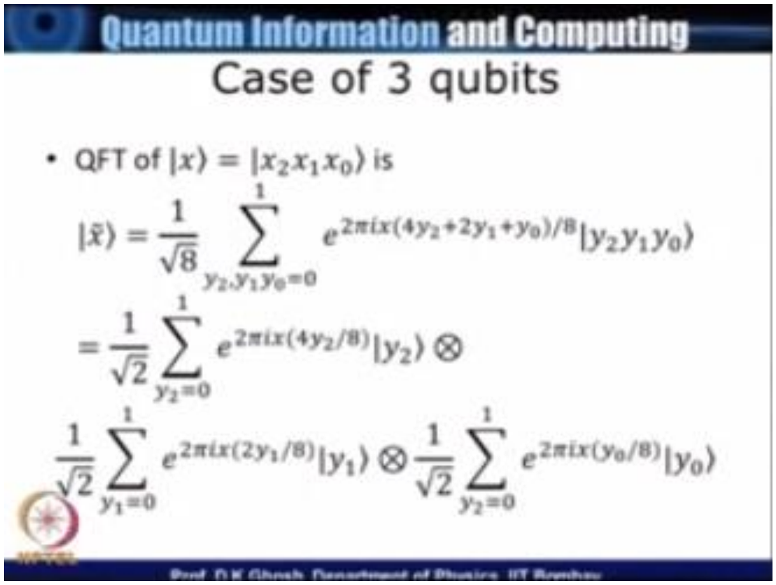

So let us look at the first term this is.

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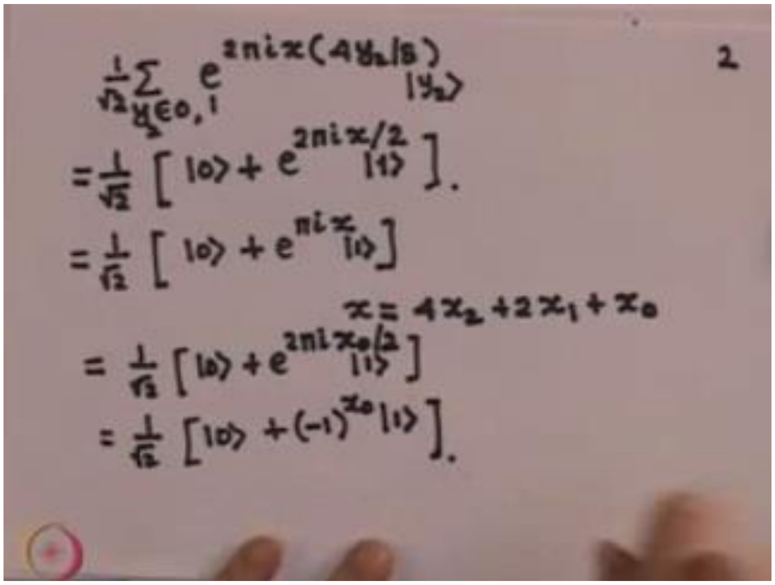

$1/\sqrt{2}\ \sum y_2 \varepsilon 0_1\ e^{2\pi ix(4y2/8)}$ let us split this term for y_2=0 the phase factor is 1 so this is $1/\sqrt{2}$ [|0>+ if y_2 =1 so I get here $e^{2\pi ix(4y2/8)}$ is nothing but 2 |1> this the first on there, and let us try to see what this term actually is, now this term is $1/\sqrt{2}$ [|0> now you notice this is $e^{\pi ix}$ but before we do that let us write down expand x and see what is it, so $e^{\pi ix}$ |1> since $x=4x_2+2x_1+x_0$ notice what I am getting here. I have e to the power this is the case for which I have x_2 x_1 x_0 I have to write down and each one of them can take values 0 and 1.

Irrespective of whether it is 0 or 1 you notice the following that I get $e^{4\pi ix2}$now $e^{2\pi i}$ as well $4\pi i$ each will be equal to 1. So therefore, both for x_2 and for x_1 taking the value 0 and 1. The prefactor here will become 1 and a phase factor which depends only on x_0, so what I get is this that this is equal to $1/\sqrt{2}$[|0>+ $e^{2\pi ix0/2}$ well it is πix_0 same thing 1 and this is equal to $1/\sqrt{2}$[|0>(-1)^{x0} |1>] so this was the first term. Let us look at what we get for the second term, so the second term.

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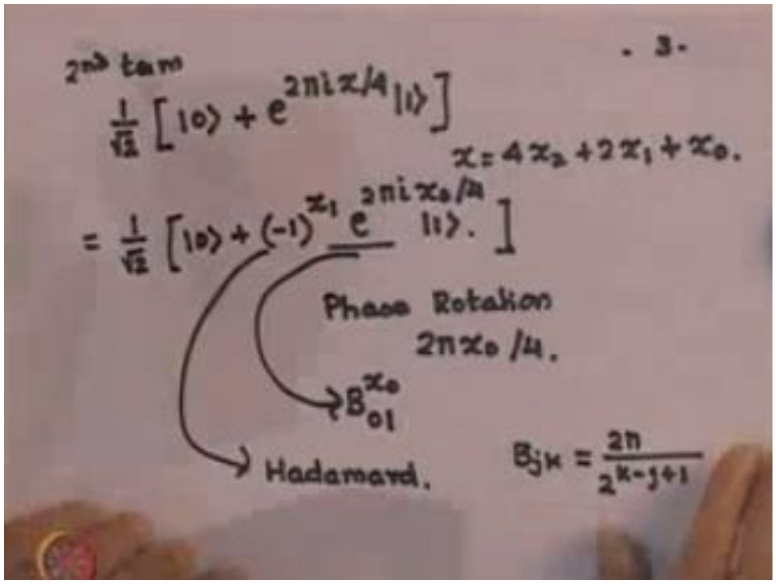

I had $1/\sqrt{2}$ [|0|>+ $e^{2\pi ix/4}$ |1>] once again since $x=4x_2+2x_1+x_0$ so this term i get $e^{2\pi ix}$ as before if $x_2=0$ or $x_2=1$I still get this pre-factor to be equal to 1. But I have also these terms here, now look at what happens to this term, so I have 2πi but then I have $2x_1/4$ so that I get $e^{\pi ix1}$ now if $x_1=0$ then of course it is equal to1, but if $x_1=1$ then I get -1, so I get $1/\sqrt{2}$[|0>+$(-1)^{x1}$ I still have a phase left, I have a phase left which is $e^{2\pi ix0/4}$ |1>] so how did we implement this, so once again this is the Hadamard transform along with a phase rotation of 2πi of $2\pi x_0/4$ we saw that these selective phase rotation.

Why selective because unless $x_0=1$ this phase rotation is not there, so I need a controlled phase rotation when x_0 is the control so that I write down B, let us put the control over it now I need to find out how much is this rotation and for that I had defined a controlled B_{jk} gate, where the phase was given by $2\pi/2^{k-j+1}$ with k greater than j and I want this to be $2\pi/4$ so therefore, you notice that if i take j=0 and k=1 I get $2^{1=1}$ that is 4 so therefore this is $B^{x0}{}_{01}$ so this is the way this will be implemented so Hadamard by this is Hadamard that leaves us with the third term.

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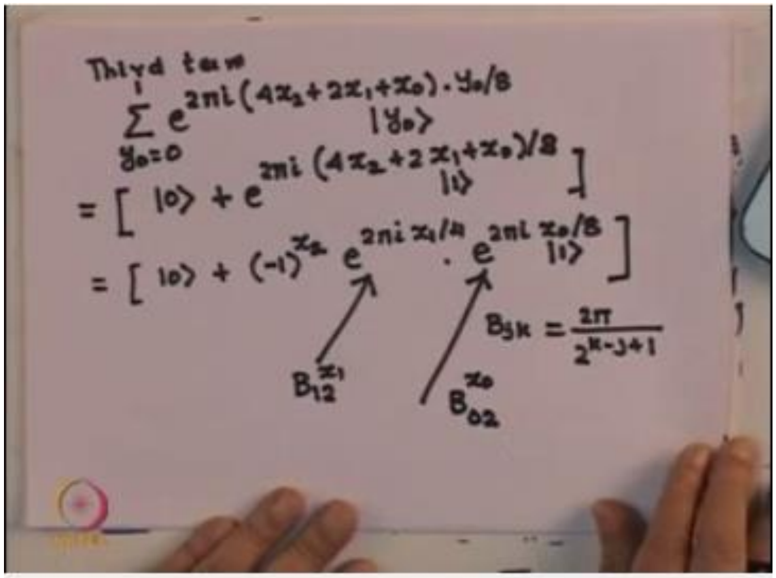

So we said y0 = 01 $e^{2лix}$ and let us expand the x (4 x 2 + 2 x1 + x0) into y0 / 8 y0and as before I split it up into two terms y0 = 0 and y0 = 1 when y0 = 0 of course I get a 0 + $2^{2лi}$, 4 x_2 + 2x_1 + x_0 by 8 so let us examine this again that this time you notice I have got here well once again if x_2 = 0 then of course I do not have anything from here because $e^{0 = 1}$ now if x_2 = 1 then I get $e^{лi}$8/8 so this term gives me 0 + (- 1)^{x2} let us come to x_1 once again if x_1 = 0 I do not have any phase contribution.

But if x_1 = 1 I have a phase term which is $e^{л2лi\ x1\ b/\ 4}$ and again for the third term what I have is 2лi x_0 /8 1 now let us look at what these are so this as before is ahead over transform on the first qubit which is the x x2 cubed now this one again is a control phase rotation and this time recalling that B_{jk} has a phase of 2лi / $2^{k - J +1}$ so this phase one has a control which is x1 because unless x_1 = 1 I do not get that and it is B_{12} because if j = 1 k = 2 i get 2л/4which is what i expect here and the this term.

Is the control is x_0 in this case be x_0 this time it is 0 so if you collect all these together then what you are getting is.

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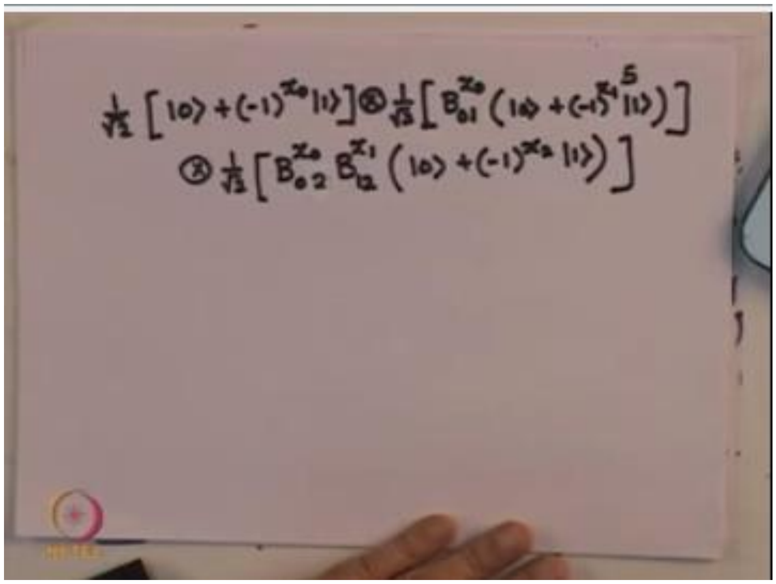

$1/\sqrt{2}$ 0 + - $1^{x0\ 1}$ we have seen that this is $B_{01}{}^{x0}$ 0 + - $1^{x1\ 1}$ into 1 /√ 2 again and this at 2 controlled B_{jk} get one with a control x_0 and the second one the control $x_1$now I had commented it on it last time.

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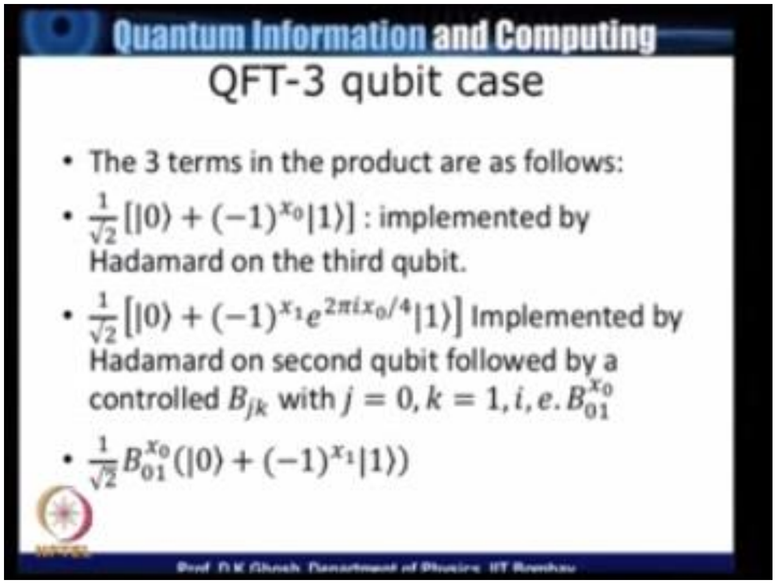

But look at this slide which summarizes our results completely so the first term is very simple I have a Hadamard gate on the third qubit now this is this is important to realize that the Hadamard gate is appearing on the last qubit x_0 and so if I do that now to begin with then I cannot use the value of x0 before application of these gates to be used as a control so I can do it the second one is has a Hadamard gate with respect to the second qubit but again x_0 being control there is a phase rotation.

And this further for this can be implemented on the second qubit by a controlled B_{jk} as we explained.

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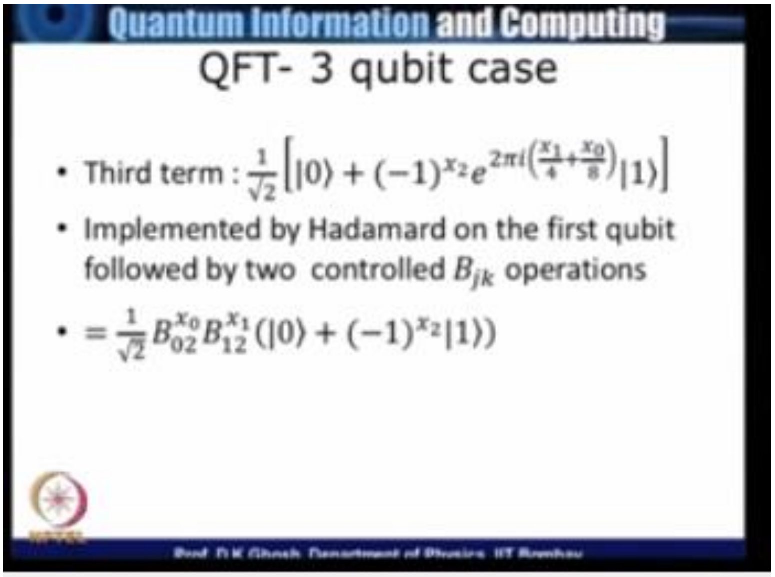

Little while back the third term as we have seen is a high demurred on the first qubit and along with two controlled B_{jk} operation what it tells us.

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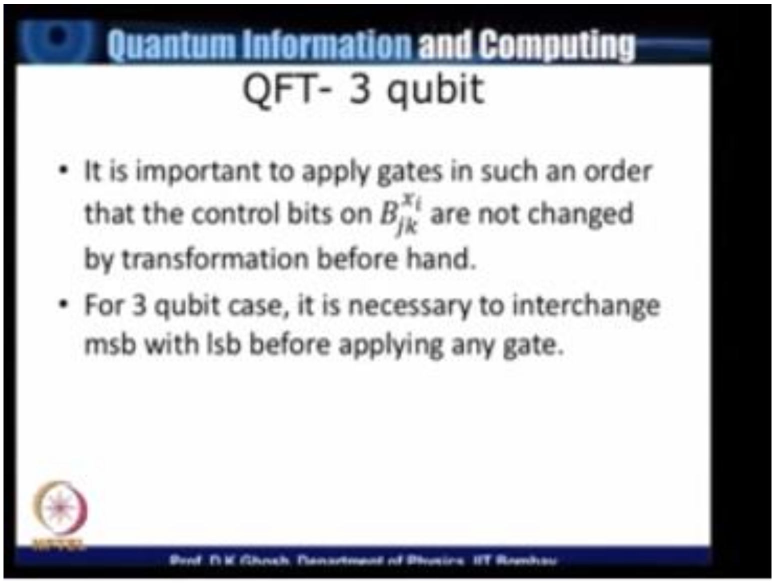

Is the following that we need to apply the gate in such an order that the control bits on B_{jk}^{xi} are not changed by transformation before they are applied that is I cannot change a particular Xi and later and expect the same old Xi to be used in the controller. Now what it requires is the following that in case of three qubits you interchange the first and the third qubit.

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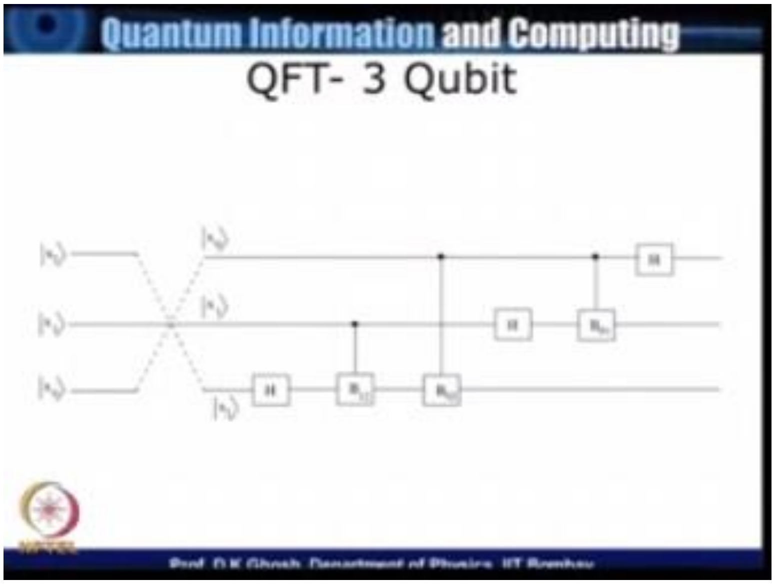

And this slide here explains to you how see the thing is this that I have so this is what I start with I start with x_2 x_1 and x_0 and the second qubit remains without change and I inter change the locations of x_0 and x_1 x_2. Now so therefore on this line I now carry x_0 on this line I now carry $X_0$the middle line still remains x_1 as expected. So now what I am doing now is pass the Hadamard now when I am doing the Hadamard gate I am actually doing a Hadamard on x_2 which is my first term in the expansion that I show you.

After that I need a x_1 as the control and applied a B_{12} the controlled b_{jk} gate I told you and finally on the same x_2 I use X_0 as the control because this line is now x_0 and apply a controlled b_{02} gate so therefore x_2 is now done and when I did that I did not change the values of x_0 and x_1 in the process because this is what I retained. Now comes the action on my bit x_1 but remember nothing has been done to x_0.

So what I do is first apply the Hadamard gate on x_1 which is what I required and then with the X_0 as the control I use the D_{01} gate. And having done all that I finally take up x_0 and apply the Hadamard gate and what I get here now are my Fourier transform of the three qubit case.

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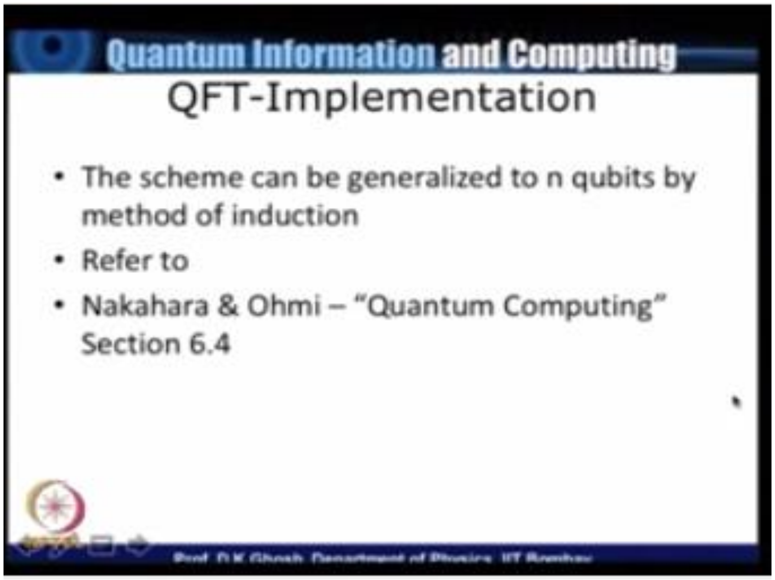

Now what we will do now for n qubits, now in principle this can be generalized to n qubits by means of a process of induction. The algebra happens to be a little complicated so therefore I will not go into it but let me sort of tell you what you actually do. What you actually do is to realize that the process can be essentially performed by a Hadamard followed by a sequence of b_{jk} operation on that qubit.

But before I do that I must previously permute the original states. In this particular case of three qubit you realize that if I permitted x_2 x_1 and x_0 and reverse them the middle one remains the same, but for n qubit it becomes a little more complicated and I permute them and keeping in mind that the order in which the control of the control b_{jk} gate will appear will be different and with that in mind I can implement the free transform corresponding to any number of qubits.

So in this lecture what I have done is to tell you that the Fourier transform can be implemented by a unitary operation and by using basic gates which are my Hadamard gates and the controlled b_{jk} gate each one of them can be implemented by a unitary operation, with this we have completed our requirement for taking on the factorization problem. In the next lecture we will be using these concepts that we have developed for an algorithm which is known as source factorization algorithm.

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