NPTEL

**NATIONAL PROGRAMME ON
TECHNOLOGY ENHANCED LEARNING**

IIT BOMBAY

**CDEEP
IIT BOMBAY**

Quantum Information and Computing

**Prof. D.K. Ghosh
Department of Physics IIT Bombay**

Modul No. 01

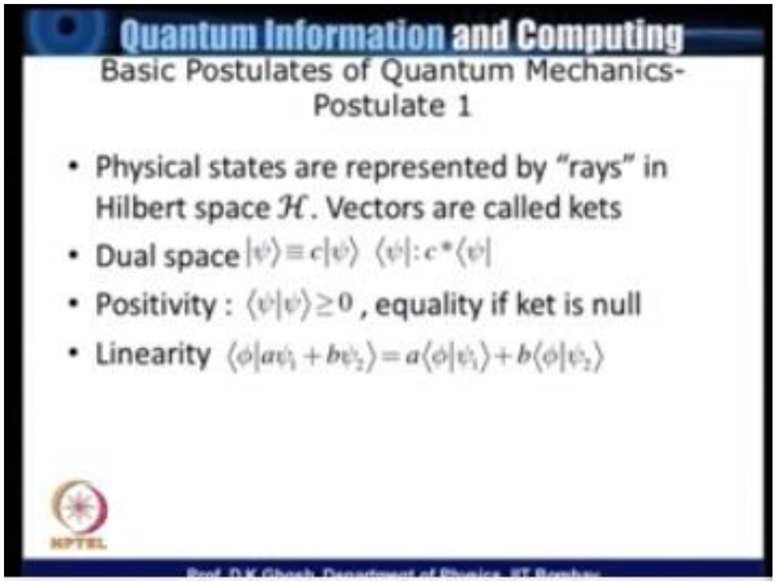
Lecture No. 02

Postulates of Quantum Mechanics - I

In the last lecture we had talked about the need for quantum computing and what makes this upcoming new principle of quantum computing that exciting. We had also talked about what is the scope of this short course of 20 hours that will have. The very basic of quantum computing the requires that we must have a good idea about the basic postulates of quantum mechanics as well as a reasonable exposure to the linear algebra.

Now what I will do in this and the next lecture will be to talk about the basic postulates which is basically the philosophy on which the principle of quantum mechanics is based, and I will also take you through the essential linear algebra that is required for understanding this course and following it. So what we do is this that there are nothing like a postulate numbers which are sacrosanct but these numberings are entirely mine so the first postulate of

(Refer Slide Time: 01:30)



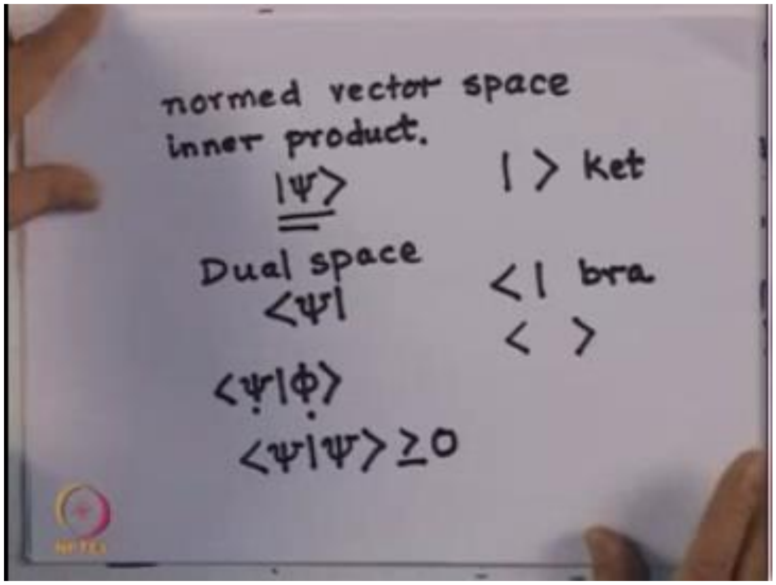
Quantum mechanics is the physical states are represented by rays in Hilbert space which I represent by the script H and these the vectors in the Hilbert space are known as kets. So let me explain what is this array and talk a little bit about what is a Hilbert space.

(Refer Slide Time: 02:01)

Now an Hilbert, a Hilbert space is a normed vector space in which the concept of an inner product is defined. Usually this space is infinite dimensional and it has another property that the space is complete in the sense it is closed. However in quantum computing we normally deal with finite vector spaces as a result of which the closeness or the completeness of the vector space is automatically satisfied.

The inner product as we will see is very much like a scalar product of two vectors with some minor differences which I will point out. And the concept of a norm that is a length essentially is defined in this space. So this is an abstract vector space in which we define vectors or the vectors represent the physical states of the system.

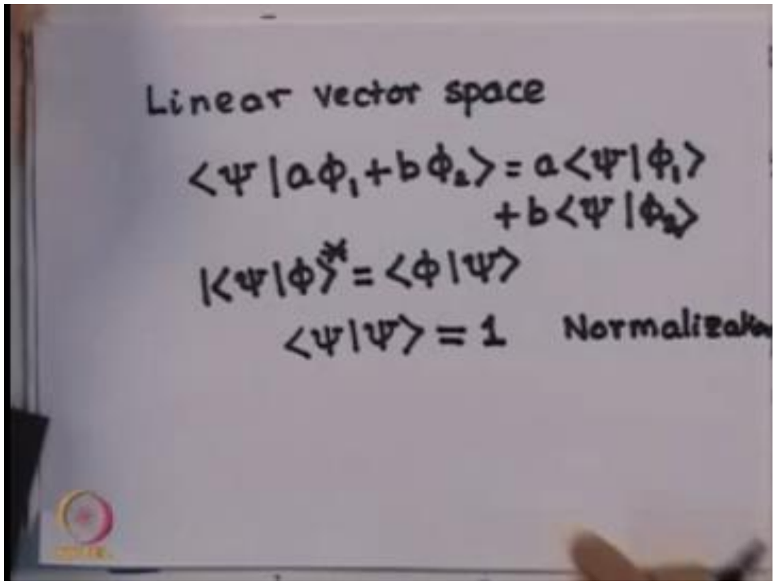
(Refer Slide Time: 03:18)



Accepting that we will represent first these states by this symbol due to Dirac this is called a ket. And so Ket's is the vector standing for a physical system in this space. Now the corresponding to this space corresponding to this space this I said is called a ket there is a dual space which is written like this and this symbol is called bra by direct. As you can see that bra and ket are two components of bracket which is what would complete this situation.

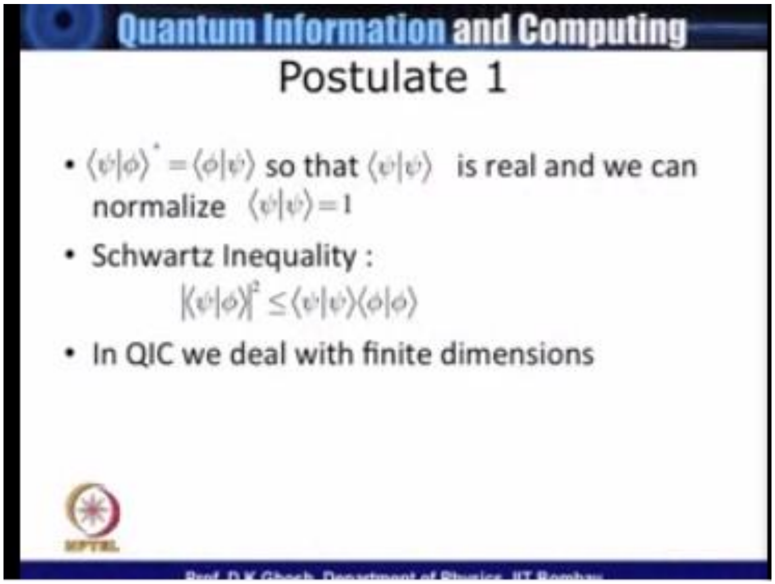
So we define an inner product between two vectors in the Hilbert space let us call one of them the Ket to be ψ and another bra to ψ and the product of ϕ ψ we will see what is the minor difference between a scalar product on this one is the inner product defined in this space. Now the second point is that the, if you look at the inner product of the bra corresponding to ψ with the vectors ψ itself now this is greater than or equal to 0 and it becomes zero only when ket ψ is a null vector. The other rough point that is there with that is that this space is linear.

(Refer Slide Time: 05:05)



It is a linear vector space, now in the sense that if you take the inner product of let us say ψ with aϕ1 + bϕ2 what you get is a times inner product of aψϕ1 + b ψϕ2. So that makes it a linear vector space. Now supposing I look at a quantity like ψϕ okay ψϕ*, ψϕ* is equal to ϕψ. Then and so therefore, we can normalize the whole things, because ψψ is always real and it is conventional and we will see why to take this ψψ = 1 this called normalize.

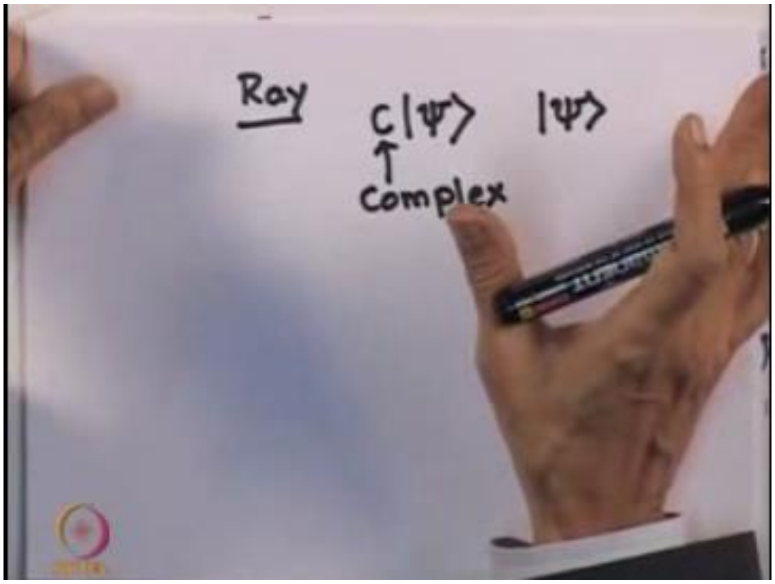
(Refer Slide Time: 06:26)



There are certain inequality satisfied in this or one of them is known as the Schwarz inequality and Schwarz inequality is I have just stated it for your for the completeness of this lecture, we will actually not be requiring it so far as this course is concerned. So we have defined a linear vector space I have called it an abstract space known as the Hilbert space and we also define as its dual.

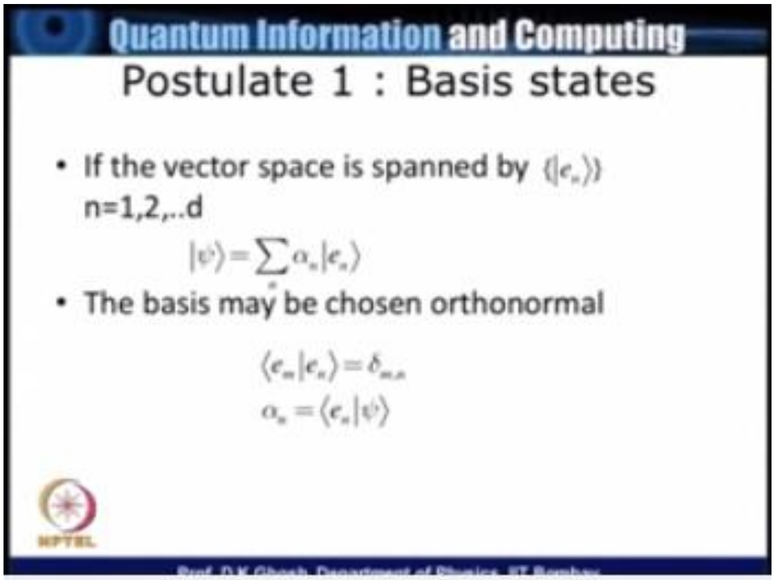
Now in the first slide I made a statement that the physical systems are represented by a ray in the vector space.

(Refer Slide Time: 07:04)



Now this is because in reality or in practice there is no difference between a state Cψ and ψ where C is a general complex number. Now as a result Cψ and ψ represents the same physical states and so therefore instead of a physical system being represented by a particular vector in the Hilbert space it is represented by a one-dimensional subspace of the Hilbert space for which any multiplication of the vector ψ gives me the same represents, the same physical state.

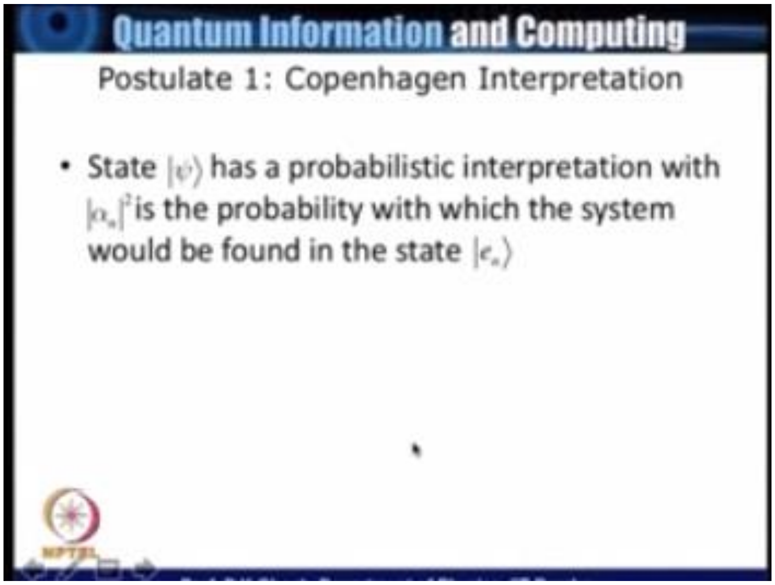
(Refer Slide Time: 07:58)



So this is what is meant by a ray, let us suppose I have a basis in this vector space and let me say that this space is spanned by the vectors which I suppose it is D dimensional vector space I have spanned e_n so that any arbitrary vector can be written at the linear combination of this basis with the coefficients being α_n and remember that the difference between a normal vector space and this space that I am talking about is our space is over a field of complex numbers and so therefore all my numbers could be complex so this α_n are in general complex.

Like we are doing an ordinary vector space I can choose this basis to be orthonormal. So if I choose the basis to be orthonormal then I can take the definition of the bases as e_m, e_n in a product to be equal to δ_{mn} and so therefore if you take this basis in terms of that it is trivial to see that αm are given by the inner product of en.

(Refer Slide Time: 09:12)

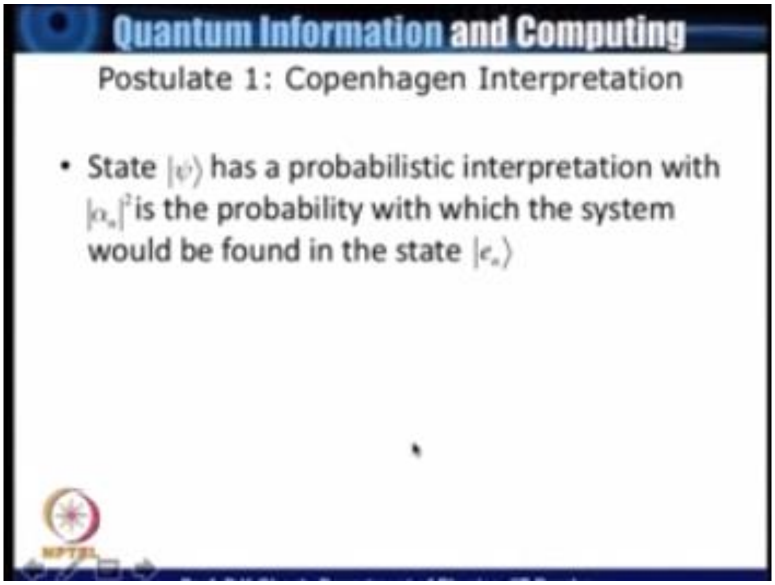


Now I go over now to what is popularly known as the Copenhagen interpretation of quantum mechanics Copenhagen as you know is a place in Europe where in the days when the quantum mechanics was being developed late 19th century to early 20th century lots of physicists had gathered there and they had been responsible for formulating quantum mechanics and get or discuss its philosophical interpretation.

And this interpretation of quantum mechanics which is considered as the standard interpretation is known as the Copenhagen interpretation. Now according to Copenhagen interpretation the state vector which I have sent is a ray ψ it has only a probabilistic interpretation. Now we will as we go along will realize this is a rather important point that they, when we are talking about for instance, a physical system and suppose you want to make a measurement of that system.

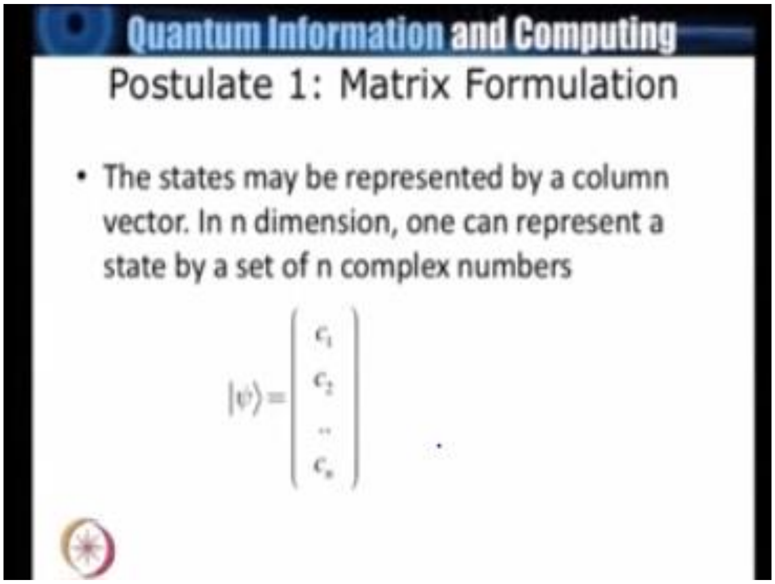
When I say we want to make a measurement of that system I could be trying to measure for instance, its energy, its angular momentum or whatever physical property you feel like. Now so therefore, if I want to make a measurement of this system represented by a $\psi\phi$ the best I can predict is that what is the probability with which I will get a certain result, this we will see caused a lot of philosophical disagreement and one of the prime opponent of this interpretation was none other than Albert Einstein who never believed that the Copenhagen interpretation is correct. We will have occasion to discuss Einstein's objections to this through this course.

(Refer Slide Time: 11:17)



So as we said it has a probabilistic interpretation.

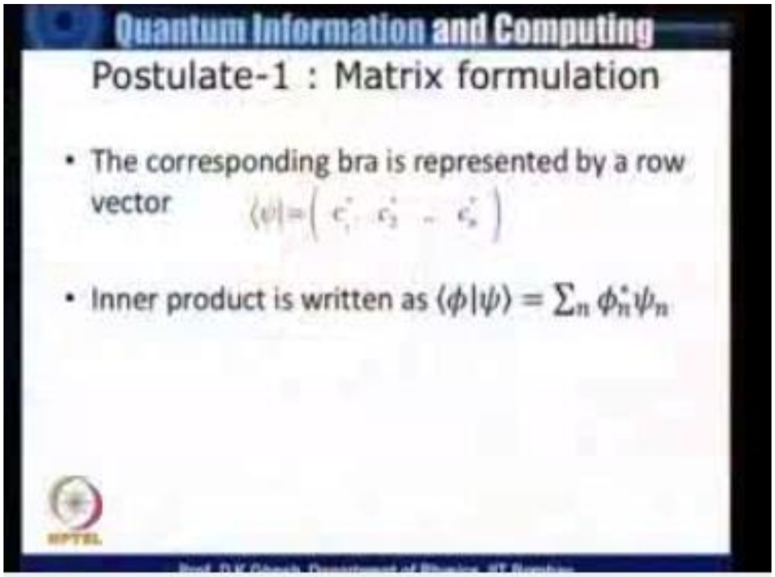
(Refer Slide Time: 11:20)



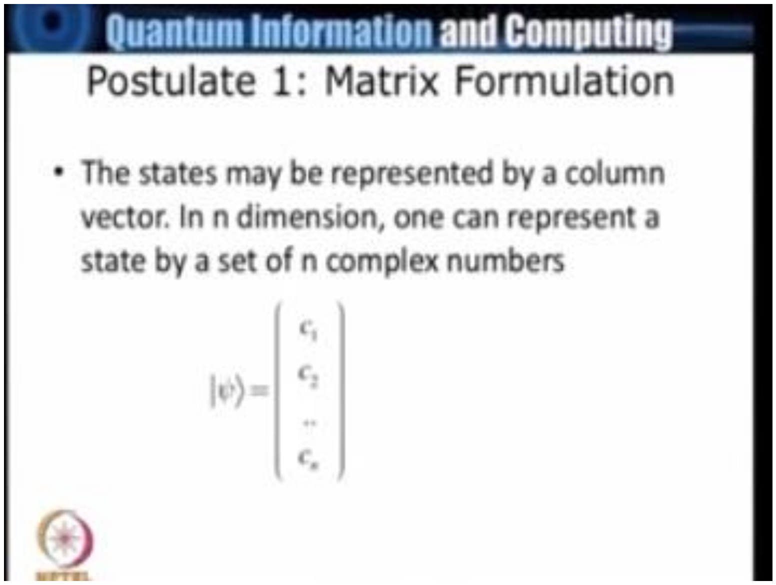
Now let me then go over from totally abstract statements that we have been making to something to which you can relate. Now it turns out that any vector can be given a matrix representation now that is fairly simple because even if you take an ordinary vector in three dimension, now ordinary vector in three dimension is completely specified by giving three numbers these are for instance they could be it is XYZ component Rθϕ components or whatever you feel like.

Now so therefore this set of three numbers for a three dimensional vector decides or defines my vector what is the difference in this case number one is the dimensionality could be arbitrary and though we will be mostly talking about low dimensions, finite dimensions. The other differences that this set of numbers are in general complex numbers, so therefore I can represent my ψϕ by a color vector which is essentially a n/1 matrix of having complex elements. So that is what is shown here.

(Refer Slide Time: 12:44)

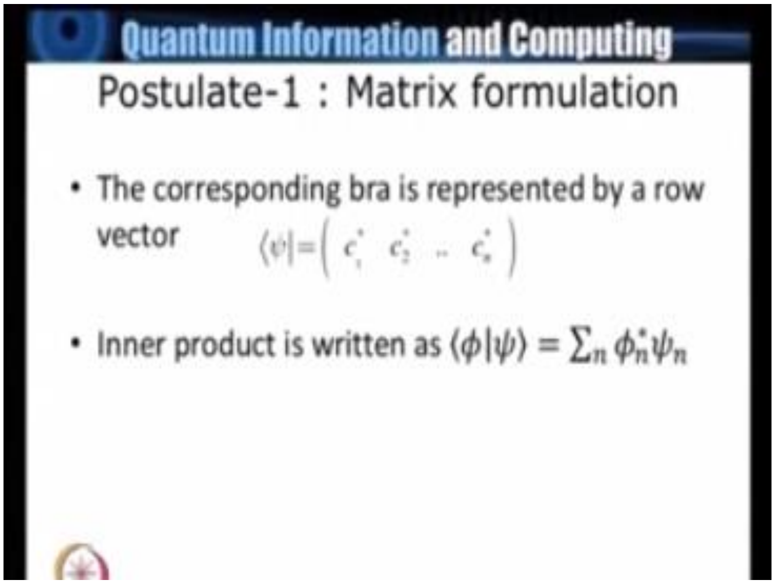


(Refer Slide Time: 12:45)



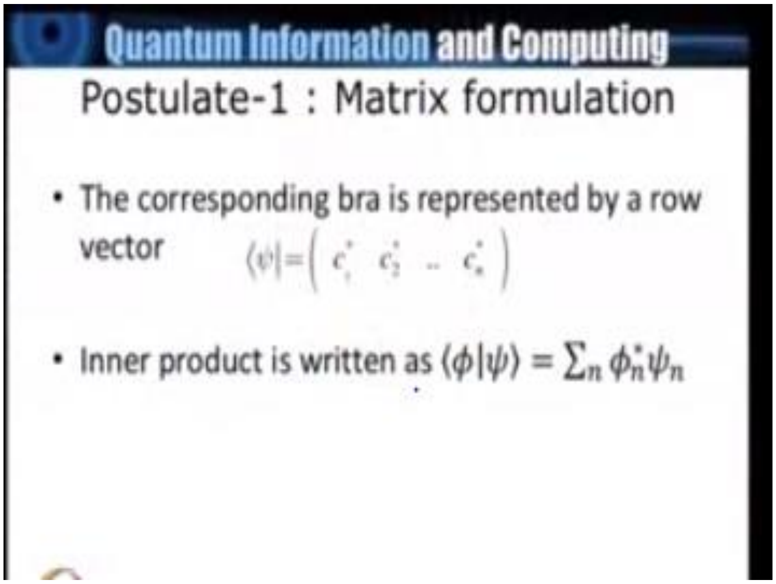
So ψ decno ψ I have represented by a column vector having components C1 up to CR. Now what about the dual space.

(Refer Slide Time: 12:57)



The dual space then would be written now there is a difference here, dual space corresponding to the ψ that I have written down is instead of being a column vector is a row vector, but the components of the row vector are complex conjugate of the components of the ψ vector. So instead of the column vector C1, C 2 etc., that we had written down.

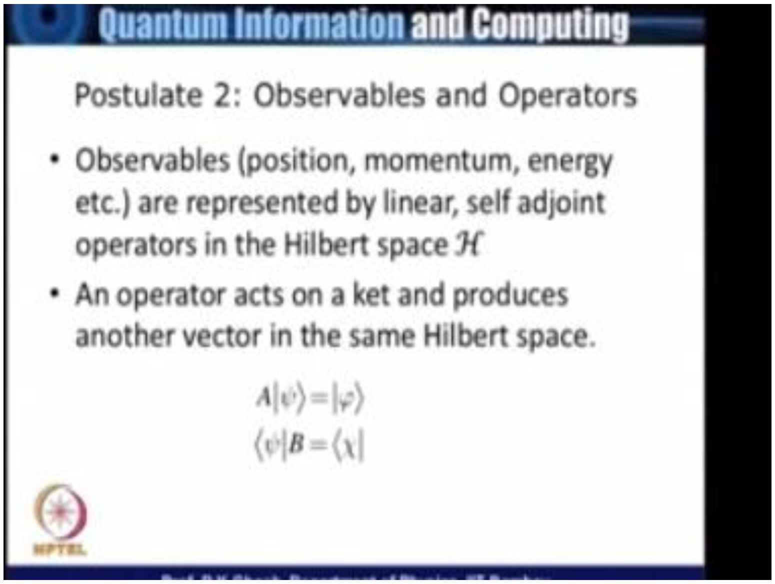
(Refer Slide Time: 13:29)



You can see in the slide that it is given by C1*, C2* now what is then the difference between the inner product or the scalar product for an ordinary vector space on this. So remember if I had an ordinary vector space let us suppose a vector with component A and another vector with components B1, B2, B3 then I will write down the scalar product of these vectors as A1, B 1 + A2 B2 + A3 B3.

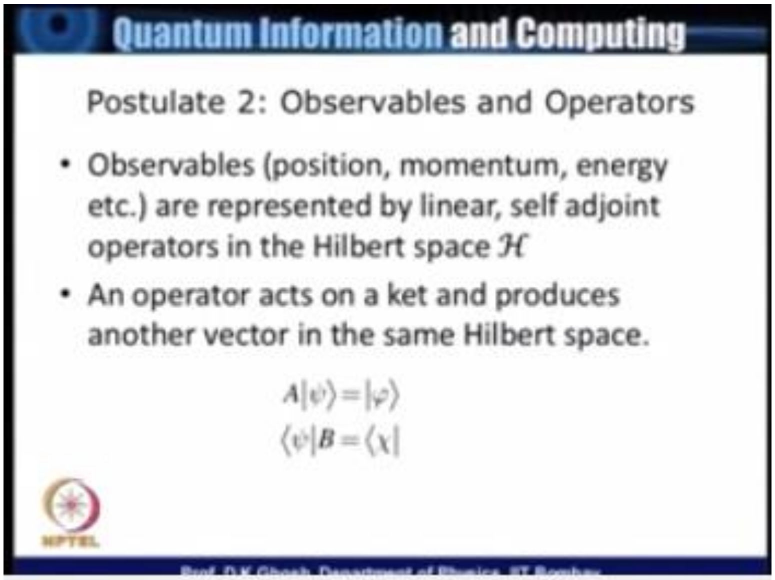
In this case it is very similar accepting that the left-hand quantity which actually is in a dual space is to be written as sum over n$\phi_n{}^*\psi_n$ that is take the complex conjugate of the components of the ψ vector ϕ so that is the only difference.

(Refer Slide Time: 14.28)



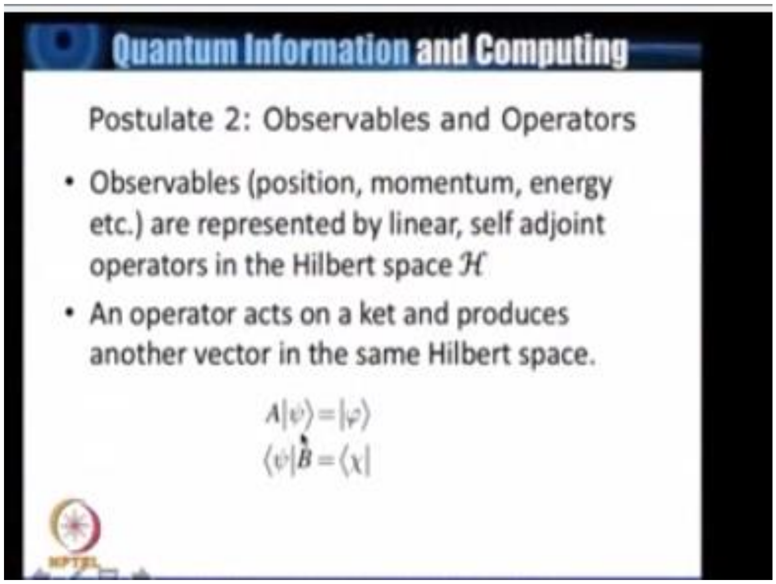
And with this I can carry out the algebra in terms of the matrices only and matrix manipulation becomes much easier. Now with this let me go to the second constituent of quantum mechanics. Now we have already defined the vector space, now we also need to define operators in that vector space because it is the operations that we will do in a vector space which will take us from one vector to another.

(Refer Slide Time: 15:07)



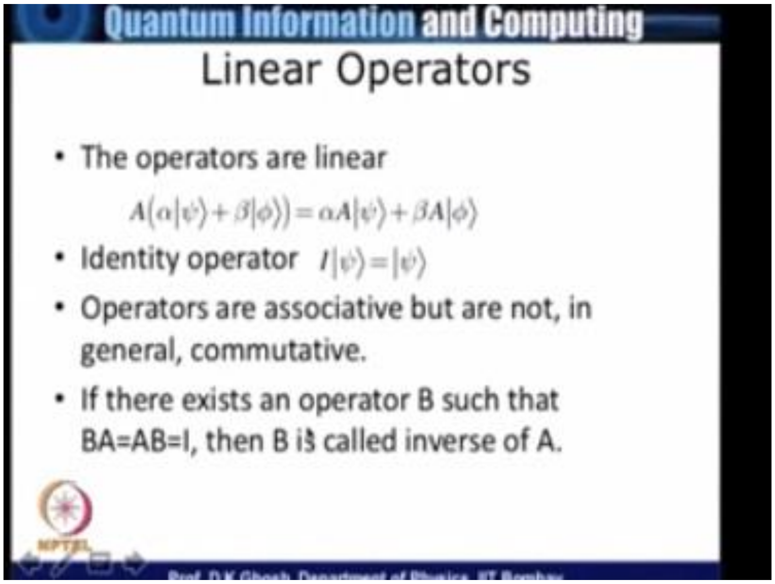
So look at the second of the postulate and this says we have certain observables. What is an observable as I mentioned that an observable could be things like a position, momentum, energy etc. whatever you would like to measure a property of the particle. Now these are represented by the adjective is linear, self ad joint operators in the same Hilbert space. So basically an operator acting on a given vector of the Hilbert space gives us or takes into another vector in the same space. So this is written in this slide as.

(Refer Slide Time: 15:51)



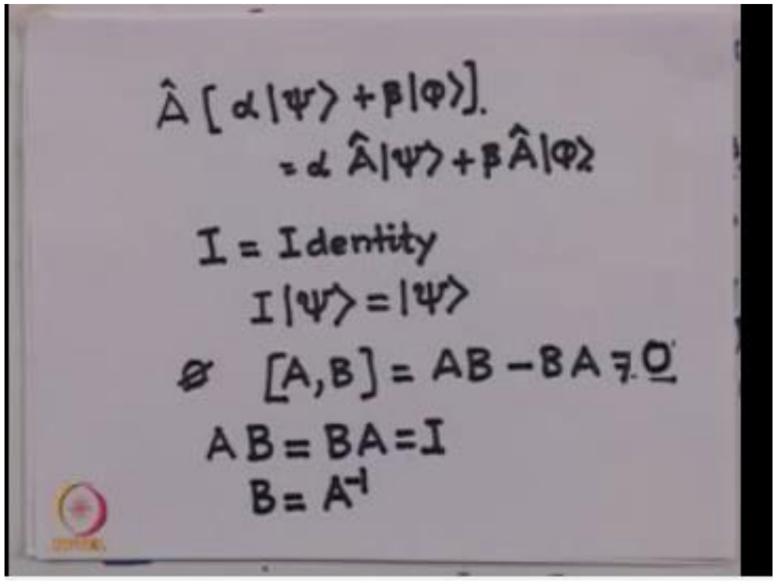
Aψ the operator A active on ψ gives me a vector ϕ. Now can I do the corresponding linear operator in the bra space the answer is yes, here is the operator B which acts to its left on a vector in the dual space giving me another vector in the same dual space. So the relationship is identical.

(Refer Slide Time: 16:16)



Now the next point is these operators that we are talking about are linear operators. Now linear operators as you have learnt earlier what does it actually mean.

(Refer Slide Time: 16:34)

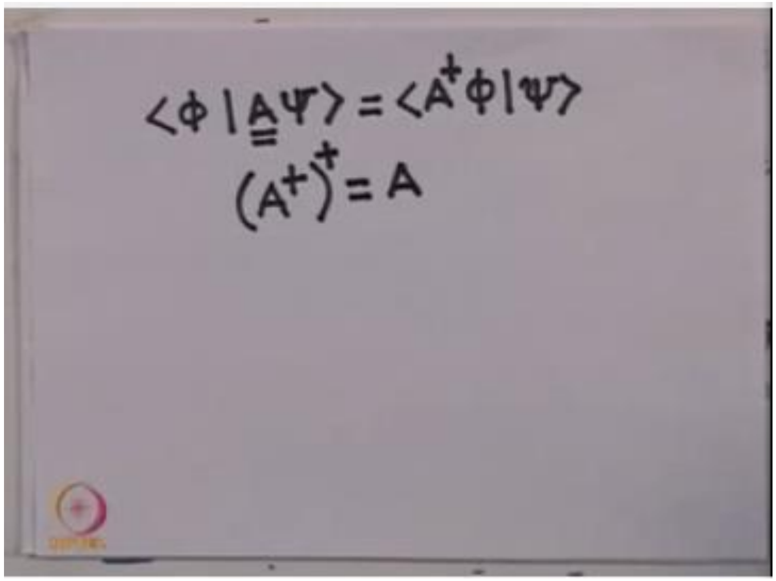


A linear operator means that an operator A acting on a linear superposition of two vectors let us suppose it acts on $\alpha\psi + \beta\phi$, α and β are complex numbers. Now this implicates as $\alpha A\psi+\beta A\phi$. So this is what a linearity of an operator means. Now in this space I define an operator called an identity operator, what does an identity operator do an identity operator acting on any state vector ψ gives me the same vector it does not do anything actually. The point that is to be noted here is that these operators though they are associated in the sense.

If you add A with B+C or add A+ with C their result is the same. But these operators in general do not compute. So these operators A and B, so in general A, B commentator which is as defined as A times B – B times A in general not equal to an identity. Now suppose there exists corresponding to an operator A supposing there exist corresponding to the operator A another operator B such that A times B is equal to B times A is equal to I the identity.

Then B is called inverse of A and is represented as A^{-1} for this is the operator relationship. Now since we deal with a slightly different form of the inner product that is our inner products are with respect to a ket vector and a bra vector. We define an ad joint of an operator the ad joint of an operator is defined in the following way.

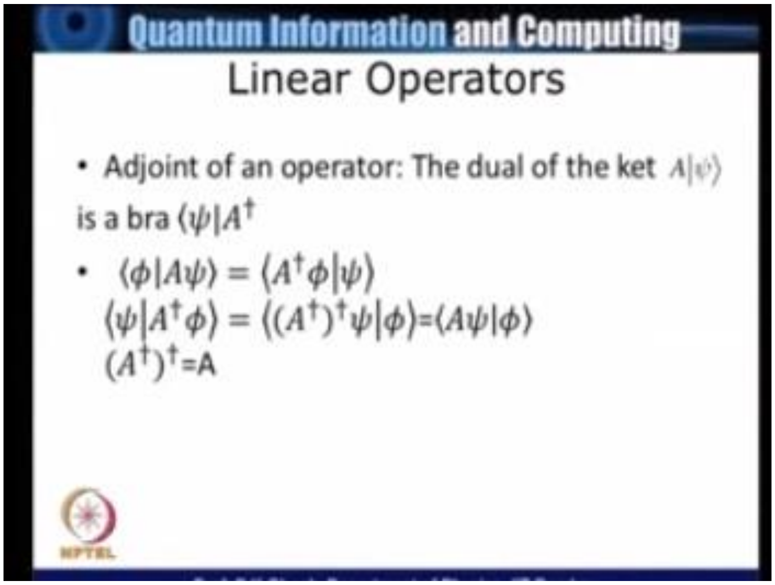
(Refer Slide Time: 19:01)



That is, suppose I take this inner product or scalar product of ϕ with let us say a vector Aψ. Now this is then identical to this scalar product of the ad joint of A which is usually represented by A with a sign like this very similar to + but it is spilt with a long stand which is called a A^{+}, A^{+}ϕ it's scalar to ψ. Now it is very trivial because of this, when an operator comes from this ψ to this ψ you just that operator changes it's dagger okay.

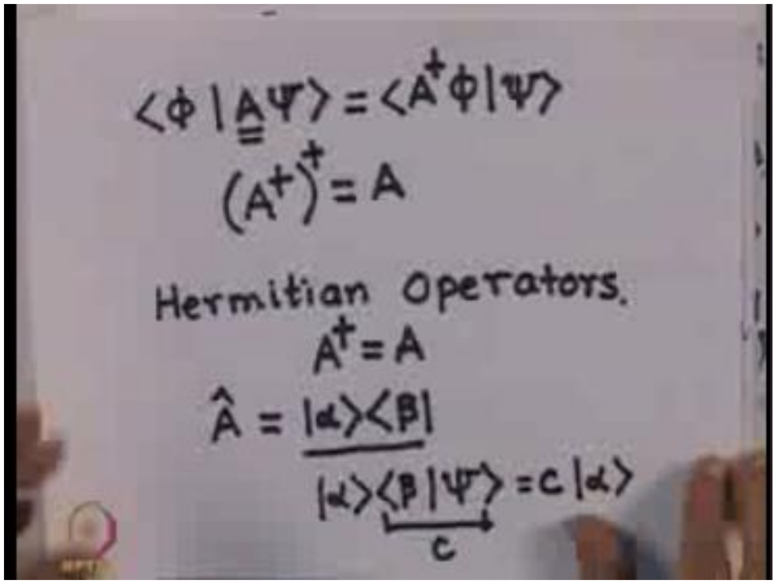
And as a result you can immediately see that if you take it twice $(A^{+})^{+}$ that gives you A itself okay. And at the trivial proof you show this slide.

(Refer Slide Time: 20.04)



Now one class of operators which are going to be extremely important for our discussion, these are the operators which represent physical variables every physical variable in the Hilbert space is represented by an operator which are known as self ad joint operator or also called hermitian operator.

(Refer Slide Time: 20:37)

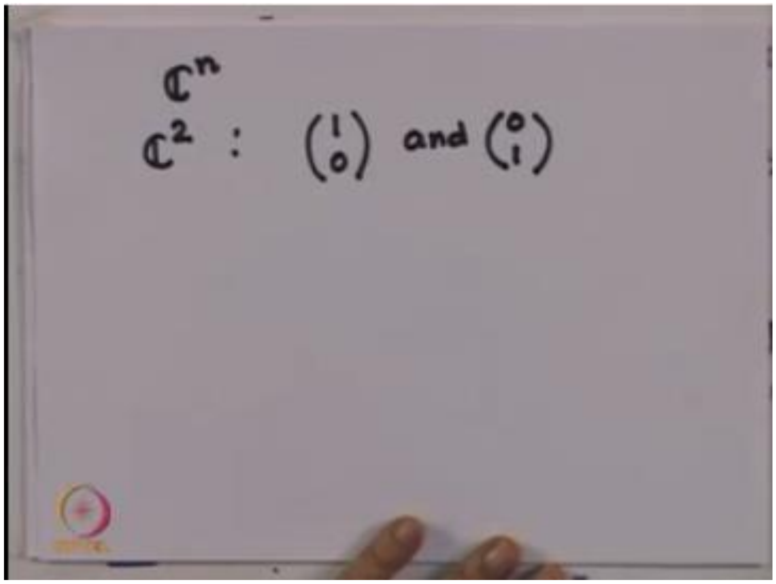


So the hermitian operators have this property that A^{+} that the ad joint of A is A itself. I will give you the matrix representation of these things and then we will realize how it is fairly easy to understand these in terms of matrices. Now since we said that an operator acting on a vector gives me another vector in the same space a general representation of an operator could be of the form that it is a ket followed by a bra.

Now this order is very important and you can see why, that if this quantity a ket followed by a bra acts on let us say any state, let us call it ψ so you notice that because ψ is a scalar product and hence it is a number. So I get a number times this α so this is equal to some C times α. And we have said that α and C times α have no different physical significance. So this is, so a general representation of an operator is to write a ket followed by a bra.

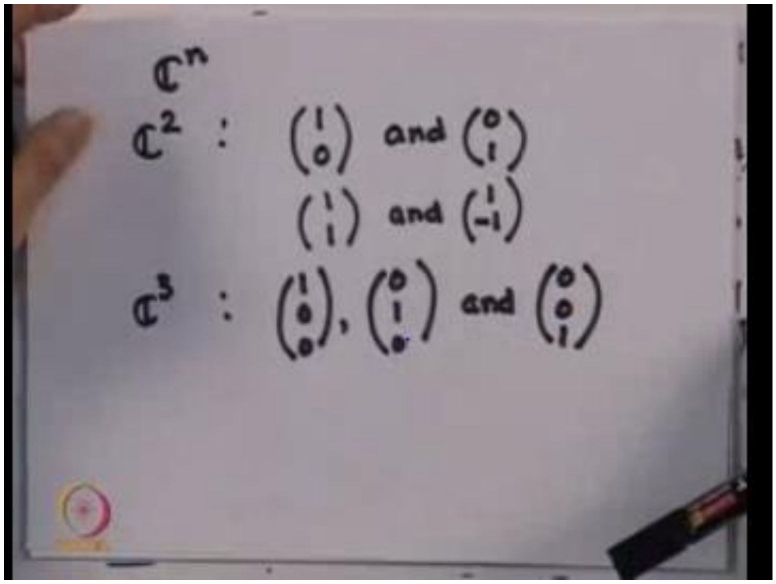
Now as I told you that it is much easier to handle the linear algebra by means of representation of the vectors by a set of numbers or in the form of a vector being represented by column matrix. Now since most of our discussion will take place in two dimensions for a reason to be understood later. Most of my examples will be in two dimension.

(Refer Slide Time: 22:46)



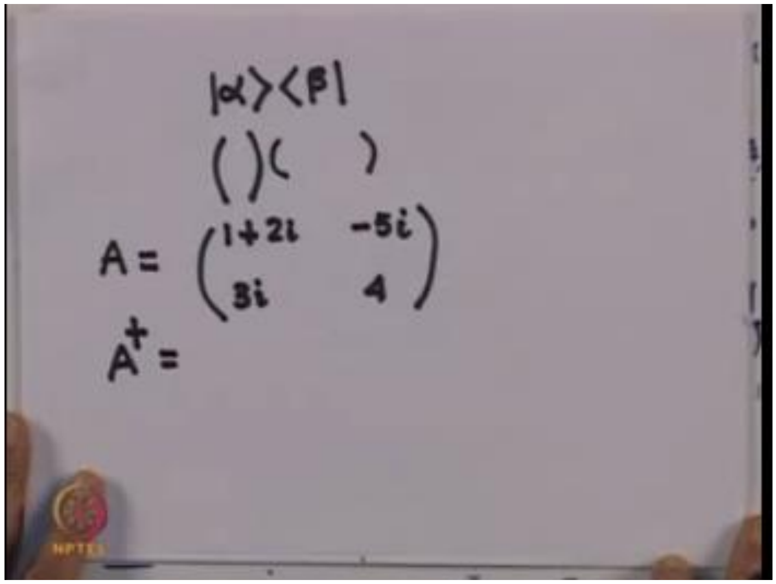
And a complex vector place a space in n dimension is usually represented by C^n of which our important space is usually C^2. So one of the bases you can immediately see in C^2 is for example, 10 and 01 any, any vector in this space can be written as a complex number times the first one plus another complex number times the second on this obvious. Now this is not a unique representation.

(Refer Slide Time: 23:23)



For instance, I could also have a basis in which I have 11 and 1-1. The difference is of course that while these two vectors are automatically normalize these two vectors are not you can easily normalize them by putting one over square root of this square plus the square it just happens to be 1 over square root of two. Now suppose you want to go to the space C^3 then my vectors could be 1 0 0, 0 1 0, and 0 0 1 and like this you can write down for any dimension the basis. Now let us look at them that what happens to an operator.

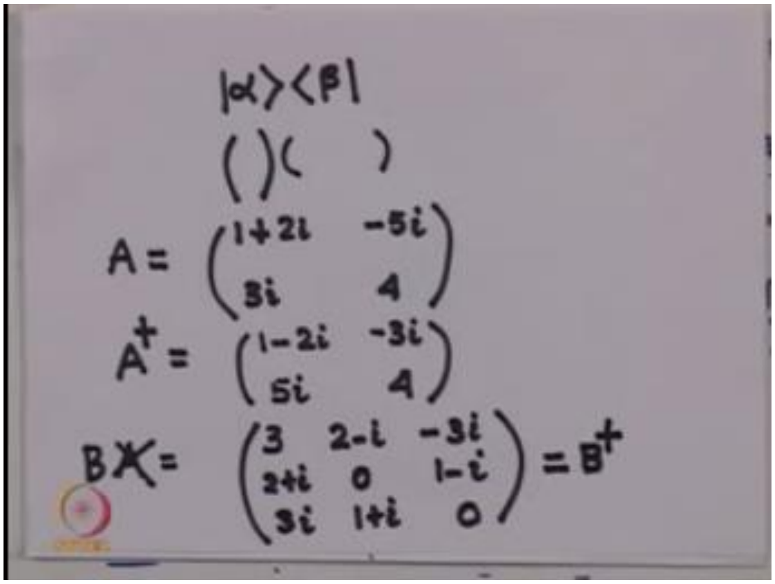
(Refer Slide Time: 24:22)



Now remember I told you that the general representation of an operator is like this ket followed by a bra. So if a cat is represented by a column vector and bra is represented by a row vector this form is essentially what is known as a matrix direct product. So if this is for example a column vector having two elements, this is a row vector having two elements this product will then become a two-by-two matrix. So take for example, a matrix A I am just writing down to an arbitrary matrix.

So this matrix is 1+2i, -5i, 3i and 4. Now what is if it is A, what if it is ad joint, now in order to find the ad joint what you have to do is take the transpose of the matrix that is interchange its row and order of the columns and having done that take the complex conjugate of the elements.

(Refer Slide Time: 25:37)



So this will then become that in place of 1+2i I will get a 1-2i for of course will remain for because these are diagonal elements, so transposition have no effect on them. But because of transposition -5i would have come here, but then when I take the complex conjugate this becomes a 5i, 3i would have gone there but because of complex conjugation this becomes -3i.

So this is the ad joint of A now as you can see that this ad joint of A is not equal to A itself, but supposing you have to look at a matrix like this just giving you another example. So let us call it instead of A let us call it B, suppose we write it 3, 2-i, -3i, 2 + i, 0, 1-i, 3i, 1+i, 0 you can observe some symmetry which is there. Then this matrix and its ad joint are the same, in other words this is a hermitian matrix.

So we will continue with the matrix representation of linear operators. So what we have said so far is if A and B happen to be vectors in the Hilbert space of dimension D then ket A followed by a bra B has the dimension D/D okay. And the matrix product here is known as either a Kronecker product or a direct matrix product.

NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING (NPTEL)

NPTEL
Principal Investigator
IIT Bombay

Prof. R.K. Shevgankar

Head CDEEP
Prof. V. M. Gadre

Producer
Arun Kalwankar

Online Editor
& Digital Video Editor
Tushar Deshpande

Digital Video Cameraman
& Graphic Designer
Amin B Shaikh

Jc Technical Assistant
Vijay Kedare

Technical Assistants
Pratik Sathe
Bhargav Sri Venkatesk M.

Sr. Web Designer
Bharati Sakpal

Research Assistant
Riya Surange

Sr. Web Designer
Bharati M. Sarang

Web Designer
Nisha Thakur

Project Attendant
Ravi Paswan
Vinayak Raut

NATIONAL PROGRAMME ON TECHNO,OGY ENHANCED LEARNING (NPTEL)