Special Theory of Relativity
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Lecture - 9
A Three Event Problem

Hello, in our last lecture we have discussed the velocity transformation formulas. We eventually started from Laurence transformation putting the transformation into a differential form, they are eventually involved and worked out the velocity transformation. We also discussed an example of relative velocity to emphasis what we mean by relative velocity, in classical mechanics sometimes we are not we are a bit casual in defining relative velocity. But, relativity one has to be very careful that when we see a relative velocity it means we have to make a transformation to another frame and find out the velocity of the particle. And this relative velocity would never exceed this speed of light.

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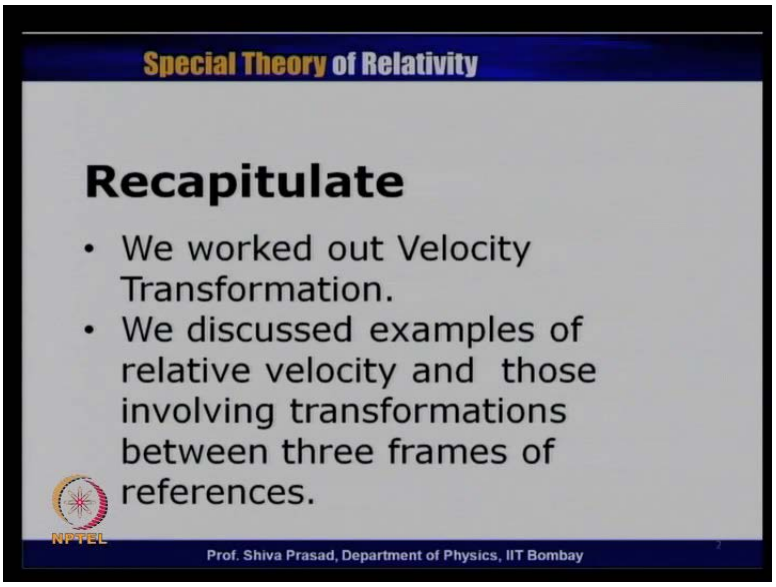

Then we also discussed a particular example in which there, three different frames of references were involved, we have been earlier doing generally two frames of references. So, to make problem little more difficult we had involved three different frames of references, and saw that how one has to take care of looking to transformations from one particular frame to another particular frame of reference. But so long we do things in a proper way we should not have any difficulty. So, this what I have written that let be

recapitulate, we worked out the velocity transformation, we discussed examples of relative velocity and those involving transformations between three frames of references.

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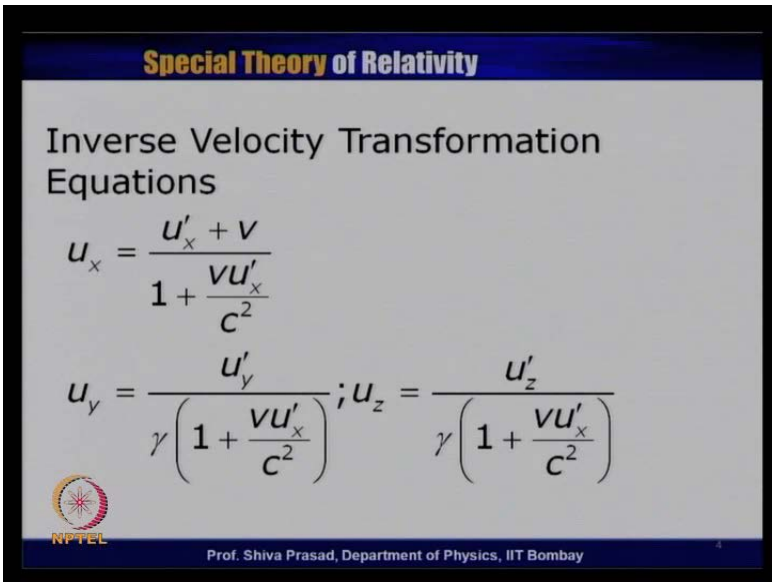

This is what is the velocity transformation equation. Just to recall again I mention that v is the relative velocity between the frames of references, u x is the velocity of a particular particle we need not be constant, it is the x component of velocity of the particle. We have suppose to know u x, u i, u y is the y component of the velocity of the particle as measured in S frame and u z is the z component of the velocity has measured in S frame. Then the velocity of the same particle is measured in S prime frame of reference, the components will be given by the x component will be given by this equation, the y component will be given by this equation, the z component will be given by this equation.

What is interesting here is that if you look at u y prime it also depends not only on u y, but also depends on u x. Similarly, u z prime not depend just not depends only on u z, but also depends on u x. So, this particular factor u a which is involving u x is appearing in all the three transformation equations of all the three coordinates, all the three components of velocity u x, u y, u z or u x prime, u y prime, u z prime. We also discussed a inverse velocity transformation, it means if the velocity components are given in S prime frame of reference, how to find out the velocity components in S frame of reference. As we have always been telling the prescription is very simple that instead

of v use minus v, change prime quantities to un prime quantities and un prime quantities to prime quantities. So, this is what basically we had discussed in our last lecture.

In today's lecture we will discuss one example which is little more complex little more complicate complicated, involving multiple events and multiple frames of references. The idea of discussing this particular example is to see what type of care we must do or what is way we should attack this problems, when there multiple events and multiple frames. As I have been telling that first thing that one has to look is that we consistent in your frame of references, we should not mix up the frames of references, which many times in classical mechanics we are not very careful.

So, we should picturized the scene looking purely from the point of view of one frame of references, then the chances are less that we will make mistake. The second thing that we do is to identify the events, what are the important events. Try to write the coordinates of the events including time, time also we are treating as a coordinate. And these coordinates must be have written in a particular frame of reference, let us try to find out whether we have got all the coordinates, if we have not got the coordinates whether we have quarter equivalent information in some other frame of reference.

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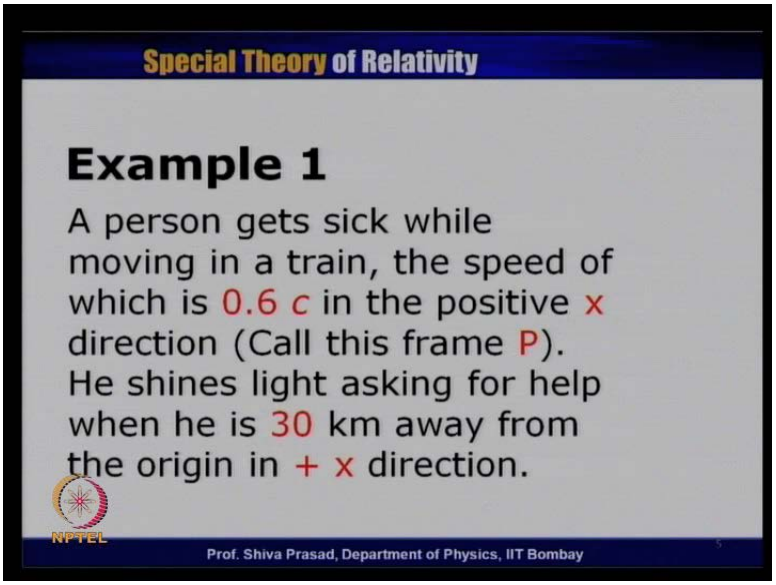

Then we keep on making transformation and eventually we fill what we called as an event table, it means all the events should be known an all the frames of references, there coordinate should be known an all the frames of references. So, once we have defined

these events, the coordinates of the events must be the known in all the frames of references that are of interest to us.

So, let us look at this particular example, the example statement is little longer let us go slowly and try to understand it. A person get sick while moving in a train of course, in because we are dealing with special theory of relativity everything moves in a constant velocity here. There is nothing like an nobody an stops everyone this moving in constant velocity, because that is the way we solve the problem. Because, must be if a person stops or anything happens like that you know, then we it is no longer way in permive of special theory of relativity, because it does not involved a inertial frame.

So, all our velocities are constant, they never change as for as the examples concert. So, there is a person who has got sick while he is sitting in a train and of course train is moving with a constant velocity and it is a relativistic velocity. And this speed is 0.6 c in plus x direction, as seen by an observer in a given frame, let us assume that is the ground frame or arch frame, which we also assume to be inertial. Now, this person who get sick I am calling this particular person as P frame just to be making little more clearer, so that you know when we talk of S prime, S double prime or things to be commotional more involved.

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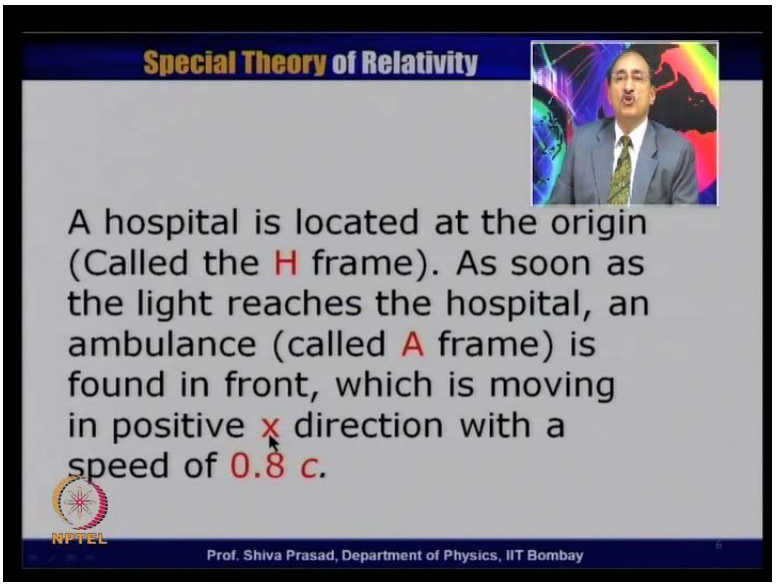

So, let us put in a specific symbols, because we talking of person, so I am calling this as a P frame. Now, this person once he get sick he sense a distant signal by emitting light.

So, he emits light asking for help and at that particular instant of time when he get sick at he said the signal, he is 30 kilometres away from the origin in the plus x direction, that is what we have written. A person get sick while moving in a train the speed of which is 0.6 c is obliviously relative to ground. In the positive x direction call this frame P, he shines light asking for help when he is 30 kilometre away from the origin in plus x direction.

Now, there is a hospital which is located at the origin, which let us assume is the again the ground frame. And we called this particular frame of references H frame, because this is the hospital frame, which is actually same as earth frame if you want to call it. So, hospital is located at the origin and as soon as this particular person receives the light signal, he tries to look for an ambulance. And he finds by that time is fortunately that there is just an ambulance which is just passing by him, I am going in the direction of the sick person, but with a little larger speed 0.8 c. So, he instructs a person that or he instructs the ambulance that he go and help the person that is what is the problem?

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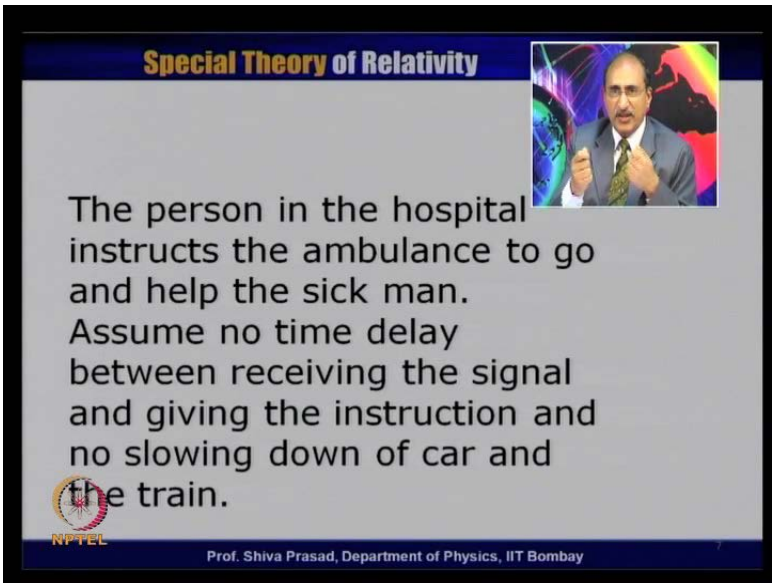

A hospital is located at the origin called the H frame, as soon as the light which is the hospital and ambulance called A frame, a person sitting in ambulance I am calling that is an A frame. So, there are three frames, there is a P frame which is a person's frame, who send the signal, there is H frame which is hospital frame, which I am calling as a earth frame and there is an ambulance frame a person sitting in a ambulance, which we are

calling as A frame. So, an ambulance called A frame is found in front which is moving in plus x direction with a speed of 0.8 c, see earlier speed of the person 0.6 c.

Now, the person in the hospital instructs the ambulance to go and help the sick man, see go and help that person. Assume more time delay between receiving the signal and giving the instruction and no slowing down of car and train. So, let us assume that as soon as a signal is received instantaneously without any time delay, he could instruct the driver of the ambulance that go and help the person.

And the ambulance person immediately rushes towards the sick, anyway he is he is anyway going and there is no slowing down anywhere. So, whatever are the speeds they are always constant because that is what is necessary to maintain that all the frames of references that I am talking are inertial frames of reference. So, no slowing down and no time delay in conveying the information from the hospital frame of reference to the driver of the ambulance frame of reference.

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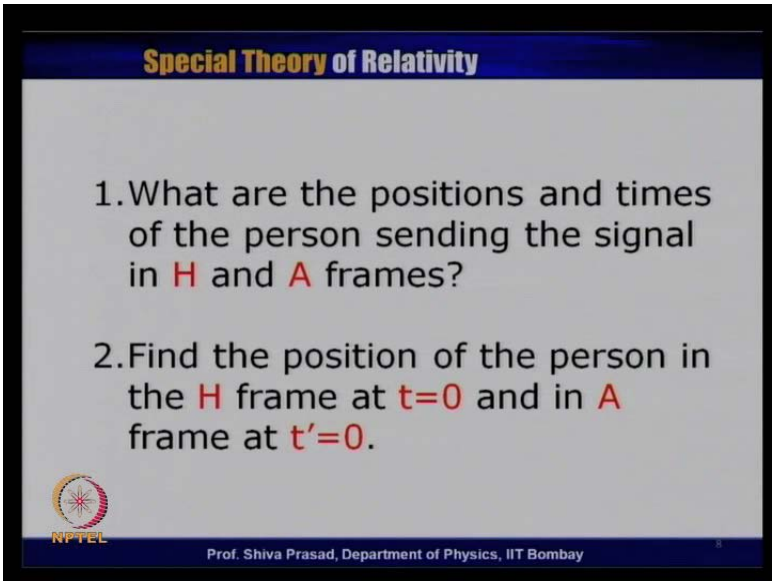

These are various questions which have been asked, question number one; What are the positions and times of the person sending the signal in H and A frame? That is a hospital and the ambulance frame of reference what is the position? Some of they may be simple question, some of they may be little more difficult question, let us see.

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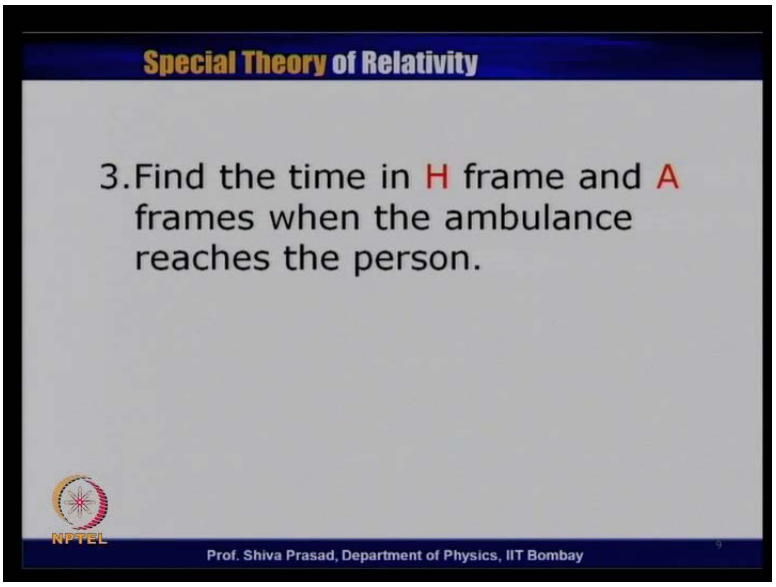

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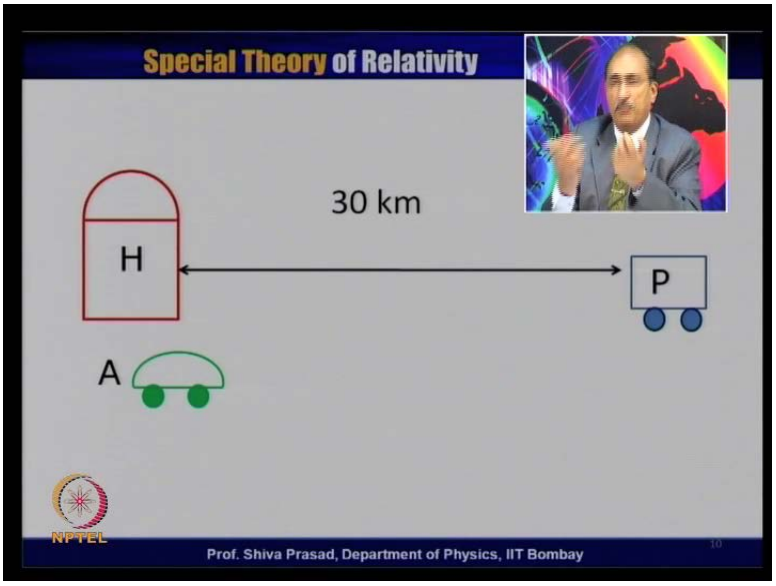

The second question which has been asked; find the position of the person in H frame, which is hospital frame at t is equal to 0 and in A frame, which is the ambulance frame at t prime is equal to 0? Number three; Find the time in H frame and A frame when the

ambulance reaches the persons? Eventually the ambulance reaches the person and they have to find out what is the time in H frame and A frame. This is basically the problem that we are going to work it out.

So, I have sort a picturized this particular thing in this particular transparency. So, this particular person this is sort of a generic figure, so just to understand we will draw a specific figures relevant to a particular frame of reference little later. So, this is hospital which is suppose to be situated on ground and this we call as H frame. This is my car, which is moving with a ritualistic speed of 0.6 c going towards the right which I am calling P frame, because the person is sitting in this particular train and that is way he get sick.

Let me call this is my ambulance which I am calling is A frame, which is also moving to the right with a speed of 0.8 c of course, this information of 0.8 c this speed of 0.6 c both are given in H frame, it is a ground frame that is the way we have described this particular problem. Now, we have to ask where we have to answer various questions as w have discussed just now. So, this is just a picture to give our idea and the distance was 30 kilometres at the time when the light signal was sent from this particular person towards H, which you called as a diastral signal just to give the information that is a something wrong with me please come and help me.

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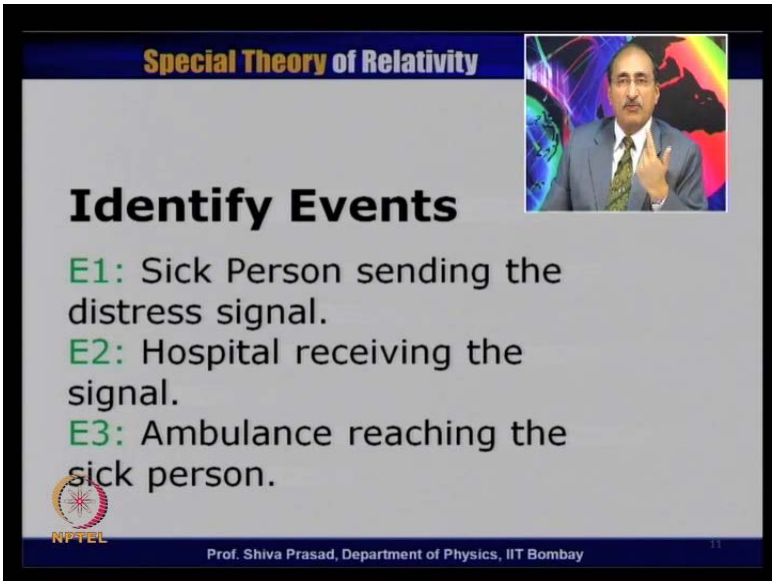

As I said our first job in all these problems is to identify the events, which are the events which are important in this particular case and then eventually start looking and filling the event table by putting to coordinates of all these events. So, if you look at this statement of the problem and think little bit, you realise that they are three events, which are important in this events, in this particular problem, that three events which are important in this particular problem.

First which I call as event number one; is the sick person sending the diastral signal. So, the person is sick, so first thing that he does is to send the diastral signal he sends a signal, that is the event number one. Then the signal is received by the hospital that is event number two, so hospital receiving the signal is event number two. Of course as we have said that is the same time the instruct the ambulance. So, instructing the ambulance excreta everything occurs exactly there is the same time.

So, they also they are all included in event number two. So, event number two you also find that the ambulance is just in front of hospital that is also an event number two. So, event number two compromises of various things in that cells. Now, event number three is that actually the person who is sick, he is actually able to the ambulance is the reach that particular person. So, that is my event number three that is ambulance reaching the sick person. So, these are the three important events as for is this particular problem is concert, sick person sending the diastral signal event number one, hospital receiving the signal event number two, then ambulance reaching the sick person is event number three. Now, let us choose one particular frame of reference and try to write the coordinates of all these three events in that particular frame of references.

If you realise that in this particular problem most, in fact all the information has been given in H frame that is the hospital frame or the ground frame. So, it is much easier to write the events in this particular frame itself. So, what I will do to make it systematic it is much better to go systematically by solving relativity problems. What we will do is to write the coordinates of these three events in H frame, the hospital frame that is a ground frame. So, that is the way first we will do, then we will look into other frames and then we will go ahead with a typical problem.

So, let us first try to write the coordinate of event number one in H frame the hospital frame. It has been given this particular problem that the light signal was sent by the

person when he was at a distance of 30 kilometres. So, obviously and of course we have said that hospital is situated at the origin. So, the coordinate of this particular event, the x coordinate of this particular event is obviously 30 kilometres that is very, very clear. We should also look at about the time aspect because as per the question is concerned to this, is nothing specifically has been mentioned about in the problem over time. But we have to choose an origin for time and looking at the Lorentz transformation, we have to choose the time when the origins coincide as we have all this said.

So, it is quite simple to realise that most of the problem is given between H and A frame the hospital and ambulance frame. And it so happens that when the light signal is received in H frame that is also the time when ambulance is passing by x sight. So, let us call that time as t is equal to 0 in H frame and that will also auto automatically turn out to be t prime is equal to 0 in A frame that is the ambulance frame. So, time is chosen at that particular step.

But we realise that though the hospital receive the signal at time t is equal to 0, the event must have occurred before, because light to finite time to reach this particular person. See as we have seen in our earlier lecture that it is not important when I get the information what is the of importance at what time the signal or what time the event to place. For example, if somebody is departing from railway station or one particular or by flight and somebody comes and tells me that to gate to some of your relative is going away and somebody has gone to see him of. Then he comes back he says oh flight was in time, no really departed at 9 o clock, it does not that mean you got the information even you would obviously did not get the information at 9 o clock, you might have got about 10 o clock or half past 9.

But it does not mean that the event occur at 9 or half past 9 or the event occur at 10 o clock, you know that event occurred actually at 9 o clock. So similarly, was I have received the light signal in my watch time t is equal to 0, but I also realise that this light we have taken finite a bound of time to come from 30 kilometres. So, this event must have occurred earlier. So, when I have to find out the time of this particular event I have to take this particular factor also into concentration.

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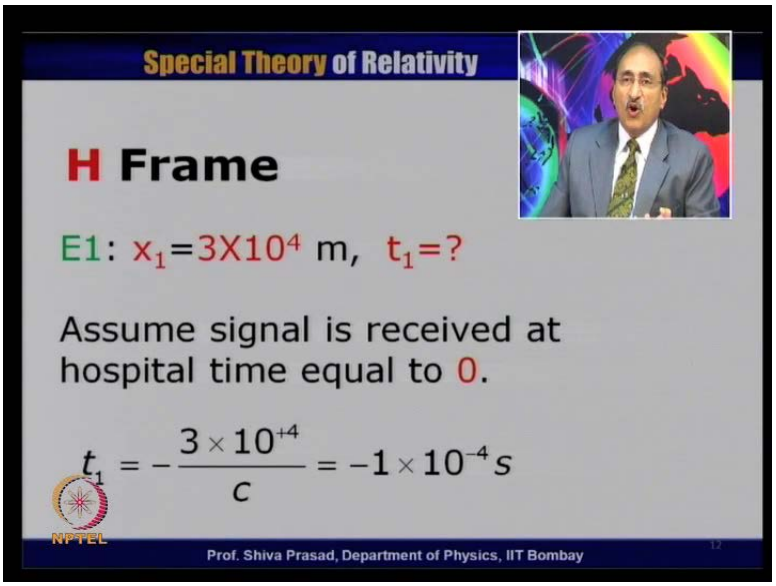

So, this is what I have written in this particular transparency that in H frame event number one occurred at quadrant is equal to event number one, so I am calling this as 1. A 30 kilometres which in s i system turns of to be 3 into 10 to power 4 meters. So, the x coordinates of the event number one is 3 into 10 to power 4 meters. The time for this particular event has to be found out by finding out, how much time light took to travel to the hospital.

As I know the distance is 30 kilometres, the light must have taken 30 kilometres divided by its speed that mush time to reach there and obviously this event would have occurred in negative time, because time was 0 when the light signal was received at the hospital. So, there for this time t 1 will be equal to three into 10 to power 4, which is the distance at which the event occurred divided by the speed of light. And if you calculate this number assuming c is equal to 3 into 10 to power 8 meters per second, you get this time t 1 as minus 1 into 10 to power minus 4 second, it means this event must have occurred at time 1 into 10 to power minus 4 second before the light released. So, this is the point which has to be realised at what we have to look is at that time when the event actually took place and that time is a negative time even though the person at the hospital receive the signal at time is equal to 0.

So, I have found out what is t 1. Now, let us look at E 2 the second event, for which to find out x coordinate and time coordinate is very simple. Because, when he receive the

signal obviously it was at its origin, he was in the hospital, the person whose work the person who received the signal at the hospital, he was obviously sitting in the hospital and therefore, was at the origin. And we have already said that when he received the signal time was equal to 0. So, the coordinate x coordinate of the second event is 0 and the time coordinate of the second event is also 0, it is the most simple that is what I am written here.

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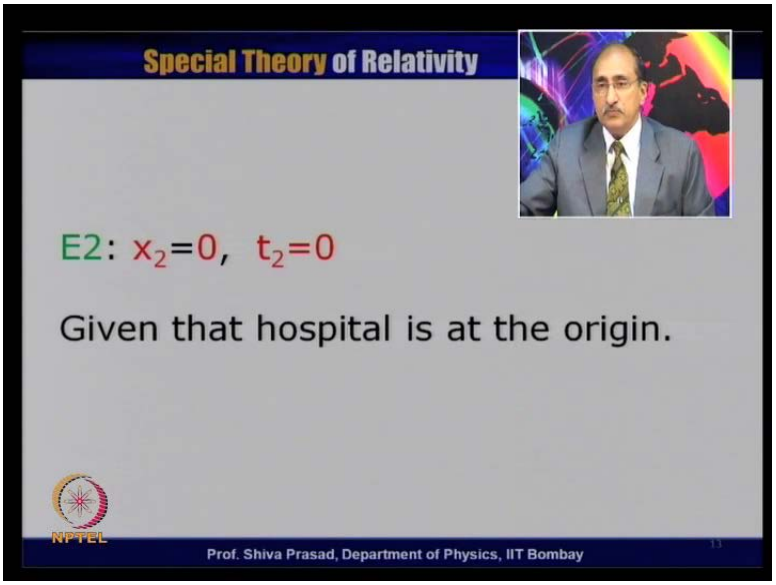

E 2 is equal to x 2 is equal to 0, the second event so I have put two here, second event so I have put two here. So, x 2 is equal to 0, t 2 is equal to 0 is the coordinate for the event number two. Of course we have said they have given in the hospital is the origin, which has already been given in the problem.

Now, let us come the event number three, this is little more tricky, but remember everything has been given in the H frame. So, it is not that all the tricky in fact you do not require lots of transformation here. So, I still do not know what at what value of x the person actually caught the ambulance actually caught the sick person or ambulance met the sick person, also we do not know the time we have to work it out. I realise that during the time light was approaching the hospital this person must be moving to the right.

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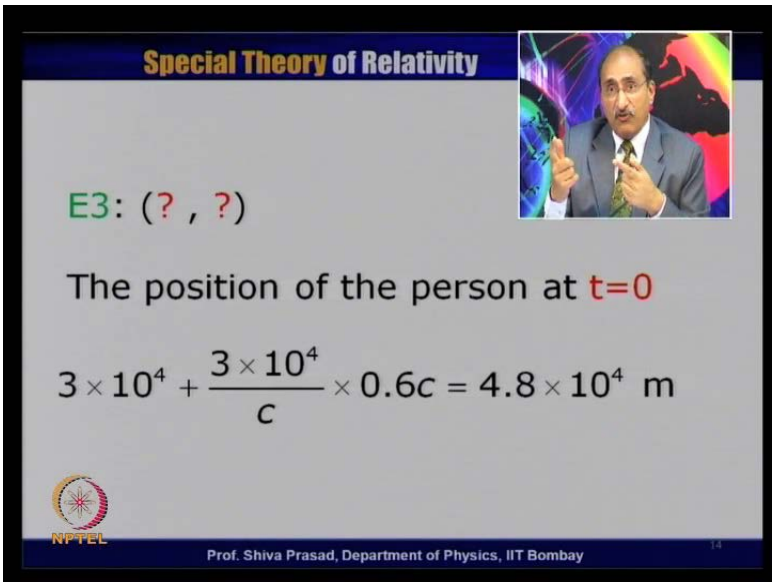

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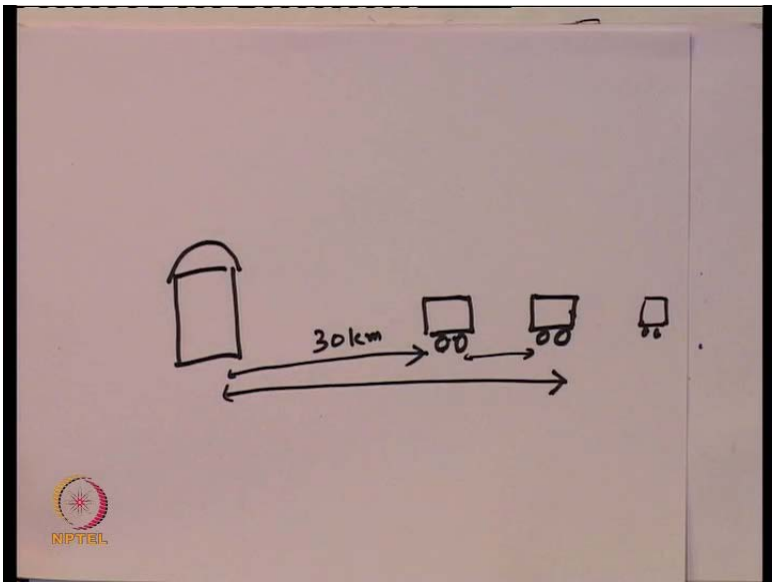

Let me just draw a figure to explain this particular thing. So, if I say that this is my hospital and a t is equal to 0, this is where was the my train, this distance for 30 kilometres. But light took certain amount of time to reach from here to here, during this particular time this particular train compartment would have gone ahead. So, at time t is equal to 0 this particular train would be somewhere of the right hand side. So, this is the situation that this particular person sends a light signal, it took certain amount of time for light to reach here.

During this particular time this particular train compartment move to the right and be somewhere here. Then there was an ambulance, which went and try to catch this particular person by the time this particular person in the ambulance goes and catch this person in the train. This event obviously would have occurred at a for the further value of x, because this person continuously keeps on moving with a speed of 0 point 6 c. So, let us first find out the position this particular position at time t is equal to 0, where the person was situated at time t is equal to 0, remember this distance was 30 kilometres.

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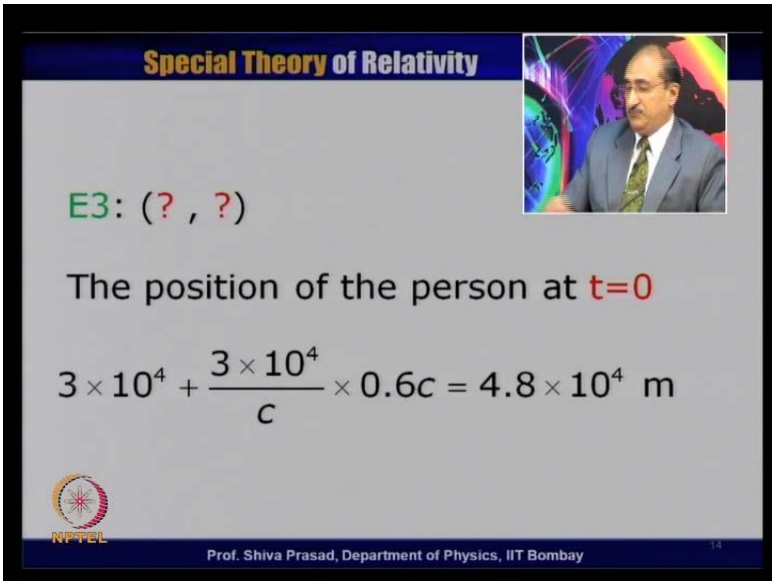

So, if this distance was thirty kilometres all I have to find out that, during the time light travel from here to here, how much this person went from this side to this side? I have already known the time that light took to reach here this train is moving with a speed of 0.6 c in this particular direction. So, 0.6 c is multiplied by the time will be this distance that is what I have calculated, added this to 30 to get the distance at t is equals to 0. So, that is what I have written, the position of the person at t is equal to 0, is this original 30 kilometres plus the time that light took to reach the hospital multiplied by the speed of the person which is 0.6 c. If you just add these numbers in terms of to be 4.8 into 10 to power 4 meters, it means at that instant of time when the light reach the hospital this particular person was at a distance of 48 kilometres, his coordinate was 48 kilo meters.

But remember this still not x 3, because x 3 event involves a catching the person it will take still further more time before the ambulance will be about to reach, because this is

the time when ambulance was the still at the origin of H. It will still take further of certain amount of time to for the ambulance to reach this person. Therefore, obviously what I have calculated here s 48 kilometres is not x 3, I have to still find out x 3.

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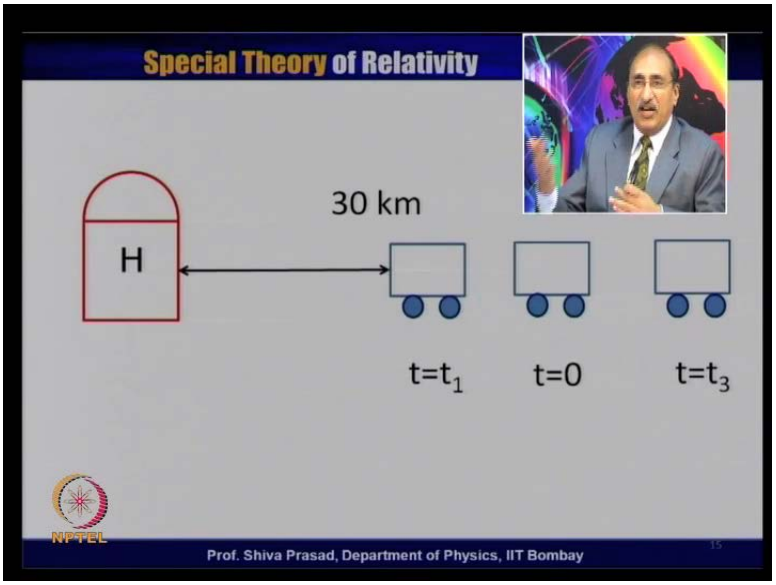

So, this is the picture which I had drawn on the paper, same thing has been drawn here at time t is equal to t 1, he was at the distance of 30 kilometres, at time t is equal to 0 it had got 18 kilometres further off. So, this distance was 18 kilometres and at really time t is equal to t 3 when this ambulance will come here and catch him the distance has to be further off. So, I have to find out this time t 3 and I have to find out this value of the coordinate, not this value of the coordinate. So, this is I have just found as an intermediately step, but eventually the event is this person being caught by the ambulance and that will occur at a different value of x. So, let us try to find out that particular value of x.

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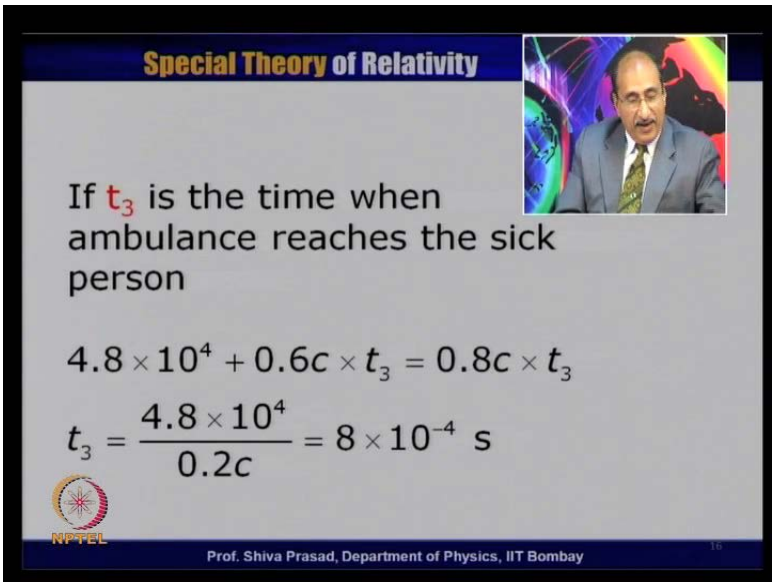

Let us assume the t 3 is the time that ambulance reaches the sick person, of course this time is also an H frame remember. And this particular moment everything is being talked in H frame no other frame, I am not confusing with any I have written in the first transference the H frame and all those things are being talked only in H frame. I am not looking at any other frame, I am not even thinking about any other frame the A frame or the P frame, I am only thinking about H frame try to do all calculations in H frame.

Hence, I try to describe into different frame then I will think only that particular frame, I will not think in terms of H frame, this is something which is very very important not to confuse between the frames, when we are trying to work out the problems of special theory of relativity. So, let us assume that time in the watch of the person sitting at the hospital must t 3 when the third event of place it means when ambulance reach this sick person.

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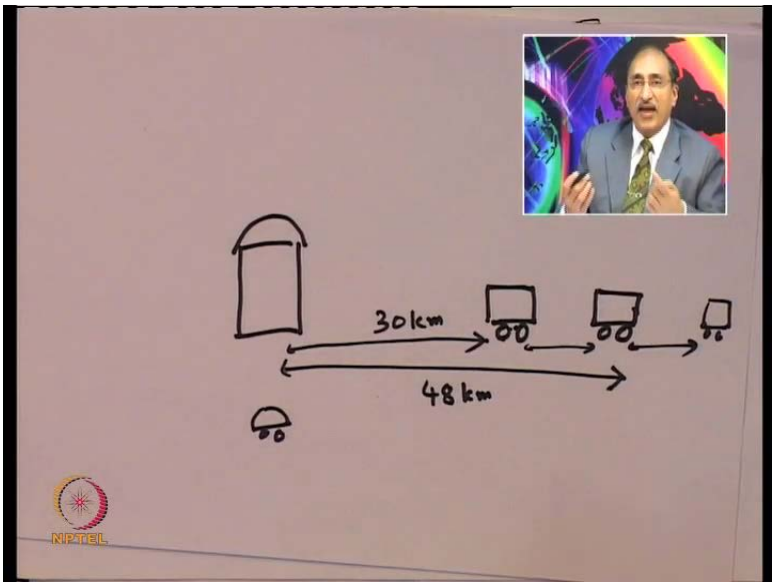

Already, we had agreed that the this distance was 48 kilometres and at this particular time the watch of this particular person show out time t is equal to 0. And if time t is equal to t 3, it means during this particular t 3 time this person was also going in front with the speed of 0.6 c. While this particular ambulance which is here was travelling to the right with a speed of 0.8 c and in the same time t 3 the distance covered by this particular person this particular person in the sitting in the ambulance will be same, will be this to this.

And the same distance in the same time I am sorry, in the same time this particular ambulance will travel a distance from here to here. Original distance was 48 kilometres, so 48 kilometres plus in t 3 time whatever distance this particular train travels that will be this distance. And that distance must be same as a distance travelled by this particular car in time t 3, of course this car travels with the speed of 0.8 c. Hence, this distance travel by car in time t 3, this distance travelled by the train in time t 3 plus original distance 48 kilometres must give the must be equal, and therefore I can evaluate t 3. This is what I have written in this particular transparency.

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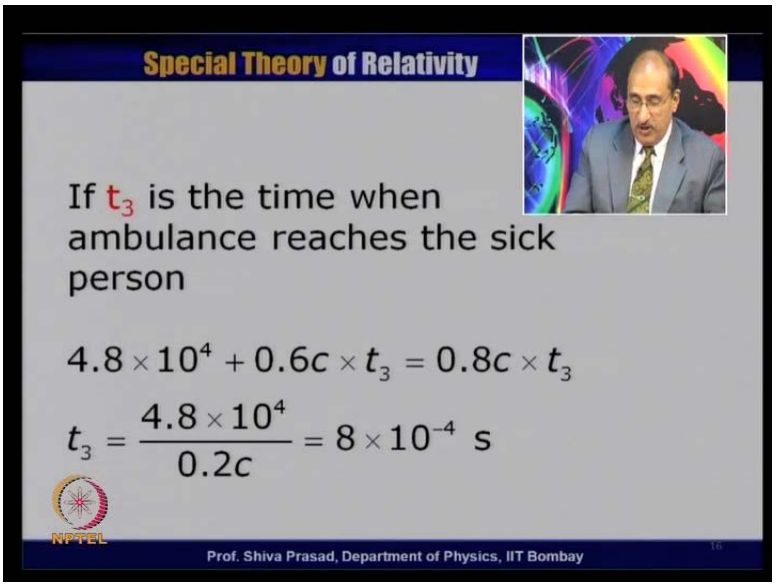

If t 3 is the time taken when ambulance reaches the sick person, then original distance 48 kilometres plus the distance moved by that train in time t 3 will be 0.6 c multiplied by t 3. Because, this p this 0.6 c must be equal to because as for as the ambulance has concert it also travels with the same time t 3, what with now a speed of 0.8 c must be equal to 0.8 c multiplied by t 3. So, from this you can calculate the time t 3, if you take this to the right hand side 0.8 c minus 0.6 b c becomes 0.2 c.

So, which comes in denominator and then I calculate time t 3, which turns out to be equal to 8 into 10 to power minus 4 second. So, this is the time for the event number three or this is the time when a person is actually caught by the ambulance. So, t 3 that is why I have written this as t 3 this is the time for third event, which is taking prise a time a denoted for minus 4 second or if I have to find out the x coordinate.

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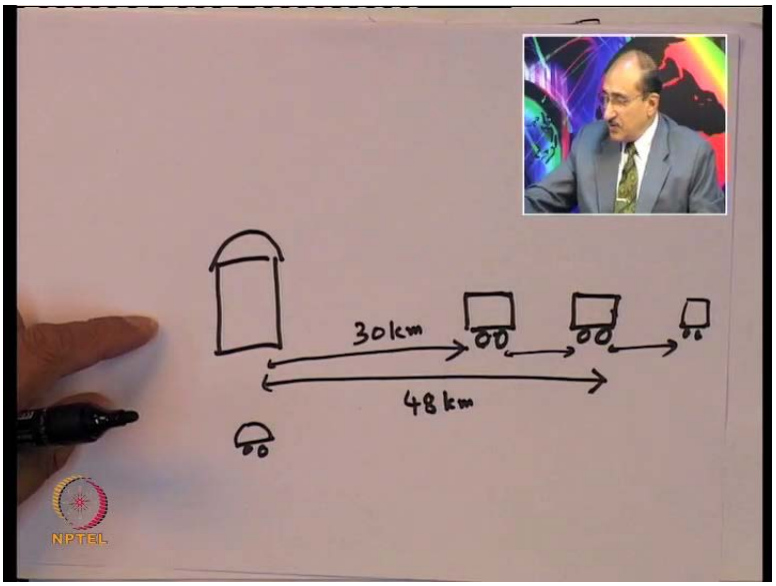

If I have to find out this particular distance from the origin, this will be just t 3 multiplied by this speed or 48 plus 0.6 c multiplied by t 3.

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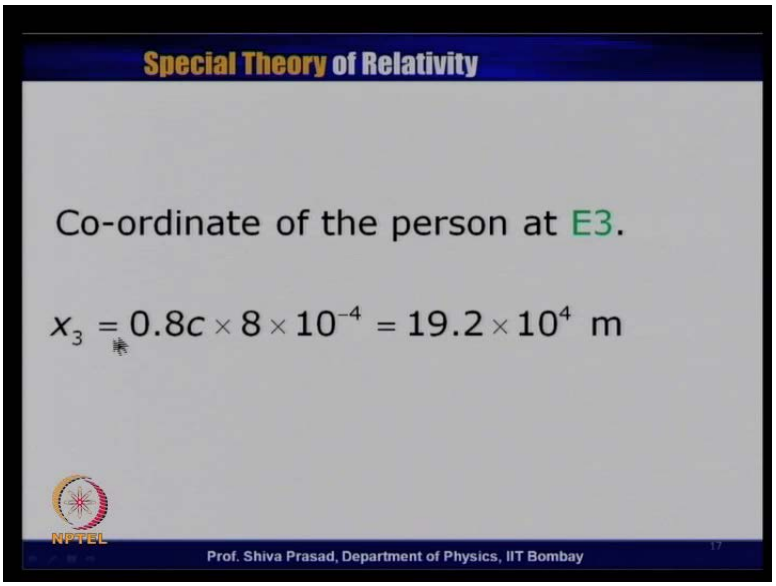

So, n if the method will give me the coordinate x 3, that is what I have done in the next transparency. The coordinate of the person at the time of event three that is the ambulance catching this particular person sick person, occurred at a time 0.8 c is this speed of the ambulance multiplied by the time, which we have just now calculated which was here 8 into 10 t power minus 4 second, this what I have calculated.

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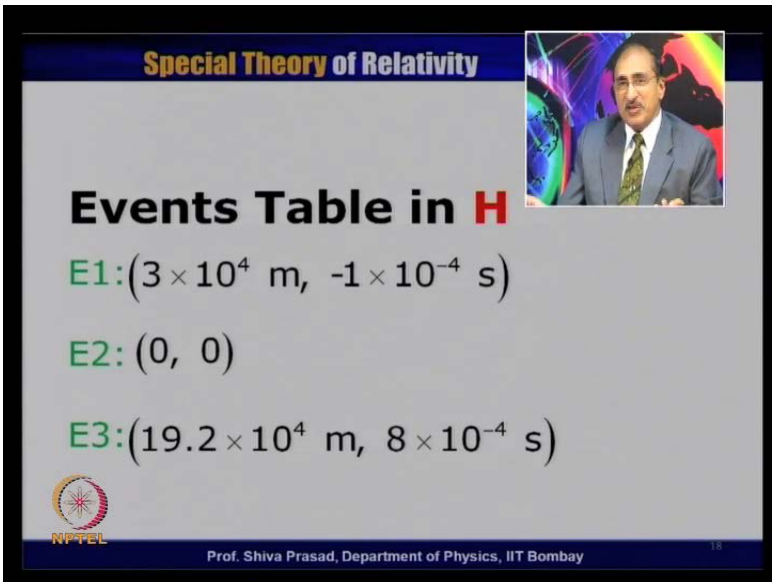

If I multiply this by this, I find out the distance at which this event or the coordinate at which this particular event would have occurred, which is 19.2 into 10 to power 4 meters, which happens at 192 kilometres quite far off. So, at the distance of 192 kilometres or a deco ordinate of x 3 is equal to 19.2 into 10 to power 4 meters even number 3 occurs. Now, I have full this particular event table in H frame the hospital frame. So, which I have written here the minus transparency, this is events table in H frame of references.

Event number one occurring at a distance of 30 kilometres and it time of minus 1 into 10 to power of minus 4 second, e two simple 0, 0, e 3 occurs at a distance of 19.2 or at coordinate it is better to sat coordinate, it occurs at a coordinate of 19.2 into 10 to power 4 meters and at a time of 8 into 10 to power minus 4 second. So, have filled the event table in H frame of references, once I have got all these information in one frame of references then it is a very simple job to transform these information to any other frame, all you have to do is to use Lorenz transformation.

Now, let us try to fill this particular table in A frame the ambulance frame because H and A obviously satisfy all the conditions or all the criterion which we have imposed for using the Lorenz transformation, just simply use the Lorenz transformation. And I get the all these events into a free, that is what I have been doing here.

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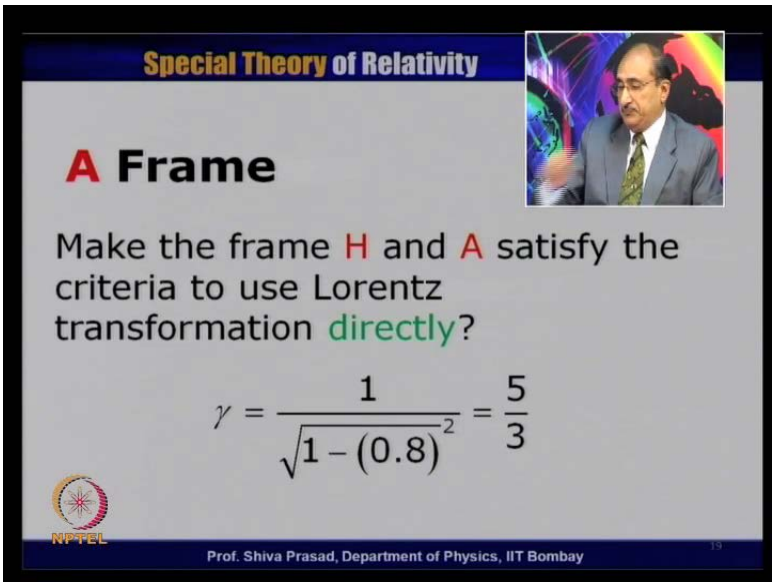

I go to A frame, now stop thinking about H frame start thinking in terms of ambulance frame. Make the frame H and A satisfy the criteria to use Lorenz transformation directly, which essentially mean that I have to put t prime is equal to 0 exactly at the same time, when the origins were coincident and also I assume that that was the origin of the ambulance. Because, I have to transform for these two frames so I have to use the relative velocity between these frames, which is a relative velocity between hospital and the ambulance frame.

Because, the velocity is already been given in h frame, so same is the relative velocity because that is how we have define relative velocity, so v is 0.8 c. Substitute in this expression for gamma, c square will cancel with the c square this gamma turns out to be equal to 5 by 3. So, the gamma that I am going to use in this particular equation will be 5 by 3 and v will be equal to 0.8 c. Because, I am going from H frame to A frame and the relative speed between these two frames is 0.8 c.

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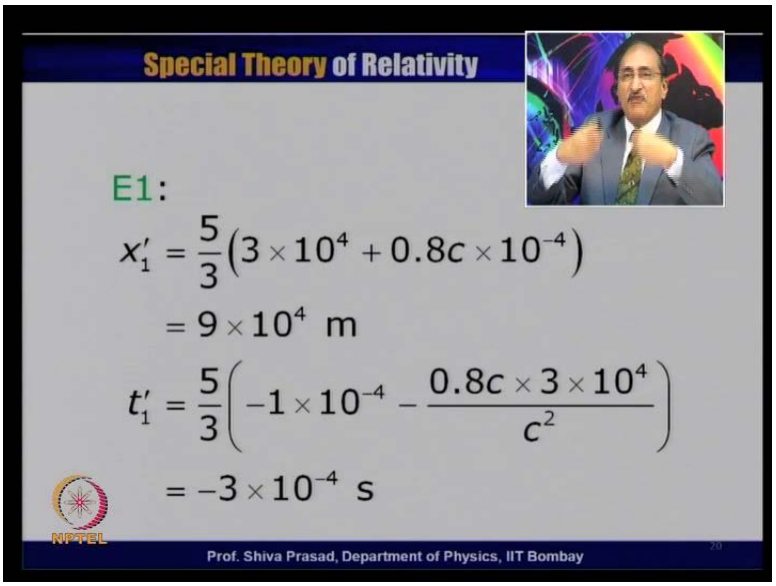

Even the first event it occur a distance of 30 kilometres. So, this is x minus v t, goes t is negative minus ten to the power of minus four. So, this becomes plus, so 3 into 10 to power 4 plus 0.8 c into 10 to power minus 4 multiply by gamma, so gamma x minus v t. That is what is going to be me x 1 prime, which turns out to be equal to 9 into 10 to power 4 meters, it means according to ambulance frame this event occurred at a coordinate of a 90 kilometres, 9 into 10 to power 4 metres. Same two time transformation t 1 prime will be equal to 5 by 3, the time which is minus 1 it is gamma t prime minus v, x upon c square.

So, this is t which minus 1 into ten to power minus 4, v is 0.8 c, x was 30 kilometres 3 into 10 to power 4 meters divided by c square, substitute the value of c you will find out that this time is minus 3 into 10 to power minus 4 second. So, according to an observer in ambulance also this event occurred before this particular person had reached the hospital. We must trying to drive towards a hospital and this particular event of diastral sending it has been sent occurred a 3 into 10 to power minus 4 seconds before this particular person in the ambulance was to reach the hospital.

Better (( )) the hospital to reach, because as per these concert he will feel that hospital is approaching towards him or the hospital to reach him, to we sort of again mix up the word we say this person will reach hospital. So, that is the way we visualise if I am sitting in a car, I always visualise that I am going to reach to this particular destination.

But actually what I visualise when I am sitting in the car that the hospital is coming to me, thought we are somewhat casual in speaking and saying that I am going to reach the hospital. As per as event two is concerned if you want to do a lots of transformation you can do it, but it is really not necessary. Because, I know that when x is equals to 0, t is equals to 0, x prime has to be 0, t prime also has to be equal to 0.

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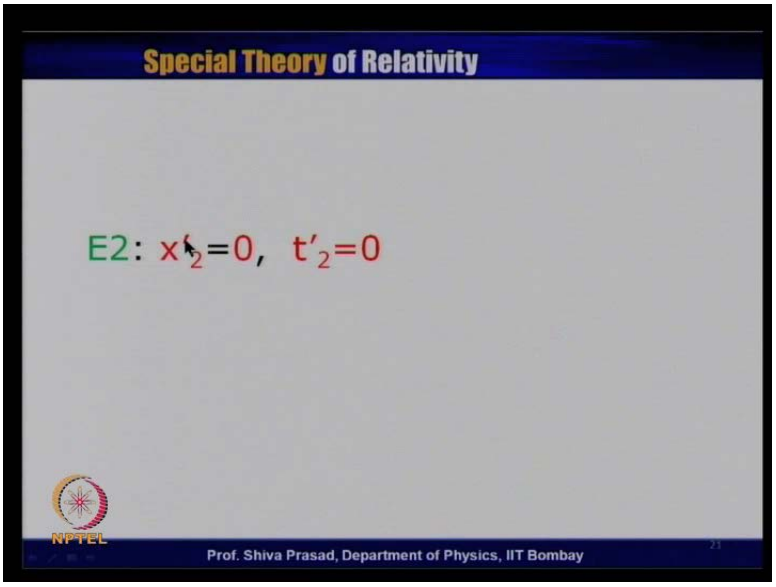

So, I just put E 2 as x 2 prime is equal to 0, t 2 prime is equal to 0, which I must remind you that you know this prime side thing I have used for the A frame for the H frame I have not use frame. Actually we will be discussing later about the P frame then I will be using double frame, so just to make it clear. Now, let us look at event three; let us try to found out the coordinates of event number three in the ambulance frame. Strictly speaking if one is smart and one has understood things quite well.

To find the coordinates of event number three in ambulance frame, one need not have to use Lorenz transformation, but can find it by a shorter rout. But let start do that at the moment, at the moment let us just try to write Lorenz transformation and were find out or work out the coordinates of event number three in ambulance frame. Later we will discuss which, is the shorter way of solving this particular this particular part of the problem of just finding out the coordinates of E 3 in ambulance frame.

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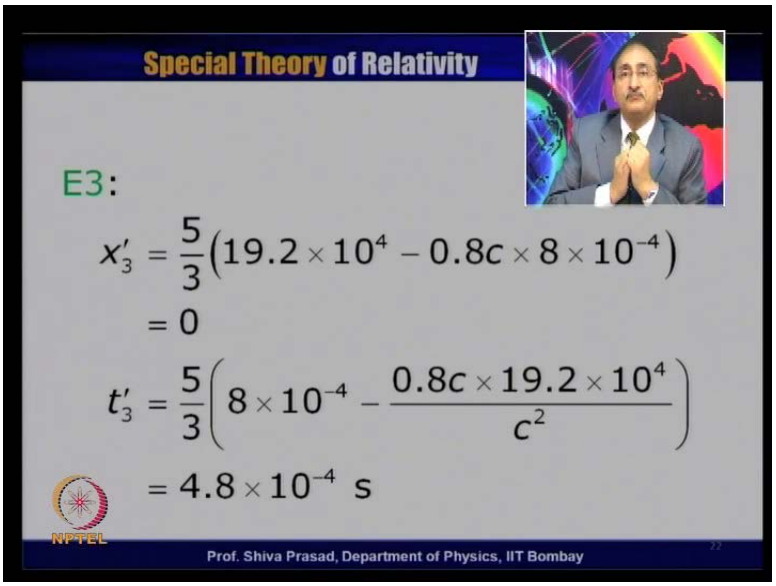

So, let us look at the transparency for event number three, I have to find out x 3 prime and t 3 prime. x 3 prime gamma x minus v t, gamma we are already calculated 5 by 3, for x 3 that is the coordinate of the event in hospital frame of reference it occurred at 19.2 into 10 to power 4 meters, these 0.8 c time was 8 into 10 to power minus 4 second. So, it is a gamma x minus v t, it so happens just calculate this number you will get it equal to 0. Let us look at the time t 3 prime, t 3 prime is equal to gamma 5 by 3, t is 8 into 10 to power of minus 4 second, minus v which is 0.8 c relative velocity between the frames, x 19.2 into 10 to power 4 minutes divided by c square, gamma t minus v, x divided by c square.

If you calculate this number distance have to be 4.8 into 10 to power minus 4 seconds. If you look at this particular value of E 3, probably you would have realise that the shortcut that I was trying to mention. See if you are really sitting with respective an observer in the ambulance, what you would feel that the car was approaching towards him? The car means the that time compartment was approaching towards you, like hospital was initially approaching the hospital pass by, then you have the train compartment it just coming towards him and then you catch them that is the event number three.

So, according to an observer in the ambulance, hospital was approaching him, the compartment train compartment in which the sick person is sitting it is also approaching towards him. So, event number three occurred when you are sitting there and what is

event number three, event number three is sick person and this particular person in the ambulance meeting together, their combining at the same point. So obviously, this particular event occurred exactly when he was sitting here.

Similarly, event number two, event number two is the event when the light reach this particular person, person sitting on the hospital. And exactly at the same time the ambulance was also passing by. So, this event number two also coincided with this particular ambulance, coinciding ambulance coordinate coinciding with the hospital coordinate. As you can see here also, if you could the transparencies x 2 prime was equal to 0, c prime is equal to be 0,

Both the coordinates are note to be same because both the events are occurring exactly in the same position. So, third person sitting in the ambulance, the hospital reaching him occurred when he was it is origin, obviously the train reaching him will also occur at when he was at its origin. So, without working out Lorenz transformation I could have immediately guess the answer that x 3 prime should be equal to 0. Now, you would have realise that I could have calculate t 3 prime also very easily, because just now we said that event number two occurred when x two prime was equal to 0, event number three occurred when x 3 prime was equal to 0.

It means both the events occurred in the same position, if both the event occurred at the same position then the time interval between these two events must be the proper time interval. Remember it is in the A frame that both the event occurred in the same position. So, time interval is proper in A's frame of reference not in hospital frame of references because at hospital frame of reference this particular catching is done at distance quite for away, while the second event occurred at x is equal to 0. But in ambulance frame both these events are occurred a exactly at the same position.

Therefore the time interval t 2 and for event number two and event number three, the time difference between these two is a proper time interval in ambulance frame of reference. Therefore in hospital frame of reference it must have been dilated. See if I know the gamma values knowing the time interval which I have already calculated, I have already calculated t 1, t 3 I can always sorry t 2 and t 3 I can always find out t 2 minus t 3 or minus t 2. And from that particular time interval I can always find out t 2

prime minus t 3 prime or t 3 prime minus t 2 prime by just using the gamma. That is what I have written the next transparency.

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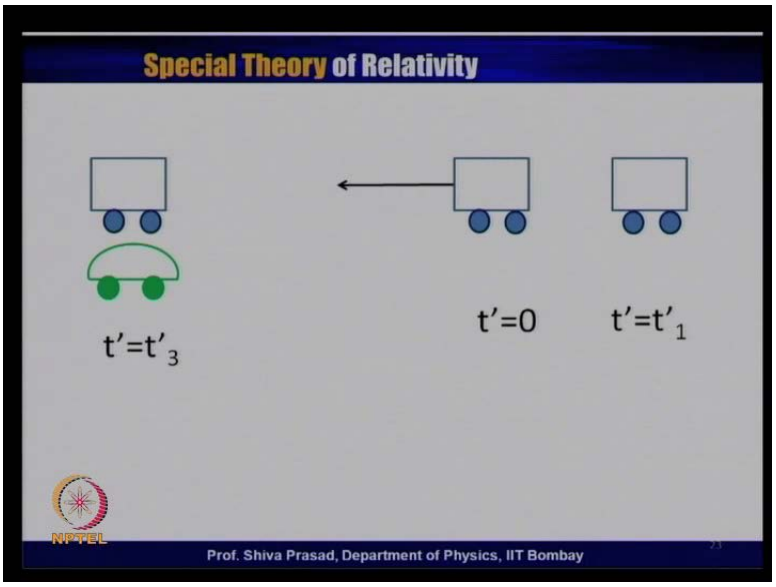

It is just a picture showing whatever I have said that this is somewhat different from the scene which was being looked for by a person in the hospital frame of reference. As for as the ambulance frame of references concert, the first event occurred furthest off, Remember just contrary to the hospital frame of in hospital frame of reference the chorology of the events were different. The first event occurred when he was closest to hospital, second event occurred when he was further away from the hospital, third event occurred when he was still away from the hospital.

But in the case of ambulance is different in the case of ambulance first event occurred this particular train compartment was furthest far from there. When the second event occurred that particular thing has come nearer to him and the third event occurred at the time it was just close to him, it was just at the same place where he was situated. So, the distance as the event events keep on coming keeps on decreasing in the frame of reference of the ambulance.

Also I would like to realise, I would like to mention that as per as the event number two is concert at that time the origin of ambulance was coinciding with the origin of the hospital and not with the train, train was still further of him. So, this is this picture which is showing these behaviour that at first event this particular frame was further of him and

suppose to be moving towards him. And second event when it occurred, but still not at the origin it was further far from him.

And it was a third event when this particular train compartment has reached this particular ambulance. Again I repeat it is important that we should be clear about our frames of references, we should be able to translate our thinking with respect to an observer who is sitting in that particular frame of references, then we are not likely to make an error.

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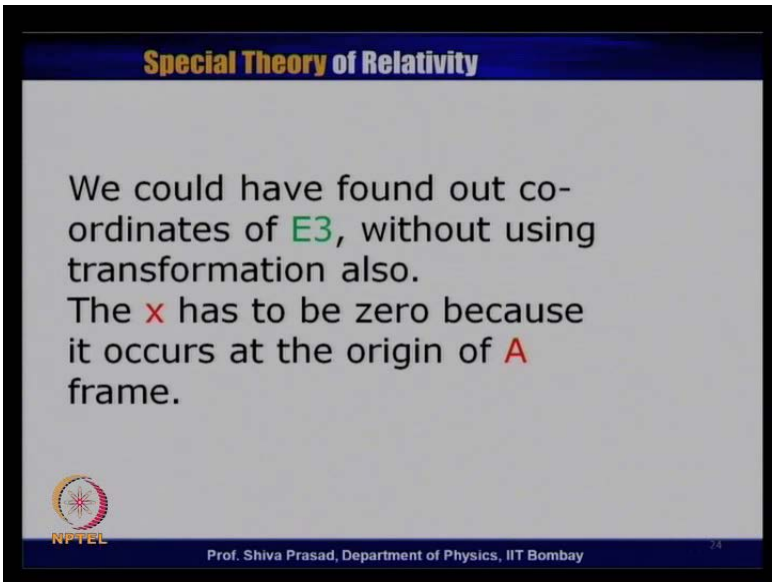

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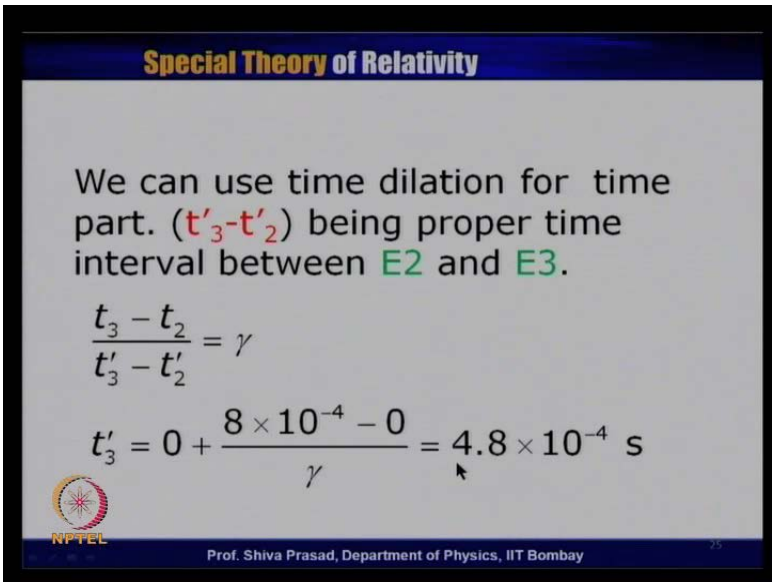

So, this was the shortcut I was describing. We could have found out the coordinates of E 3 without using transformation also, the x a has to be 0, because it occurs at the origin a frame, this is what we have to just describe, that it has to o occur at x is equal to 0. So, nothing surprising I need not to have worked out the Lorenz transformation equation. Second thing to use time ablation, we can use time ablation for the time part we realise that t 3 prime minus t 2 prime, it means the time difference between third event and second event as seen ambulance frame of reference being proper time interval between E 2 and E 3. It is actually in the ambulance frame that this particular time interval is proper.

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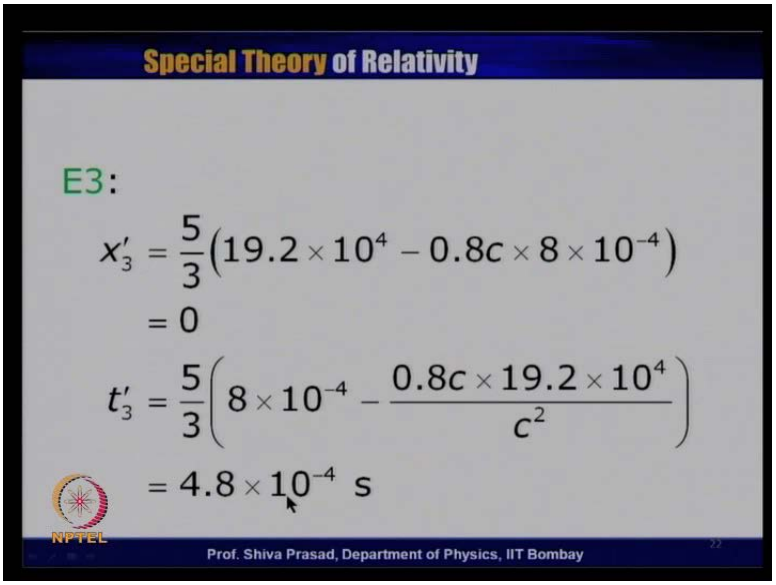

There for in this particular frame of reference which is the hospital frame of reference t 3 minus t 2, must be gamma multiplied by this particular time interval, because this time interval is proper, so t 3 minus t 2 must be a dilated time. So, if I calculate from this substitute this particular values, I just now calculated t 3 minus t 2 prime, which was 8 into 10 to power minus 4 for t 2 and t 2 prime times were 0 anyway.

So, this is equal to 0, this is also equal to 0, I substitute in this particular expression I get t 3 prime is equal to 4.8 into 10 to power minus 4 second, which is exactly the same result which we had obtained here. So, we can very easily see that we get the same result without actually using the Lorenz transformation we can just use the time ablation formula still get the same result.

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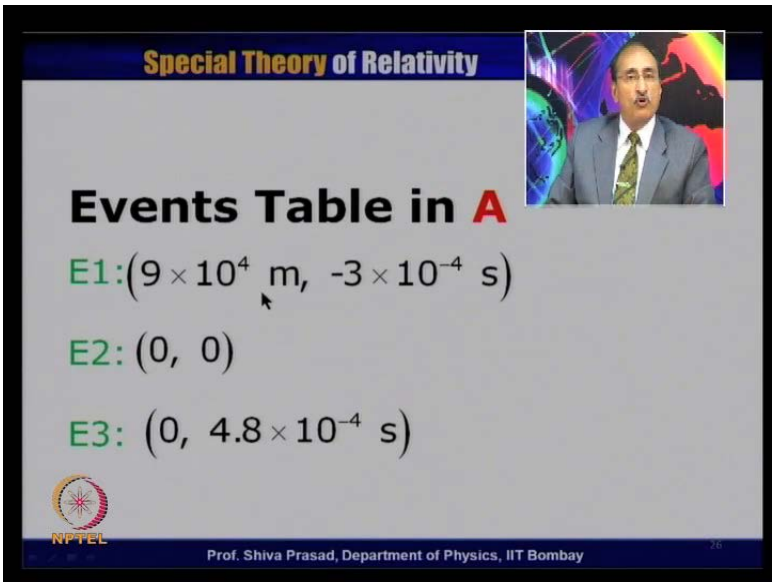

Now, I know all the events and I have filled the event table in the ambulance frame of reference. So, this transparency shows the event table in A frame. Event number one occurred at 9 into 10 to power 4 meters and at a time minus 3 into 10 to power minus 4 second, event two occurred at x is equal to 0 t is equal to 0, event number three occurred at x is equal to 0 and at time at 4.8 into 10 to power minus 4 second. So, why event table is filled both in H frame and A frame. Now, let us look back at the question see that weather we have answered all the questions, if there something still remaining.

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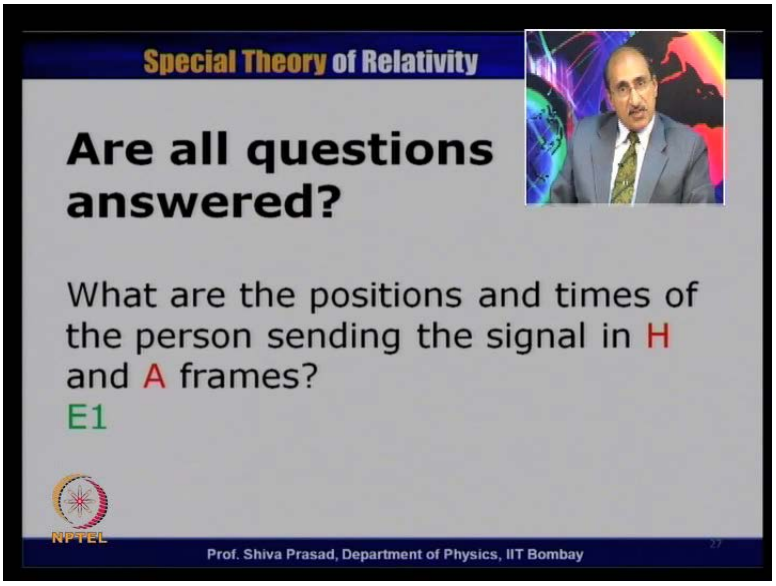

Are all the question answered? this is a first question which you have asked; What are the positions and times of the persons sending the signal in H and A frames?

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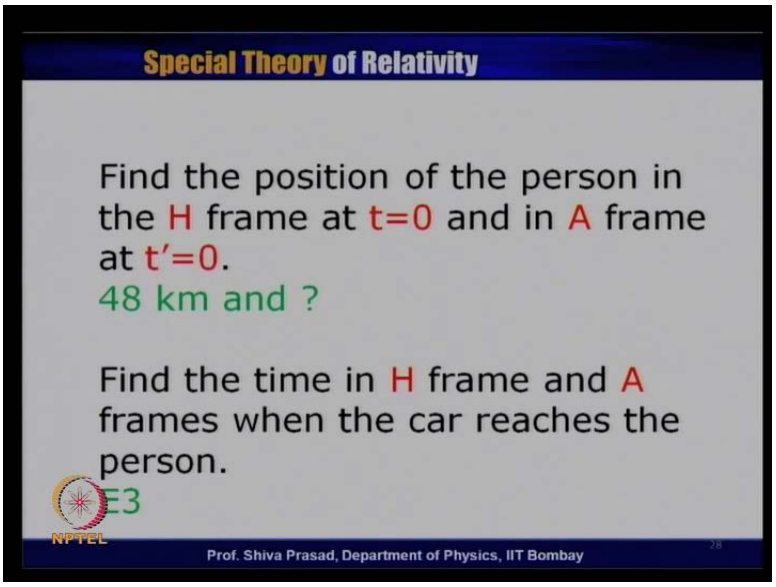

It means basically when event number one occurred event number one occurred I have already known the coordinates I know the value of x, I know the value of time both in H frame both in A frame. I have to just pick those values and plug it here that is the answer.

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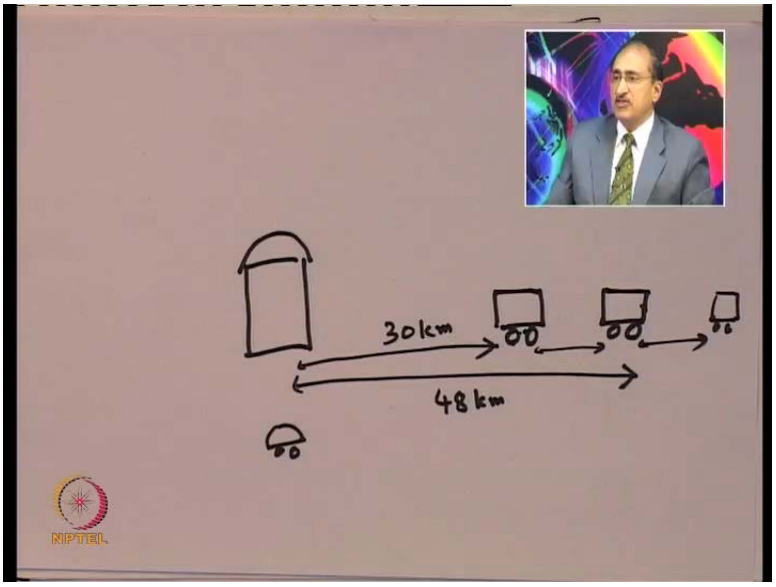

Second part; Find the position of the person in the H frame at t is equal to 0 and in A frame at t prime is equal to 0? See at H frame at time is equal to 0 I had calculated the interview position remember, we go back to this picture here.

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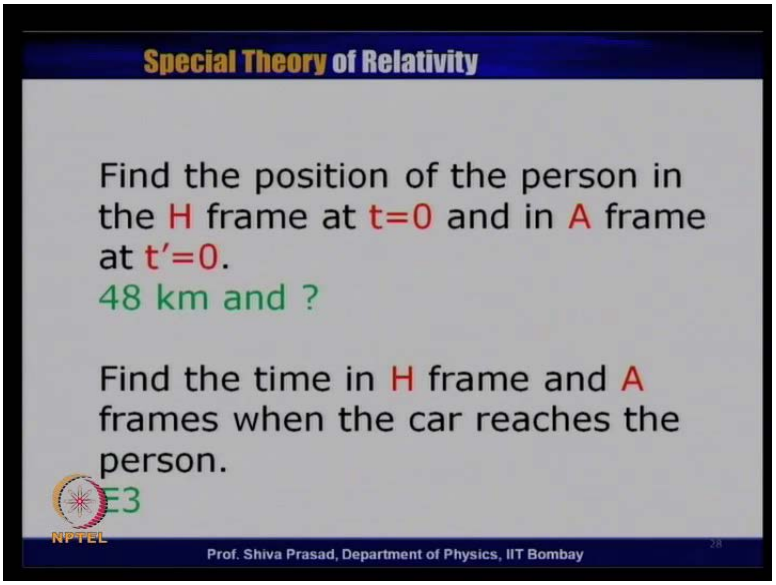

I had already calculated the time t is equal to 0, this distance is 48 kilometres. See originally it was 30 kilometres during that time light took to come to here this has moved further way by 18 kilometres. So, this distance is 48 kilometres. So, this I have already calculated, as an intermediate step both those was not really related to an event but we have already calculated, so this distance is forty eight kilometres. But what is the position of the person in A frame at t prime is equal to 0, I have not calculated yet. Because, this particular event whatever we are talking is occurring at different time and different values of x, so this need not be correct, I have to still calculate this particular value.

Third thing; Find the time in H frame and A frames when the car reaches the person? When car reaches the person or the ambulance reaches the person, this car means ambulance, when this ambulance reaches the person obviously this is time E 3, this is event E 3. So, whatever is the value of x that we have calculated, we have calculated both in H frame and A frame. So, we have to just pick up those numbers plug it here, event number three tells me the solution of the last part of the problem. We have to still find out what is the position of the person at t prime is equal to 0.

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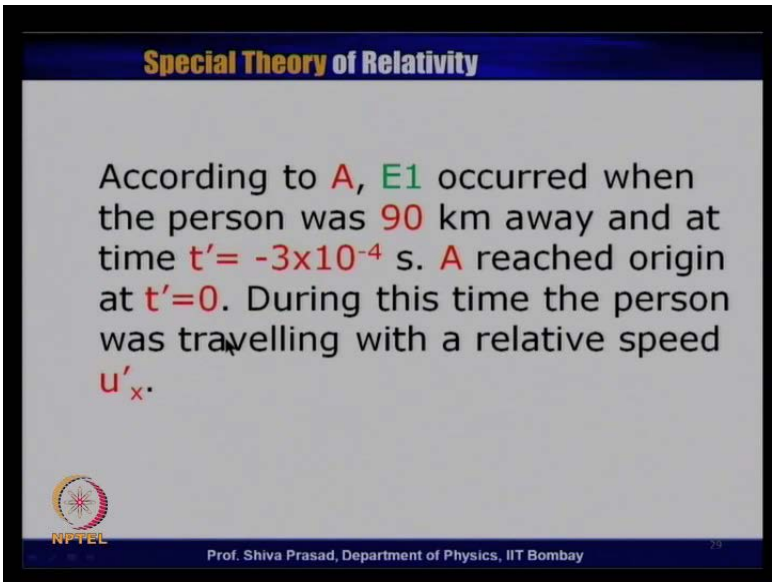

Let us go back here, according to A that is an ambulance person, event occurred the event number one occurred when the person was at 90 kilometres away at time t prime is equal to minus 3 into 10 to power minus 4 seconds. Remember these were the coordinates of event number one in ambulance frame. So, it means the distance of that particular person who had send the light signal was 90 kilometres, but the time in the watch of that particular observer sitting in the ambulance was minus 3 into 10 to power minus 4 second.

The problem is simple, I am looking only at the a frame, question is that, if there is a particular train which is at a distance on 90 kilometres away, from me at a time minus 3 into 10 to power minus 4 second, where it would be at time t prime is equal to 0. I can find out if I know, what is the speed of this particular train related to me? So, I have to do a velocity transformation, if I do a velocity transformation I can find out what is the speed of this particular train which is coming towards me.

Then, I can find out how much distance it would have travelled in time 3 into 10 to power minus 4 seconds, subtract that time that that distance sorry, from 90 kilometres I will find out where this particular person will be situated at time t prime is equal to 0. So, during this time as I have said that person was travelling with a relative speed of u x prime, let us calculate u x prime.

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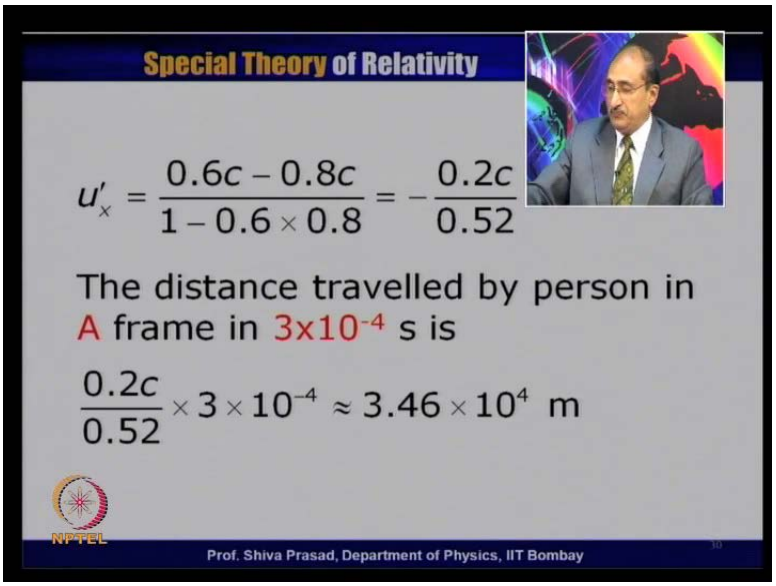

Apply a velocity transformation formula. I have to go from a frame of hospital to the frame of ambulance relative speed v is 0.8 c. u x this is a speed of train, which I want to transform I want to find out what is the speed of train in ambulance frame of reference. So, u x is the speed of train as seen in the hospital frame of reference u x. So, this is u x 0.6 c minus v, which is 0.8 c, 1 minus u x into v divided by c square. So, there was a c here there was a c here that cancels with that c square.

So, you get this value of u x prime, which is minus 0.2 c divided by 0.52 in this there is negative sign here, because as for as observer in ambulance is concert the train is approaching to him it is not going away. So, the direction is minus x it is toward minus x direction that is why there is a negative sign. Now, the distance travelled by the person in A frame as we have just now discussed. During this particular time until the time what shows t prime is equal to 0 in his watch will be this particular speed with which this train is moving or coming towards him, multiply by the time that I am talking, which is 3 into 10 to power minus 4 second, it is comes approximately equal to 3.46 into 10 to power 4 meter.

So, this is the time, this is the distance that the compartment would have moved in ambulance frame of reference toward him. The original distance at time 3 is equal to 10 to power minus 4 seconds was 90 kilometres, I have to subtract this distance of 90

kilometres to find out what will be the coordinate or what will be the position of the person in the frame of reference of the ambulance at t prime is equal to 0.

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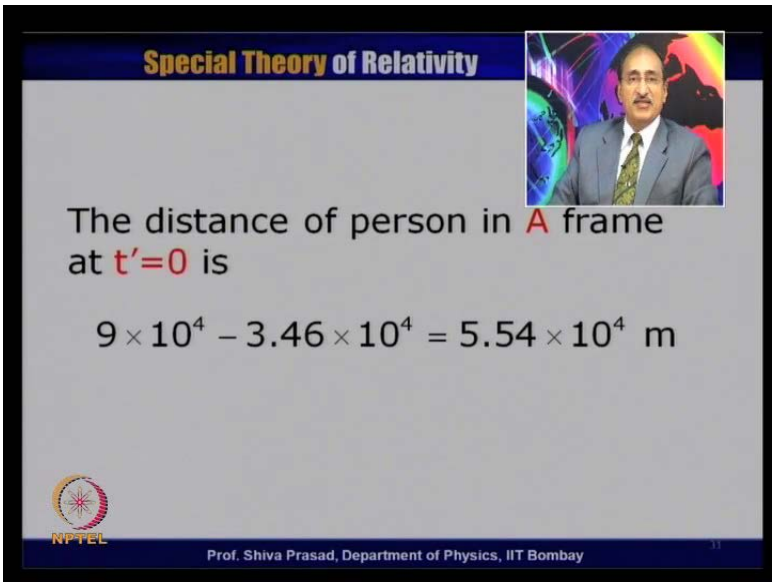

That is what I have done in the next transparency. The distance of the person in A frame at t prime is equal to 0, was the original distance 9 into 10 to power 4 meters minus the distance that has been travelling that this train had travelled in the frame of reference A in the time 3 into 10 to power minus 4 seconds, I get this distance approximately equal to 5.54 into 10 to power 4 meters.

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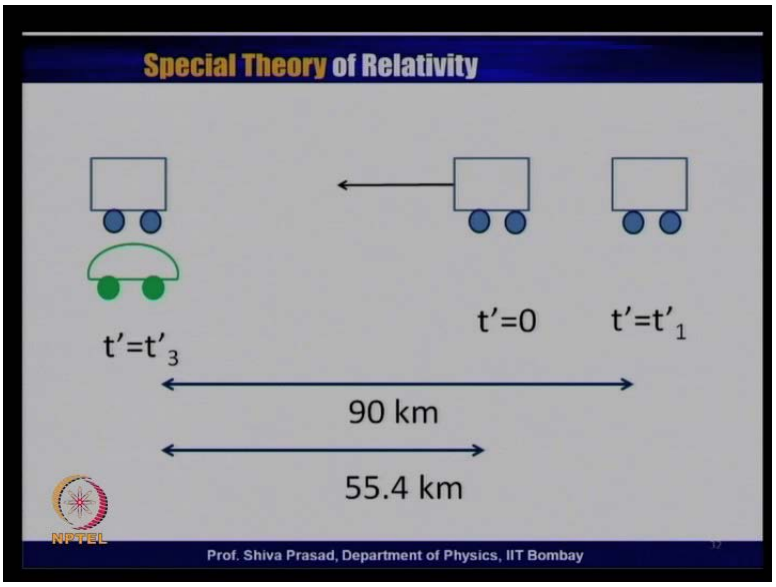

So, this is the situation as far as ambulance frame is concerned, at the first event the distance of this particular train 90 kilometres, at the time of second event t prime is equal to 0, this distance was 54 point 55.4 kilometres and at the third event this particular distance was 0. So, I have calculated all the distances, sometimes it is E becomes easy then we picturise the thing. Remember if I would have just I would have just like to make two points, if you would have just transform the first part I will not give the same result as 55.4 kilometres, because that particular times are very different, so that is one thing.

The second part is second point that I wanted to emphasise is that when I calculated the time intervals or have been calculated the coordinates of all the events in hospital frame, I never use Lorenz transformation. Lorenz transformation was not required because all the information was given in the frame of reference, in principle this problem could have been a standard classical mechanics problem. Other than the fact that I have used that speed of light is same, other than that it is not required anything special. If all the information has been given in my own frame of reference I do not require a transformation, only when I go to different frame transformation is needed.

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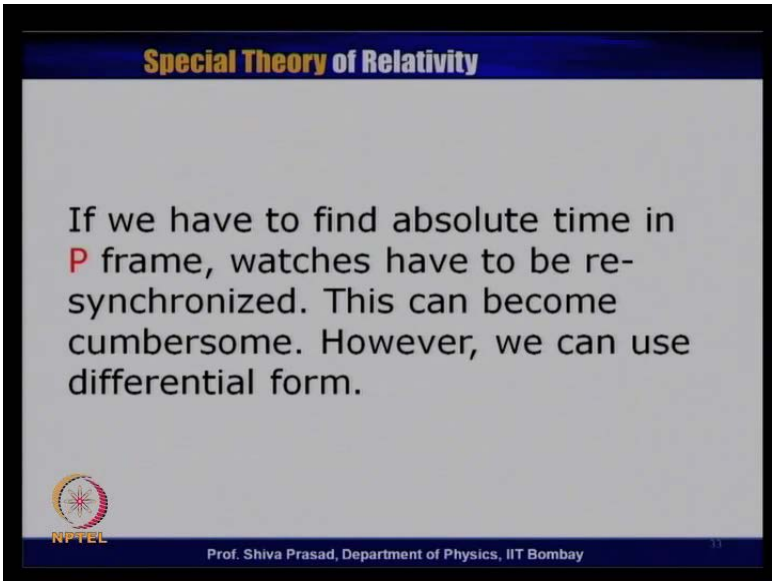

Now, let us though has not been asked in this particular problem, let us spend a little bit of time in finding about things about P frame the persons frame the person who was sick. If I have to find out the absolute times I will somewhat in trouble, it is not really a

trouble, but what if one wants one can do it. But there where the situation has been described, this person never matches either the origin of ambulance or the hospital at any time and if I have to use Lorenz transformation directly, then I have to resynchronise the watches.

So, that the times becomes equal to 0, for example, if I want to transform from hospital frame to this particular frame then I have to make their time 0, when their origins were consonant. Or if I have to transform from P frame to the ambulance frame, again I have to make time t is equal to 0, when their origins were consonant, this can become somewhat combustion. But I use in the differential form if I take the differences then I need nit have to bother, because that particular position of the origin does not matter if I am taking differences, because that cancel out. So, if I have to ask or if I have to get some information in the person frame of references the sick persons frame of reference, if it is given in the differential form I need not re synchronise my watches I can still get the answer.

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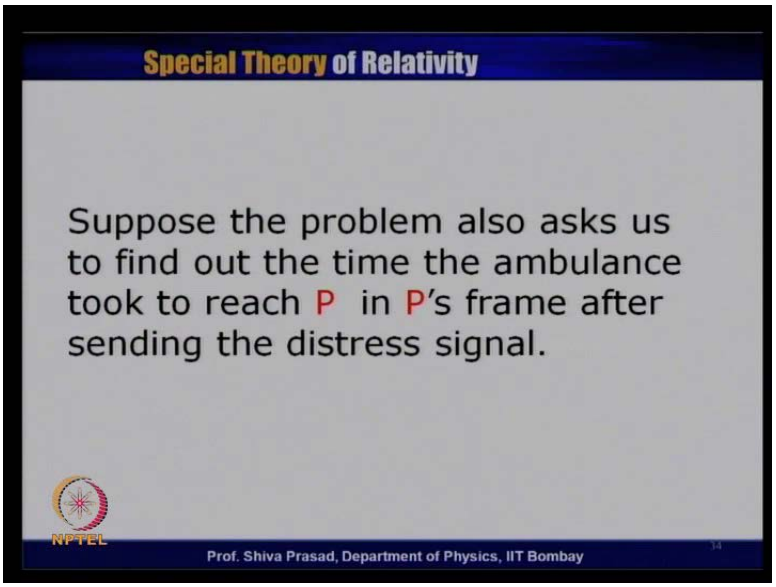

Just one quick example about this thing; suppose in the problem we had also asked that find out the time in the amb that the ambulance took to reach P in P's frame after sending the diastral signal. So, look s imagine that you are the sick person sitting in the train and you have sent a signal to get some help. Now, you have said signal let us say at a given

time, how much time after I really got the help, it means the ambulance reached. So, I ask for help after how much time I got the help.

So, if I have to ask this particular question, remember I am still talking event number one; that person I am now imagine that I am the sick person, I am sending the signal, I send the signal that was event number one, remember same event. Event number three ambulance reached me this is event number three. Now, probably would I have realised that if I am the sick person I am the person who is sending the signal. Obviously, I am always sitting at by origin, when I received the ambulance again I received at the origin.

These two events in the P's frame occurred at the same position, there for time interval between event number one and three number one and three, this time difference is a proper time interval in P's frame. So, in the hospital frame and ambulance frame this must be dilated, if I know this particular thing it is very quick then I can find out the answer.

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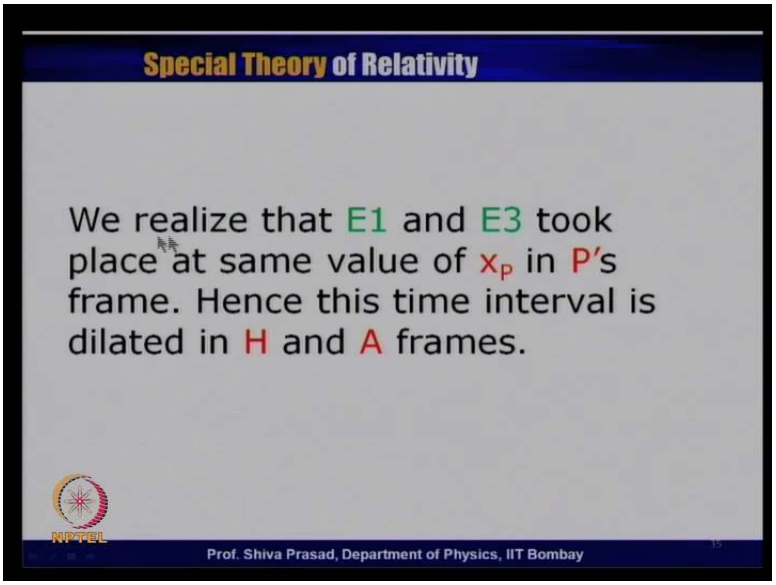

I can choose because anyway I have found out the time differences between E 1 and E 3, both in H frame and A frame, I can use any of these things, use corresponding gamma values and find out time difference in P's frame.

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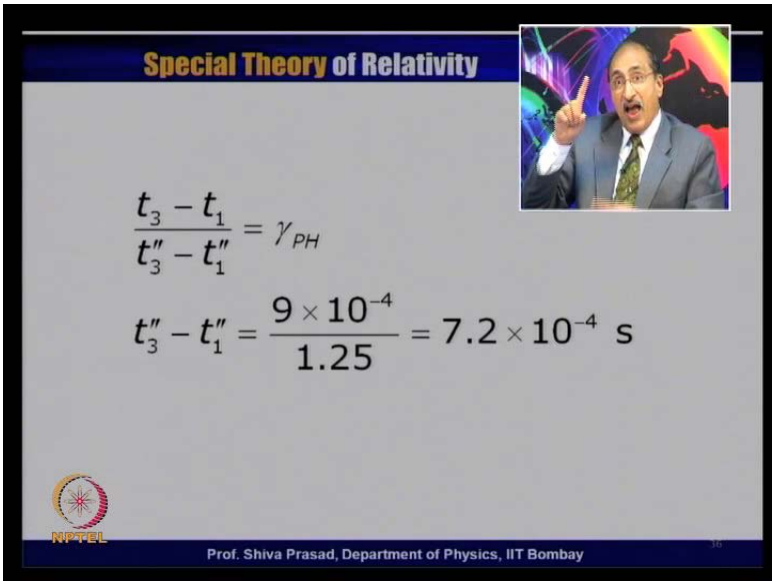

So, let just take one quick thing let us assume the hospital frame. What I have written here we realise that event E 1 and E 3 took place at the same value of x p, it is hypotheses just to p in P's frame. Hence, this time interval is dilated in H and A frames, for P frame I have put a double prime. So, t 3 minus t 1 this is a proper time interval, t 3 minus t 1 this is in the hospital frame of reference must be equal to gamma, please let us very careful that which gamma I am using, because I am going transforming in three different frames.

It is between the persons and the hospital because t 3 minus t 1 is in the hospital frame of reference. So, I have to use gamma by using the relative velocity between P and H, which was anyway given to be equal to 0.6 c, the person was moving with a speed of 0.6 c in H frame. And we know that gamma P H we have done in many problems is 1.25. So, this t 3 12 prime minus t 1 double prime, which is the time difference between event number one and event number three, in the persons frame of reference in the sick persons frame of reference will be given by this time interval. If you calculate time difference between event one and event two, one was a minus 1 another one was a 8, so it becomes

9 into ten to power minus 4 divided by gamma, which is equal to 7.2 into 10 to power minus 4.

So, this is the proper time interval and this the time interval which will be measured in the sick persons frame of reference, which I am calling as P frame of references. Just to check if I multiply this by the gamma, gamma between A and P person and the ambulance. Then I must get back the time interval which I have calculated between E 1 and E 3, in the ambulance frame of reference just to check if I have thought everything right. To use that particular gamma I have to use the relative velocity between the persons frame and the ambulance frame of references. So, I have just written just to confirm we can find the time interval in A frame using this proper time interval between E 3 and E 1 in P frame persons frame.

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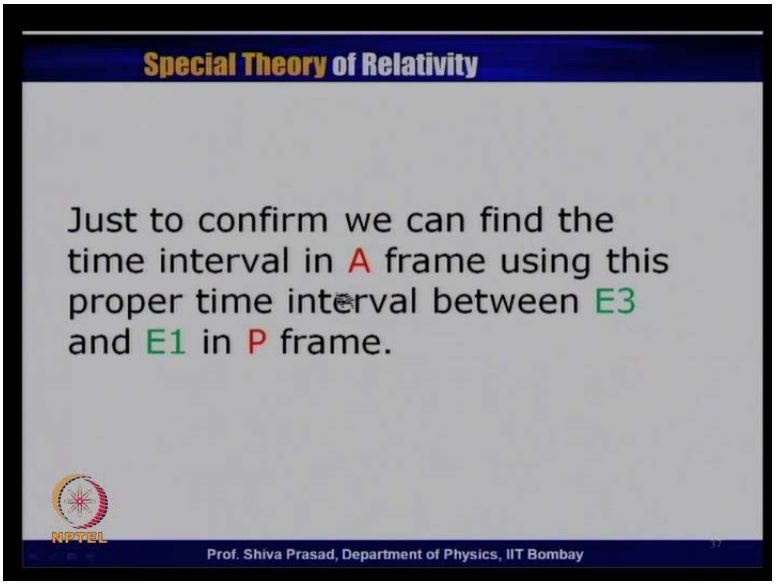

Exactly the same thing, this particular time interval proper I have to find out t 3 prime minus t 1 prime, I have to multiply by gamma P A. At calculated the relative velocity is 0.2 divided by 0.52 c, c square cancels out, gamma P A turns out to be equal to 13 divided by 12. Therefore, t 3 prime minus t 1 prime will be equal to this proper time interval multiplied by 13 divided by 12, which is equal to 7.8 into 10 to power minus 4 second. If I go back to my event table this is what you do have seen, 4.8 and 3 this is minus this time difference is actually 7.8 into 10 to power minus 4 second, this is what I have expected.

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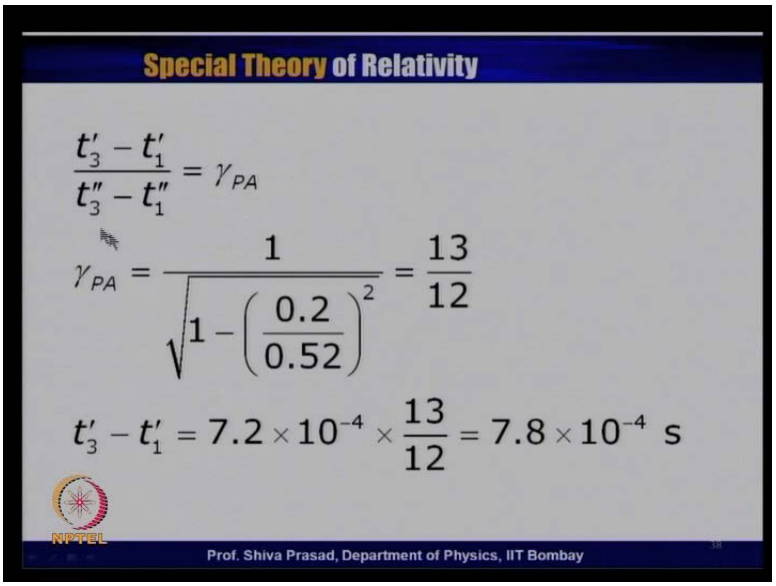

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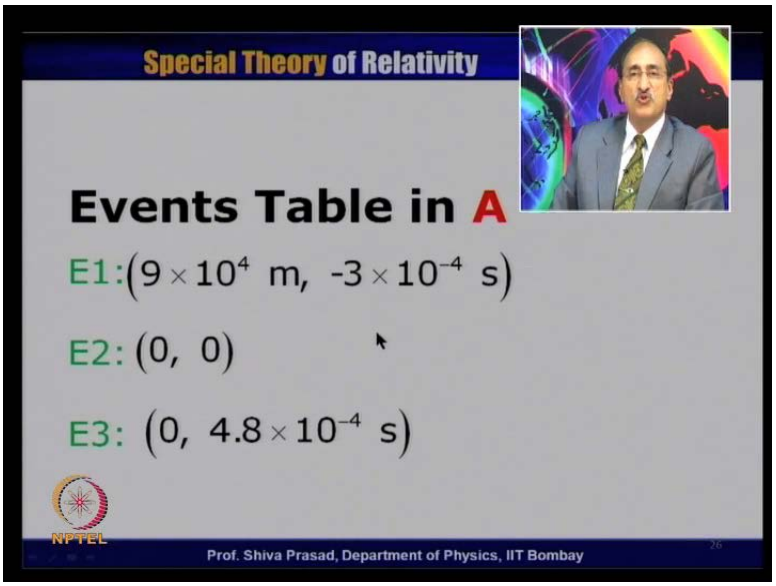

This matches with whatever we have evaluated using Lorenz transformation. In the end I would just like to give my summery of course, there is this particular summary.

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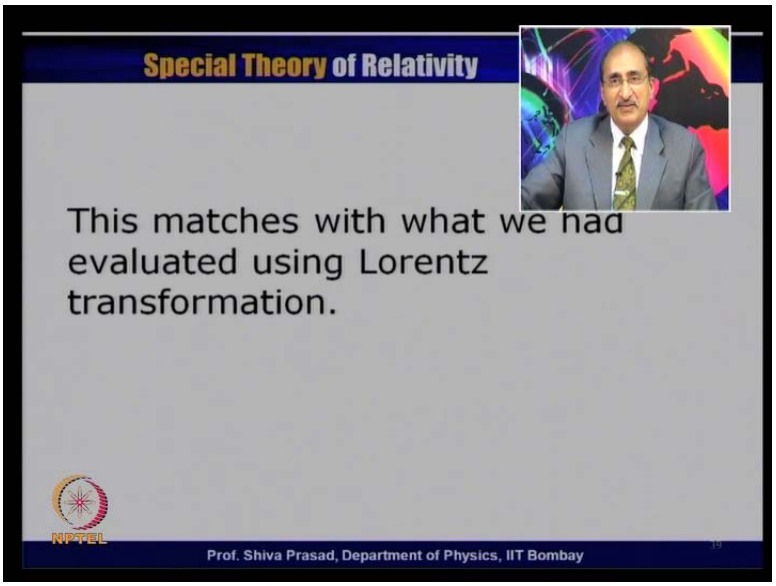

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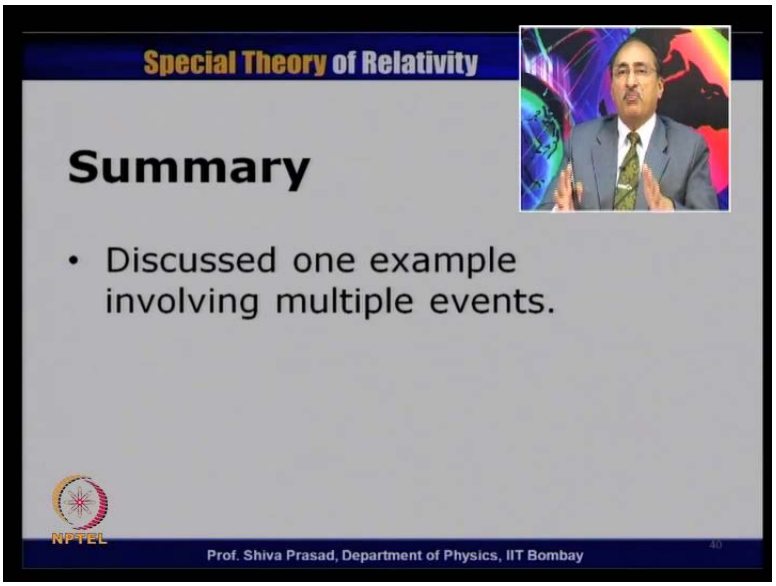

We have only solved one particular problem, which is a long problem involving three different frames of reference and three different events. So, we discussed one example involving multiple events.

Thank you. .