Special Theory of Relativity
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Lecture - 8
Velocity Transformation and Examples

In our last lecture, we had discussed some examples of which involved the use of Lawrence transformation; we specially discussed the problem of mu meson shower which provided an experimental support to special theory of relativity. Classically, it was never understood that out of the mu mesons which are created at upper end of the atmosphere. So, many of them are able to reach earth though their life time was so small they should have decayed in between. It was special theory of relativity which could explain this behaviour on the basis of time dilution that provided in a real good experimental support to special theory of relativity.

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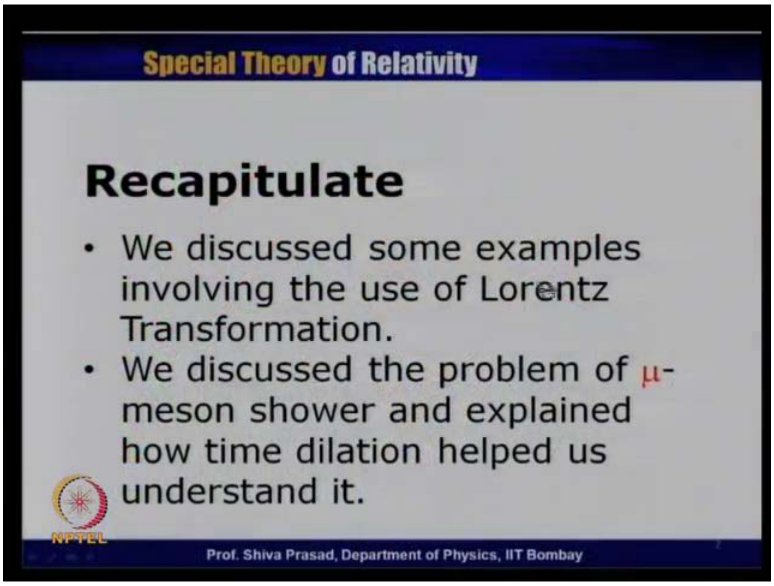

So, this what we have written, we discussed some examples involving use of Lawrence transformation, then we discussed the problem of mu meson shower and explained how time dilation helped us to understand it. Now, let us go ahead to a slightly new topic which is the velocity transformation the idea is as follows.

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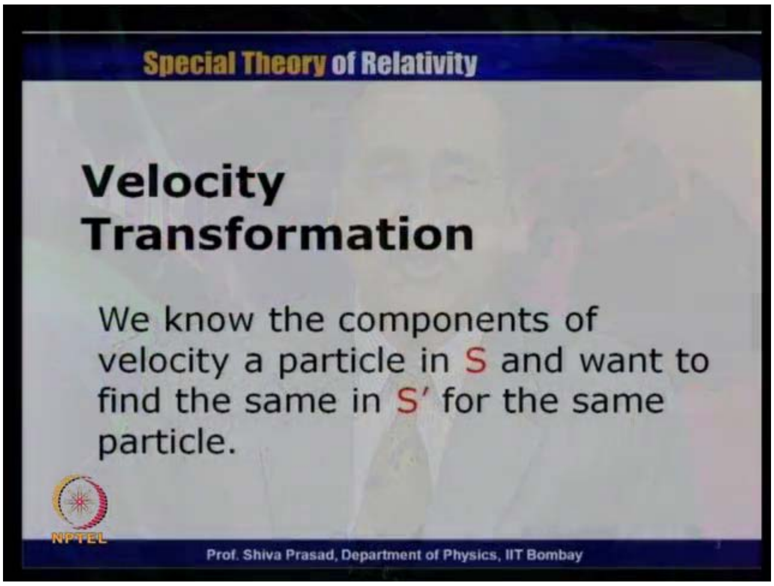

You know the velocity of a particular particle in S frame and you want to find out the velocity of the same particle S being seen in S prime field. So, let us be clear that there is one particular particle which is moving somewhere in an observer S is trying to observe that particular particle. Similarly, an observer in S prime is also is trying to observe the C particle the particle is neither fixed in S nor fixed in S prime, it could be anywhere and in fact that particular particle need not move with a caustic velocity.

It could you will be accelerated particle, it does not matter what we are saying is that instantaneous velocity of this particular particle is being measured by both the observers. One observer in S, another observer in S prime and of course both hand S and S prime observers are in their respective frames of references which are inertial. So, this is what we call as a velocity transformation, we know the components of the velocity of a particle in S and want to find the same S prime for the same particle. So, the equations which will relate these velocity components in S to the velocity components in S prime would be called velocity transformation.

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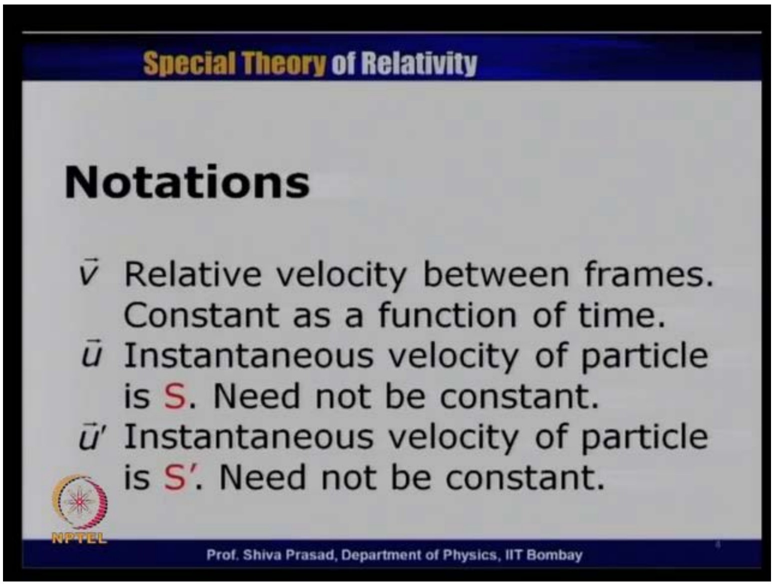

Let us put some notations because these are the things which has to be very clear c symbol v, we will reserve for the relative velocity between the frames because both the frames are inertial. Therefore, v has to be constant, v cannot change as a function of time otherwise at least one of the frame is non inertial, so v is constant is a function of time. Now, this observer S tries to observe the behaviour of a particle, let us write it here, so this S is an observer.

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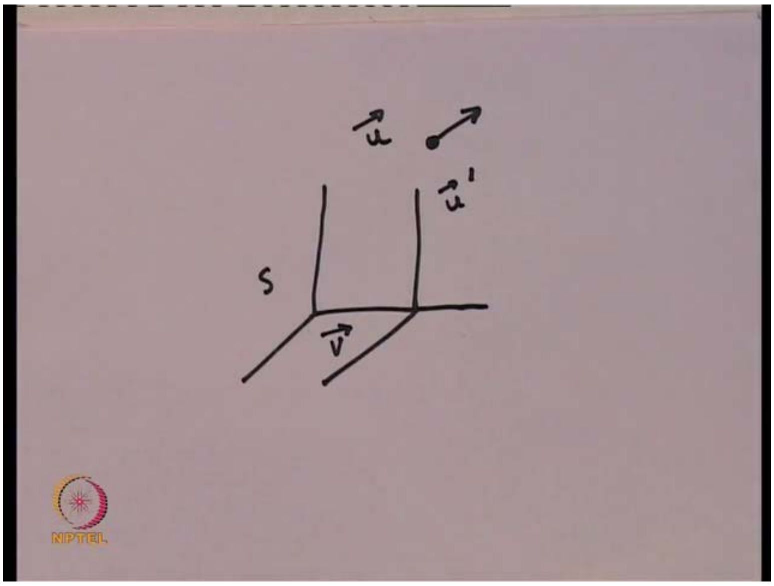

This is S frame and there is some particular particle which is moving in some arbitrary direction and the observer in S prime frame should tries to observe the same particle. So, whatever is the velocity measured of this particular particle as framed that will be called u. Whatever is the velocity measured of this particular particle in this particular frame will be called u prime while the relative velocity between these two frames will be called v. So, these are the symbols that we will be using in this particular lecture in this particular series of lecture repeating.

Here, v is the relative velocity between the frames which is constant as a function of time u is the instantaneous velocity of the particle. Even if it is accelerated, we can always find out instantaneous velocity, this need not be constant. While u prime is instantaneous velocity of the same particle, which is being observed in S prime frame of reference and this need not be constant.

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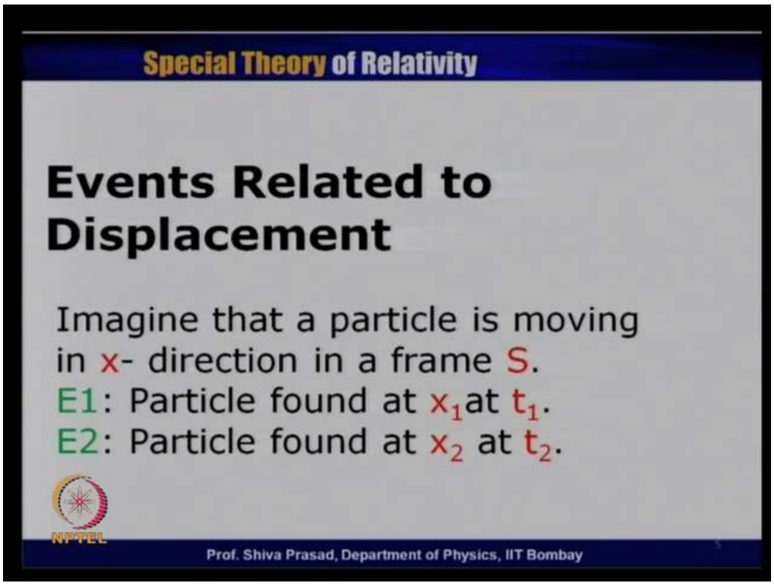

Let us talk about the events related to displacement; we have generally agreed that this special theory of relativity it is important to talk in terms of events. So, what we are saying let us assume for the moment that the velocity is only along the x direction, we will generalise it later. So, particle is moving along x direction in S frame and E have to find out it is location at two different times, so let us suppose this particular particle at time t 1 is found at a coordinate x 1. This I call as event 1 in this same particle is observed at a time t 2 at co ordinate x 2, then this particular event of the particle b at x 2 at time t 2 is called event 2. So, these are my two events event 1 particle found at x 1 at even number two particle found at x 2 at time t is equal to t 2, these are my two events.

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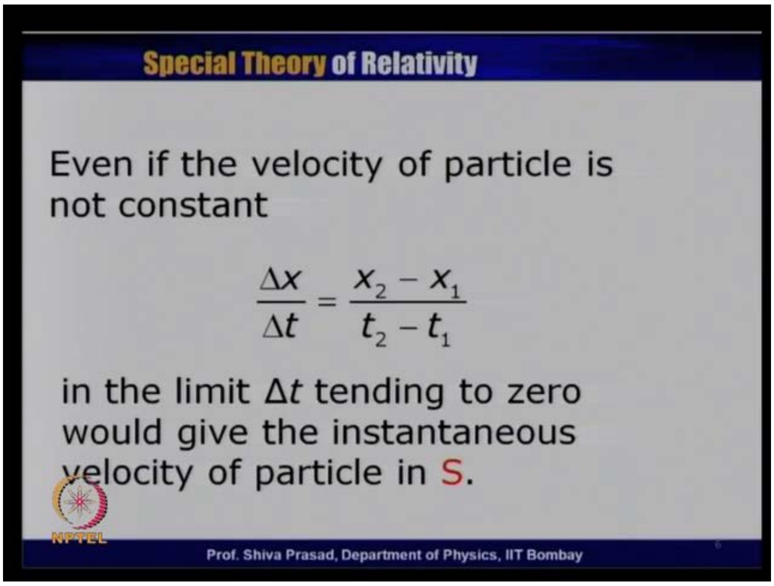

Now, you would agree that you have to define these continuous velocity of the particle, what I must do I must take x 2 minus x 1 which is the difference of the x co ordinates divide by the time difference. If the velocity of the particle would have been constant then x 2 minus x 1 divided by t 2 minus t 1 would have any way given me the velocity of the particle. If the particle velocity is constantly changing then this time limit should be as small as possible in order to find out instantaneous velocity.

So, what we say that we define delta x as x 2 minus x 1 delta t as t 2 minus t 1 and in the time limit that delta t tends to 0 this quantity delta x divided by delta t will tend to become the instantaneous velocity of the particle. This is what I have written even if the velocity of the particle is not constant delta x divided by delta t defined as x 2 minus x 1 divided by t 2 minus t 1 in the limit delta t tending to 0 would give the instantaneous velocity of the particle.

Now, let us assume that the particle is having a general motion in three dimensions, it could need not move only around the x direction it could move in any other direction. So, similarly exactly in the same way I can look at the displacement along the x direction in a given time displacement along a wider action in the same time displacement along the z direction in the same time. Divide these things to get instantaneous velocity components of this particular particle.

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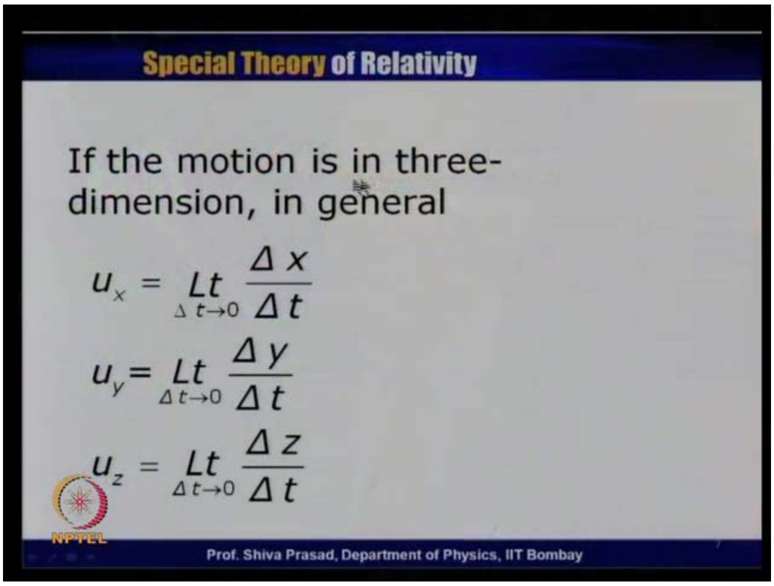

So, this is what I have defined in the next transparency if the motion is in general in three dimensions, then u x will be defined as delta t limit of delta t tending to 0 delta x divided by delta t. Similarly, u i will be defined as in the limit delta t tending to 0 delta y divided by delta t and u z will be in the limit delta t tending to 0 delta z by delta t. These are the standard classical definitions of the instantaneous velocity and I will not think that we should have any problem in understanding the only thing which we have to realise.

If the instantaneous velocity of the same particle is to be measured in S prime frame of reference, then of course delta x has to be measured in S prime's frame of reference, but no because also the time is relative. Therefore, this time also has to be measured in S prime frame of reference see like the observer in S prime cannot calculate the instantaneous velocity by taking delta x he has to take delta x prime.

Similarly, instantaneous velocity cannot be calculated in S prime frame of reference by using delta t it has to be delta t prime. This should be used an observer in a particular frame of reference has to be consisted all the information must be used in his own frame in his, or her own frame of reference to evaluate things like velocity.

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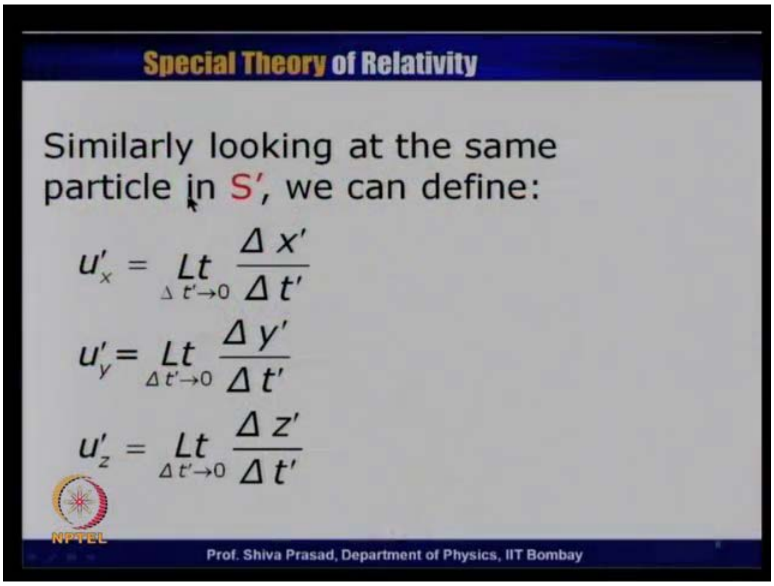

Hence, we define exactly in a same fashion t velocity components of a particle as u x prime. I have put a prime because it means this is being measured in S prime frame of reference in the limit delta t prime tending to 0 delta x prime divided by delta t prime.

Similarly, in y prime will be the limit delta t prime tending to 0 delta y prime divided by delta t prime u z prime in the limit delta t prime tending to 0 delta z prime divided by delta t prime I repeat I have been using here delta t prime. Here I am using delta t prime and not delta t because all these observations are being measured in S prime frame of reference.

Of course, we agree that if delta t tends to 0 delta t time will also tend to 0, now what is my problem my problem is to find a relationship between u x prime u y prime u z prime in terms of u x u y u z or the other way if I know u x u y u z. I want to find out what is u x prime, what is u y prime what is u z prime this is what will be called the velocity transformation.

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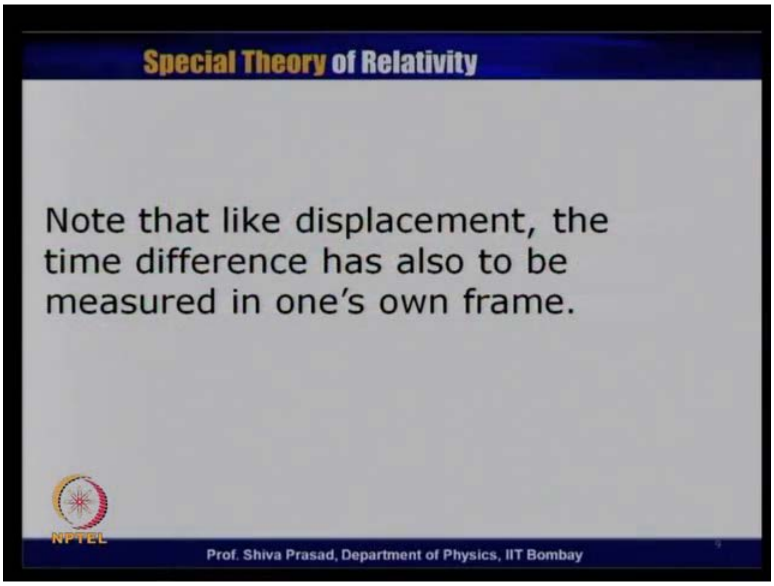

For doing that, let me just read it here what I have said note that the displacement like time difference has also to be measured in one's own frame this is what I have incised.

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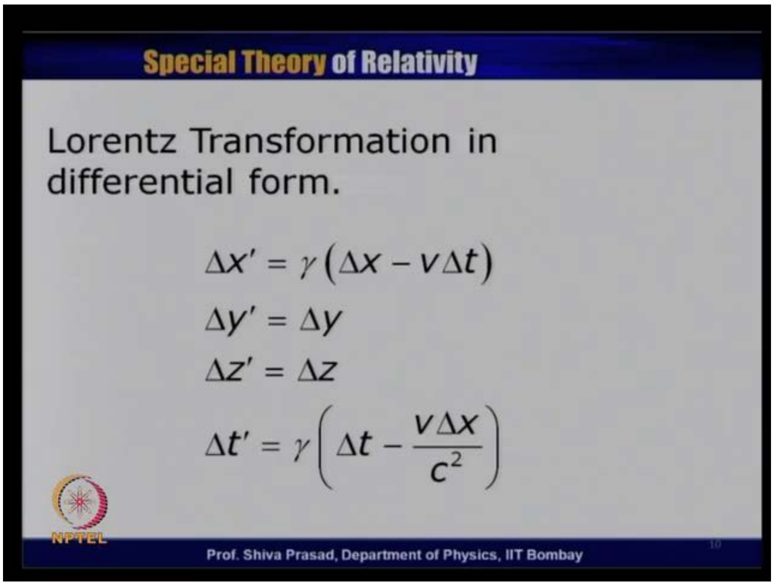

For doing velocity transformation, I would first like to write Lorentz transformation in differential form though we have being using it, but we have not been talking about it in detail. So, let us spend a little bit of time to understand it.

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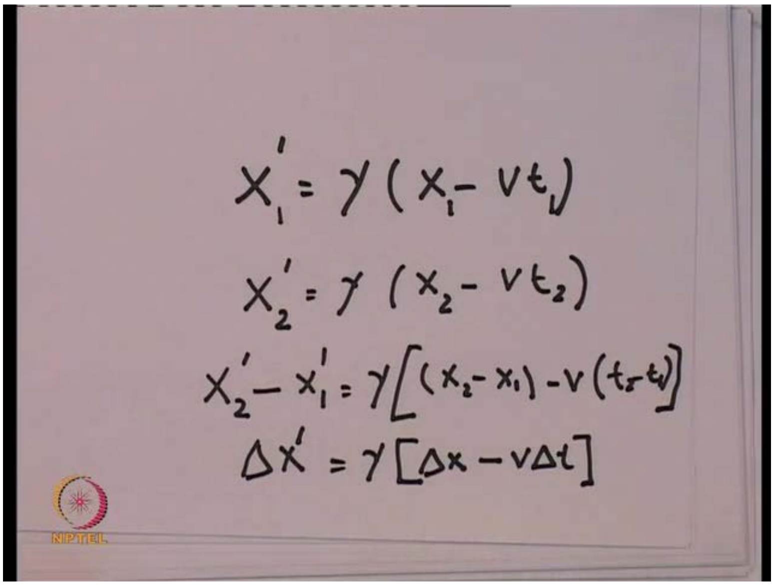

Similarly, earlier written let x prime should be equal to gamma x minus v t this is the first equation of the Lorentz transformation while the two events I can write this as x 1 prime this I can write as x 1 and this I can write as v t 1. Similarly, is there that is the second event we write this as x 2 prime is equal to gamma x 2 minus v t 2, now here of course x 1 x 2 t 1t 2 have to be measured with respect to a fixed origin. All these conditions we have defined earlier, now what I say let us take the difference of these two if I take the difference of these two I will call it x 2 prime minus x 1 prime will be equal to gamma x 2 minus x 1 minus v t 2 minus t 1.

So, this becomes in the difference in the value of x, so I can write this as delta x should be equal to gamma this I can write again as delta x this should be delta x prime this is delta x minus v delta t. So, this is what I call as writing the Lorentz transformation in the differential form the advantage here is that this depends only on the differences. Therefore, the origin of x and origin of t mean not be defined because that things get cancelled out.

Eventually, we are just looking at the differences, so similarly, I can also find out using the time transformation equation then equation relating to delta t prime and delta t. The basic idea is that put just the deltas in terms of all these variables in front of all these variables. So, this is what I have written in this particular transparency where I have used Lorentz transformation in differential form.

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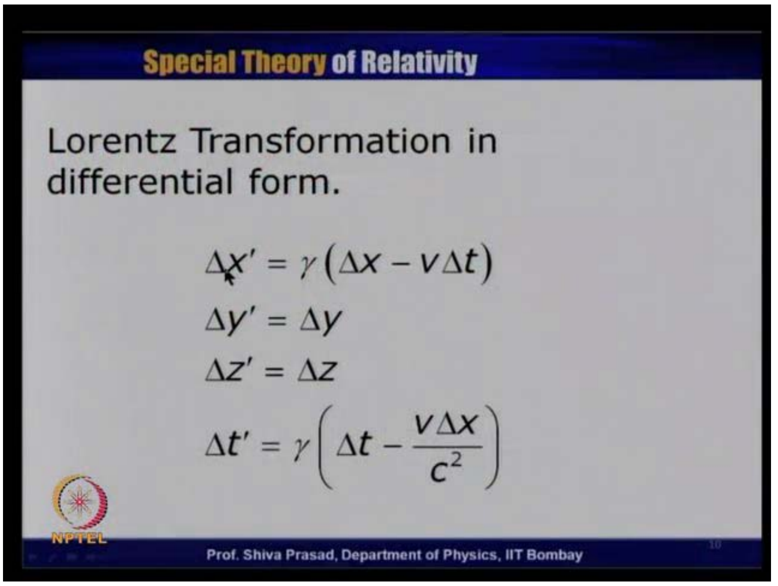

So, delta x prime is gamma delta x minus v delta t thing which we just now discussed delta y prime is equal to delta y delta z prime is equal to delta z and delta t prime will be gamma delta t minus v delta x upon c square. So, this is what is called writing Lorentz transformation in differential form, now in order to calculate a relationship between u x prime and u x, what I shall be doing is using this particular delta x prime use this delta t prime.

I divide these two take the limit delta t prime tending to 0 this will give me u x prime and let us try to work out with these equations so that I get a relationship of u x prime in terms of u x. Similarly, I will use delta y prime and divide by delta t prime then somehow manipulate these two equations in order to relate u y prime to u y. Finally, delta z prime divided by delta t prime which in the limit delta t prime tending to 0 will give me u z prime.

Manipulate these two equations to get the relationship in terms of u z, so let us first look at these two equations delta x prime and delta t prime. So, I just divide delta x prime by delta t prime delta x prime is gamma delta x minus v delta t delta t prime is gamma delta t minus v delta x by c square. So, next transparency what ill have is just this equation divided by this equation.

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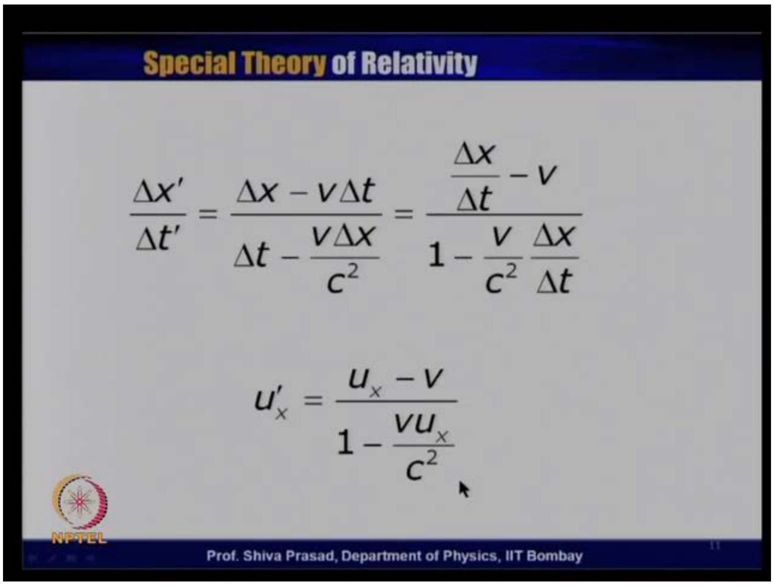

This is what I have written here this is delta x prime divided by delta t prime is equal to you remember delta x prime was delta x minus v delta t delta t prime was delta t minus v delta x divided by c square. So, all I have done is just divided these two what I will do now I will divide the numerator and denominator of this equation by delta t see if I divide these two by delta t in the numerator first 1 is delta x. So, dividing by delta t this becomes delta x divided by delta t, then we have the second term v delta t if we divide this by delta t it just remains becomes v the delta t goes away.

Similarly, in denominator if I divide by delta t delta t divided by delta t will give me 1 so there is a 1 here then we have v upon c square, then delta x divided by delta t can be taken out separately. Now, both the sides I will take the limit delta t tending to 0 which also implies delta t prime tending to 0 in that case this particular equation will tend to u x prime. As we have discussed earlier this will tend to u x, as we have disused earlier and of course this will also tend to u x. So, the equation of the velocity transformation for the x component becomes u x prime is equal to u x minus v this is u x minus v divided by 1 minus v upon c square is any way there v in this quantity which is u x v u x by c square.

So, I have obtained the first equation of the velocity transformation which is the x component of the velocity. Similarly for finding out the y component transformation instead of delta x prime I have to use delta y prime or you remember delta y prime was any way equal to delta y, but delta t prime was not equal to delta t. Therefore, in principal u y prime will not be equal to u y and that is what we see in the next transparency.

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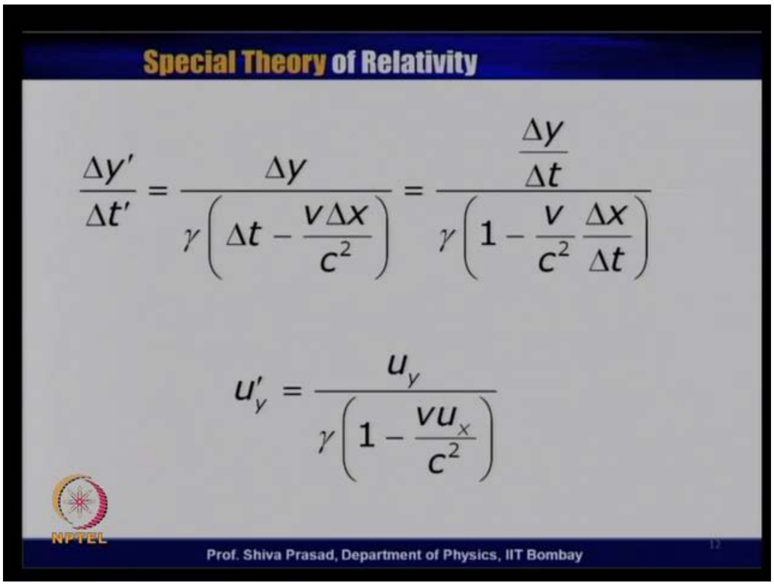

So, we have here delta y prime divided by delta t prime delta y prime was equal to y as we know delta t prime we have already written the equation earlier is same gamma delta t minus v delta x divided by c square. We will do the same trick divide numerator and denominator by delta t this numerator becomes delta y divided by delta t the denominator gamma divided by delta t becomes one minus v c v by c square into delta x divided by delta t.

So, u y prime if we take the limit delta t tend it to 0 which implies delta t prime then dent to 0 this equation this particular part tends to u y this tends to u x. Therefore, the whole equation in y prime becomes equal to u y divided by gamma 1 minus v u x upon C square. So, what we see that we do not get u y prime equal to u y as is expected because remember even classically u x prime did change when we change from x plus prime, but as far as y components were y and z components were concerned they did not change. Hence, it appeared that the particle speed of light was been different, now because remember these equations are going to be valid even for this speed of light.

So, if u x has to change u y and u z also have to change in order to make the speed of light same and that is what we have precisely seen here. The other observation which I would like to measure that u y prime this particular equation not only depends on u y also depends on u x. So, this equation is little more involved because it depends both on u y and u x. Now, finally the z component is exactly com identical to the y component and that is what we have written in the next equation all that has happened that is instead of y we have z. Therefore, we get the z component of the velocity transformation as u z prime is equal to u z divided b gamma multiplied by one minus v u x divided by c square.

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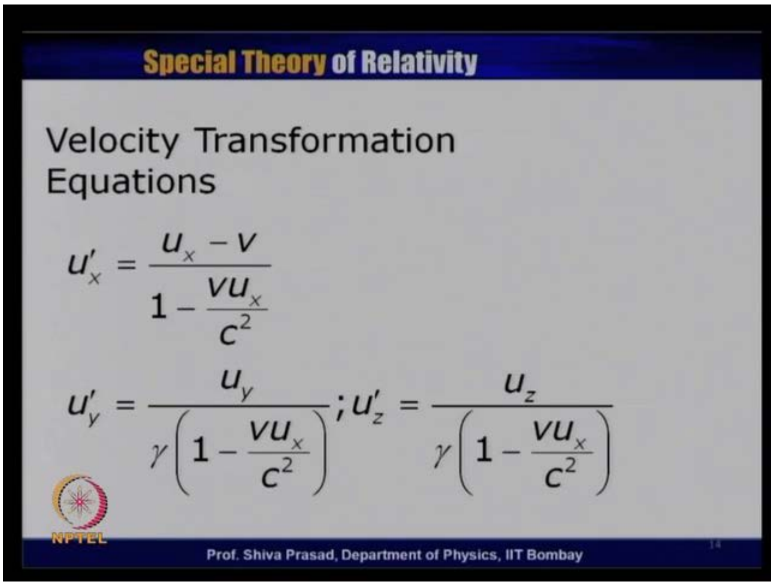

So, this is we are we have written all the equations involving velocity transformation u x prime is equal to u x minus v divided by 1 minus v u x by c square u y prime is equal to u y divided by gamma 1 minus v u x by c square u z prime is equal to u z divided by gamma one minus v u x divided by c square.

Remember, this quantity here is same quantity in the bracket here, in the denominator we do not get divide get it divide multiplied by y while here the denominator gets multiplied by y. So, what are this to remember u x prime is equal to u x minus y was a classical transformation what has happened in terms of often inclusion of Lorentz transformation. This quantity gets divided by 1 minus v u x by C square u y prime is equal to u y was the classical transformation here what is happening that we are getting gamma.

This whole quantity is divided by gamma multiplied by same quantity 1 minus v u x by c square u z prime u z divided by gamma in to same quantity one minus v u x by c square so this is what we call as the velocity transformation relativistic velocity transformation.

We can always find out inverse transformation because many times we use inverse transformation it means the information about the velocity is given in S prime of reference. We have to find out the velocities in S frame of references description is simple change prime to un prime change un prime to prime change v 2 minus v.

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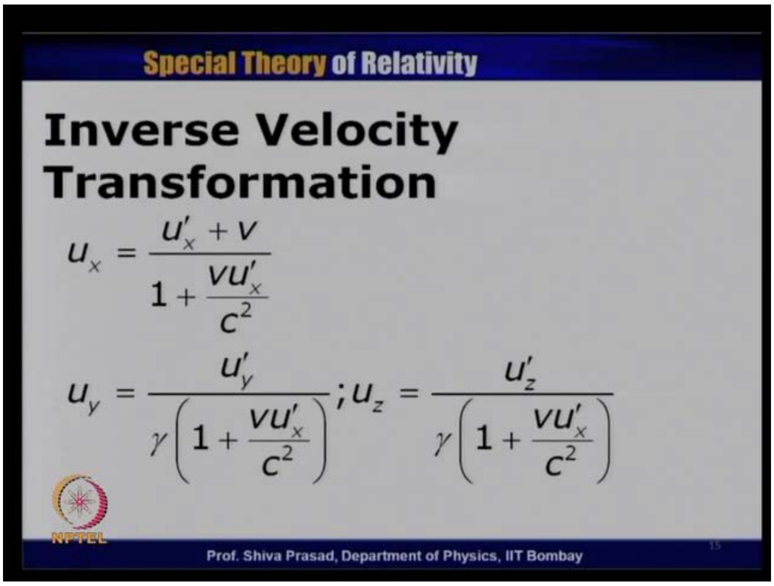

So, this is what we get as inverse velocity transformation u x is equal to u x prime plus v divided by 1 plus v u x prime by c square u y is equal to u y prime divided by gamma multiplied by 1 minus v u x prime by c square u z is equal to u z prime divided by gamma multiplied by 1 plus v u x prime divided by c square. Remember, there is a plus sign here there is a plus sign here there is a plus sign here because there is also a plus sign here because we have changed v to minus v.

It can be shown to at the moment we will not show it we will show these things little later and we will have little more discussion on this particular aspect, but one can show if we take the magnitude of the velocities. If speed of the particle is found to be less than c in S it will also be found to be less than c in S prime irrespective of v if u is equal to c in S u will turn out to be equal to c in S prime also irrespective of v.

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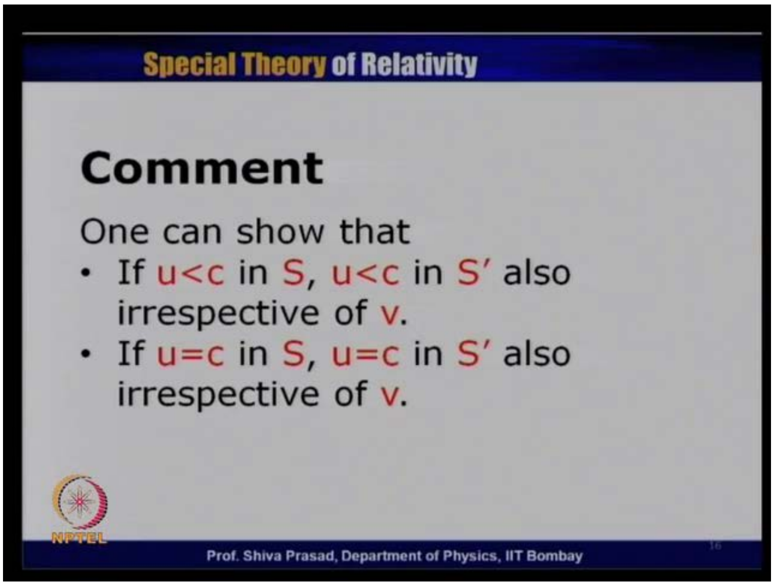

This is the thing we have always been mentioning that the speed of light should be framed independent quantity that is the pretext on which we had started the special theory of relativity. Now, let us take one simple example what we call as a relative velocity we will take one more example. We will see that classically many a times we are bit confused about the relative velocity concept here once in relativity, when we say a relative velocity it means very clearly that we change from one frame to another frame.

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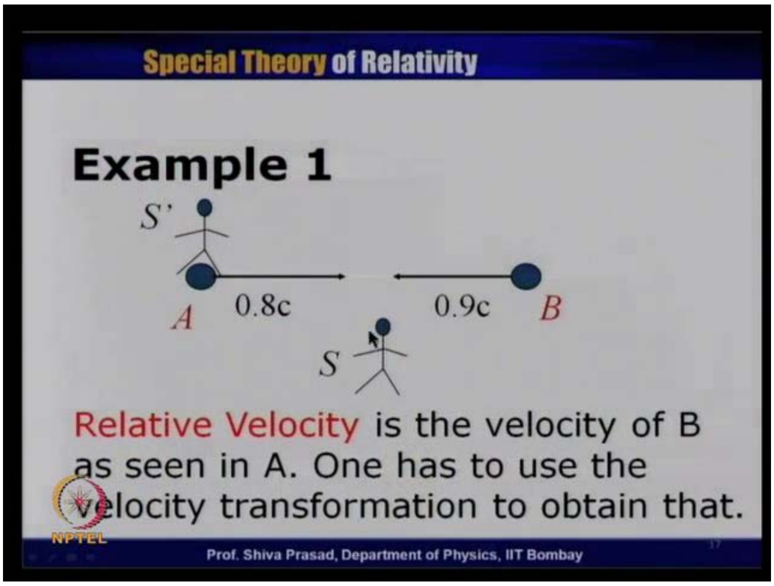

So, let us say my question is that that an observer in S frame observes two particles of course the speeds of the particle have to be relativistic. If we have to apply relativistic relativity theory this particular particle moves according to this observer S with the speed of 0.8 c. I am just calling this one this particular particle is particle a let us assume there is another particle b which is moving to the left hand side with a speed of 0.9 c both these speeds are being measured by an observer in S.

So, once I make a statement that this particular particle is moving with the speed of 0.8 c and this particular particle is moving to the left with a speed of 0.9 c. It essentially means that we are talking all those things with respect to some other observer which is this observer S. Now, if I have to ask a question what is the relative speed between particle a and particle b it essentially means that I have to change my frame of reference I go to let us say frame a find out what is the speed that an observer is sitting on a would find of particle b.

That is called relative speed there is no other meaning for relative speed of course if we wanted we could have also want from S to v and found out what is the particle speed of a in b's frame that will be a rate of velocity. As one can see very easily that this would have given me the same result other than the sign, so relative velocity is between these two particles will always be same as been observed by this particular. Either this particular observer or this particular observer, so question now is that if these two speeds are given what is the speed of particle b in particle a's frame of reference which I am calling as prime frame of reference. Once I calculate this quantity, that by definition is relative speed between A and B or if I have to go from S to S prime of reference.

We have reserved the symbol v which has been reserved for relative velocity between the frames. This v would be equal to the velocity of a as seen in S frame because I am changing from frame S to S prime this S prime frame is linked to particle a. Therefore, v must be equal to 0.8 c, so this is a vector so we can take an appropriate direction and put it vector if we want it to be more accurate.

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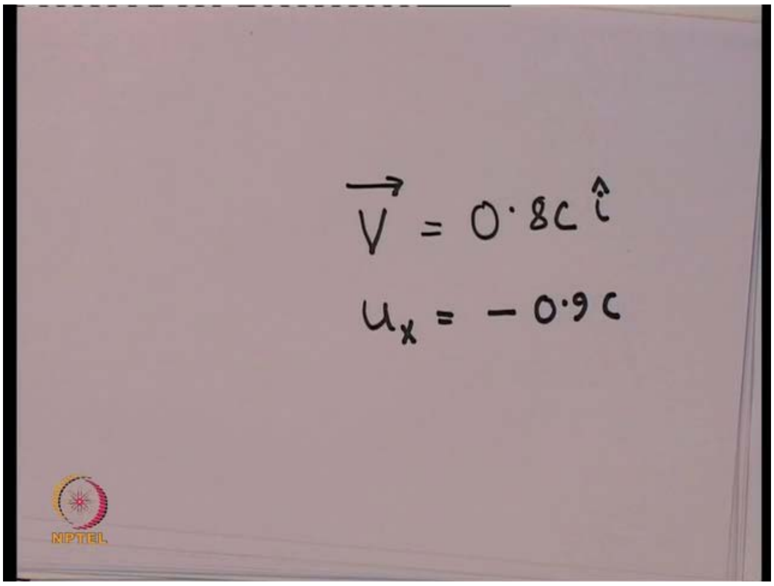

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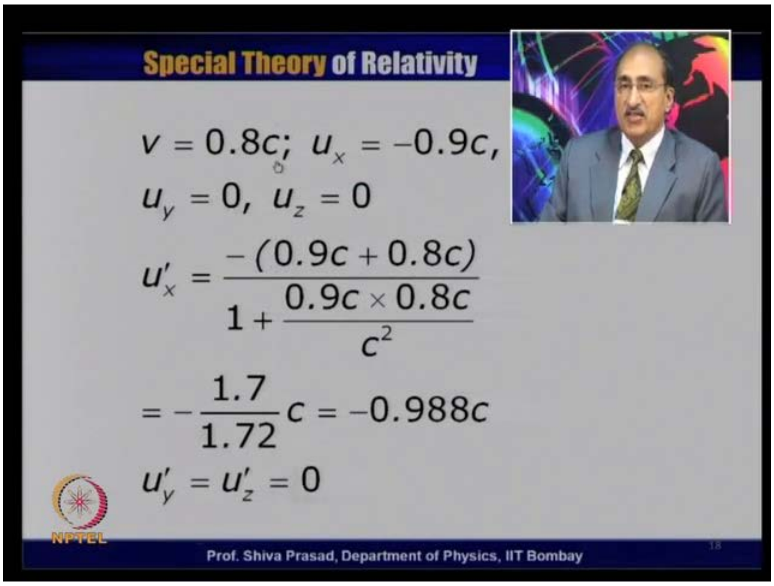

Now, this observer would say that this particular particle is moving to the left at a speed of 0.9 c and my question is, this is what is the speed of this particle as measured by an observer in S prime frame of reference. So, the particle whose speed is known in S and what I want to find out in S prime is this speed of 0.9 c and because this is in minus x direction. I will write u x is equal to minus 0.9 c. Now, question is that I know the particle velocity which is 0.9 c with a negative sign and I know the velocity relative velocity between the frames which is 0.8 c what is u x prime what is the velocity of this particular. As we measured in S prime frame of reference that is what will give me the answer of relative velocity.

So, first equation of the top this parameter transparency shows very clearly v is equal to 0.8 c as we have written here u x is equal to minus 0.9 c. That is what we have written earlier u i is equal to 0 u z is equal to 0 because there is no velocity component in y or z direction the particular is assumed to be moving only in the x direction.

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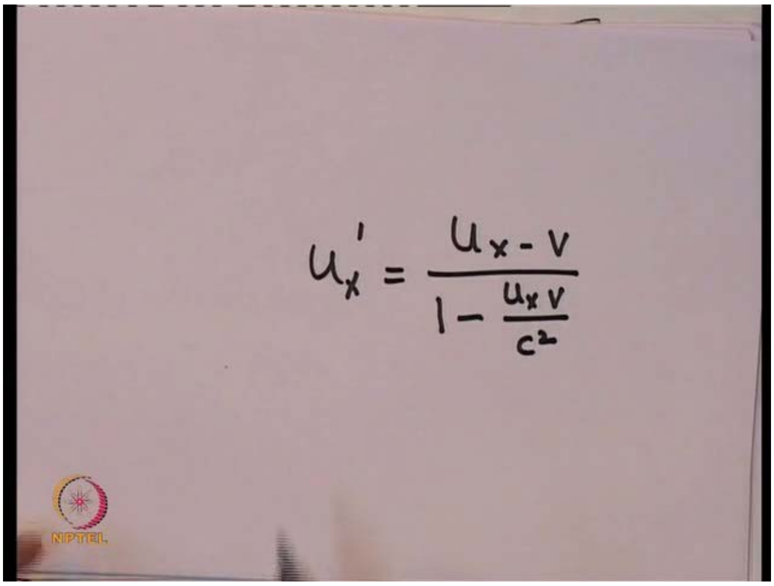

We apply the formula of velocity transformation, let me write the formula just to make sure u x prime is equal to u x minus v divided by 1 minus u x v upon c square. So, I will substitute the value of u x and v in this parameter formula to get value of u x prime here I write u x prime is equal to u x which is msn 0.9 c. Then, minus v is 0.8 c, so there is minus sign here there is a minus sign here, so this minus sign I have taken out, so this becomes 0.9 c plus 0.8 c divided by 1 minus u x. Once I say one minus u x u x is minus 0.9 c, so this becomes plus this becomes 0.9 c into v which is 0.8 c, so this is 0.8 c divided by c square.

This c get cancels with this c square get candled with the c square so you get 0.9 into 0.8 which is 0.72 to adsorbed one this becomes 1.72. So, my answer is if you add 9 and 8, 0.9 and 0.8, we get minus 12.7 divided by 1.72 c which is equal to minus 0.988 c approximately. Of course u prime and u z prime are 0, so we clearly see that the velocity components are relative velocity turns out to be smaller than c.

Even if I have change the frame of reference though classically, we would have said that the relative velocity is 1.7, but here it does not turn out to be it always turns out to be less than c as we had always expected. Let us go to another problem in which there three particular and three frames involved and therefore this problem is little more interesting form the simple problem which we have just now discussed.

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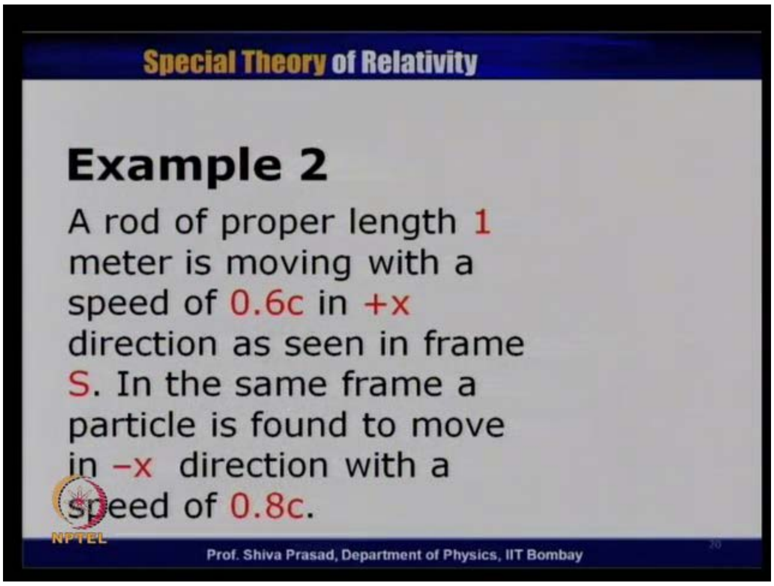

So, let us assume that we have a rod of proper length 1 meter a rod of proper length 1 meter its moving with a speed of 0.6 c in plus x direction as seen by an observer in a frame S. In the same frame a particular is found to be moving in minus x direction that is the opposite direction to the rod with a speed of 0.8 c, so this was the statement of the problem. We will see what the things that are being asked are and then show by a picture to make it clearer.

So, what has been asked in the question find the time taken for the particular to cross the rod, in a rods frame it is I am calling as prime frame most reference b in the frame S double prime of the particulars. So, there is a third frame which is the particular frame and lastly in S frame which is the frame in which all these velocities have been given.

So, there are three different frames the ground frame let us call it the S prime frame which is the rod frame S double prime frame which is the particular frame and essentially we have to find out in all these three frames what are the time taken or what is the time taken by the particular to cross ground. We realize that the speeds that I am talking are all constant, so we are actually discussing special theory for relativity. So, we can apply Lorentz transformation formulas are all the formulas derived.

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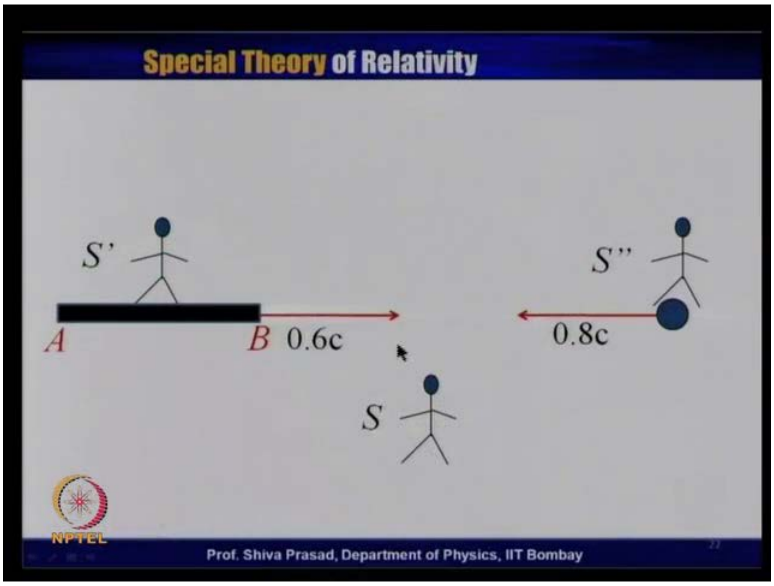

On the basis of special theory of relativity without difficulty, so this is what I have shown in this parameter picture here approximate to given an idea that this an observer S which I am calling as a ground observer. This picture has been shown with reference to this particular observer this particular rod is moving with a speed of 0.6 c in the plus x direction there is a particular which is moving in in the left hand side with a speed of 0.8 c. This particular observer sitting here I am calling as S prime observer, this observer sitting here I am calling this S double prime observer.

So, let us assume that this particular particle is slightly displaced with respect to this rod, so it is not really hitting it, so this particular particle can go over and cross this particular rod. My question is that how much time this particular particle would take to cross this particular rod.

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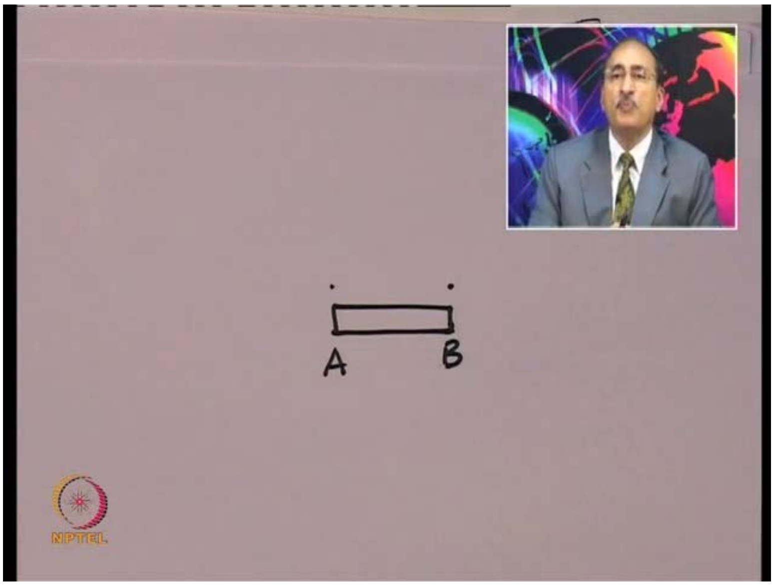

Let us define events, we will discuss about the observation in different frames of reference little later, but let us suppose this is my rod A B, so I call my even number one is that when this particular particle reach is the B end of the rod. This I am calling as event 1 when this particular particle which is here and that is event B of course how different observer so will perceive this that is we are going to discus little later, but the events are being defined in this way.

This particular particle reaches the end B and that is event number one when this particle reaches end A of the rod that is event number 2. In fact these two events would signifying would signify because this is the point when this particular particle has just enter this particular rod area. This is the time when it is just leaving this particular rod area so in principle the time difference between these two events should be the time that they particle takes to cross the rod.

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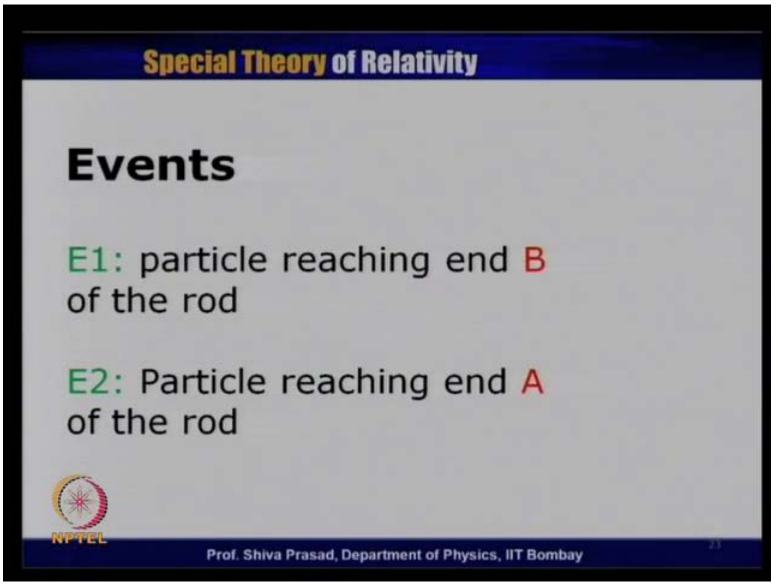

So, this is what I have written let me repeat here even number one particle reaching end B of of the rod even number two particle reaching end A of the rod.

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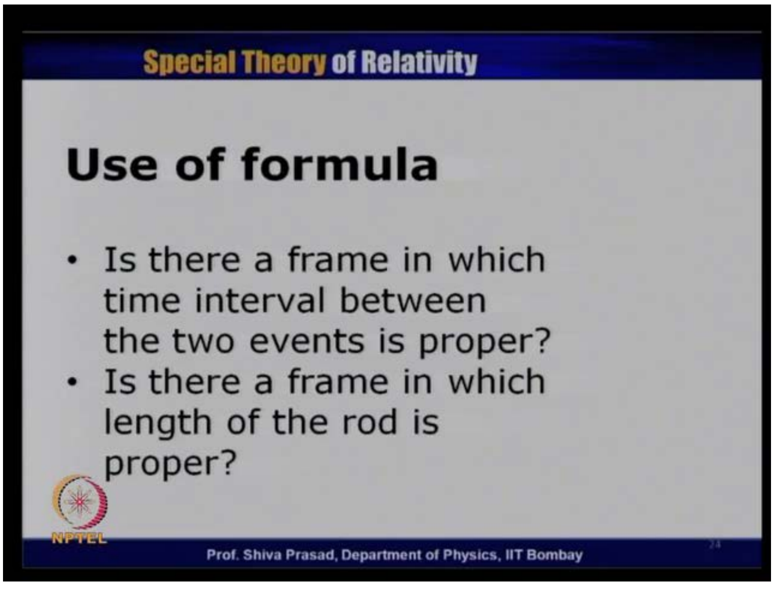

Can we apply the formula here, yes we can apply formula and this particular problem will try to solve by applying time dilation and length contraction formula, but before that we should answer the following question. Remember, whenever we are applying a formula either of time dilation or of length contraction the first question that we must always ask, this is the frame in which the information given appropriate information given is proper. So, this is what I have explain here is there a frame in which time interval between these two events is proper is there a frame in which the length of the rod is proper.

Answer to the second question is very simple because the length has to be proper in the rods is own frame will causes only in the rods own frame that the length is stationary. The rod is stationary, so length observed in the rods own frame of reference will be a proper length so it is a S prime frame of reference in which the length of the rod is proper.

Remember what we have given in the problem is that proper length that length in other frames has not been given, so if you want to find out will have to find it out by using appropriate method, but its only in S prime frame that length is proper. So, information given about the length is S prime frame of reference, now let us try to answer the question about the time interval is there a time interval which is proper out of these three frames or if you look in comparison to S frame.

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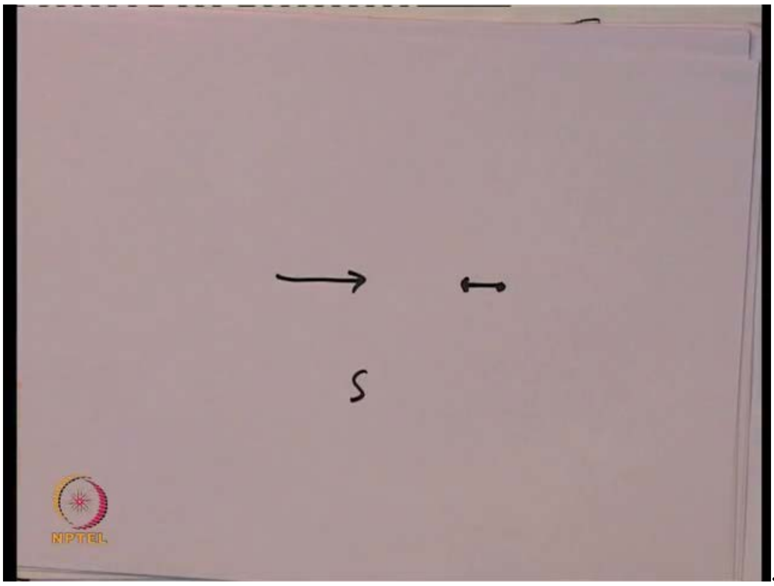

According to an observer in S frame, this particle is moving to the right this not sorry this moving to the right and this particle is moving to the left both are moving. So, in principle the event of this particular point this particular particle crossing the rod and crossing the other rod will happen at 2 different positions in S prime. Also, it will be different because according to an observer S prime, in fact the situation between S and S prime is very similar to the plate form, and the train examples that we have discussed earlier.

The person sitting on the rod will feel that the rod is stationary and the particle is actually going and coming first to the end a and then coming back coming going out of the end B. Therefore, these two events offer a different position event with respect to an observer S prime it is only in S double prime that is the particle frame in which that particular person will notice as if the rod is coming towards him. First, the end a of the rod reaches to him and then end B of the rod reaches him and therefore both the observations are being measured or being found at the same place in S double prime. So, in principle it is the time interval in S double prime frame of reference which is a proper time interval.

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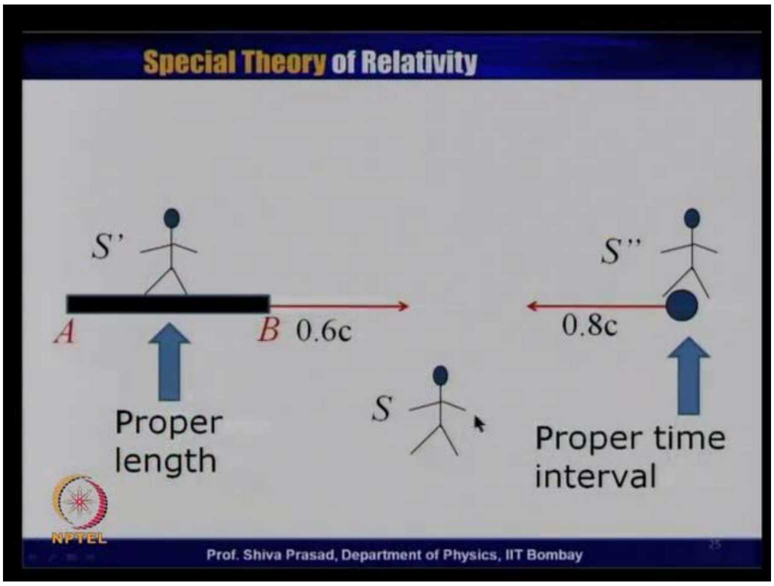

This is what we shown here in this particular transparency that it is this S prime frame of reference in which the length is proper it is S double prime frame of reference in which time interval is proper. If we consider these two if our problem was not mentioning about S frame of reference these two would be just like the train and the compartment problem.

The difference here is that we have not been given in a third frame in which this rod as well as this particular particle both are moving and we realize that in S frame neither the length nor the time interval is proper. Now, if we know the if we have to find out the time interval we should realize that we can go use a time dilation formula only when one time interval is proper.

So, it means it is possible if I know the time interval in S double prime I can use a time dilation formula and find out time interval in S prime. Similarly, if I know time interval in S double prime I can also find out time interval in S prime frame of reference if I know the relative velocity between S and S double prime, but I cannot directly go from S to S prime because none of these time intervals is proper.

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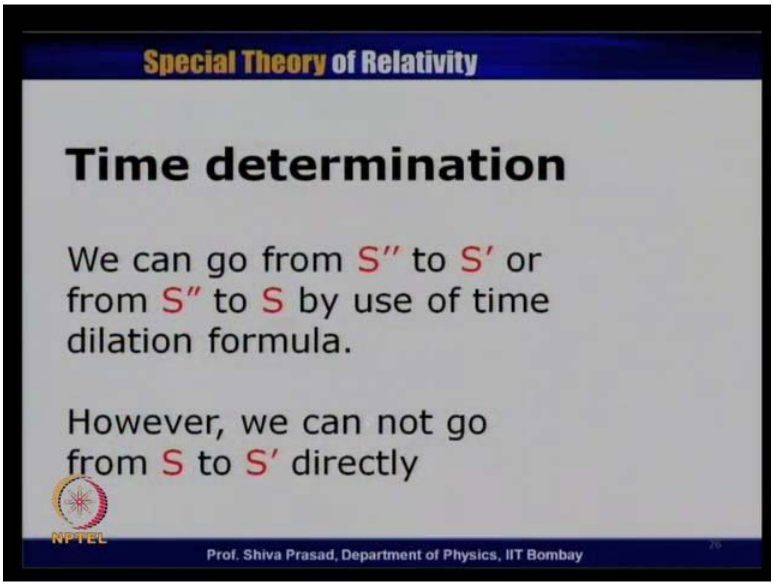

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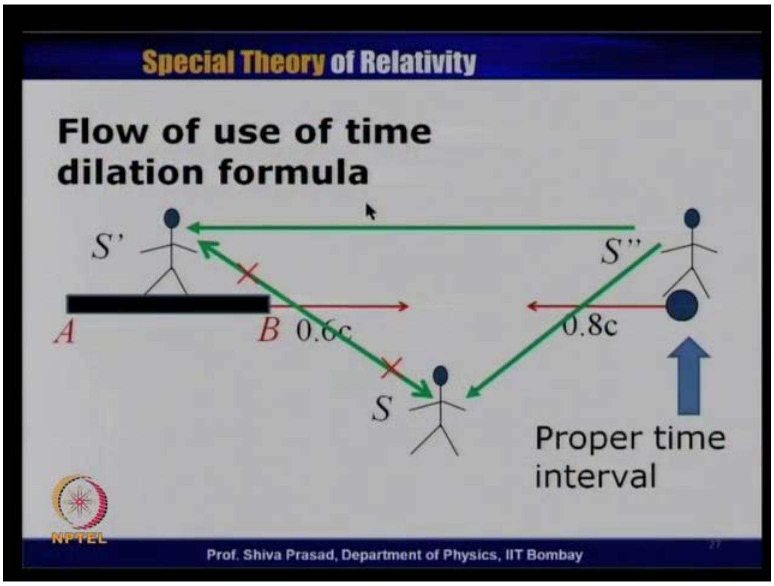

This is what I have shown in the next transparency; let me first just read it we can go from S double prime to S prime or from S double prime to S by use of time dilation formula. However we cannot go from S to S prime directly.

So, this picture shows a flow of the use of time dilation formula as we have said that time interval is proper in this particular frame of reference. So, I can go from here to here by multiplying by gamma I can find out what would be the time interval in this particular frame of reference I can from the here to here backwards also, but then I have to divide by gamma. Similarly, I can go from here to here if I know the time interval in this frame I can also find out the time interval in this particular frame by multiplying by gamma if if know the time interval at this frame I can find out time interval at this frame by dividing by gamma.

So, this particular path and this particular is open but, I cannot directly go from S to S prime so I cannot multiply the time interval in S to gamma by gamma and go to S prime. Take the time interval in S prime and multiplied by gamma to get an S prime get into S frame because time interval is neither proper in this frame nor in this particular frame. Suppose, if I have to go from S to S double prime I have to necessarily follow this particular route I must go from here to this and then from here to here.

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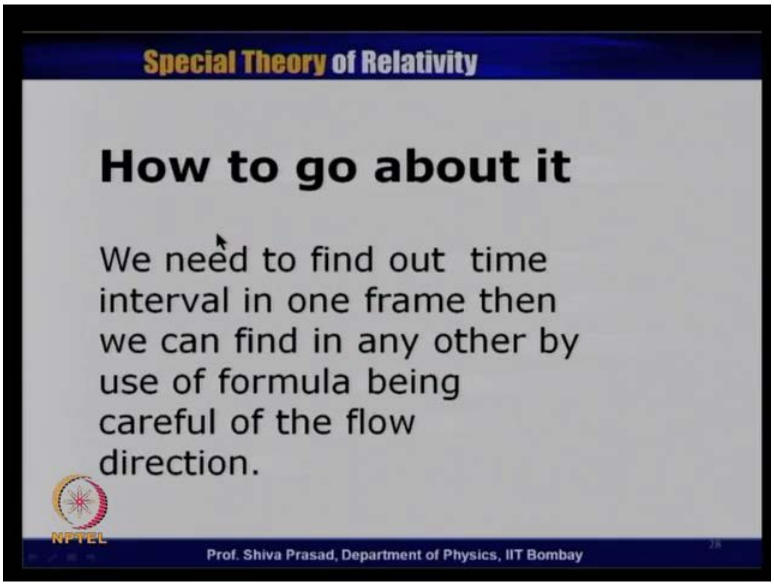

If I have to go from here to here I can go from this particular frame to this particular frame and then come back to here I cannot make a direct application of the formula. In this particular path because neither the time interval here is proper nor it is here which is proper. So, in order to apply time dilation formula one of the time interval has to be proper.

If you have understood this particular thing, then this decide how we go about this particular problem, now once we know the path once we know the flowchart then we can find out time interval. It will be any of the frames and following the flowchart we can always find out that time interval in any other frame what I have written we need to find out time interval in at least 1 of the frame. Then, we can find in any other by use of formula being careful of the flow direction that is very important that we have to be clear about the flow direction once we do that then we can find out without any error.

So, let us choose one of the frame let us try S prime frame of reference which is the rod frame of reference, so let us first try to calculate the time interval that particle will take to cross the rod in S prime frame of reference. Then, we will use this particular flowchart to find out the time interval in different frames.

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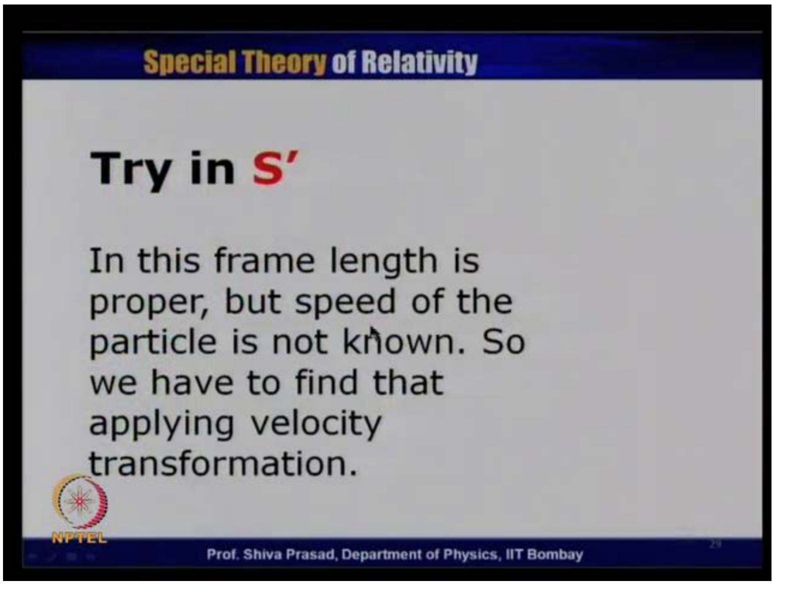

Now, let us look at this what information has been given S prime frame of reference we have already all already agreed that in this particular frame the length of the rod is proper because rod is stationary. Therefore, I know the length which is one meter which is given in S prime frame of reference this has been given to me, but I do not know any other speeds. In fact, I know the speed of S in S prime frame of reference, but immediately that is not very useful to me because as I have said that I cannot directly go from S prime to S or form S to S prime.

In any case, I have to evaluate first in this particular frame the time taken by the particle to cross the rod, therefore what I need clearly is the speed of the particle as being measured in S prime frame of reference. So, if I have to calculate this speed of the particle I come back to the problem that we have just now discussed earlier about relative velocity I have to find out the relative velocity between S prime and S double prime. So, my problem is that if I know v that is the velocity of S prime observer in S frame if I know u x which is the velocity of the particle in S frame what is the velocity of the particle in S prime frame of reference.

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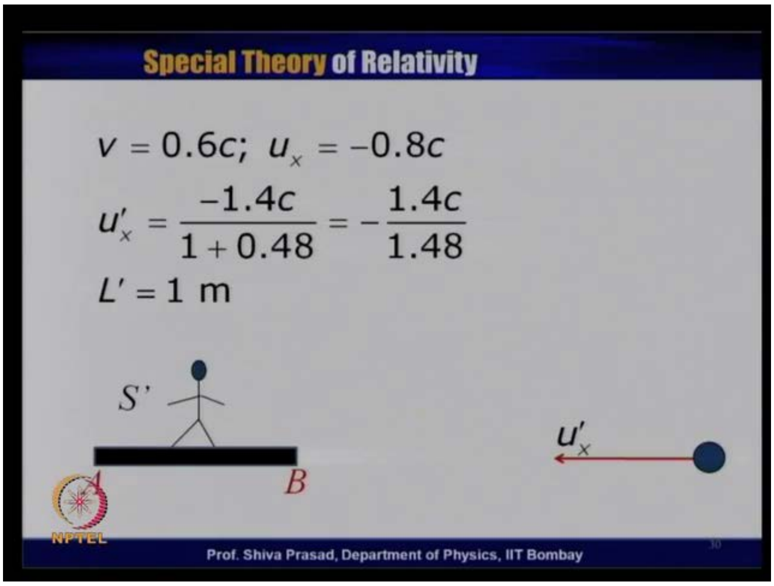

So, I come back here because I want to transform from S to S prime remember S was my ground observer therefore, v is the relative velocity between S and S prime frame of reference. These are the two frames which when wish the transformation is taking place so v cutter outs to be equal to be 0.6 c. The particle whose speed is to be observed is going to be given by u x and that is the particle which is this particular particle which is moving to the left in S even in S frame and that is minus 0.8 c which has been given.

So, we apply the same formula u x prime is equal to u x minus v divided by 1 minus u x multiplied by v divided by c square exactly the same formula and we will get this answer. We can be now probably quick let see just write this particular formula once more which was u x prime is equal top u x minus v divided by 1 minus u x into v upon c square.

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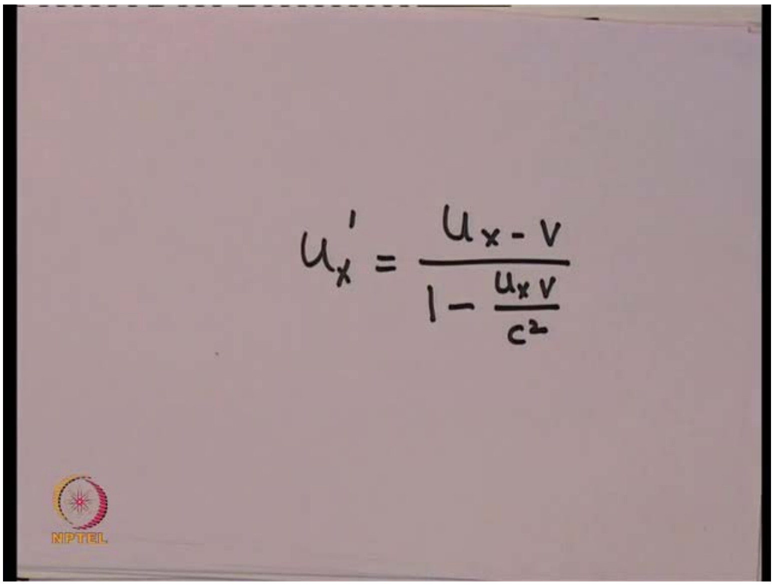

I have taken u x is equal to 0.6, c v is equal to minus 0.8 c, u x is equal to 0.6 c, v is equal to minus 0.8 c because of this minus sign this becomes plus because of this minus sign the numerator is minus and these two quantities get added up. So, my u x prime becomes minus 1.4 divided by 1 plus which is the product of these numbers 0.48 and you expand. It will turn out to be 1.4 c divided by 1.48, now what an observer in S prime would note according to him, he does not see the motion, he sees the motion of this particular particle only he does not see his own motion. Therefore, situation according to him would be as if this particular particle is approaching towards him first it comes near B there and goes and causes over point a just like train passing the platform.

So, even B takes place here even a takes place here we have already agreed that in this frame time interval is not proper, but if I have to calculate the time how do I calculate the time I know this speed of this particle in my frame. Remember, I have to be consisted in my frame I have find out this speed of this particle as minus 1.4 divided by 1.48 c. I know the length in my own frame which is 1 meter which has already been given this length force is a proper length.

So, what I have to do is to just divide this one by this speed to find out what is the time so this is length one this is this speed v x prime that will give me the time this particular particle will take to cross the rod. Say as simple problem as in it traditional classical mechanics it is in fact in classical mechanic this will hardly be called a problem that is extremely simple problem.

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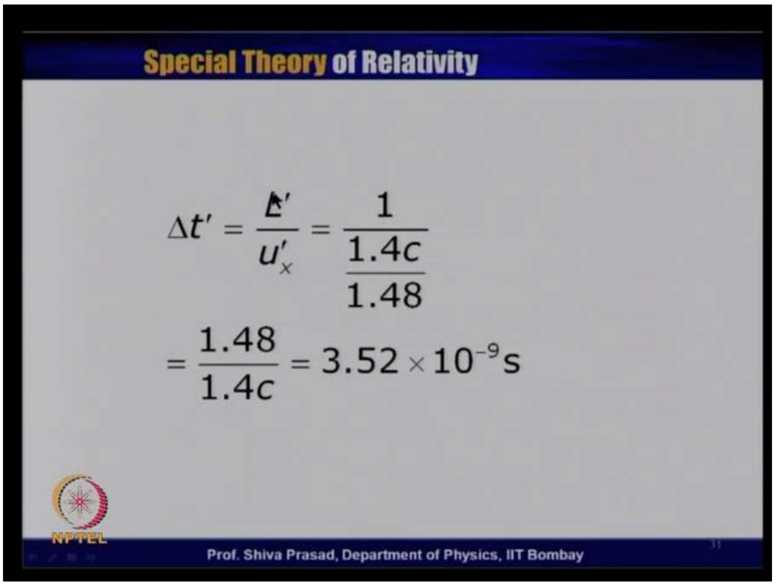

So, this is what I have written delta t prime is equal to l prime divided by u x prime this l prime is 1 meter u x prime we have already calculated is 1.4 c divided by 1.48.

So, this goes to the numerator this becomes 1.48 divided by 1.4 c taking the value of c as 3 into 10 power meters per second this delta t prime turns out to be 3.52 in to 10 power minus 9 second. So, I have find out that time interval in S prime for motion of length, now if I have to find out S double prime I realize that this delta t time is a dilated time. So, in order to go to S double prime I have to divide by gamma then I will find out its proper time interval which happens to be the time interval in S double prime frame. Once I know my proper time interval that I can always find out what is the time interval in S frame by following the flow.

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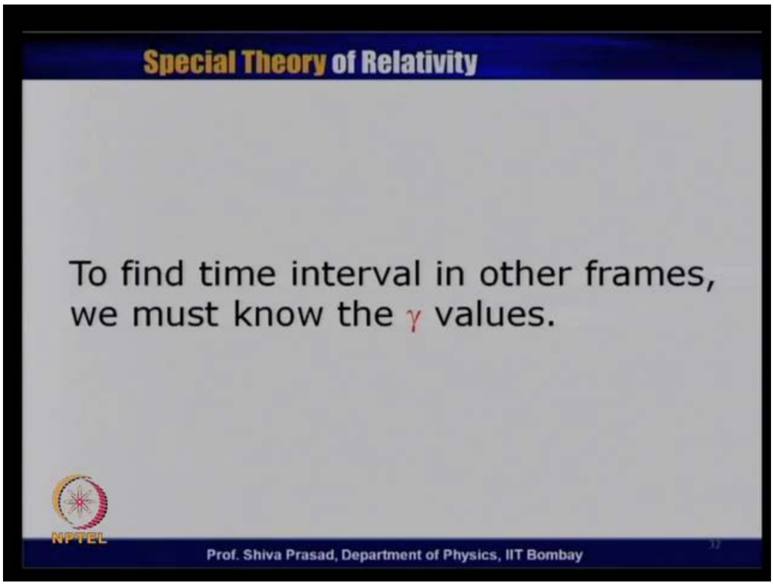

So, that is what I have written find time interval in other frames first of all I must know the relatives speeds and in fact what we are going to use is gamma. So, let us calculate the gamma value first remember there is there are going to be three gamma values because one w S and S prime another between S prime and S double prime and third between S double prime and S. So, that three gammas because they are three relative velocities and therefore we calculate we will calculate three different gamma values.

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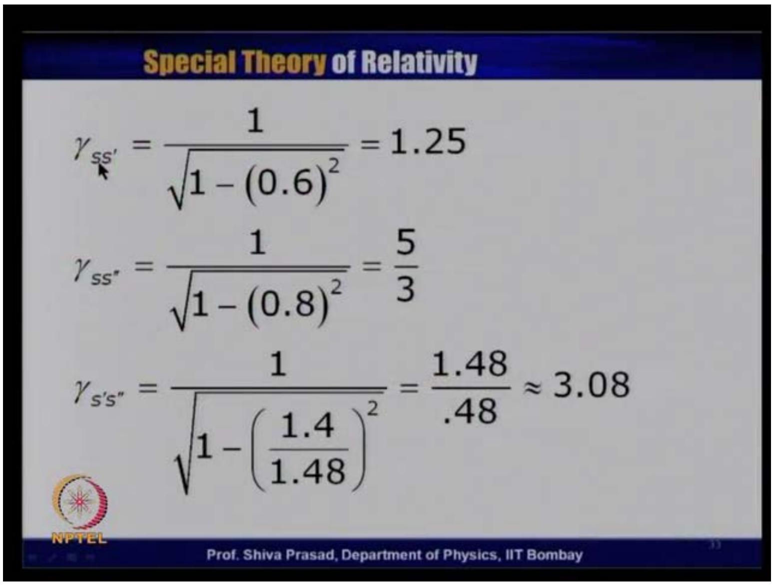

So, this is the first equation here gamma S s prime means I am finding out the value of gamma using the frames using the relative velocity between S and S prime frame most reference which is more S 0.6 c. Now, I think we should be comfortable that gamma S s prime will be given by a 1 divide by under root 1 minus v square by c square while v have to be taking as 0.6 c. So, this becomes one divided by under root 1 minus 0.6 square is 1.25, similarly, gamma between S and S double prime S and S double prime frame have to be found out by using the relative velocity between S prime and S double prime bit sorry between S and S double prime.

That is 0.8 c which has been given so this turns out be equal to 5 by 3, now we have find out the value of gamma it is in S prime and S double prime for which we had already calculated relative velocity which was 1.48, 1.4 c divided by 1.48 c square cancels out. So, this gamma S prime S double prime turns out tom be equal to 1.48 divided by 0.48 which is approximately equal to 3.08. So, I have found out corresponding to three different relative velocities three different gamma values, now all we have to do is to apply is simple time dilation formula.

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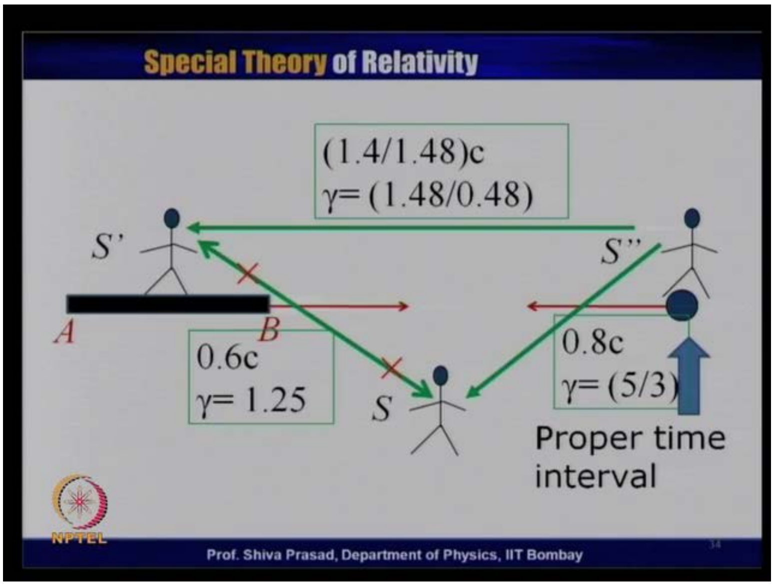

I come back to the picture here relative velocity between these two was 0.6 c, between these two was 0.8 c, between these two was 1.4 divided by 1.48 c that is what we calculated using this relative velocity I got gamma which was this 1.48 divided by 0.48. Using this relative velocity, I got a value of gamma which was 1.25 using this relative velocity which is 0.8 c. I got gamma is equal to 5 by 3 what is only in this frame that time interval is proper you agree it that I cannot use this particular route directly.

So, I will not use this number directly, but I can always go from this frame to this frame remember the flow direction is like this it means time will be actually be dilated. Here, this is proper time interval, see if I have to find out how along the reverse of the direction I have to divide by gamma if I have to go from this direction to this direction I have to multiply by gamma.

So, whatever is the time interval which has been found here if I divide by gamma against the flow of the arrow then I find out what is the time interval in this particular frame of reference. So, I can go from here first a direction opposite to the flow here in a direction towards the flow I cannot use this channel directly that is what is the idea.

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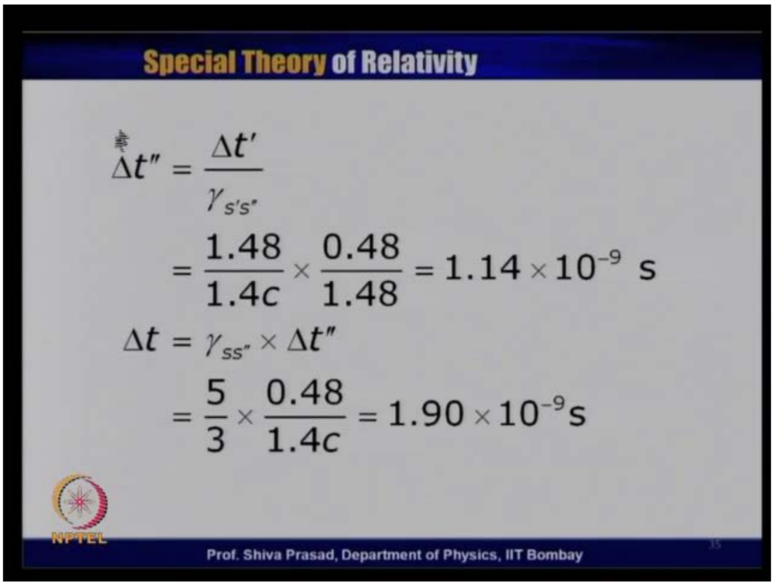

So, that is what I did I calculated delta t double prime and use the delta t prime that has just now calculated and divided by gamma S prime S double prime because we are in a direction opposite to the direction of the flow of the arrow. This was very theta prime which was calculated earlier gamma S, S prime has been calculated here if you calculate this time this turns out to be 1.14 into 10 power minus 9 second.

In order to go from S double prime to S, S frame we find out delta t in S frame I have to multiply because I am in the same direction as the direction of the arrow. So, I have to multiplied by gamma S, S double prime to get delta t double prime and this would be given by 5 by 3 into 0.48 divided by 1.4 c which turns out be 1.9 into 10 power minus 9 second.

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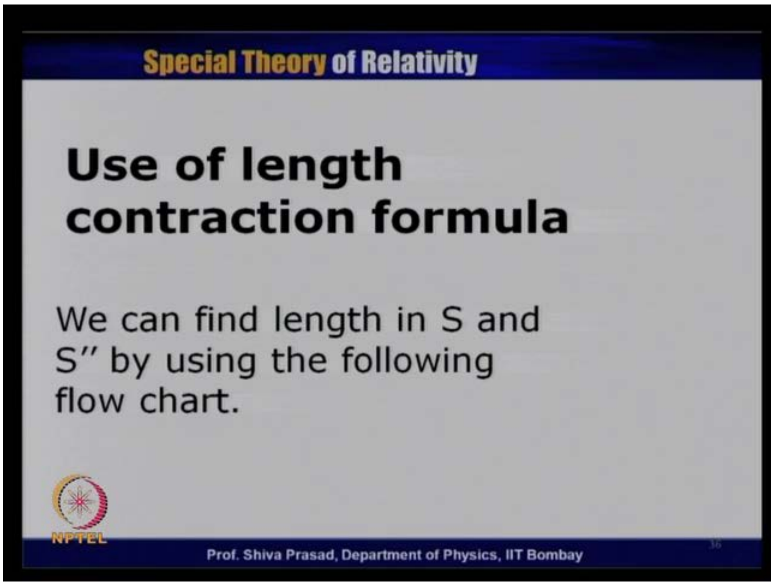

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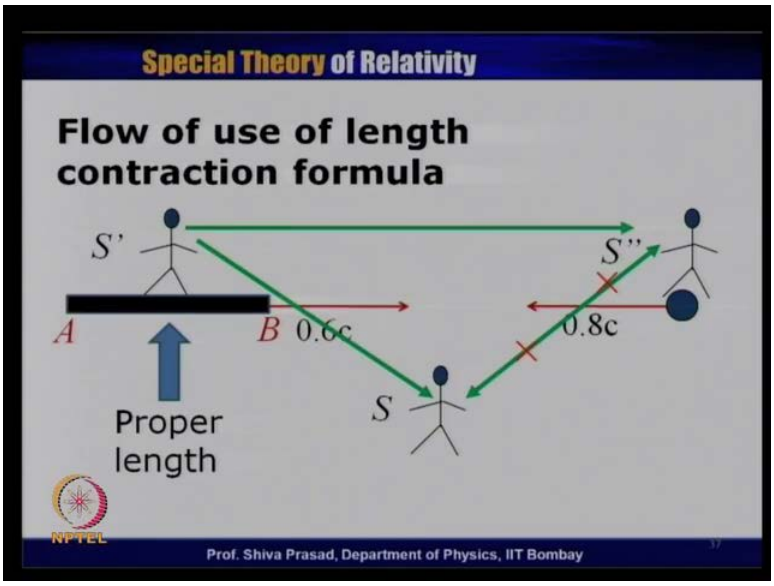

I have found out all the three time intervals as we have calculated as we are asked; now there is another method of doing it that is by finding out the length contraction formula. I do not want to directly find out or I do not want to directly use the time dilation formula, but I want to apply length contraction formula can I do that? Yes, I can do that also there is another method of calculating all these time intervals by using the length contraction formula, so let us try to move to this particular thing here.

Now, as far as length contraction is concern I realize that is only this frame in which the time interval is proper I am sorry the length is proper. So, this is the frame in which length is proper, now I can go from this frame to this frame by applying length contraction formula. I can go to this frame to this frame by applying length contraction formula, but I cannot go from this frame to this frame because this channel is blocked because neither this frame nor in this frame the length is proper.

So, I must use proper length which is given in this frame and that has to follow this path or this path I cannot use a path like this of course I can use the plot path like this from here go to here and come back here or from go from here to here and come back here. That is possible, but not the direct use of length contraction formula from S to S double prime that is prohibited because in neither of these two frames the length is proper I have. Already, know the proper length which is one meter which is an S prime frame of reference I can immediately find out I have already evaluated the gamma values.

So, problems reasonably simple I have to calculate the lengths in S double prime frame of reference in S frame of reference. Both have to be obtained by dividing gamma because appropriate gamma because I am always flying I am always in the direction of the arrow remember its contraction length is contraction. So, the formula is dividing by gamma so l double prime will be l prime divided by gamma S prime S double prime which is as we have calculated gamma S prime S double prime was 1.48 divided by 0.48 this just becomes 0.48 divided by 1.48.

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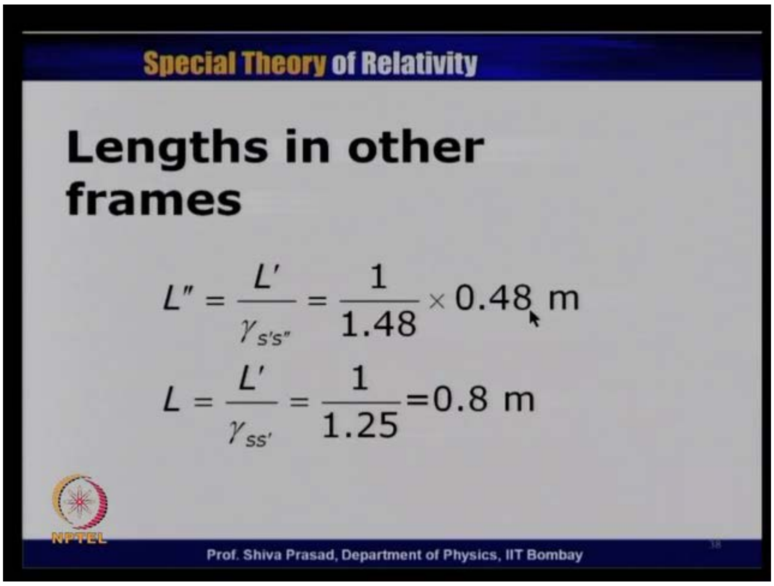

Similarly, if I have to go from l prime to l that I have to use gamma S, S prime this is 1.25, so this becomes one divided by 1.25 this 0.8 meters if I know the length in S frame as well as in S double prime frame. So, let us use this particular length first in S double prime frame of reference which is comparatively simple.

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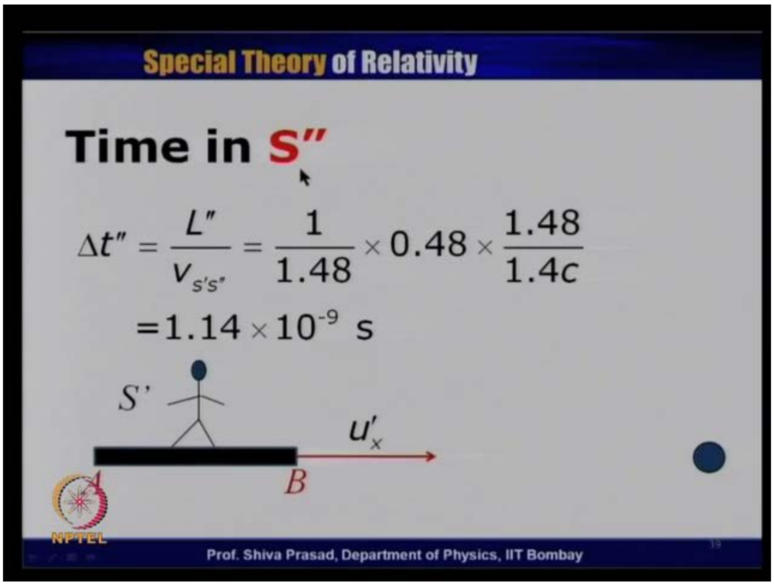

As far as S double prime is concerned it means the frame of the particle he would feel as if the rod is coming towards him first in b reaches in then in a reaches in and as we have discussed earlier. Both these events are occurring in the same position as far as S double prime is concerned, therefore this particular time interval turn out to be proper, but length is not proper in this particular case.

So, delta t prime again is a very simple I have to know what is length I have to know what is velocity with this rod is moving I can immediately find out what would e the time interval. So, the length was one divided by 1.48 and v, we have already calculated this was the length and this was the v value immediately. Substitute, you get 1.14 into 10 power minus 9 c or let us go in S frame here things are little more tricky.

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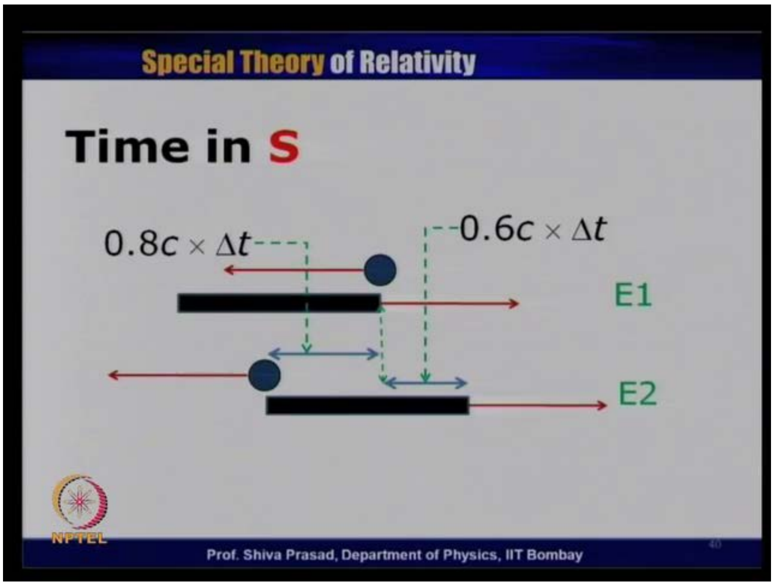

According to an observer S frame both particle and rod are moving seen S prime or S double prime there was one over advantage that either particle or the rod was retraced, but in S frame both particle and rods are moving both particle and rods are is moving. So, particular rod is moving to the left side this particular rod is moving to the right side, so let us suppose event 1 occurs it means this particular particle reaches this end of the rod this is my event 1 event 2 is when this particular particle reaches second end of the rod, but what within these two events.

This particle is moving to the left this rod is moving to the right, so situation would be that this rod by the time the event 2 occurs this rod have would have already moved to the right. This particle was any where moving towards the left so when event 2 occurs it occurs somewhere here. Now, if you realize that in this particular frame this let suppose delta t is the time interval as observed within these two events as observed in S frame.

So, during this particular time delta t this rod would have moved to the right a distance of 0.6 c multiplied by delta t while this particular particle would have moved to the left by an amount 0.8 c into delta t. So, this is the distance that this particle has travelled in time delta t this is the distance that this particular rod has travelled right in the time delta t and as you can see very clearly from this picture this length.

This distance plus this distance must be equal to the length of the rod, in case this event occurs at the first end of the rod and this event occurs at the second end of the rod. So, as far as S observer is concerned I must get 0.8 c multiplied by delta t plus 0.6 c multiplied by delta t must be equal to the length of the rod as observed in S frame of reference.

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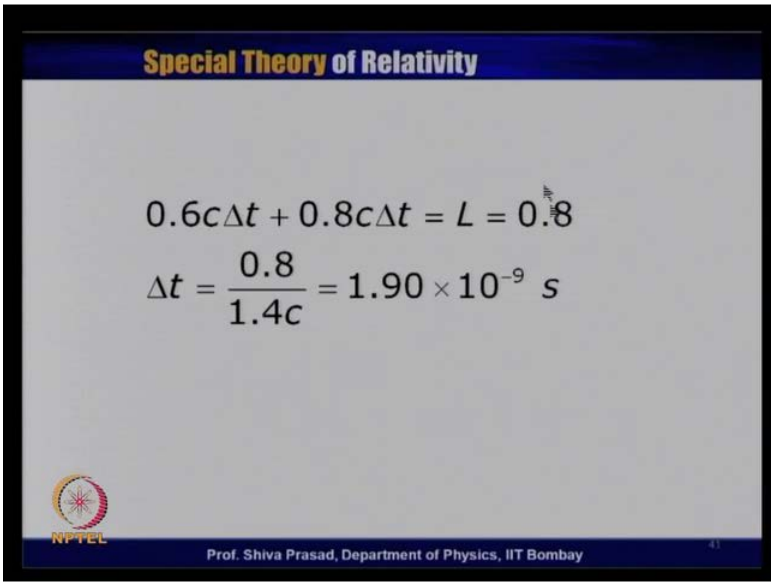

What I have written in the next transparency that 0.6 c multiplied by delta t plus 0.8 c multiplied by delta t is equal to the length which is equal to 0.8. We can add these things we will get delta t is equal to 0.8 divided by 1.4 c which really gets u 1.90 into 10 power minus 9 second.

The result that we had obtained earlier from the time dilation method only one comment I would like to make that if you would have solve the classical ease of problem probably you would have said what is the relative speed. Then, you would have calculated classically the relative speed and S 0.6 and 0.8 and divided by 0.8 and divided by 1.4 c. Now, would you say that relative velocity has increased I has has become larger than c, that is not correct relative velocity in relativity means only one thing is that you must change.

You must change your frame of reference and go to other frame of reference and then calculate the velocity what I have calculated one point four c is just a number because remember in relativity. When I am saying S frame, I have not calculated the rate of velocity I have calculated the displacement of the particle and displacement of the rod and added them to make equal to the length of the rod as seen in S frame. I have not used the relative velocity it just happened that these two by adding gives me 1.4 c, but it does not mean that this 1.4 c can be interpreted as the relative velocity for relative velocity I must change the frame this is the point which I would like to say.

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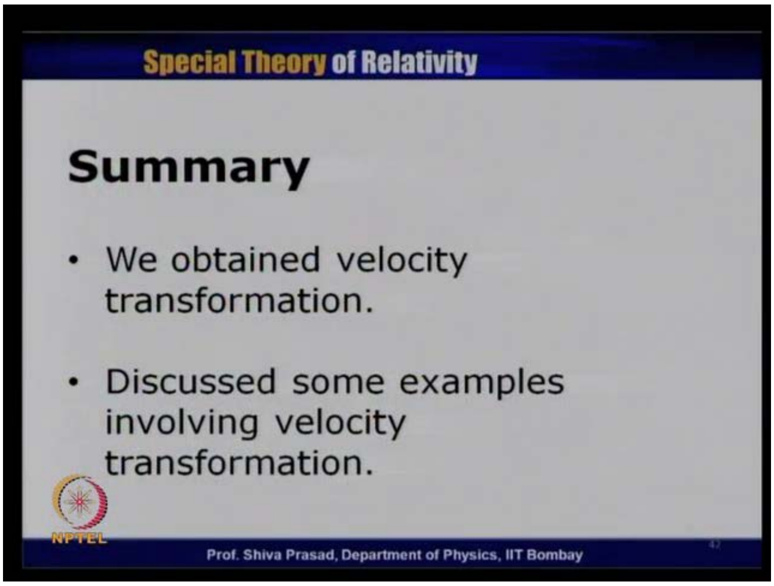

So, this was essentially what we wanted to discuss in this particular lecture, so at the end we would just like to give the summary. We obtained the velocity transformation and then eventually discussed some examples involving the velocity transformation.

Thank you.