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Quantum Infromation and Computing

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Modul No.04

Lecture No.22

Grover's Search Algorithm – IV

In the last three lectures we have been talking about Grover's search algorithm and what we have pointed out is that this is a problem requires of the order of n searches.

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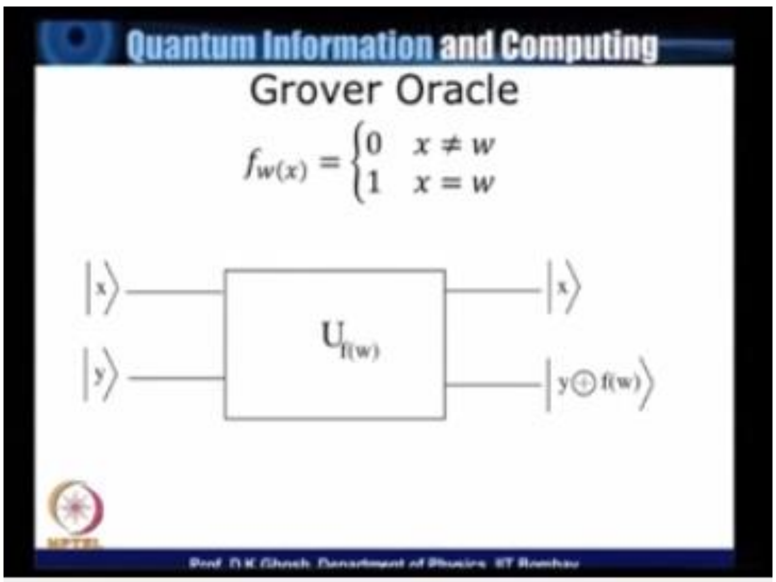

In an unstructured database to locate a particular item the schematic diagram of the Grover oracle is here that the oracle computes a function of x which is a n cubic string and when they string exactly matches certain such condition that is it exactly matches a marked stage, the function evaluates to 1 otherwise the function becomes equal to 0.

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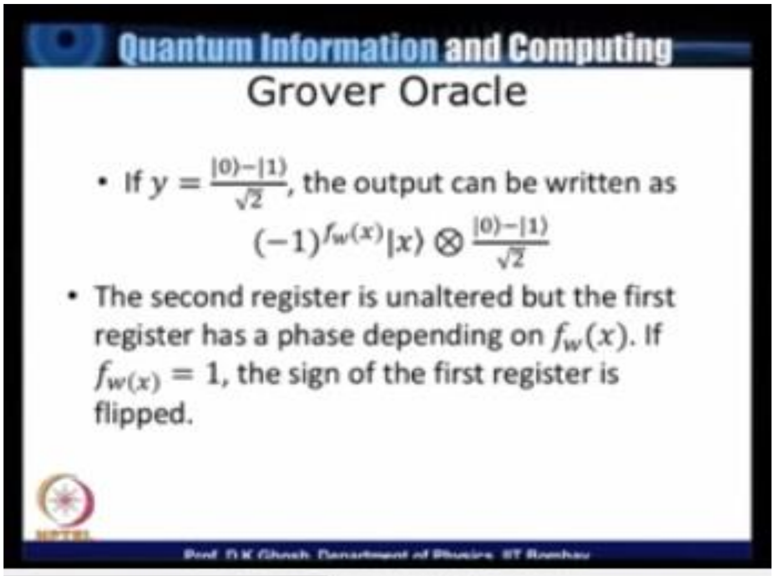

What we have seen is that we start with a linear combination of the computational bases state as the input the target is set to 0-1/ √2 which is obtained by passing a qubit 1 through a Hadamard gate and then we have found that when the target state is 0 just the out pout is the function itself when the target bit is 1 it is the complement of the function the state of the 1^{st} and the 2^{nd} register is given by this slide.

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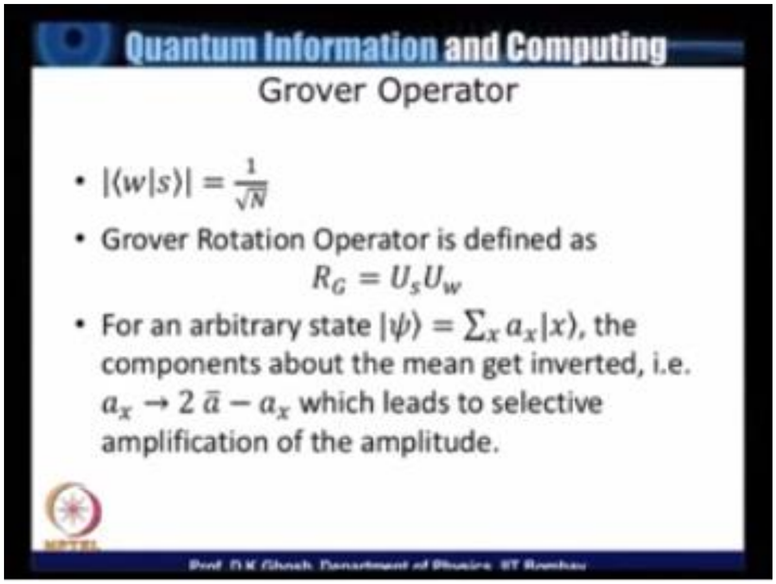

Phase which is $(-1)^{fw(x)}$ is associated with the first register and the second register essentially unchanged, so the point is this, that if the phase happens to be equal to -1 and that happens to an we are at the marked state because that fw(x) = 1 then of course sign in the first register is skipped, we had defined two operators one Uw which is given by I – 2|w><w| and Us which is two times the operator for the standard state 2|s><s| - I.

And then we had said that the Grover rotation operator is given by Us times Uw and we had seen last time that when the Grover's operator acts on an arbitrary state ψ given by sum over x ax |x>, x is your computational bases, ax is the amplitude of ψ in various computational base state, then we found that the components about the mean that divided inverter that is ax goes to 2 times mean of a, a-bar (–ax) which leads to selective amplification of the amplitude.

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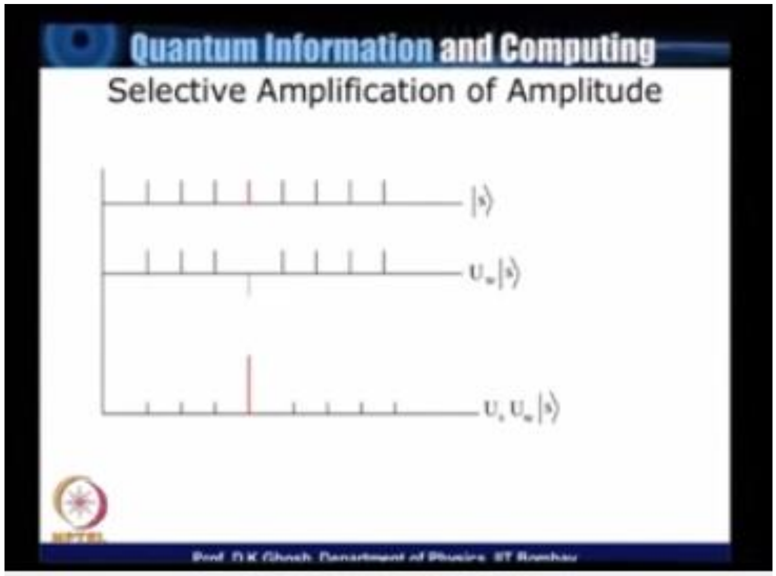

And we had worked it out for the case of a 3 qubit case and we had seen that if you meant two rotation Grover rotation than the marked state probability becomes 25 time the probability amplitude becomes 5 times that of the other computational bases.

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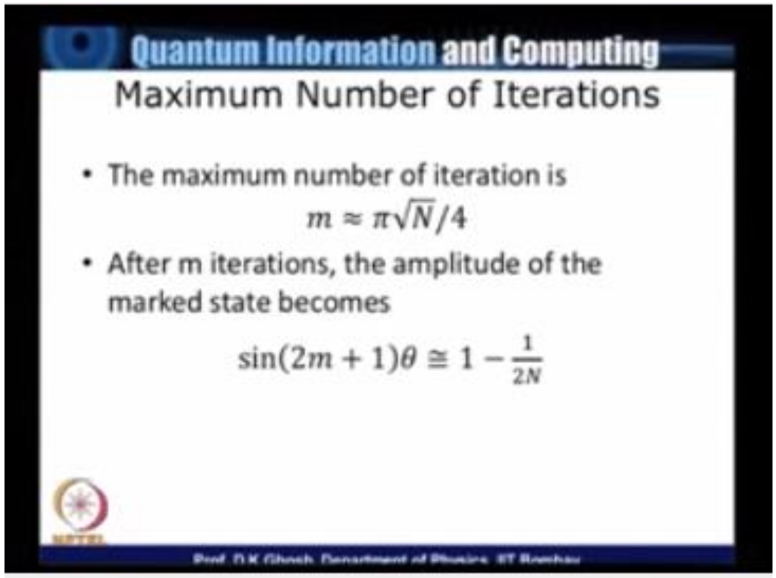

And this is the basic principle, behind locating a particular marked state, we had also shown that the maximum number of iteration is given by π times √N/4 and after N iterations we found that the amplitude of the marked state becomes Sin (2m + 1) times θ where θ is approximately equal to Sin θ for large N and is given by 1-1/2N.

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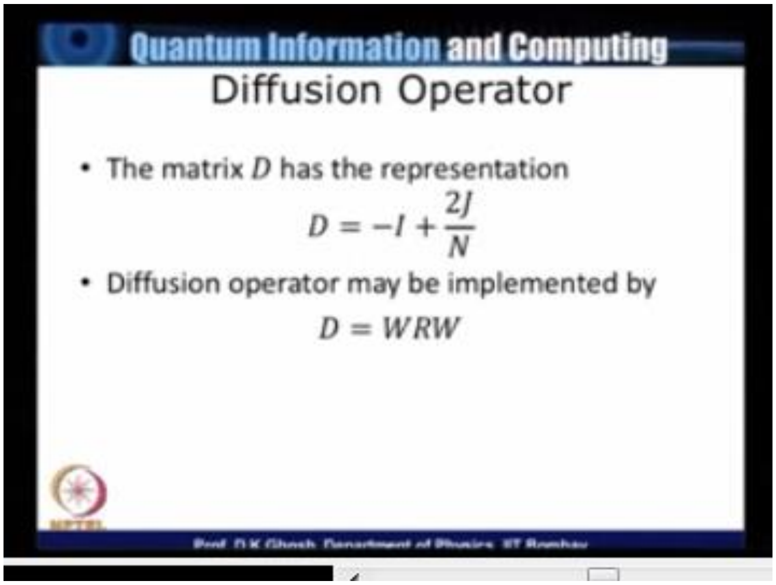

The last thing that we attempt its last time was to define a diffusion matrix and we said that diffusion matrix.

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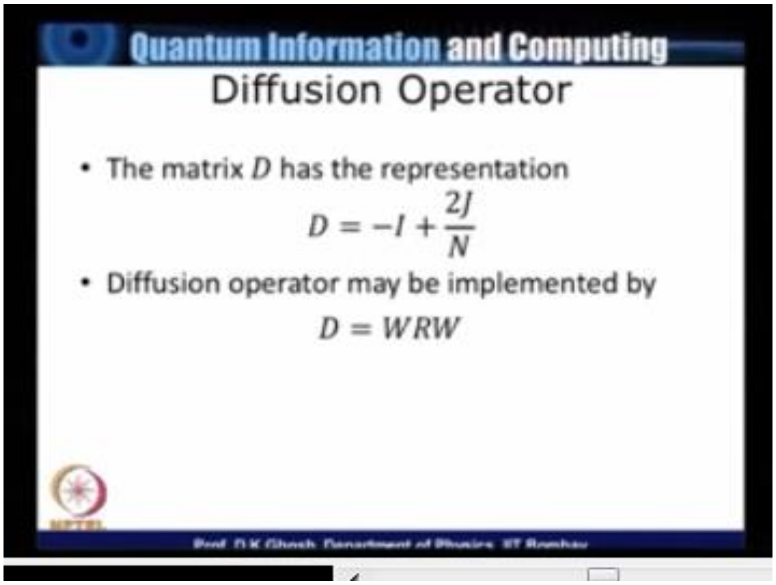

Is defined by –I + 2J/N and we made a statement though we did not prove that the reason for defining a diffusion matrix is that, that can be implemented by quantum circuit, consisting of WRW where W is the wall’s Hadamard transform and R is a selective rotation.

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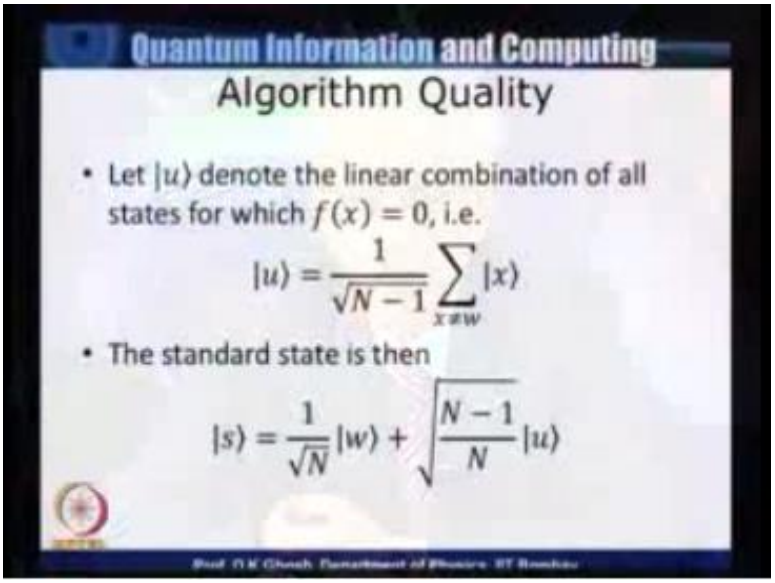

So we will now look at how this actually works and let us will attempt to complete the Grover rotation tool.

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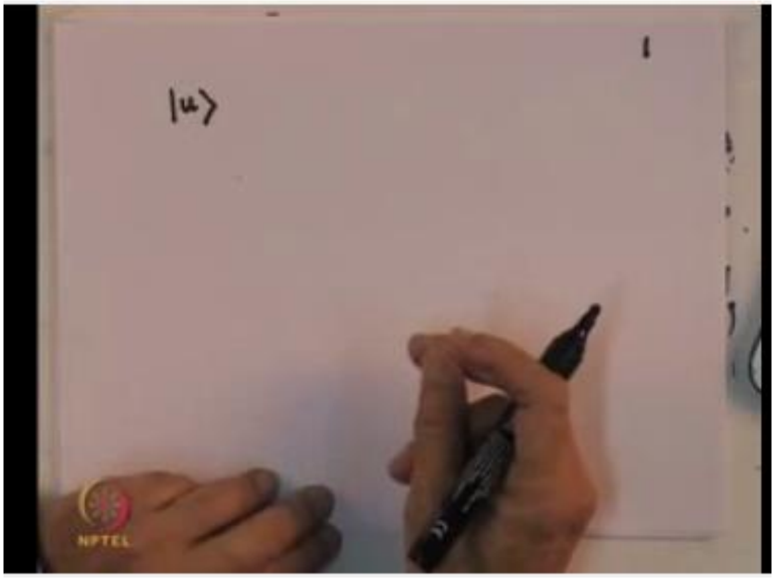

Suppose |U> is a linear combination of all the members of the basis other than the particularly marked state that is which we called as the W for which f?(x) evaluates to 0 rather than 1 which is what happens when x happens to be equal to w.

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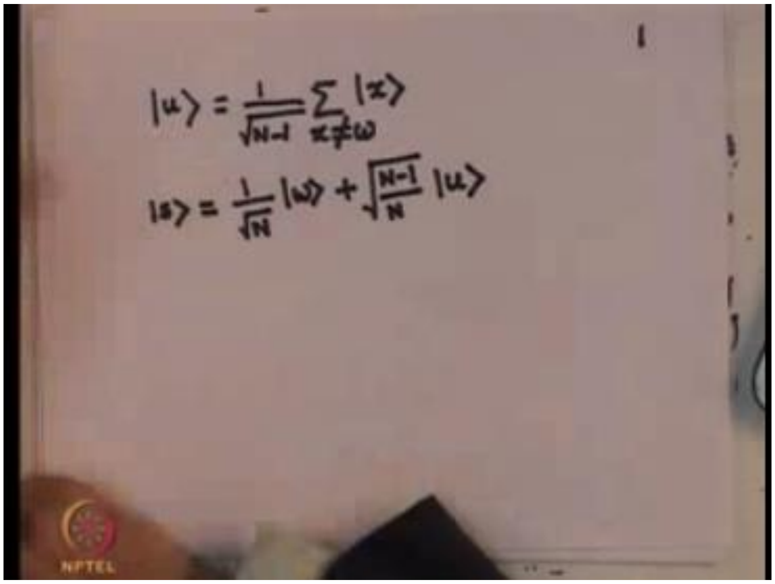

Then I define U because it has a N-1 number of items as 1/ √N-1 sum over x but then in this case is not equal to W of the computational basis x, so this is not quite the standard state but it defers from the standard state in the sense that one of the items there is missing, so as a result my standards state because its normalization is 1/ √N so this is simply given by 1/ √N |W> + √N-1/ N times |u>.

Remember we have two types of operations there one we called a the deep which acting on the states inverse the marked state, that is the marked state amplitude picks up the minus sign.

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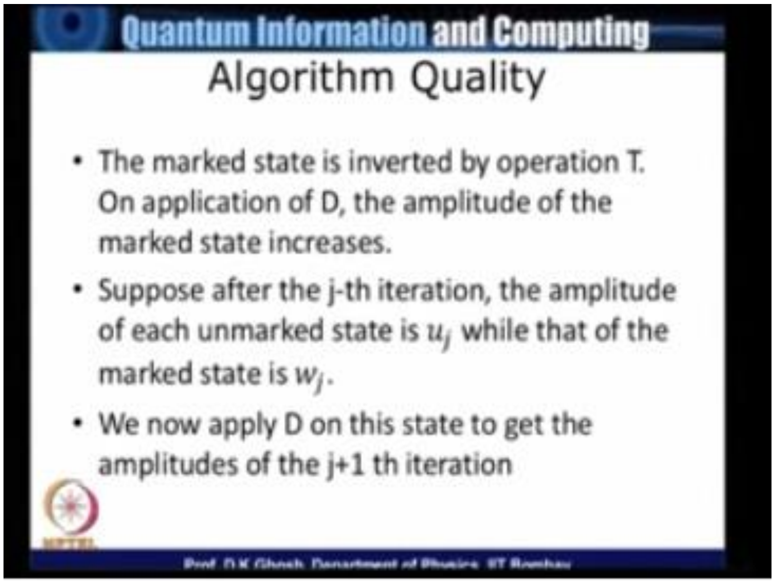

And after that we will apply the which has the effect of selectively increasing the amplitude of the marked state, so we said that suppose after J^{th} iteration.

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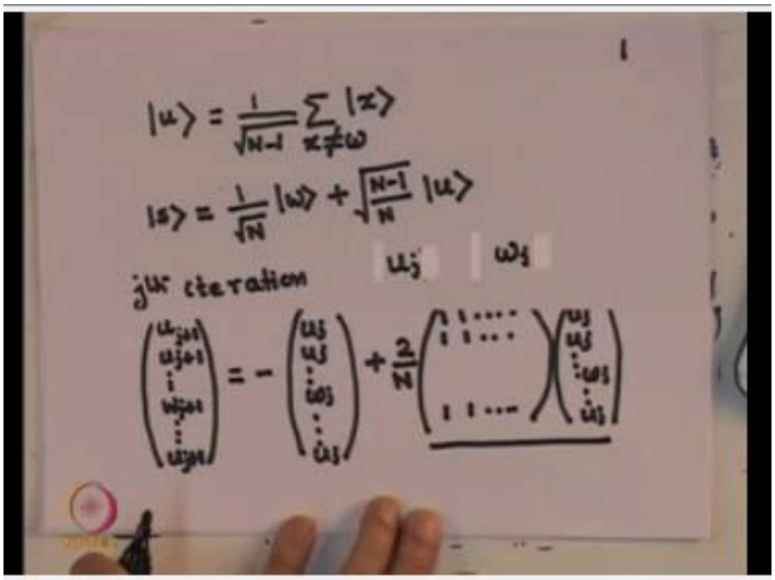

Suppose after J^{th} iterations the amplitude of the unmarked state is U_j and that of the marked state is w_j now we would be interested in knowing what happens when you apply D on such a state? So what is the result of the J + 1^{th} corporation? Why you will do it as this that remember that if I write this define each one of the unmarked state as u, so I will call u_{j+1}, u_{j+1} I do not distinguish between 1,2,3,4 for the simple reason that each of the unmarked state has the same amplitude. But I have a w_{j+1} which is what changes and then of course u_{j+1} also, so this is equal to. Now –I, so therefore $–u_j$, u_j extra up to W_j and then again everything is u_j+2/Nj if you recall my j was a matrix which has each element equal to 1.

And this then you multiples my u_js the, the j^{th} iteration values which is W_j extra. Now this is a fairly simple matrix multiplication to work out and the reason you see is this, because each element is 1, the sum of the column factor here, they all becomes n-1 times u_j and they are plus of course W_j, so then if I equate, if I equate the elements of a matrix because this will becomes a matrix equation, I essentially get two types of equations, the equations differ in the following way.

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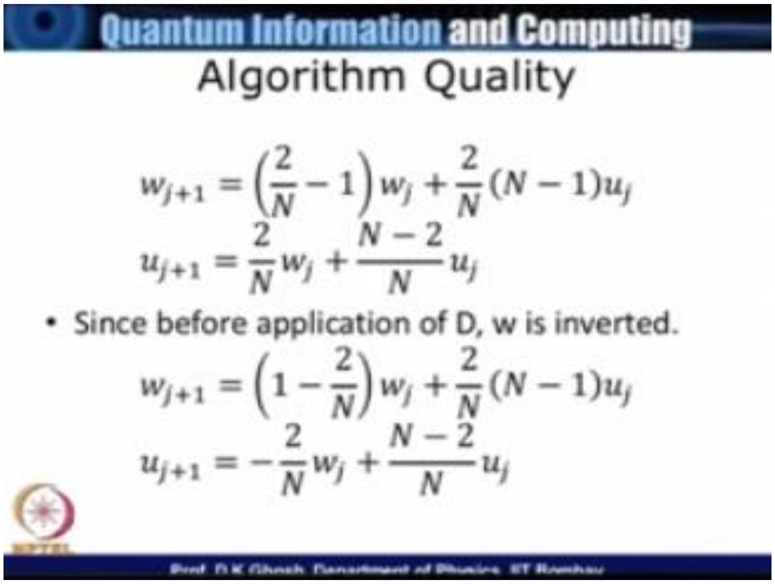

That is if you look at the slide the first equation is.

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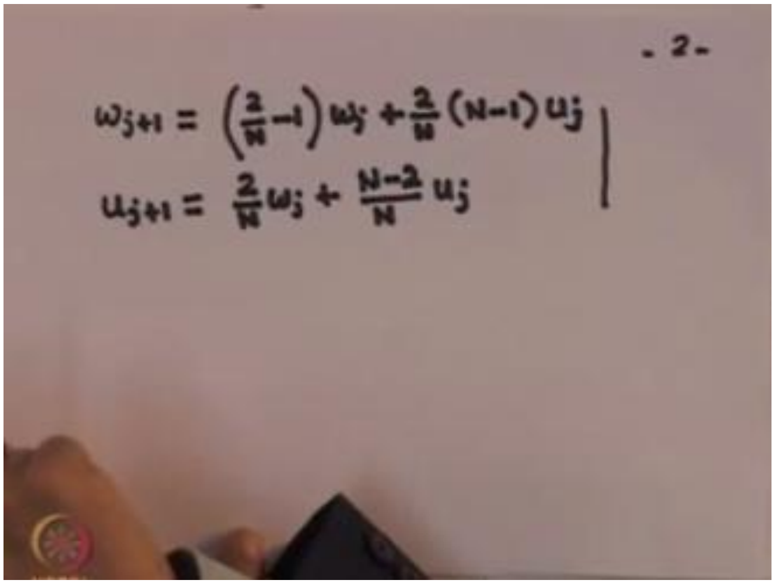

W_{j+1} is equal to, now you notice that I have got (2/N-1) acting on W_j+2/N(N-1) because there are N-1 of those u is u_j and u_{j+1} is 2/N W_j+N-2/N times u_j, so this is a simple way of just adding up the elements of a matrix and equating them. Now the only thing that we would like to comment here is this, there is a pair equation which you would like to solve, that you recall that before I calculated the j^{th} + 1^{th} I can, I had inverted the j^{th} value of W_j and that is because I had calculated W_j from j-1^{th} iteration and whatever value I got I had made it negative. So therefore, in order to relate to the previous value of W_j what we need to do is to invert the W_j in this equation so that I get

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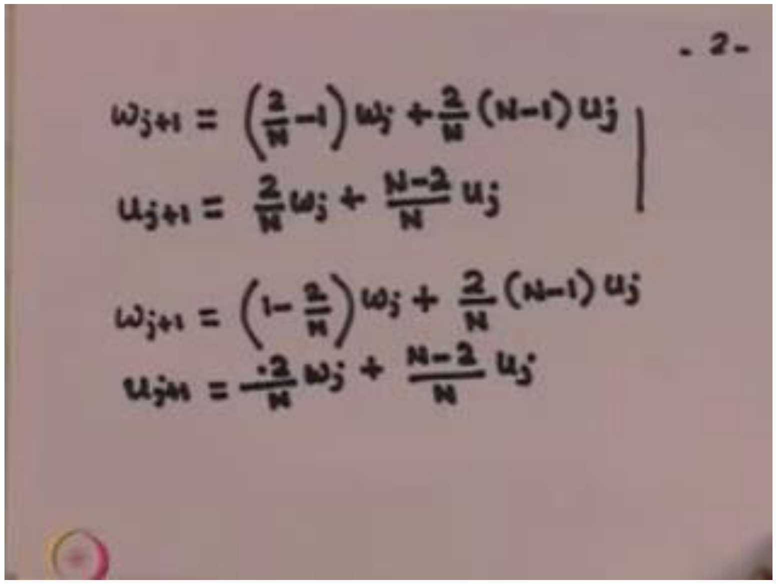

W_{j+1} =(1-2/N) W_j instead of 2/N-1 and of course the other term simply remains the way it was 2/N(N-1)u_j and u_{j+1} becomes equal to -2NW_j+N-2/N u_j, so this the pair of equation that I need to solve. In order to be back let me introduce simplification is actually fairly straightforward equation. So what I do is this instead of solving for u_j let me define.

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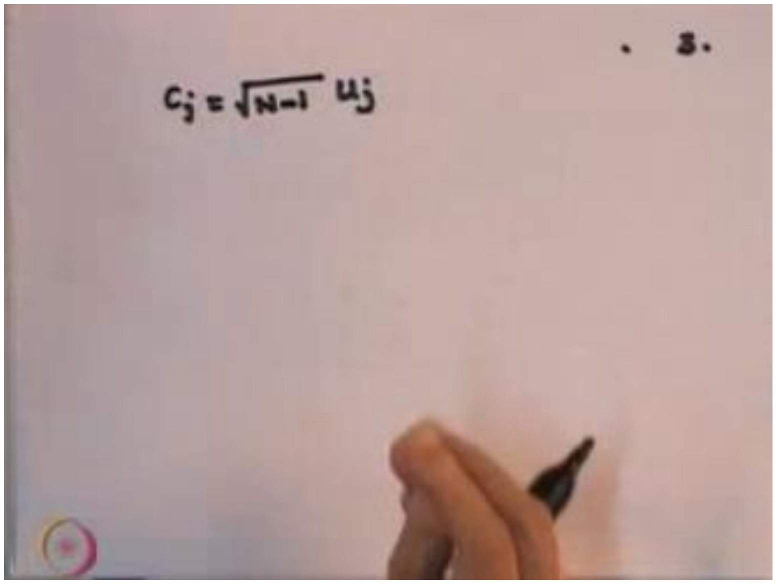

$c_j = \sqrt{N-1}\, u_j$

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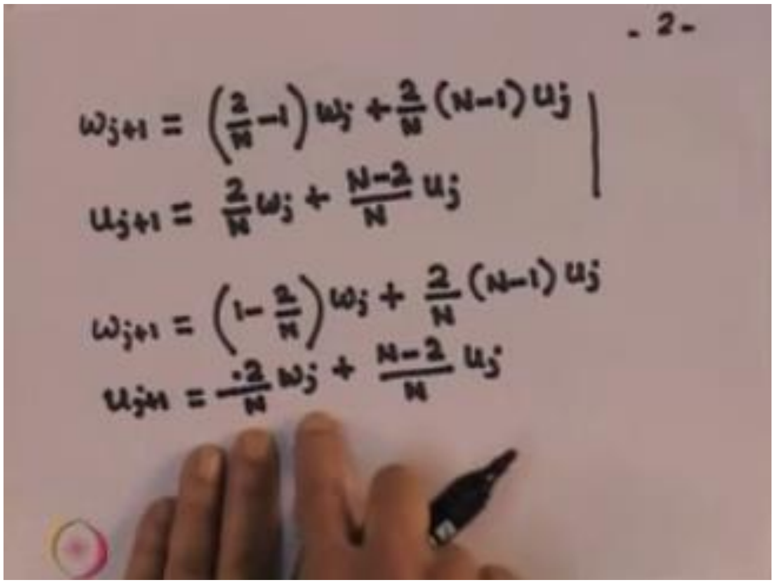

So basically if you look back at this equation I notice that this u_j will be written as $c_j/\sqrt{N-1}$ that will sort of make the numerator here $\sqrt{N-1}$ times c_j and likewise in this equation also I will simply multiply both sides with $\sqrt{N-1}$ and that will make this is an equation c_j and W_j.

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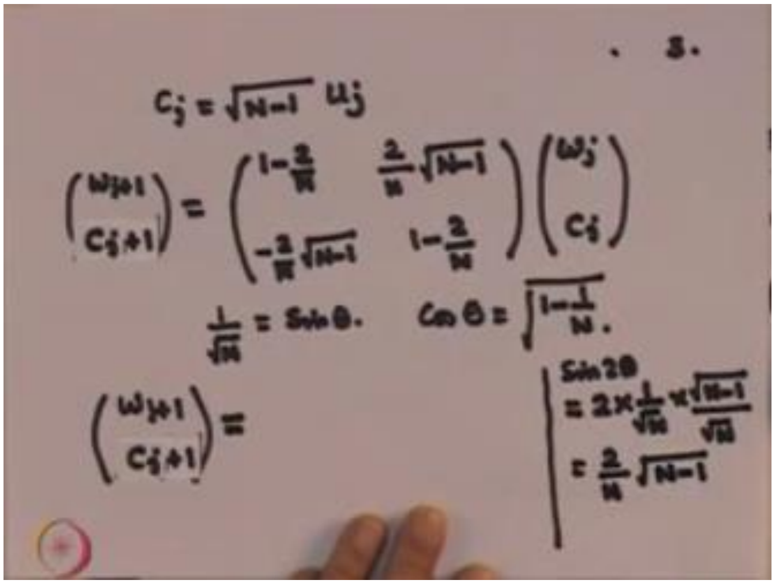

And the equations that I get then will be written as a 2/2 matrix equation which is W_{j+1} c_{j+1} equal to let me write it as a matrix 1-2/N and this is 2/N times √N-1 that is because I have defined in terms of c_j rather than u_j -2/N√N-1 and here 1-2/N this with W_j c_j. Now this is a matrix which I will be able to solve easily and let see how. Recall that we have said that 1/√N is sinΘ that gives me cosΘ=√1-1/N, so supposing I rewrite this equation in terms of Θ what I get is W_{j+1} c_{j+1} is equal to, now let us try to convert this, so if 1/√N is sinΘ and this is 1-√N what is this 1-2N you notice one thing I will just let me do the calculation on the side. Now what I get is sin2Θ, sin2Θ is 2 times sinΘ which is 1/√N, times cosΘ which can be written as √N-1/√N and that is equal to 2/N into √N-1.

Which you notice is this of diagonal and minus of this of them so therefore let us I so this is sin2θ this is – sin2θ now let us complete what is cos2θ the cos2θ and we know is $\cos^2\theta$ – sin2 so 1- 1/N – another 1/ N so = 1- 2/N so therefore we will diagonal matrix element happens to the cos2θ, cos2θ and of course we have the column matrix w_j and c now this should not have come has this apply to you the reason one what we are seeing here is that on application of Grover rotation they angle by h they it terms is twice the angle so this is exactly what happen that. W_j + 1 and w_j are related like this okay now in that case let us look at what is happening if you.

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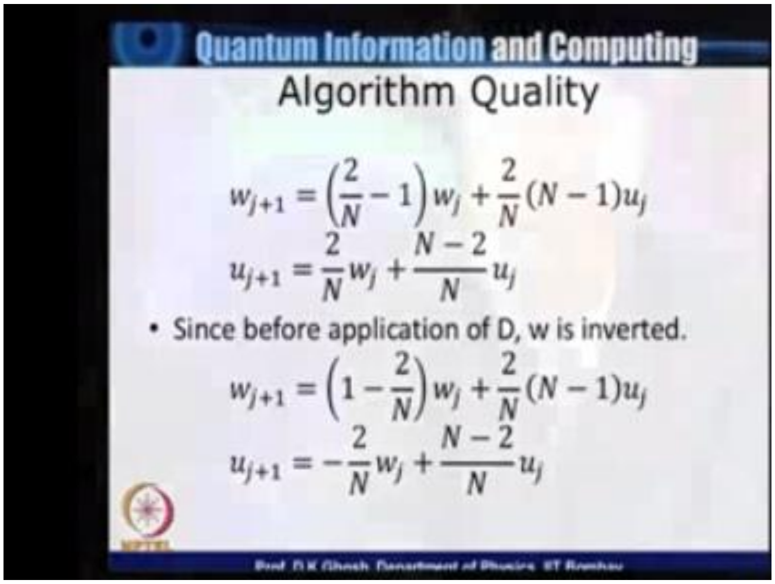

Applied this starting from the original value let say one so I would then get w so we have already worked out that w_j + 1.

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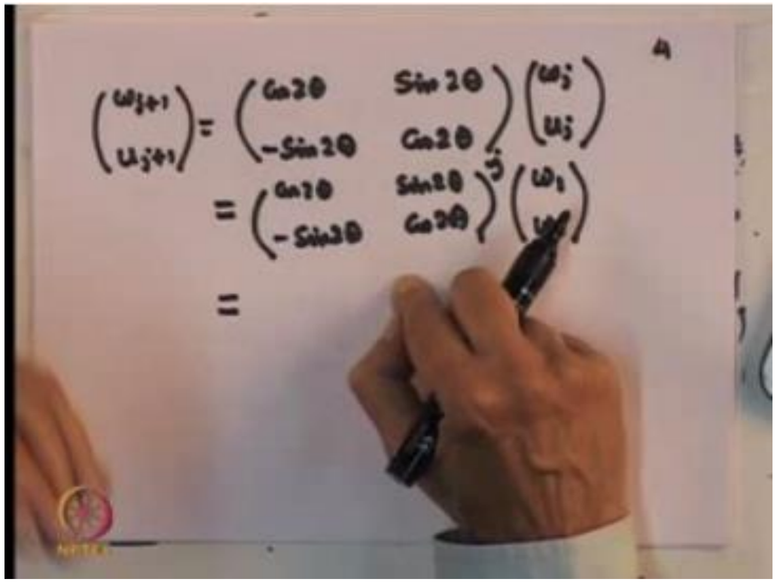

Uj + 1 is = cos2θ sin2θ is just rotation matrix sin2θ cos2θ w_j u_j now obviously I can go down in the change and so therefore what I would get is this matrix(cos2θ sin2θ – sin2θ cosθ cos2θ)^j acting on w_1 and v_1 now it is since these are rotation matrix I know that if I take a square that equivalent to increasing this angle from θ2 to for this so every kind you multiply in it the rotation matrix the rotation matrix the factor the rotation matrix is to increase the rotation in this can we check.

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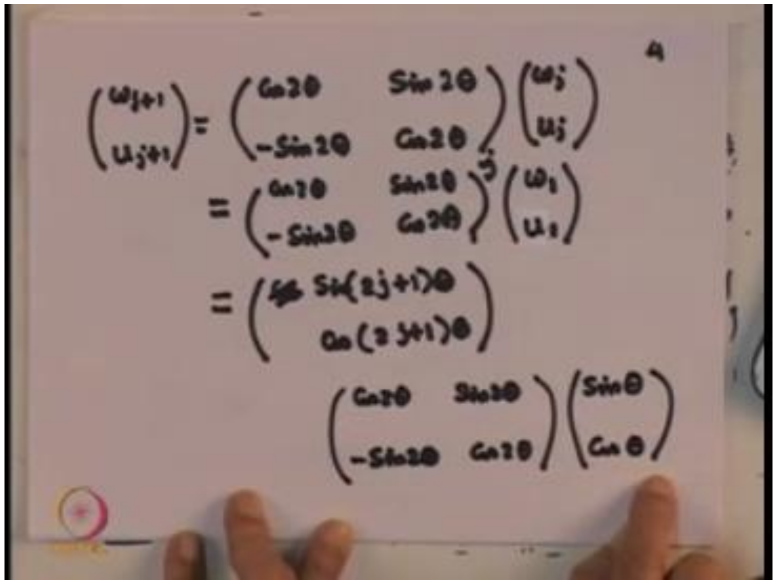

By just multiplying to consequent values of the notes so therefore this one when we raise to the power j what we you find is remember that I have had to multiply this with that and all that so I will get sin2_j + 1θ cos2_j + 1θ we have started with that remember why that +1 comes if because our starting point was sinθ cosθ in fact it is can be straight forward that suppose in would be one rotation we have cos2θ sin2θ – sin2θ cos2θ this is was sinθ this is cosθ so therefore cos2θ sinθ + sin2θ cosθ is sin3θ and sin2θ – sin2θ – x sinθ cos2θ x cosθ so that is cosθ yes trigonometric.

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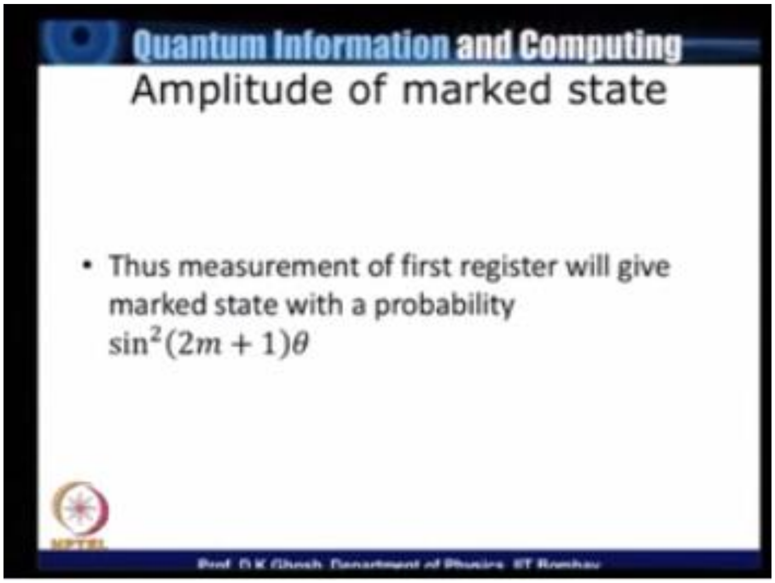

So what we said is at this stage if I measure the first register I will get the marked state if the probability implicate if sin of 2N + 1θ which means the probability is given by $\sin^2$ of 2N + 1 now it is important that you realize that since we are oscillating function so if start with the small value of θ I can gradually increase the sin2 function tell me the sin2 function has approached one or just about cross one after that your increasing the number iteration will not help between that make the condition works.

And that is what we pointed out several times that we will must have an a priori idea of how many times the Grover iteration should work and we have pointed out it is given by a number which is approximately 5/4 x √n.

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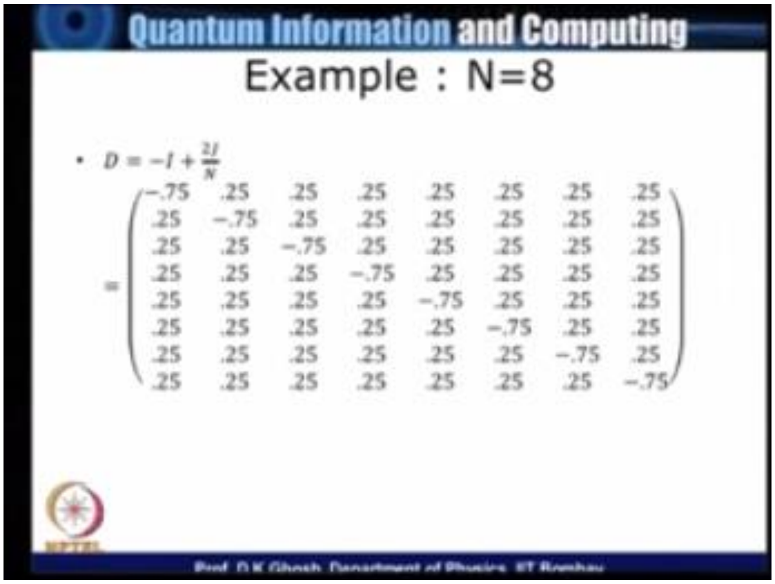

Now let us tick it then take the example of energy code and we get how the matrices look like they structure on the matrix if you remember my d was.

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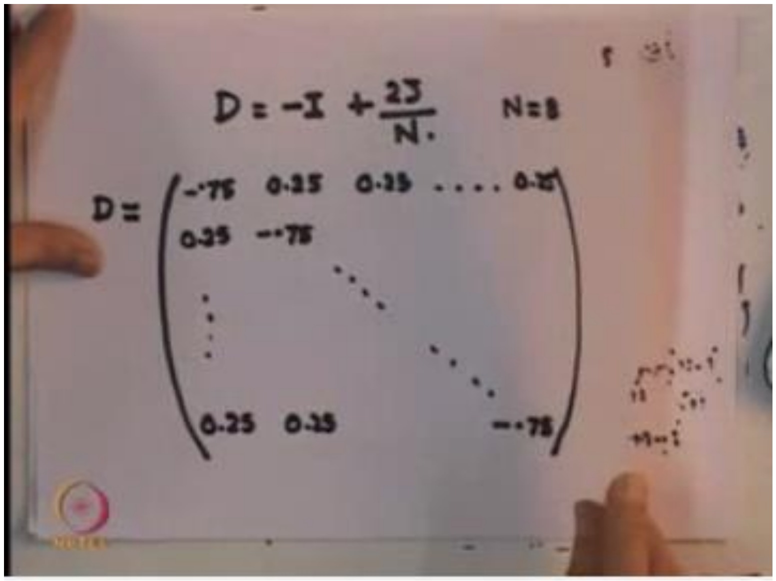

-i + 2j/N I am taking N = 8 so this is going to then 8/8 and j is very simple because j is value is one also so if I applied that and getting the whole matrix on a slide that suppose I am write down attempt to write down part of it so – I has – 1 has each on the diagonal to that I must have 2/8 which is ¼ and this identity make it is as 0 everywhere so therefore this matrix has -.75 - .75 etc. along the diagonal the full matrix is given in this the of the parallel elements because this has a half diagonal element 0 so this is +0.25, 0.25 etc right up to the eighth element 0.25.

So notice it is a fairly straight forward matrix and enumerating it if you are writing a classical computer program is standard quantum. So this is the way the matrix the look like. Now let us then look at the other matrix T.

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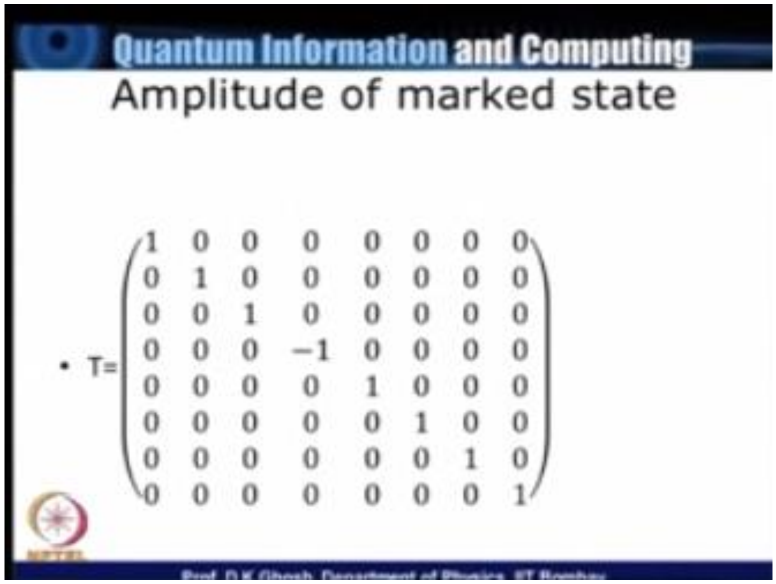

Remember what T does the this is the matrix which, what have done here is to take the matrix T also as an 8/8 matrix that what it does is to act on the bases states, now remember our bases states our bases states are 1 0 0 0 0 1 0 0 etc., so therefore if it act on a basis state T it should invert or flit the sin of the marked state which in this example I have taken to be the for the state. And the only way I can now remember that my computer short bases for the system is something like this 10000 etc eight of them.

And 0100 eight of them, but my fourth one is 0001 this is important not very much different but I am just pointing this out and the rest of them are there, but this T matrix acting on this state would convert this to -1, but acting of all other will keep the base is the same. So as a result my T matrix structure is it is the diagonal matrix which has elements 100 etc, 0100 the slides shows the full thing.

Then the fourth element here is this and rest of them again 000010 etc, the full matrix is shown here on the slide.

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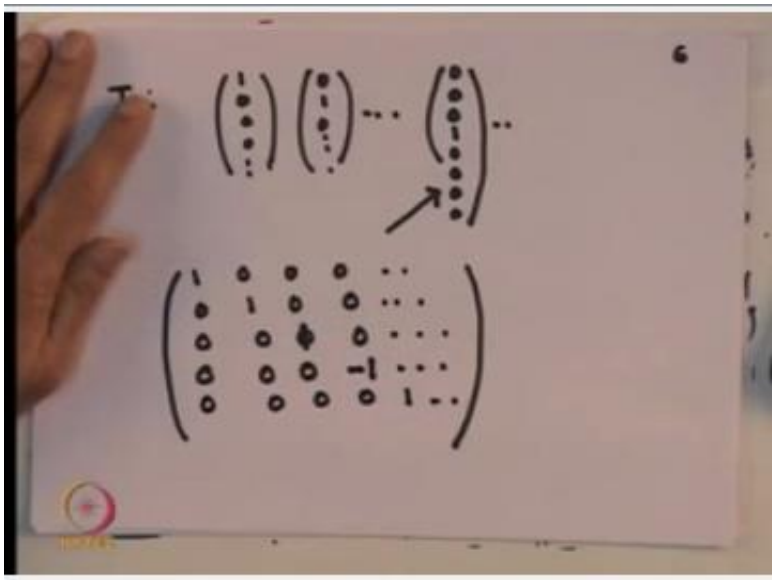

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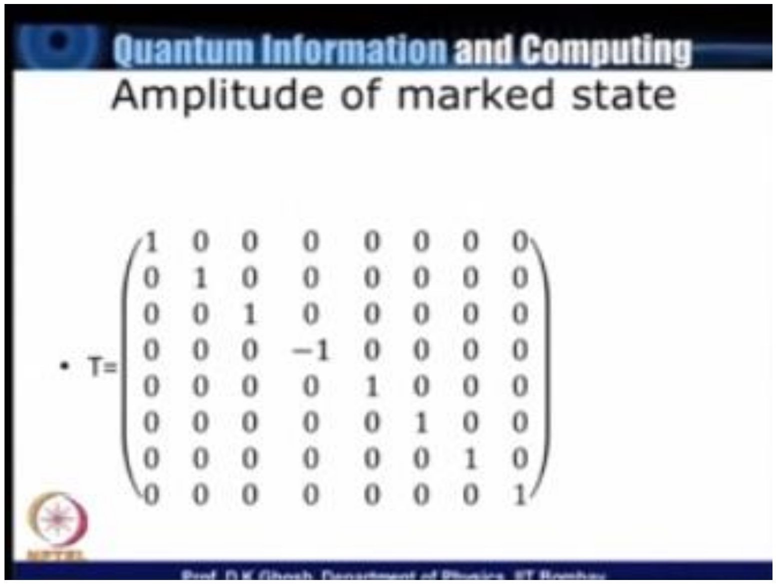

So if such a matrix acts.

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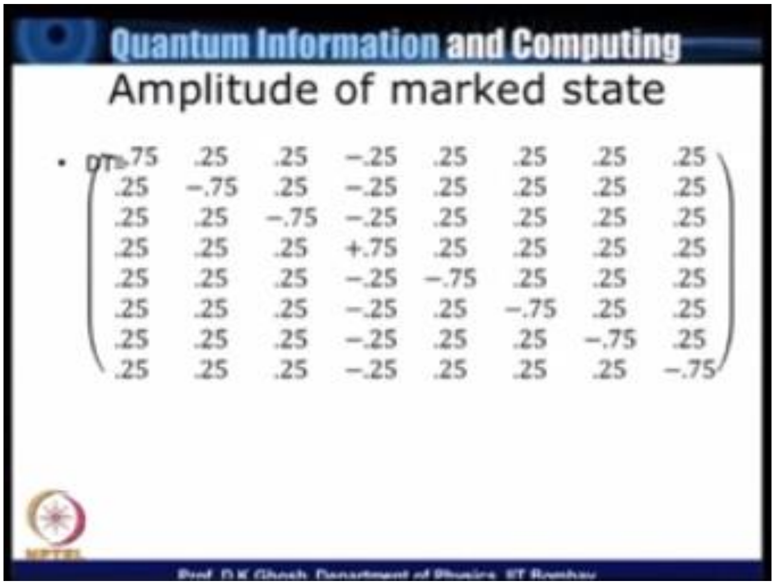

So what is the amplitude of my marked state, what is this matrix DT. Now that is the way it acts, now if you look at DT matrix then the matrix is shown in the slide that look at what it will happen my D had -0.75, -0.75 all over. So therefore DT matrix will essentially remain the same -0.75 I had +.25 +.25 etc, etc. But when you come to the fourth row, when you come to the fourth row what will happen is will find 0.25, 0.25, 0.25 then +0.75 and simply multiplied that because there is just one element to be interfere okay.

Actually this column will have the fourth column will have a total change this will become -0.25, --.25, look at the full matrix shown in this. This is simply multiplying one matrix with the other look at the slide.

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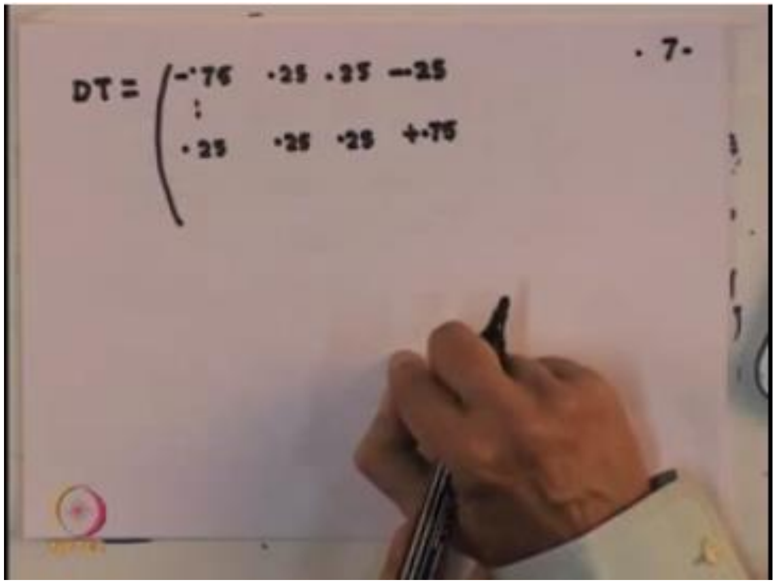

Which will give you the full matrix.

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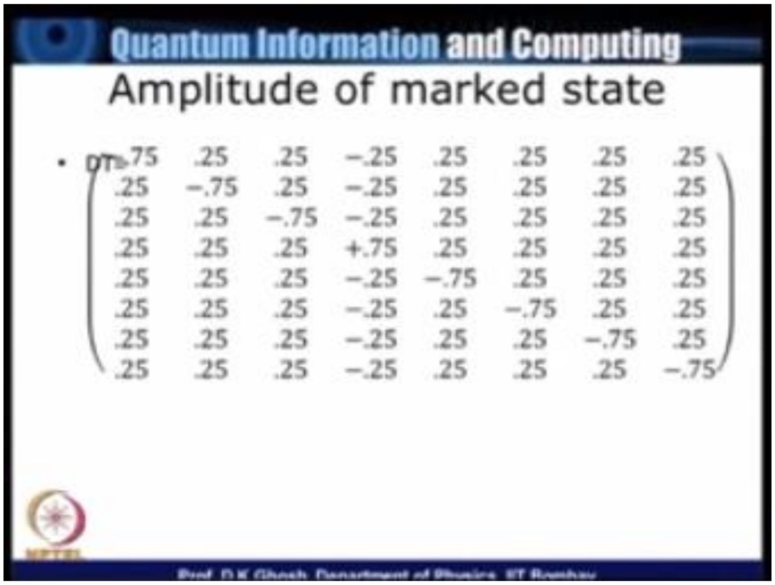

Then you note this here that my, what is happened is this column has changed, in this column the every element has been end of a -0.25 with respective D. So therefore, what happens.

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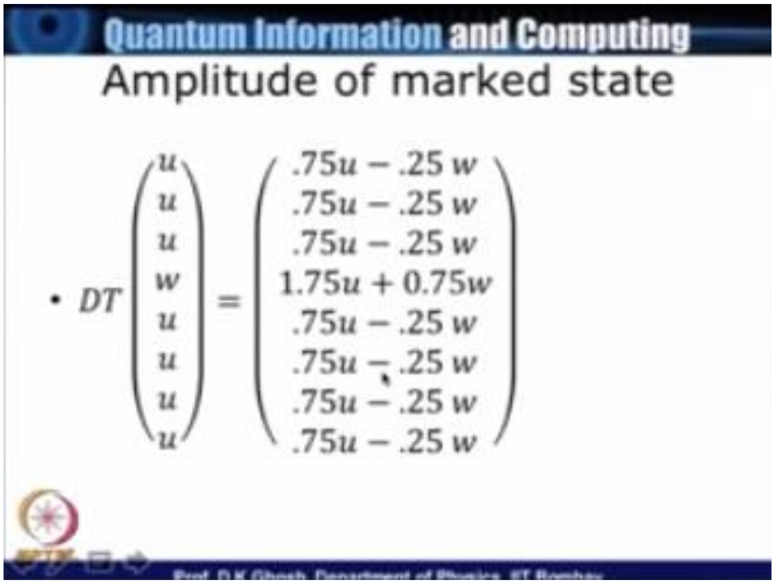

If my DT is applied on the matrix a column vector which all elements are U other than the marked element of this. Now it is straightforward I have given you the matrix DT add it up, multiply it with that matrix and what you get here is a matrix which is given by this. This matrix I will just try to complete this work here.

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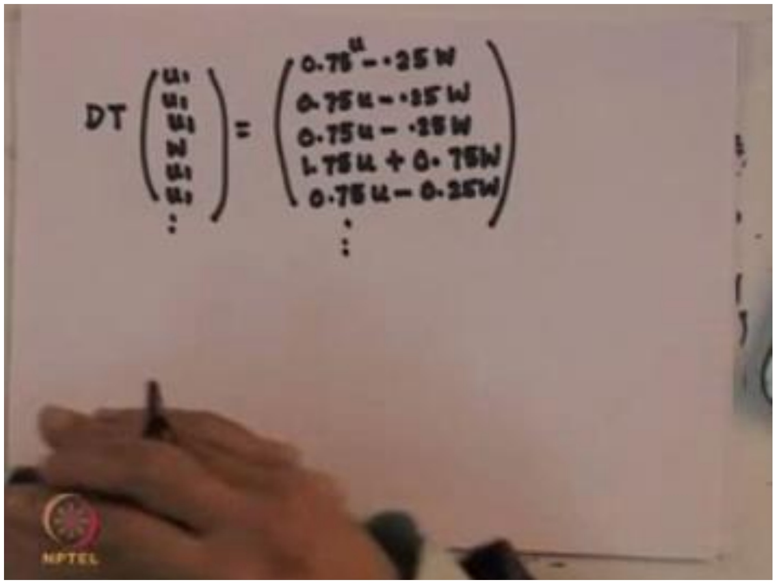

So I have got u_1, u_1 all of them are the same let me write that down w and then u_1, u_1 etc, that is equal to if you multiply you get this 0.75 - 0.25, 0.75 u -0.25W this is what you will get for all of them -0.25W.

But this element forth one which is different will give you a total different situation and again I have got 0.75 u, -0.25W etc, etc. Once again this is a matrix equation though I have written eight of them but there are only a pair which is different and this pair is simply given by this equation this on the slide.

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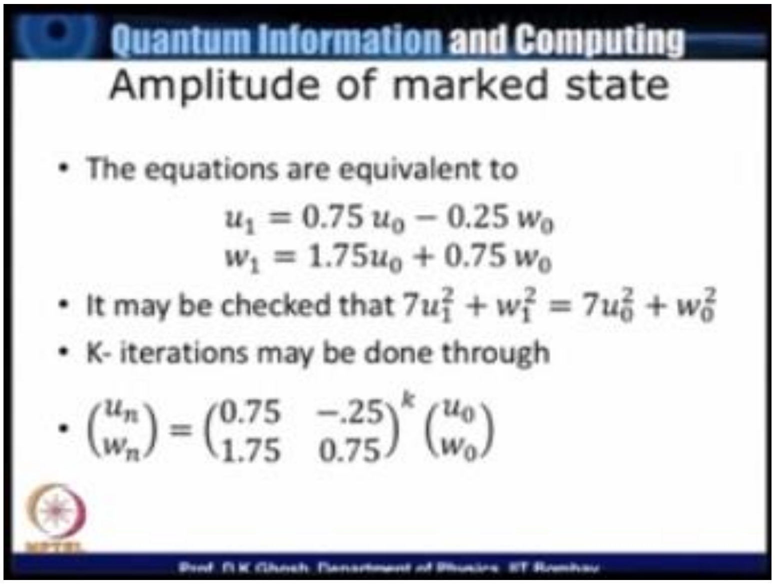

So look at that, that is why you want each one of those gives me this and this is my w_1 we could check that total norm is preserved I will doing unitary operation for several times u_1^2, because I have seven of the unmark state and w_1^2 is 7 $u0^2$+ w_0^2 and I can perform k number of iterations by simple taking power k at power of this matrix starting from in initial state.

So what we have done so far is to tell you about Grover rotations, Grovers algorithm for finding out a particularly marked state in a nonstop change database we had given it is geometrical interpretation, we had also given some simple way of understanding it and then we have seen how this can be implemented by having unitary matrices.

And in this case the diffusion and the operator which inwards the initial state. This leads to selective amplification of a marked state and this allows us to identify the marked state. I will not be proving it, but we will leave it to you as an exercise to show that if in my database I had more than one marked state but for all of them a f(x) = 1. Supposing I have half a dozen of marked state having the same character state in a very big database then the same principle can be used to identify that.

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