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**NATIONAL PROGRAMME ON
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Quantum Information and Computing

**Prof. D.K. Ghosh
Department of Physical IIT Bombay**

Modul No.04

Lecture No.21

Grover's Search Algorithm-III

In the last lecture we had given as interpretation for the Grover rotation and we had seen that the Grover rotation operator.

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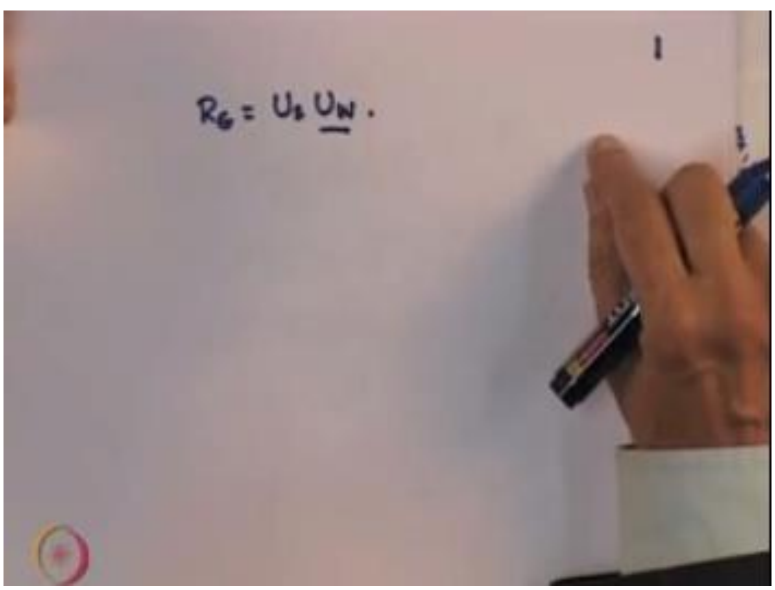

R_G is U_s times U_w, we have seen that what U_w does which to it in words they particular mark says and after that what U_s does is to inverted about it mean and this become an algorithm which one can apply several times to selectively amplify the amplitude of the mark state. I have taken the specific states of the 8 and there is a calculation which is.

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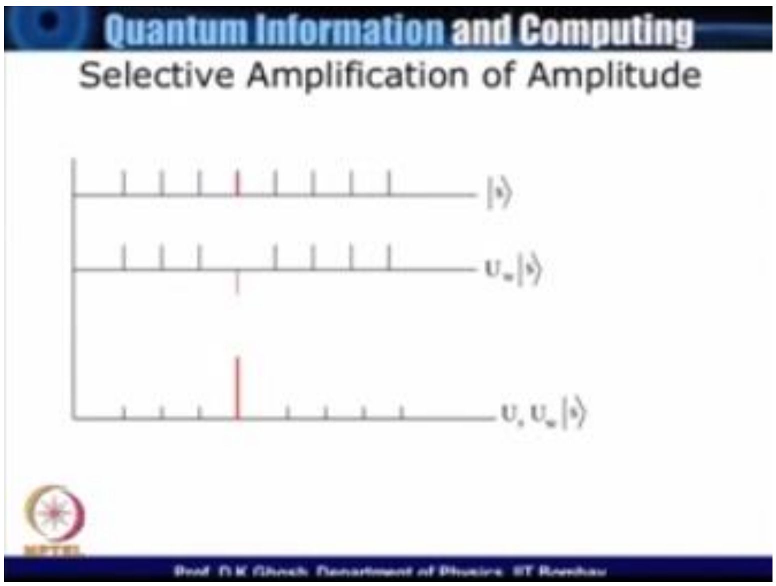

Review to calculate and let see that after one Grover iteration the amplitude becomes five times the amplitude of the inter given that the end of state.

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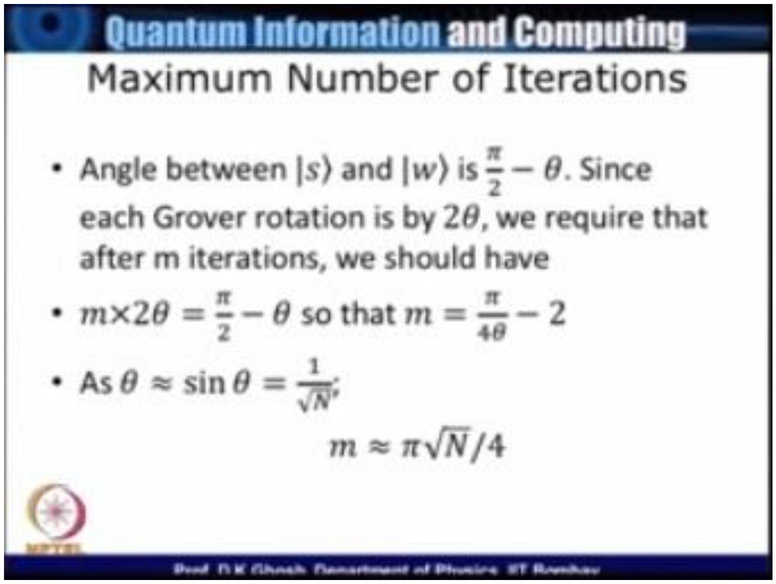

The other point that we pointed out is that the number of maximum number of iteration.

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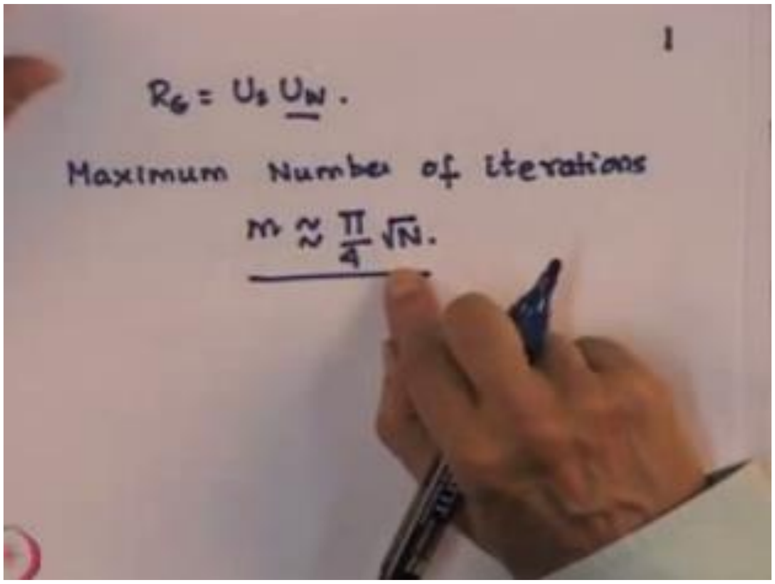

M is this approximately π/4 times √N showing that we restrict to the classical algorithm this is to detect probability. What I will do today this write to find out what is the method of we have given it is there in this spirit of being just but how do I have to do and we will see that it is possible to implements the elements that you have learned.

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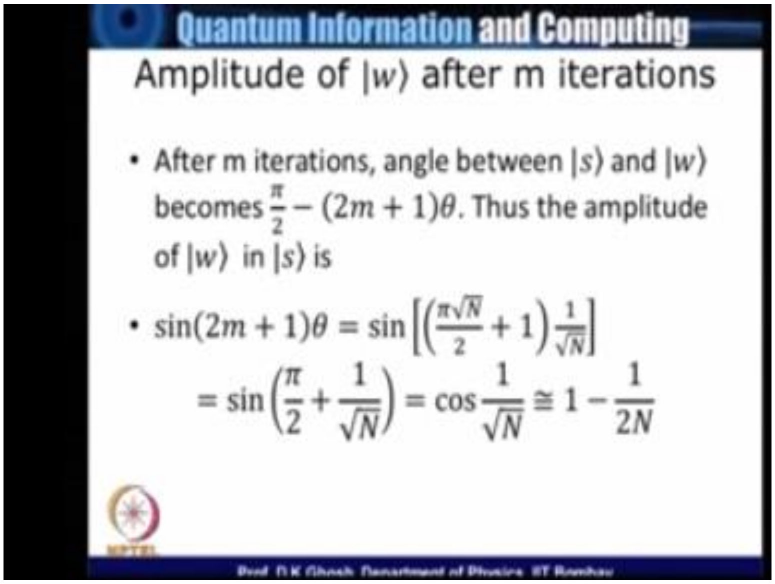

So let us support I have made m iteration if we have made any m iterations at the number by original angle what л99 is θ and after every iterations I am increasing the angle between the standard step and the marked state is decreasing by 2 1 so either that.

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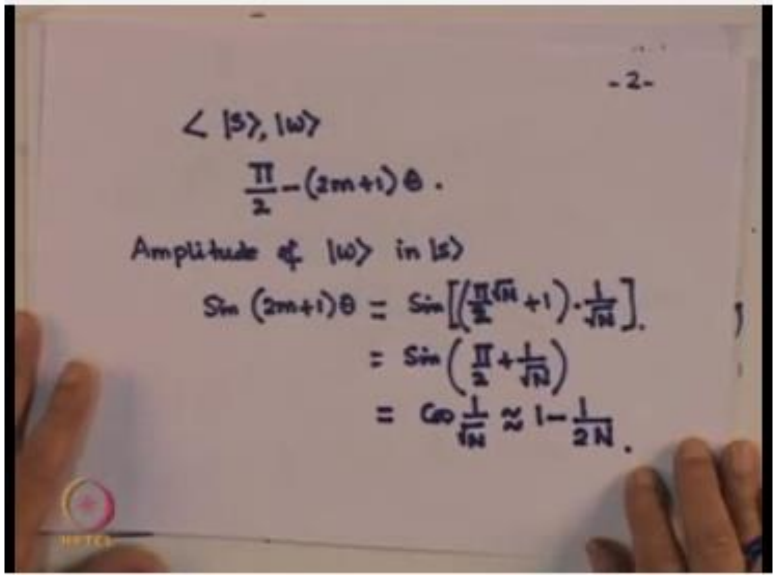

The angle between s and w after m number if iterations will become л/2 – (m + 1) so in this is the angle between s and w the amplitude of s and w the mm amplitude of w and s yes sing out this quantity (2m + 1) θ that and it will be back and I can put into that be m that I just now calculated that is sin[л/4 √n] x by 2, 2√N + 1 times θ which I go to θ so that is equal to sin (π/2+1/√N) which is cos (1/√N) and for large N this is given by 1-1/2N, this is yeah and you can I had given you a graph of an amplitude the relationship be evaluate equation for 4096 number of items. But if you have compute then you can easily do for others you will find that this is your actual value. Now let us look at the matrix implementation of these.

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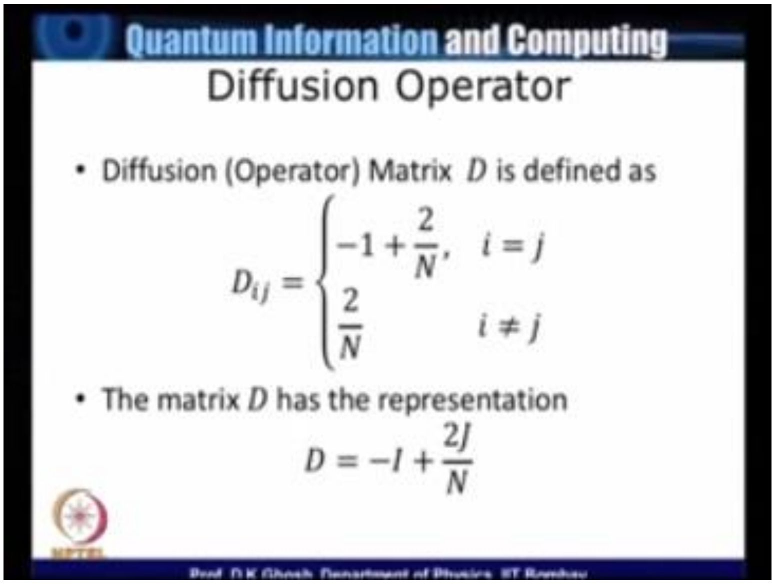

Now in order to do that, what I do is the following. I will define something called a diffusion.

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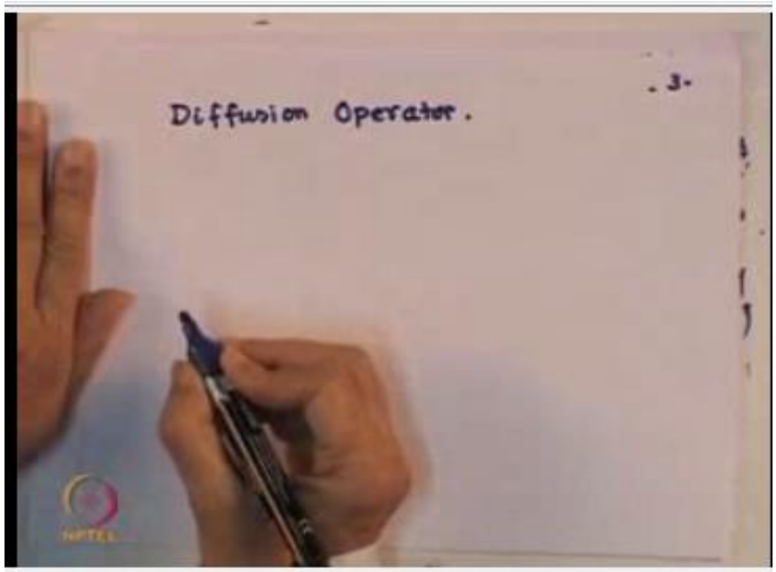

I will define something called diffusion. The diffusion operator or a matrix I will be talking about the matrix representation.

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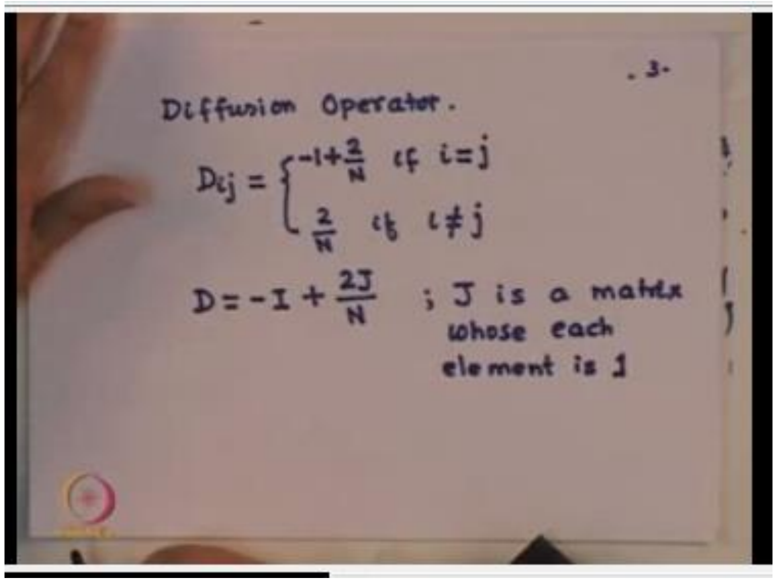

Is a matrix Mi matrix is ij^{th} element is given by -1+2/n if i = j that is the diagonal elements of that and is just equal to 2/m if i not equal to j, that turns out which once that usually say that D has the following matrix reference, D = -I + 2J/ N where J is a matrix n x n matrix whose each element is 1. Now let us see how this happens.

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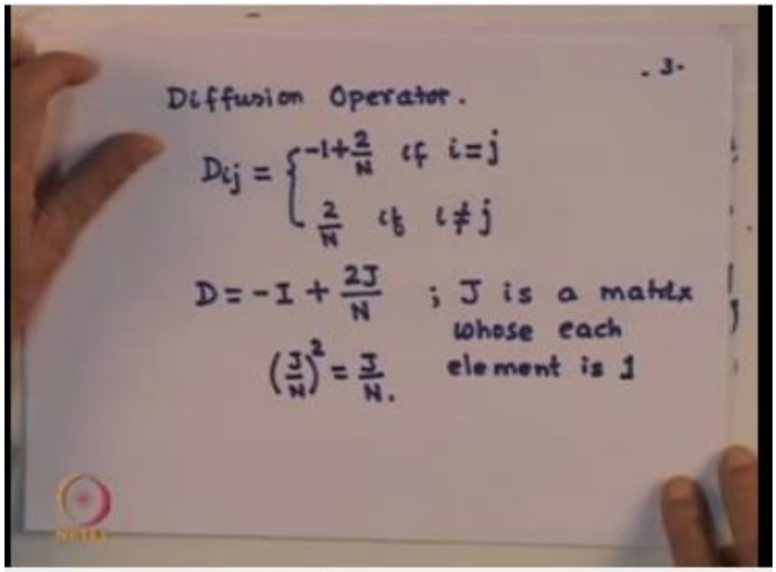

It fairly straight forward for you to prove that if j is matrix with each element is 1 then J/N happens to be a projection i or projection with that is $(J/N)^2$ is equal to 0, we just multiplying two matrix of identical matrix is whose each element a happens to be the same. Now let us look at why this D is obviously.

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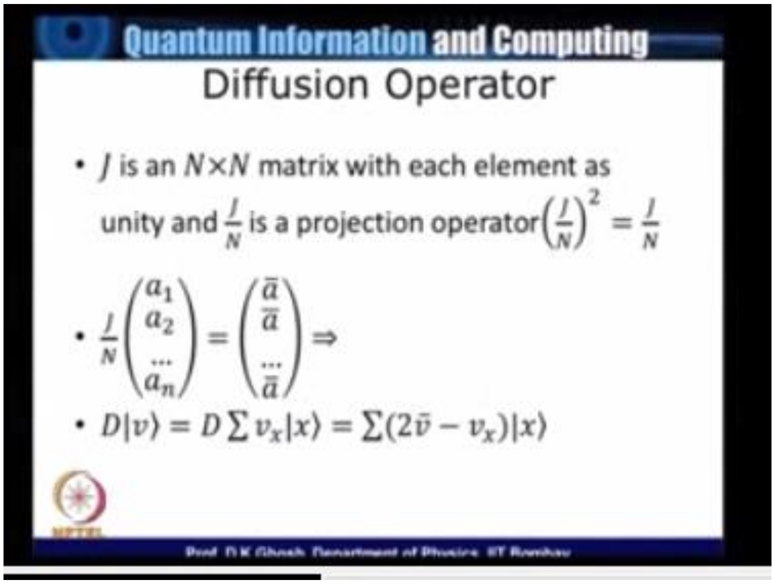

So first thing that you realize is the following.

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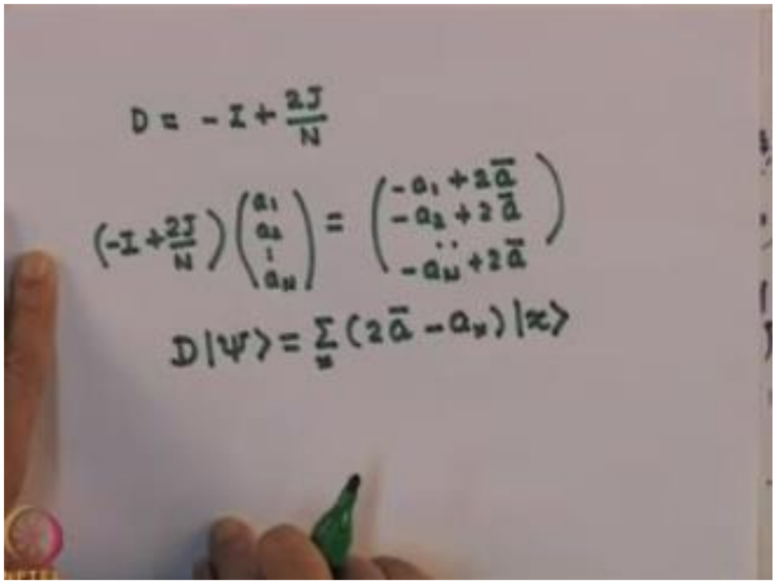

Notice my D is – I + 2J/N so when it acts on the state a1, a2 upto an what will I get, first thing to observe is, -I so I want to find out what is this, so is an identity so therefore I get a1+a2…an with a minus sign and this J if you recall is a matrix whose every element is 1, so what is this gives me is –a1+2 times the average.

Because 2J/N acting on this –a2 + 2 times the average etc, etc the right upto –an + 2 times the average. So what it means is, that this diffusion matrix acting on ψ is giving me sum over x 2 times a-bar –ax acting on x. So therefore the action of Us I can implement by a matrix and we have seen on data will be talking about later that once I have your given a matrix I can always find a circuit or a quantum circuit if you can give.

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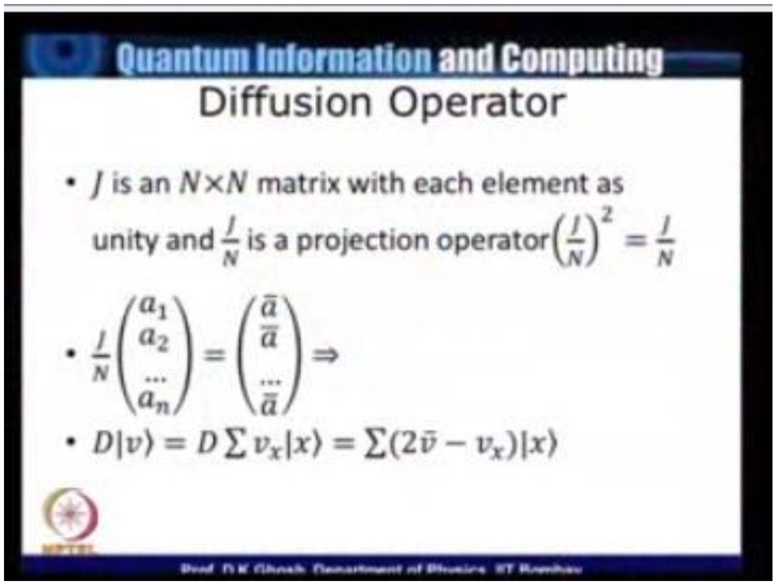

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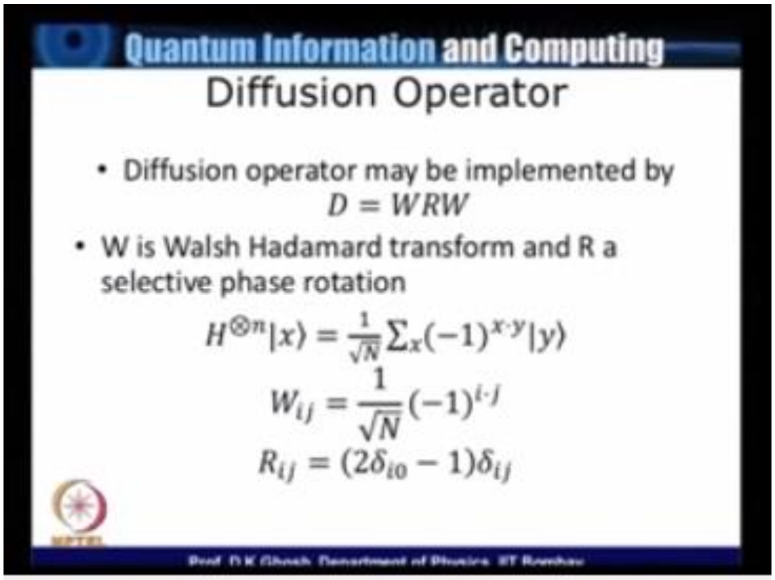

This is the statement I have made it, I will not be proving it to my lecture because it requires a bit of a argument but what I will do is, the algebra is fairly straight forward it will learn from this but what I will do is, to include this algebra details in the lecture notes that you will attempt on my lecture. So I came which as you can see from my notes.

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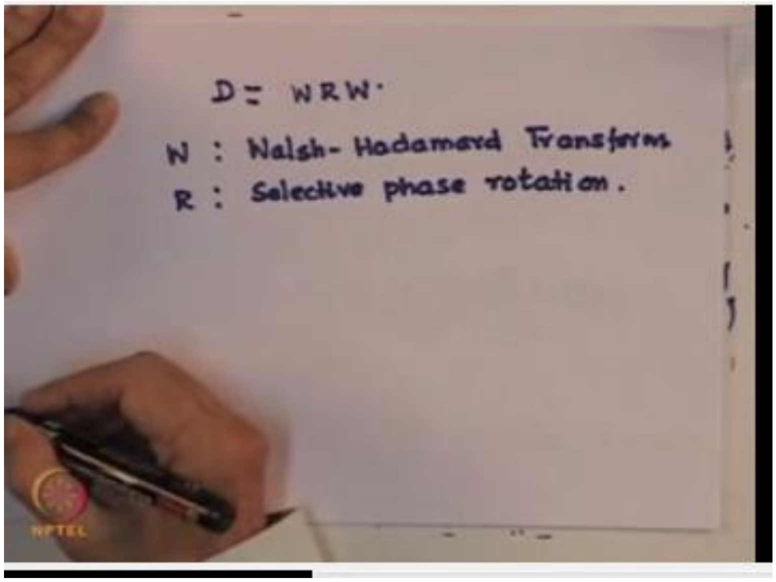

That the matrix D can be implemented by the following quantum classic, for this w is Walsh-Hadamard Transform and R is the Selective Phase Rotation, once Hadamard transform is simply the extension of the Hadamard operation on a single qubit to the case of many qubits.

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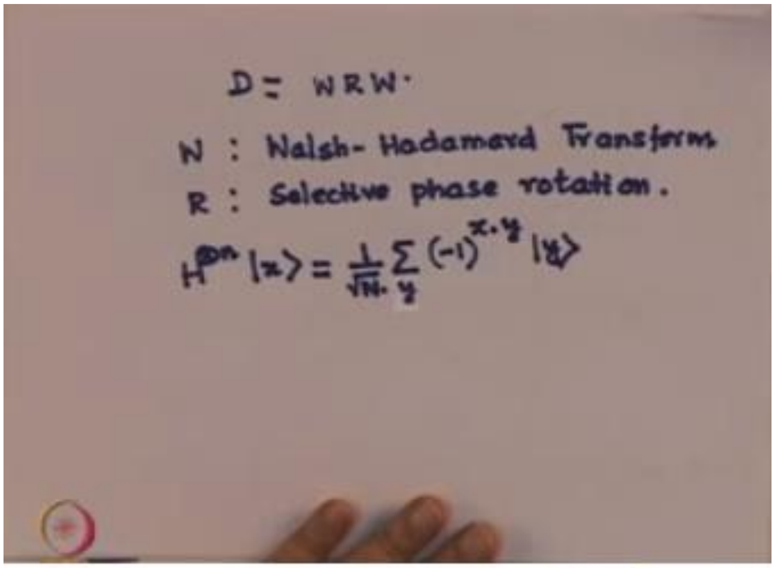

Recall that Hadamard operator acting on N qubits state |x> we giving $1/\sqrt{N}\sum y(-1)^{x.y}$ bitwise product x.y|y>, now this is simply a trivial extension for the fact that on a single qubit the Hadamard transform gives me $1/\sqrt{2}$ $0+(-1)^{x1}$ when it acts on it then will be acting on 0 it gives you $0+1/\sqrt{2}$ acting on 1 it gives you $0-1\sqrt{2}$.

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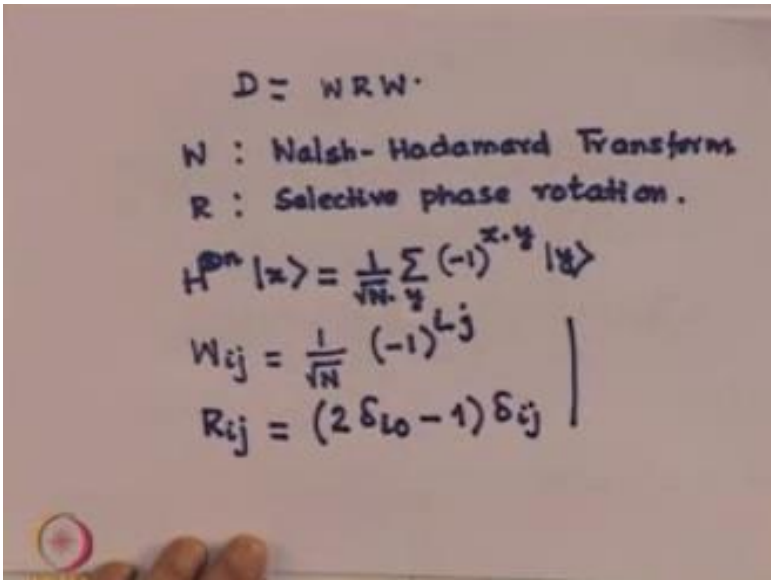

So what I call as the Hadamard transform have matrix element $W_{ij}=1/\sqrt{N}$ this is what implements this times $(-1)^{i.j}$ R_{ij} is a phase, selective phase rotation as I told you its structure is given by $(2\delta_{i0}-1)\delta_{ij}$ as I told you that I am not going to spend a lot of time in proving this but for those who are interested you can look up the comparing notes. Okay, once we have made this statement that it is possible to implement the diffusion and or the Grover algorithm to the quantum process is that we met.

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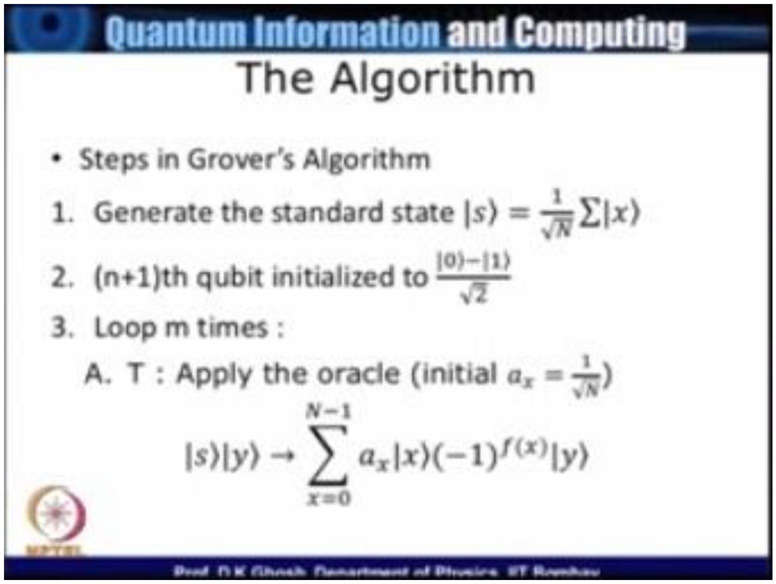

Let us talk about the algorithm which are, so first what we do is we generate a standard state, if you recall the standard state is simply uniform in your combination of the computational basis which you have thinked by passing N qubit null vector, so you are seeing with the Hadamard this we are talking as a linear. The $(n+1)^{th}$ qubit and I am selecting with which my target that is initialized to |0)-|1)/√2 and once again you were aware I can start with that single qubit 1 and then pass it through it the Hadamard.

Now I am going to apply now the Grover operation on it M number of times and M as we have seen is a quantity of the order of square element. So the way I do it is shown in the slide.

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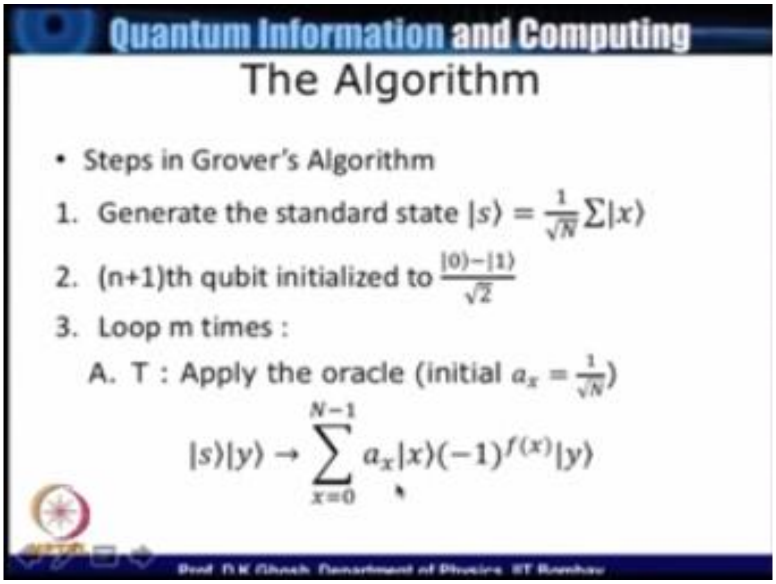

So I started with the standard state on line 1 and y on line 2, and so this is simply what happens after if it pass through the oracle we know that the oracle, we know that the oracle has calculated f(x) to the [indiscernible][00:13:55].

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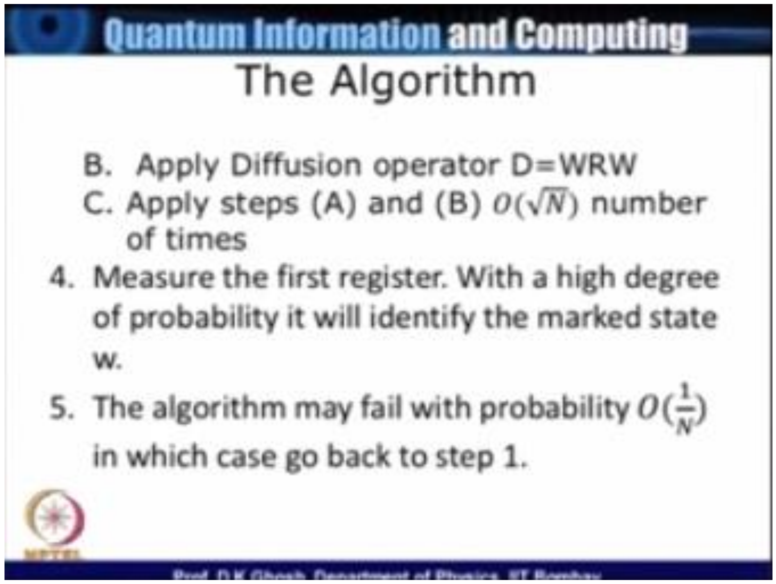

Then I apply the diffusion quietly have said can be implemented by D=WRW these two steps, step (A) and (B) I apply of the order of √N. Now if we apply the algorithm √N term we have already seen that the amplitude of the mark state is going on based selectively amplitude. So what will happen if you measure the first register now?

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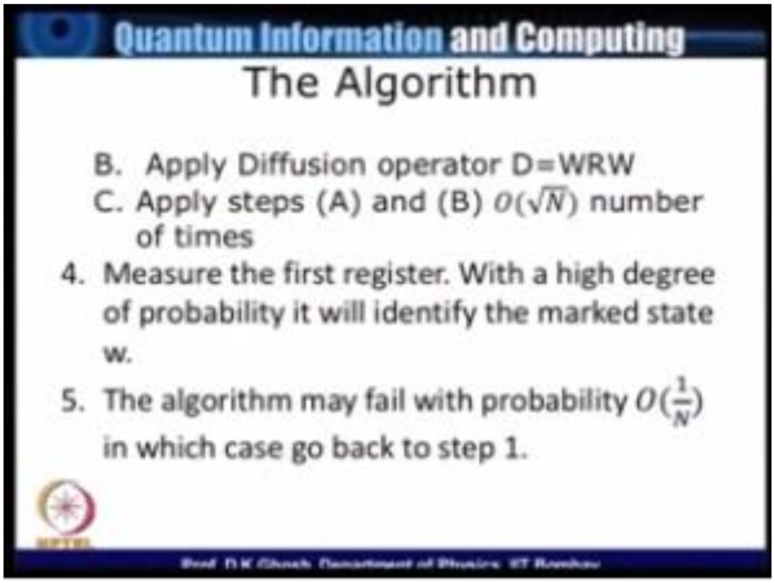

Then it is a very high degree of probability it will identify the marked state, so remember what is the idea, the content of the first ones is the collection or a linear combination of the inputs corresponding to whatever is there in my target is and in that input when I measure it I will get one of the contents with the probability defined by what is the amplitude of the linear combination in that state.

And since we have said that on application of √N number of information the amplitude of the marked state a significantly higher than the amplitude of other state so all probability even refined the mark state being flitted out by measurement of the first stage. The algorithm can okay and you said the probability of they did not show in of the order 1/N. Now it is happen then we have to simply repeat the algorithm by doing that the step one.

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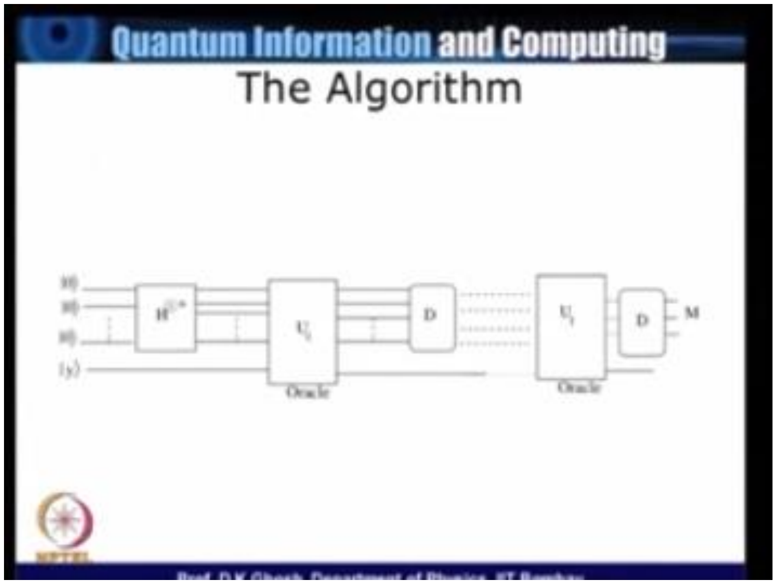

This essentially gives you what we will be actually bring and I said that these are the null state with the part the Hadamard giving me the linear combination that you copy about that this is the y is equal to substitute 0 – 1 /√2 we apply an oracle and then apply the repeat D. The reason why you are seeing in ….. is we are saying that this oracle is to be applied √M. Now remember I am not doing any measurement later that measurement will be done at the end of the transits when I know that the significant possibility of its bit.

The next thing that I will introduce is to find out what is the quality of this algorithm. In other words what is the chances of success and the chances of filling. And I already given you an idea that become the order 1/n that the algorithm will face.

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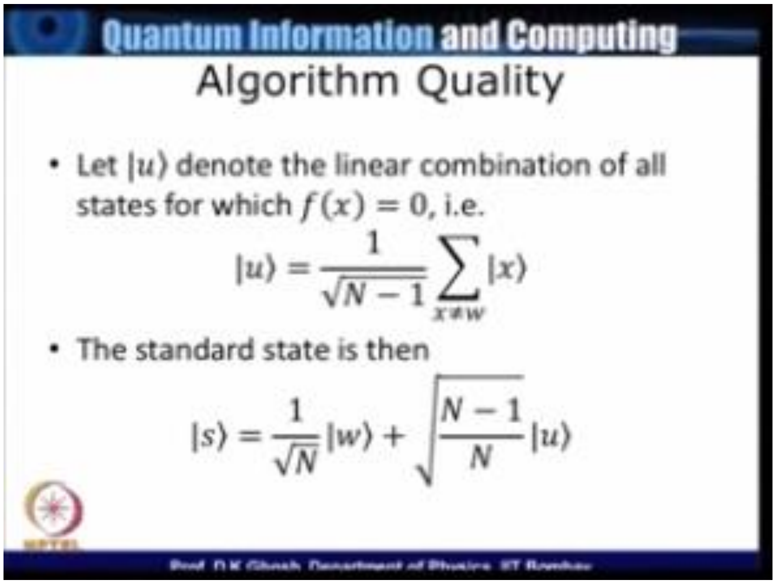

So look at what is done, we have said that I have a standard space which has a large number, very large number other than 1 of basis steps. Now if you call them by u they will say there are n-1 of them I have normalized that space now. And this timed out space is simply the 1/√Nw to times of reason and this.

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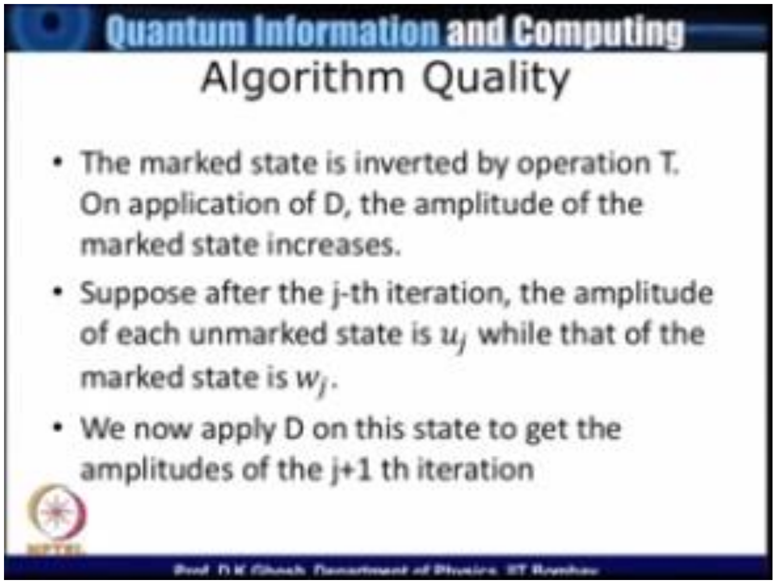

The marked state even inverted by the operation T that it is my first state the application of the D is an operation the amplitude of the marked now what we are going to do we are going to say support after get it the amplitude of each unmarked state of U_j there are number that the unmarked states that all at form so whatever is the amplitude in the computation basis and one states the and that of the marked that we get now what we have to do, that we will now apply a the diffusion operator on this with and try to find out.

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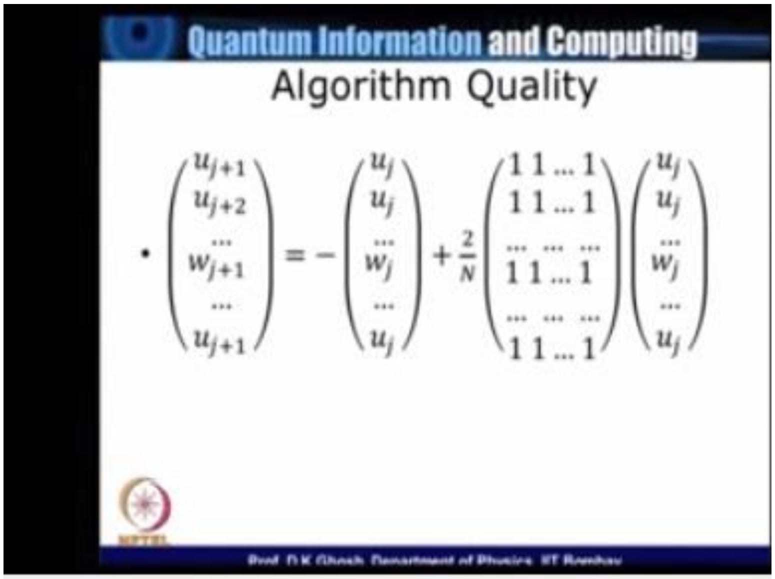

What it means amplitude of both the unmarked and the marked so of the take the definition of the deep and applying it on this.

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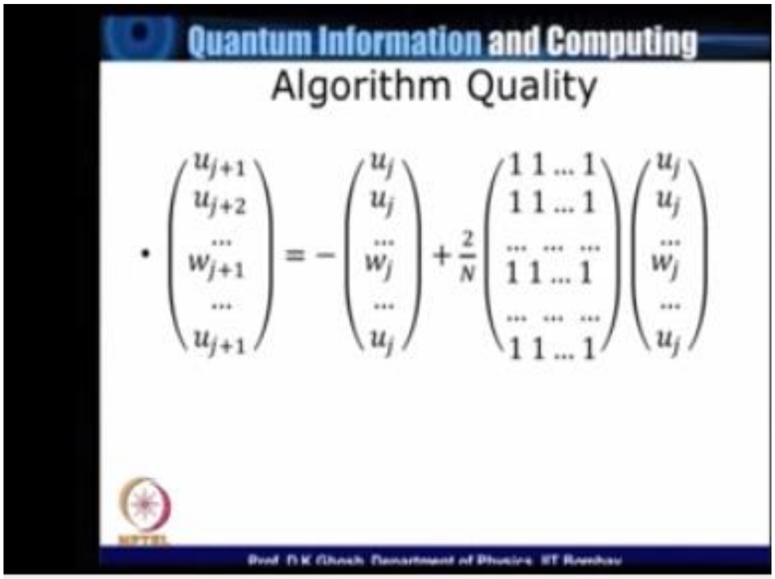

Now this is a vector which is the state of the system after j for state and so therefore, after this separation this is – i + 2/N yes that straight forward addition because these element is one then it is multiplication.

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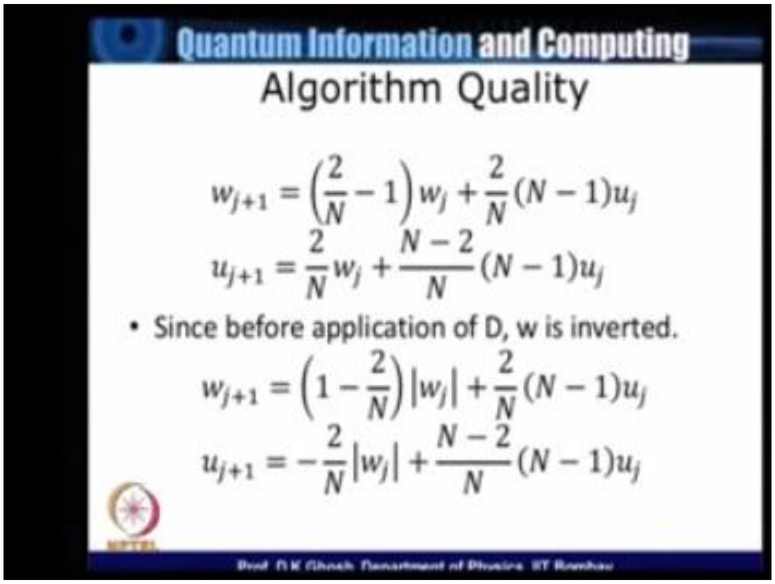

No when you have done that a go to large number of equations but because of the part that N – 1 of those equations are identically because the contribution of each of the unmarked state we mention from this is basically a pair of this pair of equations.

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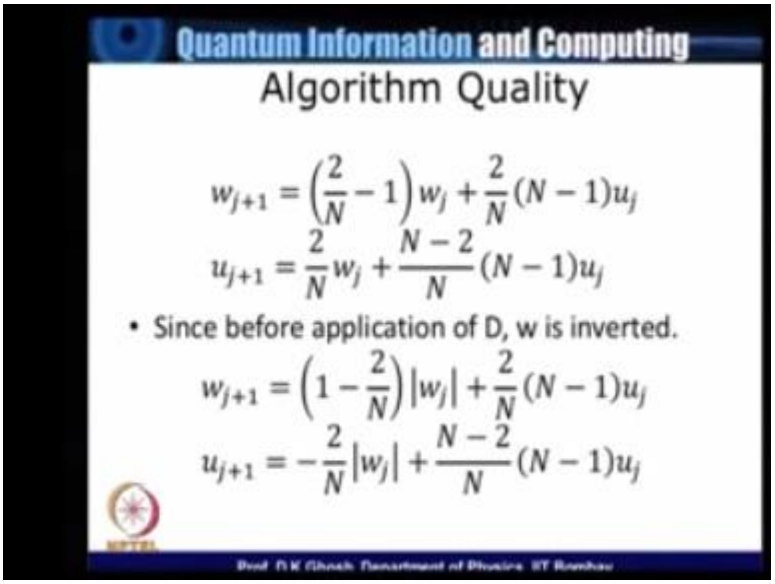

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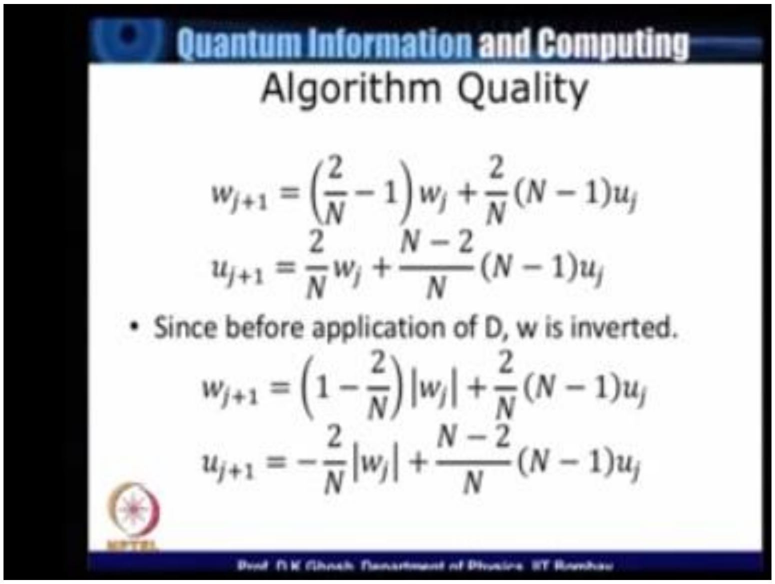

Are in this I have change the right hand style from w_j so because we have seen that what the application of the uj that is they application just have before B the w_j is in this so that can I write w have because inverted w_j have 9 has form modulus is in w and I convert this is the we will see in the next lecture that this pair of equations and we need this and we will get and at presents for the amplitudes after n of bits and that will tell us about 40 is the possibilities and structures of this having from that what we will do next.

Will be to take the specific at that at N= 8 with you will be in a position to calculate using the operation we have already done the I mean with the just have but using the rotations by to that then using the fraction that and how the meaning is inverted the amplitude and is last in that will be doing will be see how that work for that operation.

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NPTEL
Principal Investigator
IIT Bombay

Prof. R.K. Shevgaonkar

Head CDEEP

Prof. V.M. Gadre

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& Graphic Designer

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Bharati Sakpal

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