Special Theory of Relativity
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Lecture - 7
Examples of Length Contraction and Time Dilation

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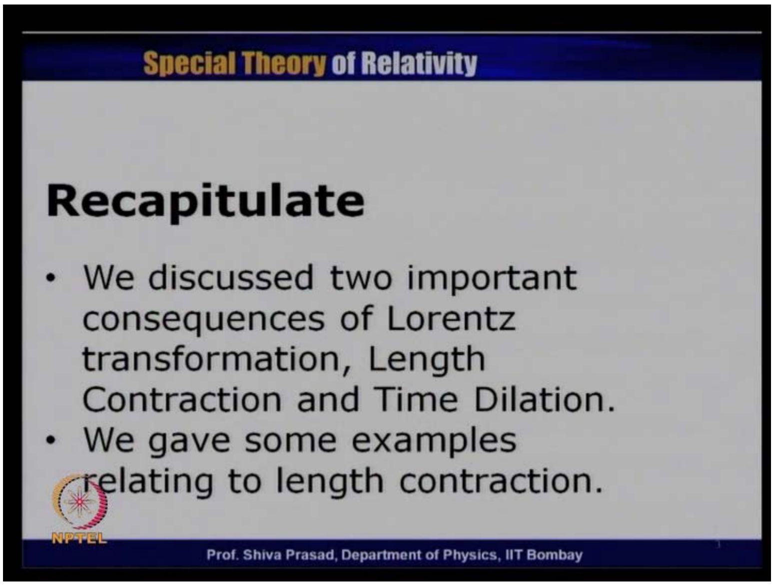

Hello, so let us recapitulate what we had discussed in our last lecture. We discussed two important consequences or I would say two most popular consequences of Lorentz transformation; one is length contraction, another is time dilation. We had defined something called a proper length that is the length of an object measured in a reference, frame of reference, in which the object is at rest; we called that particular length as a proper length. We said that if we go to any other frame of reference, the x-component of this particular length will appear to be contracted in a different frame of reference. Of course, as we all know that the direction of x is always chosen along their relative velocity between the frames. The other aspect that we had discussed was time dilation.

Similarly, we had defined a quantity called a proper time interval, which is a time interval between two events in a given frame in which the two events occur at the same position. This proper time interval appears to be dilated or supposed to be enhanced or increased, if you go to any other frame of reference, in which these two events do not appear at the same position. Then eventually we give some examples relating to the length contortion. Today, we will try to give some more examples, initially to discuss various other aspects of Lorentz transformation. Let us, let us go to one example which is probably again a simple example.

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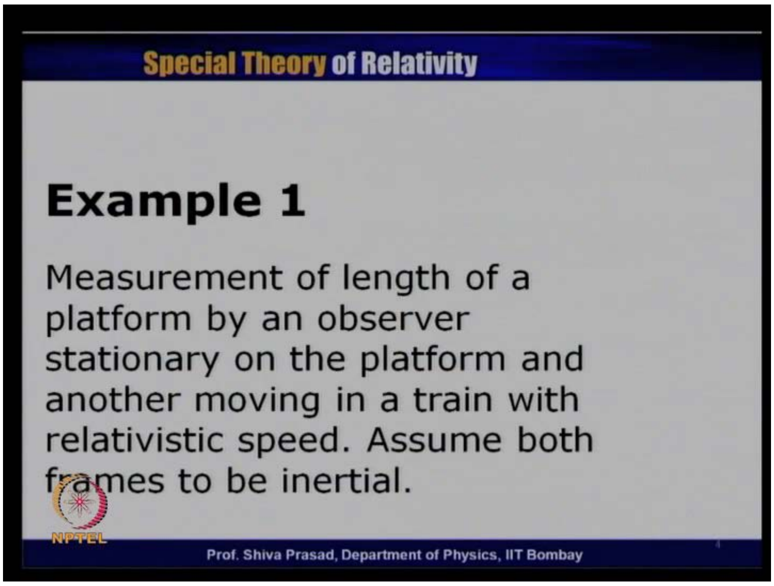

Let us assume that there is an observer standing on a platform, a train platform, the type of platform that we normally are quite aware of. Now in that particular platform a train passes by it, let us assume that platform is also on a frame of reference which is inertial. Though we all know that earth is not strictly speaking a inertial frame of reference but, we for all practical purposes we can treat it and let us assume that this particular platform is on an inertial frame of reference. And let us also assume that the train which is passing by the platform is also moving with constant velocity therefore, it also represent n inertial frame of reference. So there is no acceleration involved, so we can always treat one frame reference is s and s another is s one s prime.

Now both the observer an observer on platform another observer sitting in a compartment in a train both of them want to measure the length of the platform assuming that this particular is moving to relativistic speed. The session means that the speed is very high close to speed of light. Then let us do have some discussion about the measurement of the length of the platform.

So this is what I have written as the subject of the example. Measurement of a length of a platform, by an observer stationary on the platform and another that is another observer moving in a train with relativistic speed relative to of course, platform. Assume botht rains to be inertial.

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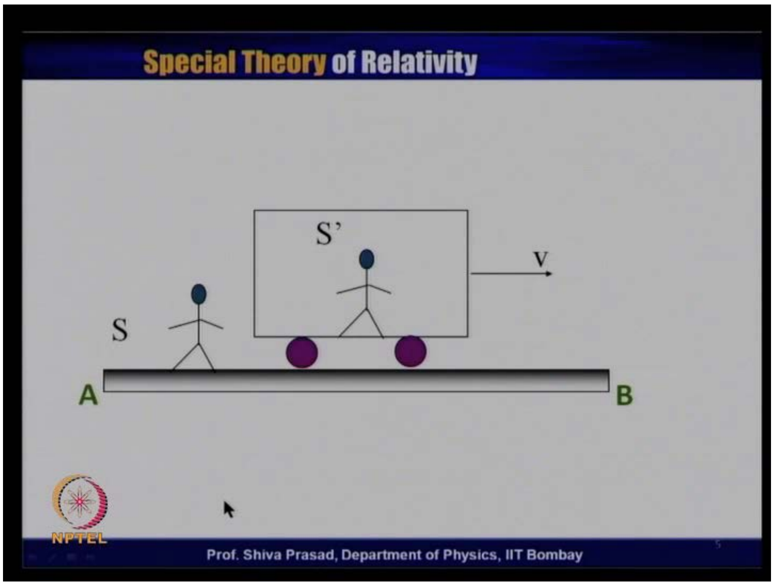

This figure approximately represents the situation in this particular case. Let us assume that is the platform, platform number you know the beginning of the platform is A; the end of the platform is B and there is an observers standing on the platform. You can assume that is origin you can assume it to be atman called an.

We can move that thing and this represents the frame of reference S. This is a train which is moving velocity v which is we are assuming to be relativistic speed very high speed. And observer is standing or sitting in a proper a particular seat in this particular compartment and which called this frame of reference as S prime frame of reference. And as we said that both observer S and S prime want to measure the length of the platform A B. Now let us define events. As we always said that the relativity is much easier to solve problems if we define events and write their coordinates including time.

So that is what we will do. We will define two events relating to this measurement of length I mean similar thing which we have done, also in the case of time dilation and (( )) dilation. So we come to the fact when we define the events, my first event is defined when the origin of S prime coincides with A end of the platform. Essentially means that whatever is origin of S prime we can take as one of the corner let us move this particular corner coincides with point A. Because train is moving a relative to B at a later time the symbol if we take for example, this as the origin of this particular S prime frame of reference it makes so difference what we take. Then this point coincides with point B of the platforms, so that is my event number 2.

So I repeat event number A is that origin of S prime coincides with A, event number B at 2 is origin of S prime coincides with end B of the platform. So these are my two events. Let us try to write the coordinates of these events in both S and S prime frame of reference.

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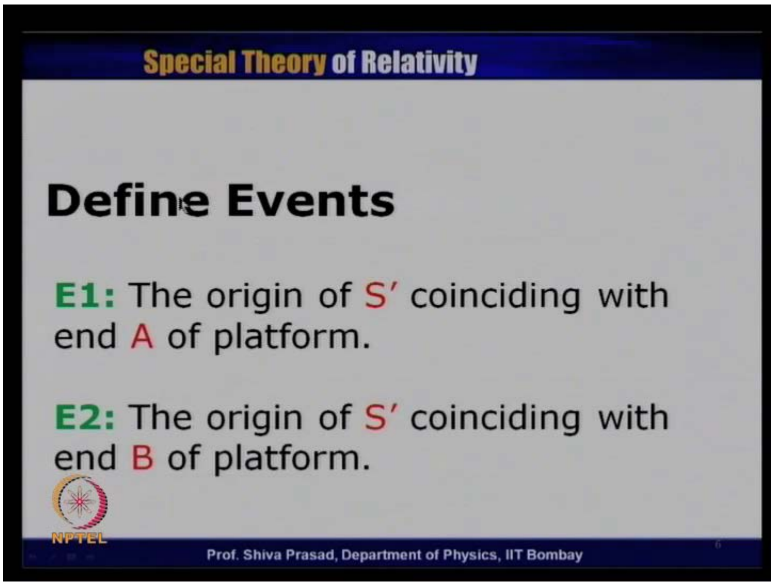

So here I am defining my events E 1 event one, the origin of S prime coinciding with end A of platform. E 2 the origin of S prime coinciding with end B of the platform. We can choose any origin in S prime. It does not make a difference, because what matters is only the difference but, for thinking proposal we can another one end or somewhere in the centre or at any other point.

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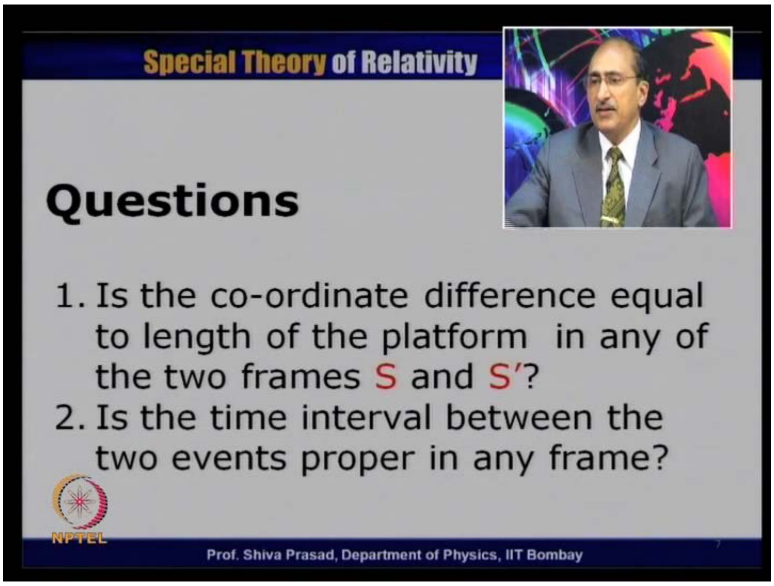

Let us write the events or before that let me raise some questions. Is the coordinate difference equal to the length of the platform in any of the frames? This is a question number 1 that I imposing. So if we take the difference of X let us assume that AB is a wrong X direction because that is what we have any way assumed. If we want to apply Lorentz transformation, because relative velocity direction is always chosen along the x direction or other x direction is always chosen along the relative velocity direction. So what is a ordinate difference that is x 2 minus x 1? Is it equal to does it give equal to other length in any of the frame of reference S or S prime? Question number 2, the time difference between these two events; it means we take the time t 1 when event number 1 occurs, time t 2 when event number 2 occurs. We take time difference t 2 minus t 1 is proper in any of the two frames that I have just now described namely S and S prime.

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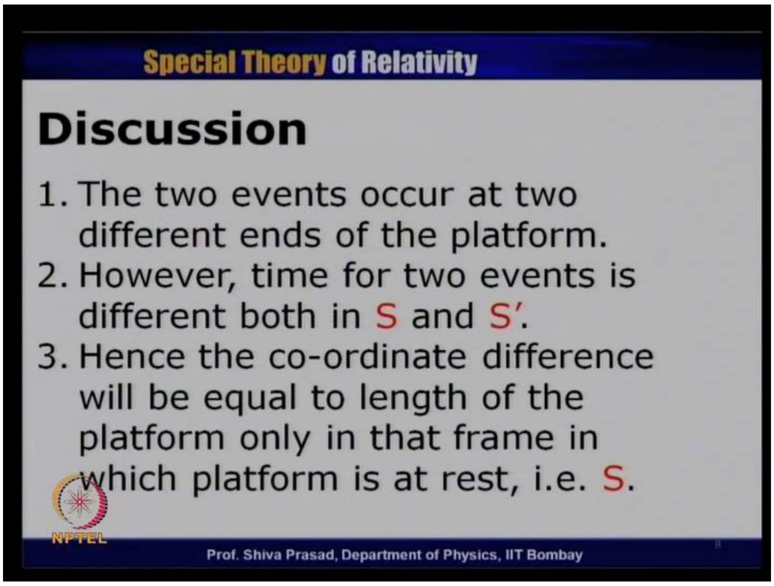

Let us have a little bit of discussion. We realise that two events occur at different ends of the platform. One occur at end A another in occurs at end B, this what we have already said. And neither in S nor in S prime the occur at the same time. So there is always you going to be a time difference between event 1 and event 2 whether it is S which observes it or whether it is S prime observes it.

And if you remember we have said it earlier that in object which is moving and if we measure the coordinates of the two ends then that is not going to give you correct length. It is going to be a coordinate differences going to give you correct length only in a situation when the object is at rest, because then only x 2 minus x 1 is corrected otherwise if there is a time difference; during a time difference of measurement the object has moved and the coordinate difference would not actually give the correct length. So we realise that at this particular situation also it is only the S frame that is the platform frame that it is only that platform frame that the platform is not moving. In the train frame the platform is actually moving and am assuming the length of the platform.

So if the coordinates of the two events has to give me the length of the platform then this platform must be raised at rest, because the two events are occurring at two different times. And that is only the frame S, so I conclude that in frame S the length of the platform will be given by the coordinate differences x 2 minus x 1. This is what I have written. Let us just read, the two events occur at the two different ends of the platform.

However, times for two event is different both in S and S prime. We just now discussed. Hence the coordinate difference will be equal to the length of the platform only that frame in which platform is at rest, that is something which I have been probably over emphasizing. But, this is something which is to be understood, because this is where people normally tend to make a mistake. They just write tried x 2 minus x 1 equal to length. X 2 minus x 1 is length only in case the object is not moving or the time should be same. So hence the coordinate difference will be equal to length of the platform only in that frame in which platform is at rest and that is the frame S.

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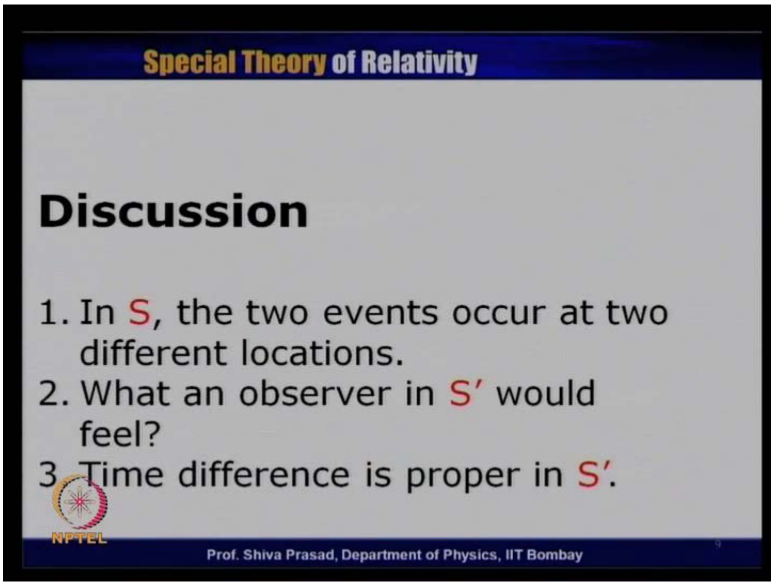

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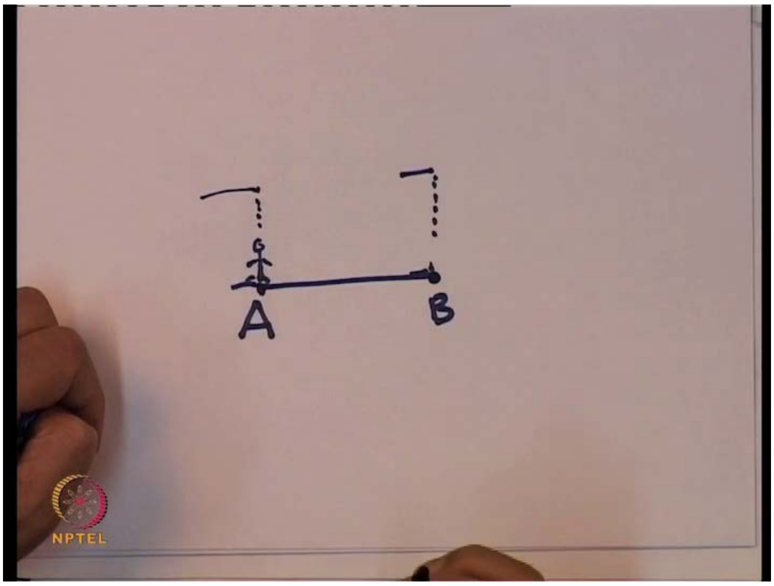

Now if you go to S frame, suppose we are standing on a platform and you watch the train coming you realise that event number one actually occurred at point A. Let us write here let say this is A this is B. So if suppose my observer was stranding here, what he would observe that train is coming from this side and passing by the platform and going to the other side. And let suppose the front end is origin of S prime, so when this particular train reaches this particular object this particular point event one occurs. When the same train comes and reaches at point B event number two occurs.

So position of event number one is A; position of event number two is B. And we have discussed the position difference of this would be giving the length of the platform. But, we realise that these two events actually occur at different location in S prime S frame. It means that the time difference between them is not proper, because for the proper frame for the time difference to be proper these two events must occur at the same position.

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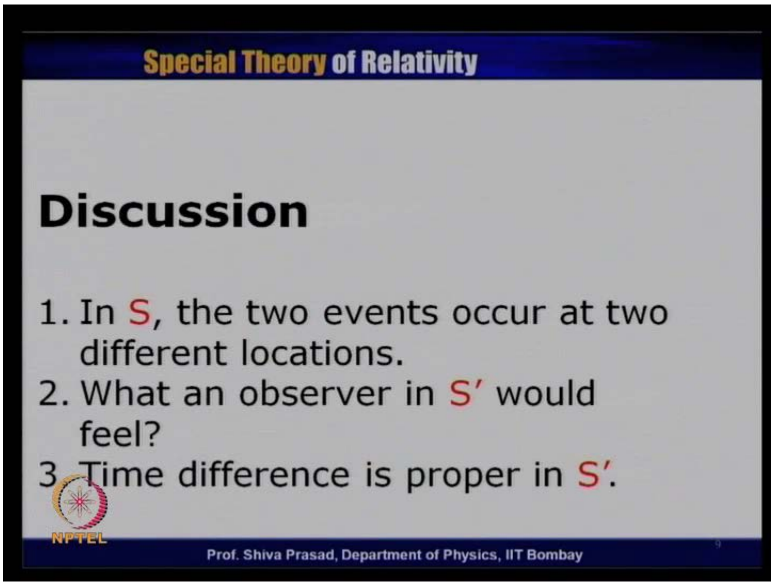

But what an observer S prime would be feeling? If you look from the point of view an observer S prime, he would see completely different situation. He would feel or she does not see his motion. Let us suppose this my origin of the train; he will feel that this particular platform A B is actually approaching towards him, because this person observer sitting in S prime does not notice his or her own motion. So according to him the platform is actually coming and going by this side. This is a very common thing if you are moving in a fast train and you see a particular station at least a train does not stop, it appears as if one end of the platform is coming towards you, then it passes by and the second end of the platform reaches you.

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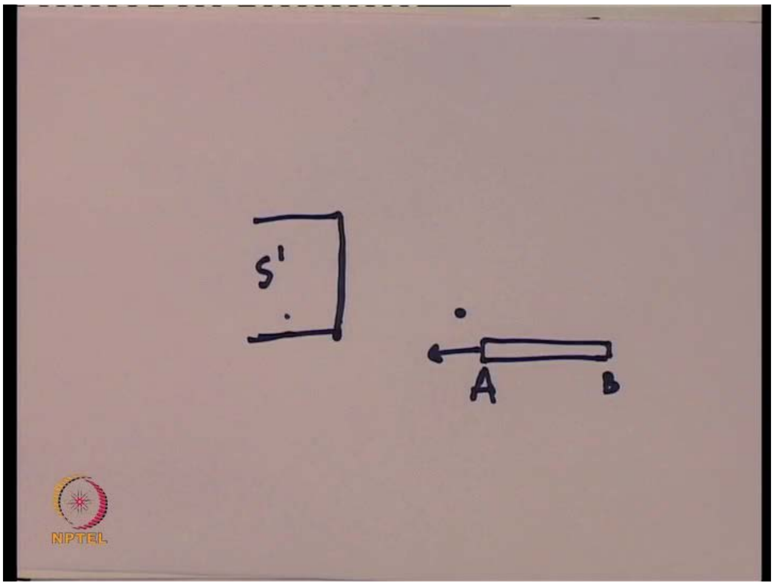

So in fact this person standing here or sitting here will observe exactly the same thing. That end A of the platform first reached him, that end B of the platform be at him. So according to an observer an S prime the first event that is A point meeting this particular point, that occurs at this particular point event number two that is point B meeting this particular point, again occurs at the same point. So as far as the observer sitting on the train is concerned he is sitting comfortably or standing comfortably at his place, he notices end of A of the platform coming towards him passing by it, then end B of the b of the platform coming towards him passing by it. He is standing at his origin or he is sitting at his origin. For him both the events point A, coinciding with him, point B coinciding with him occurs at the same position. Hence the time interval that will be measured by this observer in S prime frame of reference would really be a proper frame of reference proper time interval.

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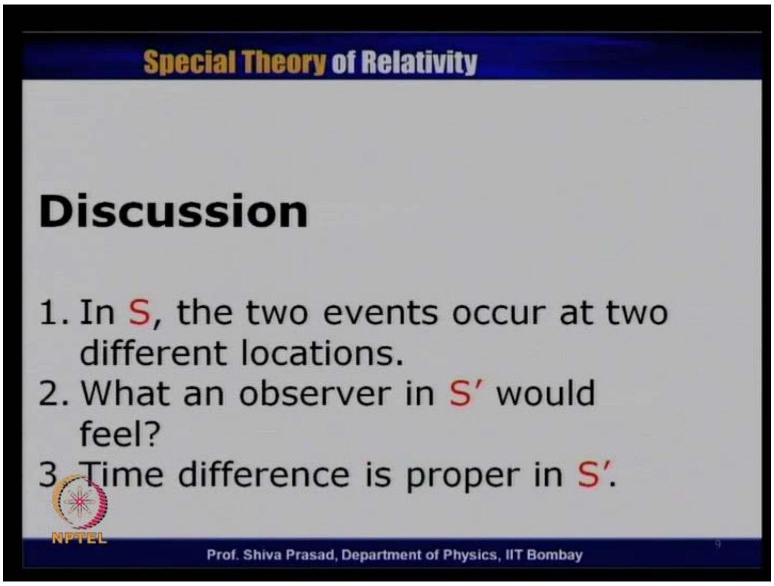

So that is what I have written in S the two events occur at two different locations. What an observer S prime would feel? An observer S prime would feel as I said that they occur at the same position wherever the person is sitting or standing. Hence time difference is proper in S prime frame of reference. So therefore, in S frame this time difference must be dilated.

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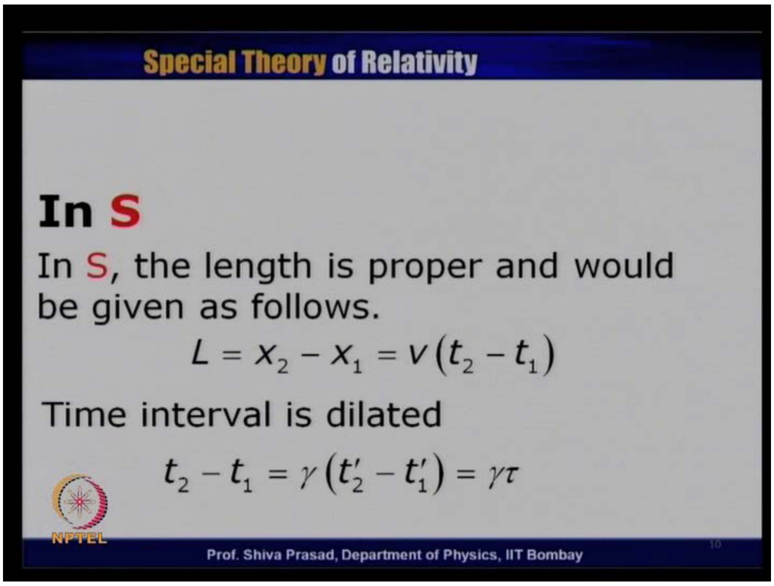

Now let us go to frame S and try to write the coordinates. We just now agreed that if we are looking with respect difference to the frame of reference S, which is the person standing on the platform the two events the difference of two events will give the actual length.

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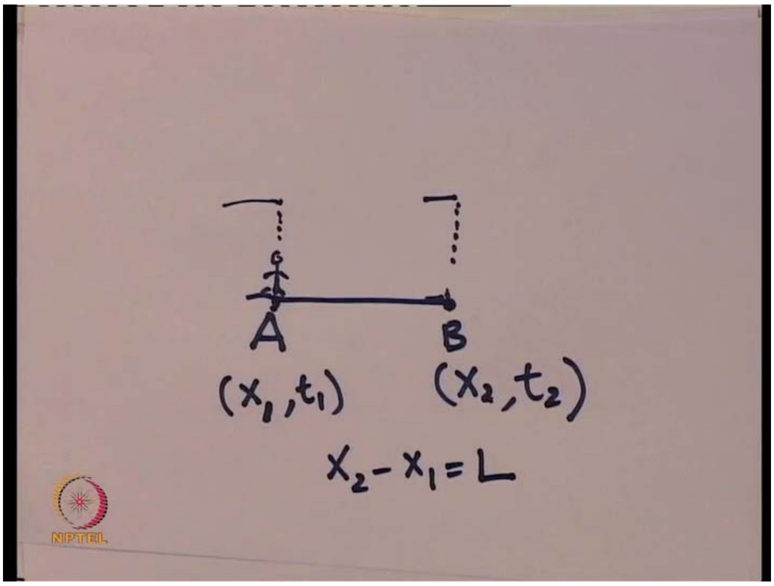

Because according to this observer this event occurs let us say at X 1 t 1 this event occurs at X 2 t 2,now clearly X 2 which is here X 1 which is here, both these events the position difference X 2 minus X 1 this minus this will actually give me the length of the platform. So I must have X 2 minus X 1 is equal to l which is the length of the platform. That is what I have written here. That in S the length is proper and would be given by l is equal to X 2 minus X 1 where l I have taken as the length of the platform as measured in S frame of reference.

However this particular observers had another way of measuring the length, provided he knows the relative velocity of the train. If he knows with what speed the train is passing by it, then what he could have done, he could have just measure the time difference between the two events event one and event two. But, for that he must to have he must have the information of speed and he must have the information of the time differences.

So if he knows the time difference t 2 and t 1, t 2 minus t 1 rather, then if you multiplies this speed of the, speed of the train, he should still be able to get the length of the platform. Or that is another way of determining the length of the platform. If I know the speed of the train and I know how much time the train took how much time the train took to go from here to here, that particular time and if I know the time difference between these two, then the speed multiplied by the time would give me the length of the platform.

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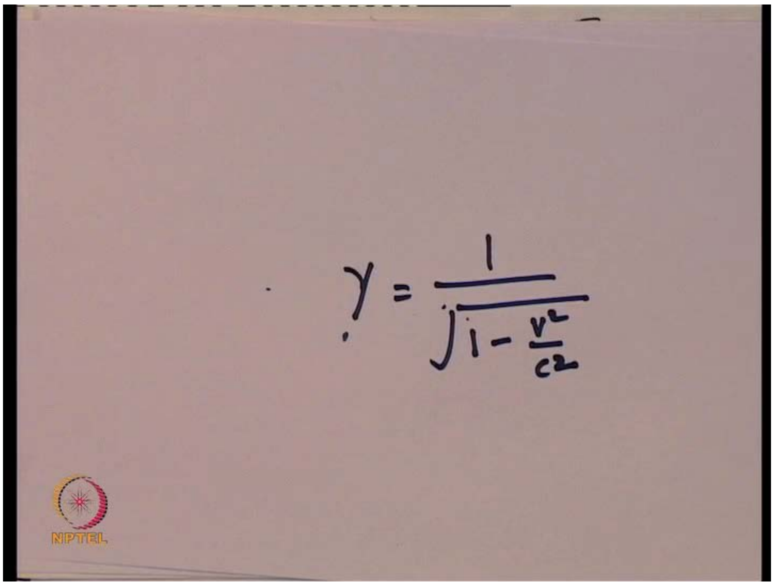

So this what I have written here that this length would also be given by v multiplied by t 2 minus t 1 which is time difference between these two events. But, as we have just now said that this time difference must be dilated, because it is in the S prime frame of reference, in which the time in interval is proper. Therefore, this t 2 minus t 1 must be given by gamma times t 2 minus t 1 prime, where as we all know gamma depends on the relative speed of the train and is actually given by gamma is equal to 1 upon under root 1 minus v square by c square.

So if I know the relative speed of the train, I also know the gamma. Once I know the gamma I can substitute in this particular value gamma and I know that t 2 prime minus t 1 prime that is a time difference measured in S prime frame of reference which is actually the proper time interval between these two events. That multiply by gamma must give me t 2 minus t 1. So this is equal to gamma times tow. If you remember we have mentioned that normally we have been writing the proper time interval between two events as tow. So this is gamma times tow.

Now let us go with respect to the frame of reference of S prime. We agreed that in S prime frame of reference the time interval was really proper. But, what about position difference, what about X 2 minus X 1?

We just now seen that, as far as an observer S prime is concerned the two events occur at the same position; it means X 2 minus X 1 must be equal to 0. If that is the reason we have called that this time interval is a proper time interval, because both the events occurred at the same position. Is I say as I say means that X 2 minus X 1 is 0. X 2 prime rather X 2 prime minus X 1 prime is 0. So if we take the coordinate difference in S prime frame of reference that coordinate difference is 0. So again I remind you that X 2 prime minus X 1 prime is not equal to the length. So because actually in the S prime frame of reference split from which is moving, therefore, just taking the coordinate difference and putting equal to length can we disastrous.

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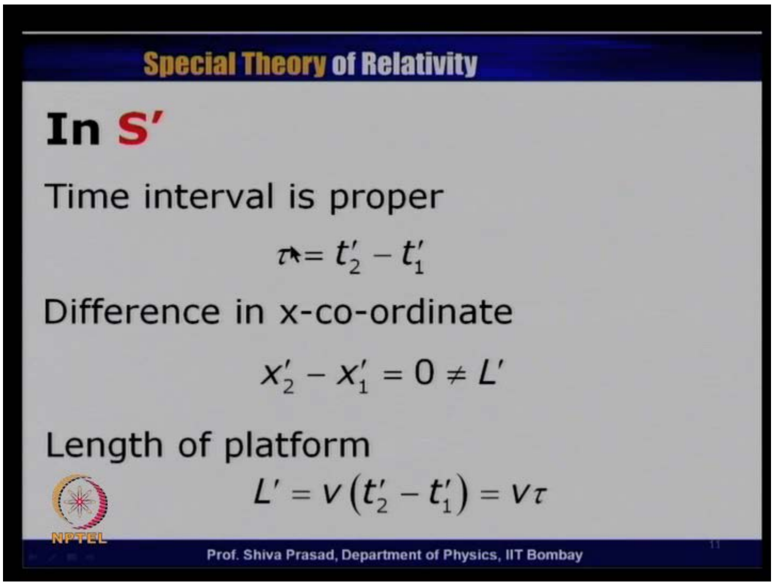

So this is what I have written here. Time interval is proper in S prime frame of reference I was actually equal to t2 prime minus t1 prime and difference in X coordinate X 2 prime minus X 1 prime is equal to 0, which is definitely not equal to the length prime; it means the length measured length of the platform measured in S prime frame of reference. Now suppose you are sitting in a, you are sitting in a train and want to find out the length of the platform. What way for example, you can adopt?

You can have a stop watch in your hand and then end A comes you can start the stop watch, when end B arise you can stop the stop watch, measure the time difference. Once you measure the time difference between these two events that would be t 2 prime minus t 1 prime and as I have agreed this is a proper time interval. And if I know what is the speed of the platform relative to me an S prime, which essentially is a same speed as an observer in S will notice of the train. If I know that particular V then I can multiply the time difference between these two objects.

These two events by velocity and I will get the length as observed in S prime frame of reference. So this is what I have written here the last line that L prime which is the length measured length of the platform measured in S prime frame of reference will be given by V which is relative velocity between the two frames multiplied by t 2 prime minus t 1 prime and as we had agreed that t 2 prime minus t 1 prime is tow which is the proper time interval between these two events. Therefore, this L prime can be written as V times tow which is t, where tow is the proper time interval or time interval measured in this frame of reference itself.

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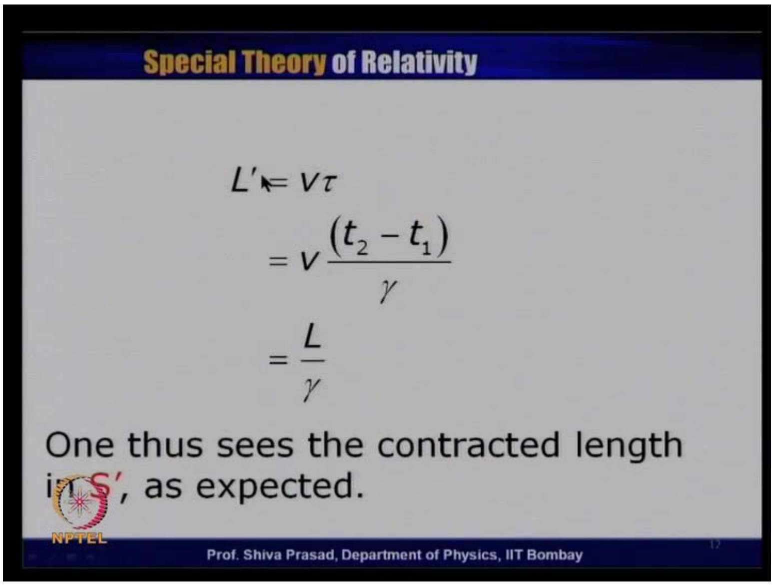

Let us just try to come sort of compare out notes with reference to S and S prime frame of reference. We just now said that L prime must be given by V times tow we have agreed that this tow is a proper time interval, therefore, what an observer in S frame that is the platform frame would observe would be dilated time interval. Therefore, this tow must be given by t 2 minus t 1 divided by gamma. Where t 2 and t 1 are the times measured in the platform frame of reference not the S prime frame of reference.

In the s prime frame of reference, where platform is at rest. So this time interval divided by gamma must give me the proper time interval, because t 2 minus t 1 was actually dilated. Tow is the shortest time interval. And if we agreed we have said earlier that V multiplied by t 2 minus t 1 is actually the length measured in S frame of reference. So I can write V t 2 minus t 1 as the length as measured in S frame. This terms out to be equal to L divided by gamma which tells that length of the platform as will be measured in a frame of reference S prime would turn out to be contracted. We have told that gamma is greater than 1, therefore, L divided by gamma will give you a smaller length. So this length will be contracted length as expected because remember L is the proper length of the platform. Had it not been proper length, then you could not be apply this particular formulae. Then we apply length contraction formulae one of the length must be a proper length. Then we applied time dilation formulae, one of the time interval must be proper time interval. Then only I can use that particular formulae, otherwise I cannot use it.

One thus sees the contracted length in S prime as expected. Now let us discuss this another example. This example in some form or the other has been discussed in many textbooks. This is actually real life example and before invent of special theory of relativity this was considered as a problem, which people were never able to understand. It is only special theory of relativity, which provided a solution to this particular issue. Therefore, it has a historical importance. There are large number of particles, some of which are stable; some of which are unstable. Unstable means they decay and change into some other particles.

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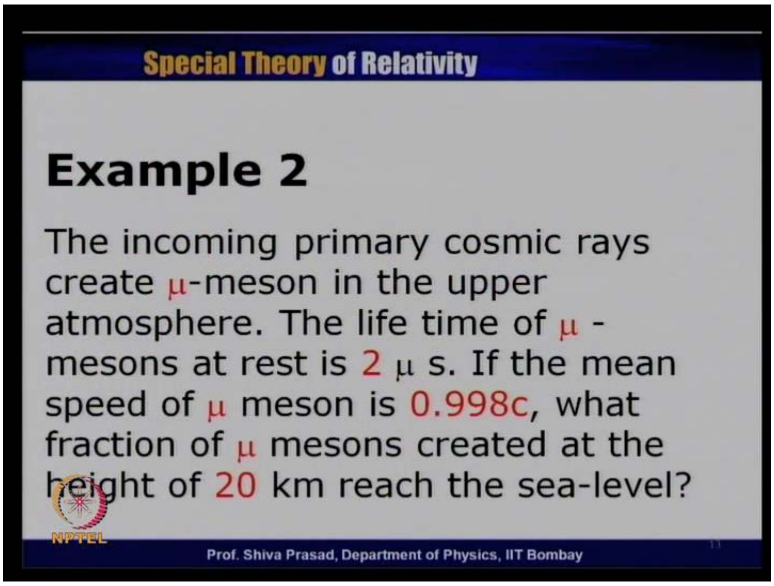

Now mu meson is one such particle. The lifetime of this particular particle is 2 micro second if this particular particle is kept at stationary is or a at rest in a particular frame of reference. What essentially be the lifetime is it means there is a statistical process of decay. It is like radioactive decay. Then there is a particular chance, you can never predict. Where a particular radio a nucleus will go and read your in decay. It may take some time it may take a different time but, you can always define something which is called the lifetime which represents in some way and approximately in you can call it in average type of behaviour.

So essentially, the idea is that if lifetime is very large then particle is much more stable; if the lifetime is very small compare in comparison to that the particle is like into decay comparatively very soon. So this particular particle has a very, very small lifetime with just 2 micro seconds, 2 into 10 upon minus 6 seconds is extremely small. Now these particles appears are created at the upper atmosphere of the i.

Now then they are created they are created with a very large speed. The speed is essentially close to speed of light, which in this particular problem we have taken to be a approximately 0.998c just to give some number approximately. So it is a very large speed essentially close to speed of light. Now we can calculate classically how much time it will take for this particular particle to reach earth. Then we can approximately estimate that out of how many of these particular particles which are created at the upper poseur how many of them would reach earth. And that number turns out to be extremely small as we just now see. But, people have found that on earth you get a very large number of mu mesons or.

What you called a mu meson shower from the upper atmosphere. People could not understand that if their lifetime was only 2 micro second, we expected very small number of these mu mesons to actually have reached earth, before that they would have decayed into something else. So how it is possible that such large number of particles can really come and reach earth in spite of the fact that their lifetime, so small. And this particular problem was actually solved by special theory of relativity by virtue of time dilation.

So let us now read the problem, the incoming primary cosmic rays create mu meson in the upper atmosphere. The life time of mu mesons at rest is 2 micro second, if it is at rest if you create this particular mu meson in laboratory at rest the lifetime will turn out to be 2 micro second. If the mean speed of mu meson is 0.998c, let us talk about everything in terms of n average. What fraction of mu meson created at the height of 20 kilometres, let us take that no they have created a distance of 20 kilometres above the means level how many of them would really reach the sea level; so that is essential the problem. Now let us assume that we focus our attention on one particular mu meson.

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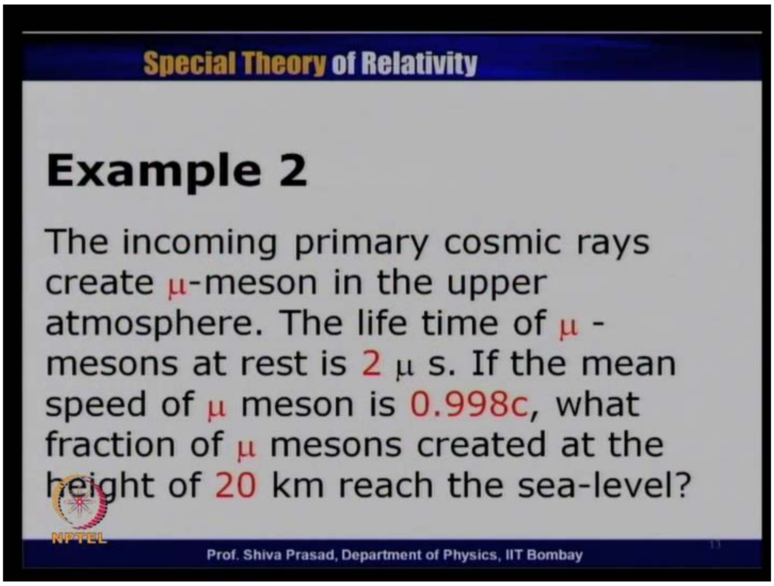

Because we are looking at the average behaviour and we will take one particular mu meson which has exactly the same lifetime as 2 mu meson few more micro second. Now if the particular is at rest then we expect that after 2 micro second it will decay. So if I this particular mu meson was created at the upper atmosphere of the earth and if there is a person sitting on mu meson or my laboratory was moving along with the mu meson assuming of course, its speed to be constant because we applying special theory of relativity.

Let us assume that mu meson has the same speed as travel through the earth, it remains constant as 0.998c. So if it is always the same then this particular observer would find that this particular mu meson would decay into micro second. In just like the train problem and the platform problem of course, my platform is from here 20 kilometres long from here to the top of the atmosphere.

Now this person who is coming along with the mu meson would conclude that 2 micro second after 2 micro seconds this particular particle will decay and because the mu meson is at rest in that particular frame of reference, so the event number one creation of mu meson; event number two decay of mu meson, they will occur at the same position and therefore, this time interval will be proper time interval. However on the earth event number one occurs 20 kilometres above, which is the upper end of the atmosphere and the event number two occurs wherever indicates somewhere much down much below down.

So according to an observer on earth this time difference between these two events must be dilated. And because c is and the speed of the particle is very close to c therefore, you would find that the time dilation affect will be very, very large. And therefore, affectively an observer or on earth would feel that the life time of the mu meson has increased and that is why he could observe a such large number of fraction of mu mesons the reaching earth.

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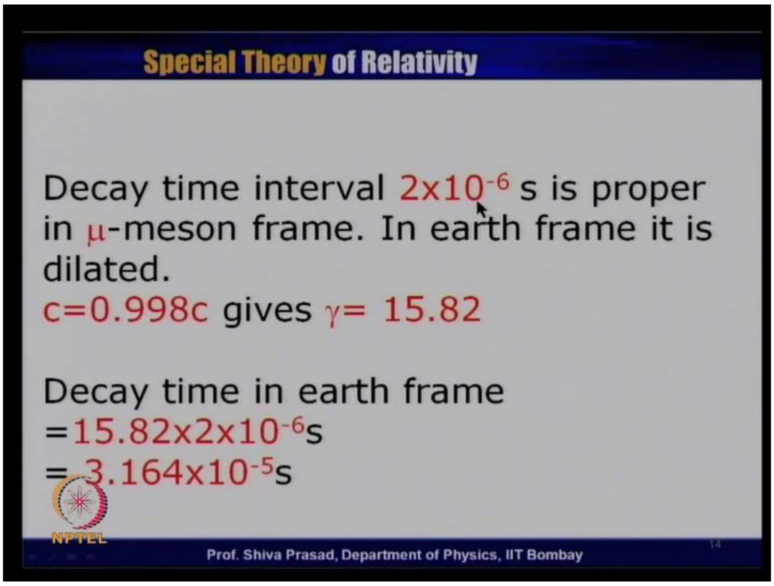

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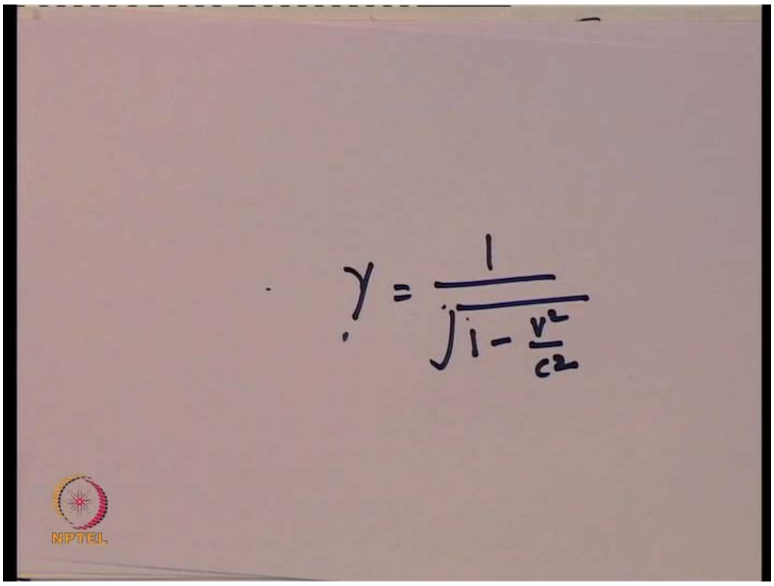

Now let us do the numerical calculation. Decay time interval 2 into 10 to the power minus 6 seconds which is 2 micro second is proper in moves on frame. In earth frame it would appear to be dilated. If we take c is equal to 0.998c and use this expression of gamma that we have just now written gamma is equal to 1 upon under route 1 minus v square by c square. We put this number of v which has 0.998c, you will get gamma is equal to 15.82 approximately 16 up of that order. It is a very large gamma 15.82.

Therefore, decay time or you can say life time of mu mason in earth frame of reference will be just this factor multiplied by 2 into 10 to power minus 6 second which terms out to be approximately 3 into 10 to power minus 5 second one order of magnitude larger 15 times larger. Means lot of difference or if you want to talk of the tow will if decay these essentially follows the same law or same rule as a nuclear decay obvious which is an exponential behaviour.

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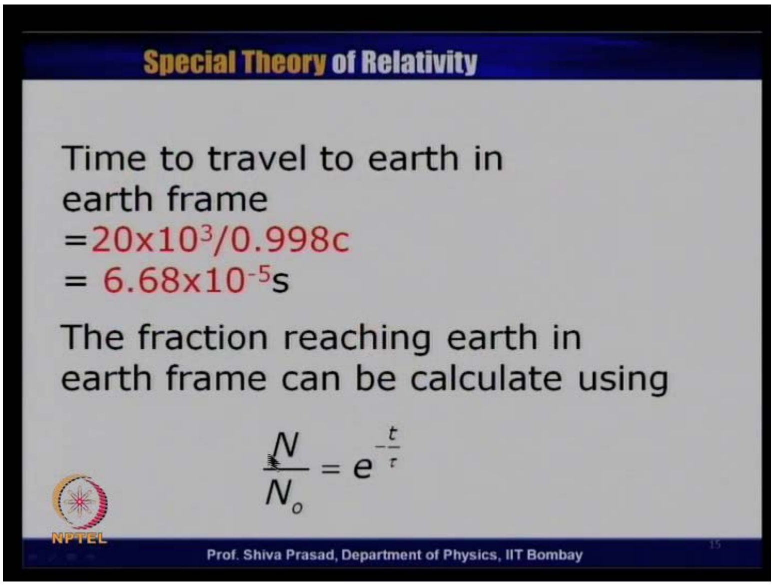

Which is given here at the bottom end of this particular transparency, that N naught is the number of particles with which we had started, then total number of particle which remain at a time t is given by N divided by N naught into e raise for minus t by a tow. Or if we calculate which is the fraction of the particles which are which will be reaching earth. Now if we take this particular life time we can calculate how much time it will take in S frame of reference which I am calling as a earth frame of reference. How much time it will take for this particular particle to reach earth, that will be 20 kilometres which is here divided by what is the speed of the particle.

So normally if this particular mu meson was suppose to reach earth, according to an observer on earth that observer must have travelled sorry this particular moves on must have travelled for time 6.68 into 10 to power minus 5 seconds approximately 7 into 10 to power minus 5 second. So if I have to find out what is the fraction remember this calculation I am doing in earth frame of reference, if I have to find out what is a fraction which must reach earth that will be N upon N naught will be given by e raise for minus t by tow therefore, t I will substitute 6.68 into 10 to power minus 5 second.

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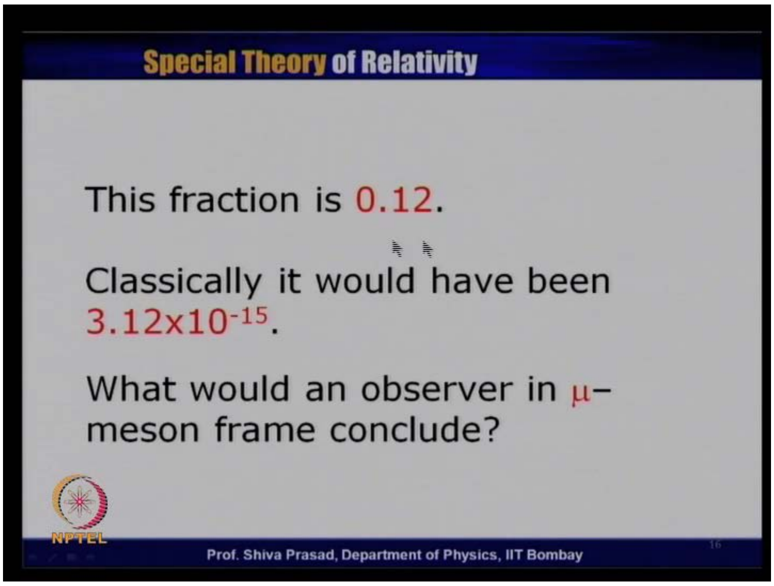

And for tow will substitute the value which we have obtained in my last transparency which is 3.164 into 10 to power minus 5 second. So I substitute these numbers and next transparency I have given this fraction turns out to be 0.12 which means approximately 12 percent of the mu mesons may be able to reach earth others would have decayed in between. Because this is sort of a statistical process in which some will decay some will decay later some will decay earlier.

But if we do the if we had done the classical calculation; it means for this particular time instead of you for this particular tow instead of using 1 sorry 3.164 into 10 to power minus 5 you would have used 2 into ten to power minus 6, then this number would have turned out to be 3.12 into 10 to power minus 15, see remember this is an exponential factor. Exponential factor is a very fastly decaying function. And you have see that how much the fraction is changed. Earlier this was a calculation which people were doing earlier before special theory of relativity and they would have said that 1 into 10 to power 15 approximately one out of 10 to power 15 mu mesons want to reach earth approximately.

I am just ignoring the factor 3 just to calculate order of magnitude. That many number want to reach earth while according to relativity 12 percent 12 out of 10 would reach earth. The number has become so large and that is what was experimentally observed which people could not understand earlier and only after special theory of relativity came people could understand it. That was considered as one of the great success of special theory of relativity. Just a question, what would an observer a mu meson frame conclude, how you identify this event in terms of these two events in terms of a mu meson frame of reference? As you can probably guess from that platform frame of reference, platform example which we have given just now.

That according to the observer on mu meson the distance between the earth and the upper atmosphere will appear to be reduced or contracted. So that observer will feel that this mu meson is had not actually travel 20 kilometres is distance but, has travelled much smaller amount of distance, that is why it has on decayed.

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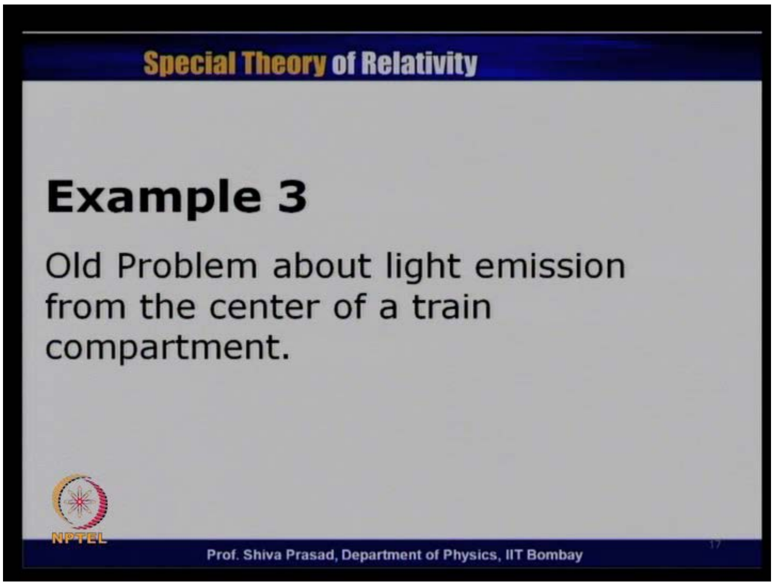

Now let us go back to our old example which we have discussed number of times in different ways. That is the light emission from the centre of train compartment. In our first time when we had discussed we had discuss that instead of light we are throwing two balls, one person sitting at the centre of a train compartment throws two ball one in a direction of the motion of the train towards the front wall, another towards the back wall. And we defined two events when the ball reaches the front wall and the ball reaches the back wall. We concluded that in the frame of reference of train which we called as S prime the two events were stationary. Then we did a classical Galilean transformation and we found out that in S frame which is the ground frame of reference also these two events appear to be simultaneous. Second time when we discuss this particular problem was when we assumed that instead of the ball somebody is shining light, one towards the front of the front wall, another towards the back wall. We concluded at that time that these two events in the frame of reference of S prime which is the compartment frame of reference that the light reaching the front wall light reaching the back wall occurs at the same time and are simultaneously.

Then at that time we discussed with reference to the ground frame and said that if second postulate of special theory of relativity was correct; it means if the speed of light is same in S and S prime frame of reference then the observer in S frame of reference which is the ground frame of reference would feel that these two events are not simultaneous. But, at that time we did not calculate time difference because we did not have Lorentz transformation.

Now that we have our Lorentz transformation. Let us actually try to calculate the time difference between the these two events, which we had discussed just now as seen in the ground frame of reference which I am calling as S frame of reference.

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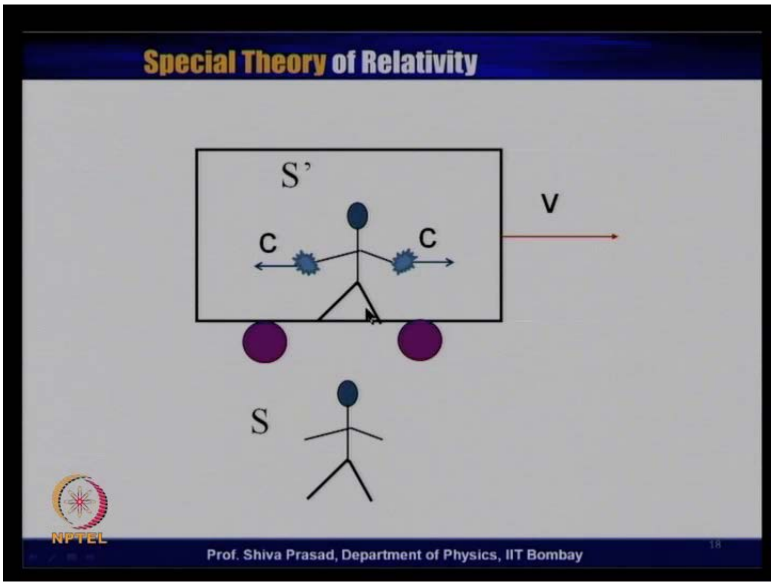

So let us look at this picture again, this is the picture or this particular train compartment this person standing at the canter of the platform, he shines the light. Let us assume this two hands are just for making picture clearer. I have put them with a large hand long hands but, let us assume that these are just at the same point and the light is emitted at this particular direction. This I am calling as a front wall, light is being emitted towards back direction opposite direction to the motion of the train and this I am calling as a back wall my event number one was light pulse reaching here; event number two light pulse reaching here, which I had we had agreed in S prime frame of reference, these two events are simultaneous.

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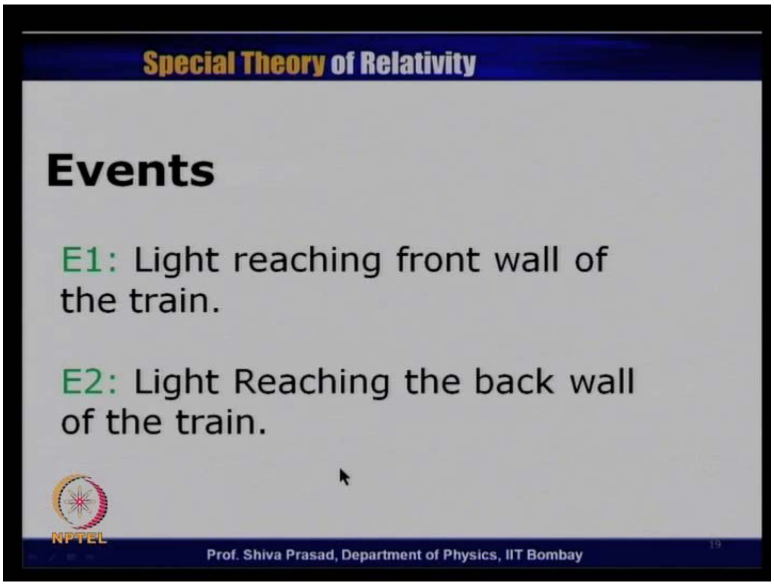

And now my question is that what would be the time interval between these two events as will be observed by an observer S standing on ground. We had earlier defined events, let us redefined them. Let us rewrite them, event number one light reaching the front wall of the train; event number two light reaching the back wall of the train. So these are my events which we had already defined. Let us again reiterate. Now what I will do because I want to do some mathematics using Lorentz transformation. Let us be clear and write coordinates of this event one and event two. And because the initial information has been given to me in S prime frame of reference which is the frame of reference of the train in fact everything has been described with respect to that observer.

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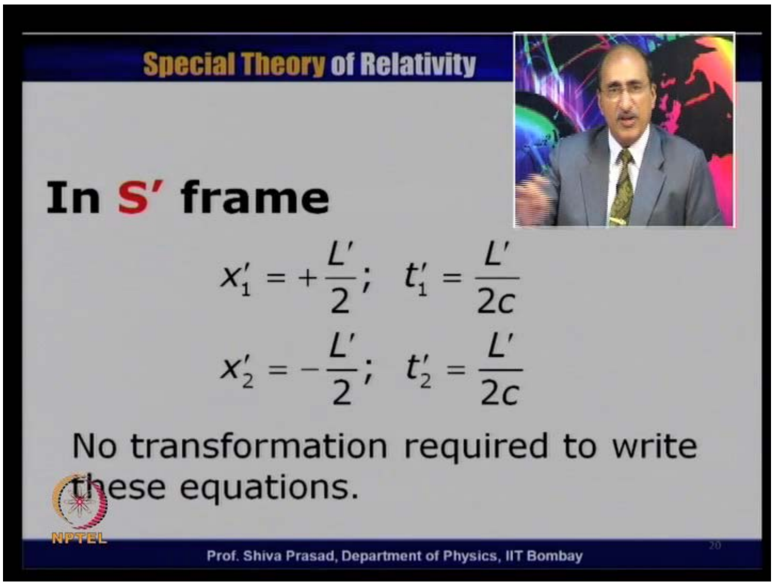

So let us first write the coordinates in S prime frame of reference which is the train frame of reference. This is what I am doing in my next transparency, so because this is S prime frame of reference or my coordinates are putting prime. Now that we know that lengths are different in two frames. So this length which is been measured the length of the compartment itself, that is in S prime frame of reference so that length also I am calling as L prime.

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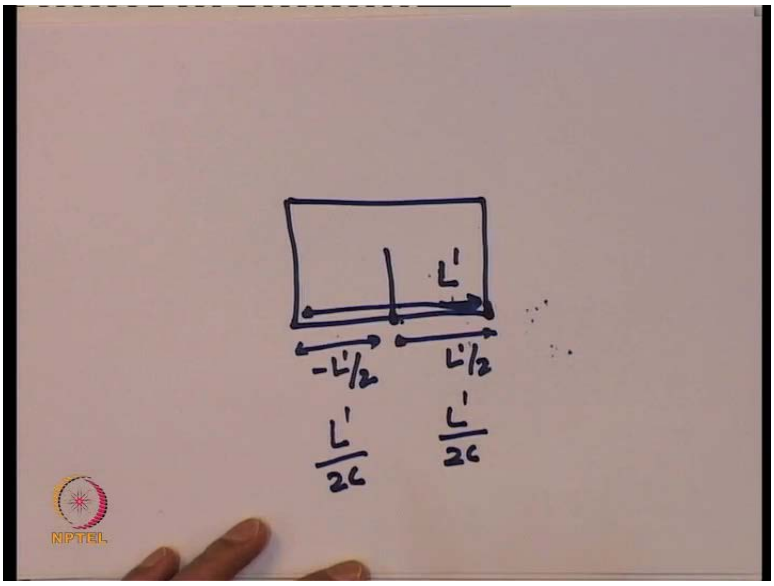

So we agree that is this is my train compartment and observer is just sitting standing on half the way, then in this particular frame of reference this length is L prime, so the first coordinate and this particular person is standing at the origin, so we assume we get that this particular event number one will occur at a distance of L prime by 2 positive side I am taking this is positive side. This is negative side and this will occur at minus L prime by 2.

So this is what I have written here that X 1 prime occurs at plus L prime by 2 ;X 2 prime occurs at minus L prime by 2. Even primetime of the event number one is a time taken for the light to travel from here to here. We know that this time this light travels of the speed of light c. So this particular time will be given by actually the length is L prime by 2 the distance it has to travel is L prime by 2. So the time taken will be L prime by 2c. So time will be L prime by 2c. Similarly, this light which has been emitted backwards also moves a distance L prime by 2 and travels with the same speed c.

So therefore, this time interval will also be L prime by 2c. So according to an observer in S prime frame of reference this event number one occurs at a vary of X L prime by 2 and a time L prime by 2c. Event number two occurs at a coordinate of minus L prime by two and a time L prime by 2c. These two times are same, these two events therefore, are simultaneous in S prime frame of reference.

This is what I have written here this particular transparency X 1 prime is equal to plus L prime by 2, t 1 prime is equal to L prime divided by 2c, X 2 prime is equal to minus L prime by 2, t 2 prime is equal to L prime by 2 c remember we have not use any transformation to write these equations. I do not need any transformation. If we have all the information in the same frame and all these basic information has been given in S prime frame of reference. I do not need any transformation.

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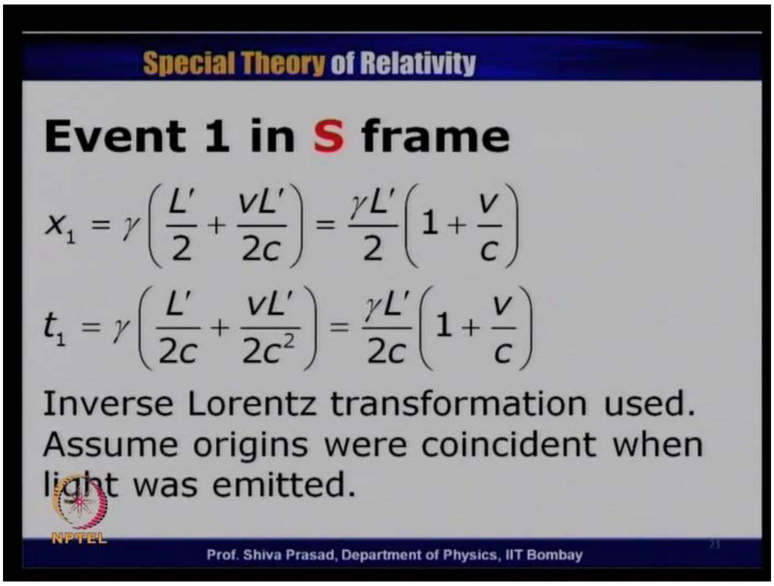

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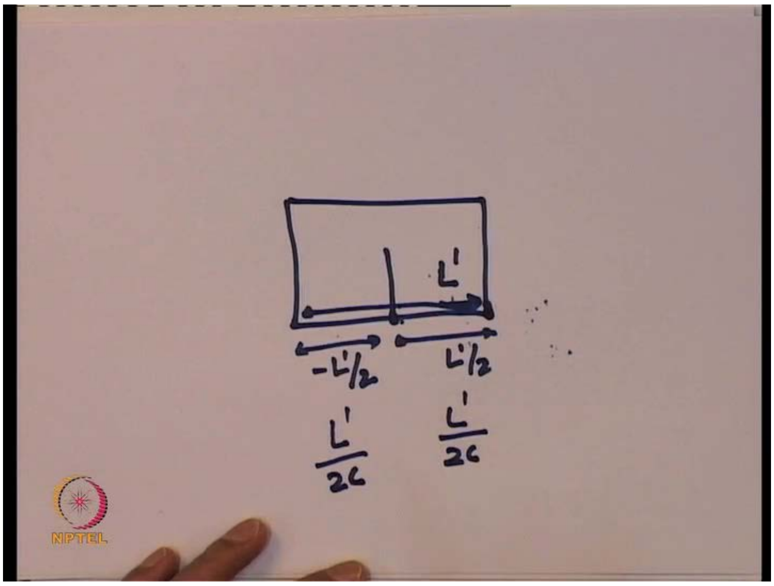

Let us try to write the coordinates of these two events in S frame. Let me first write the event number one, event number one was light reaching the front wall this occurred at L prime by 2 and L prime by 2 c.

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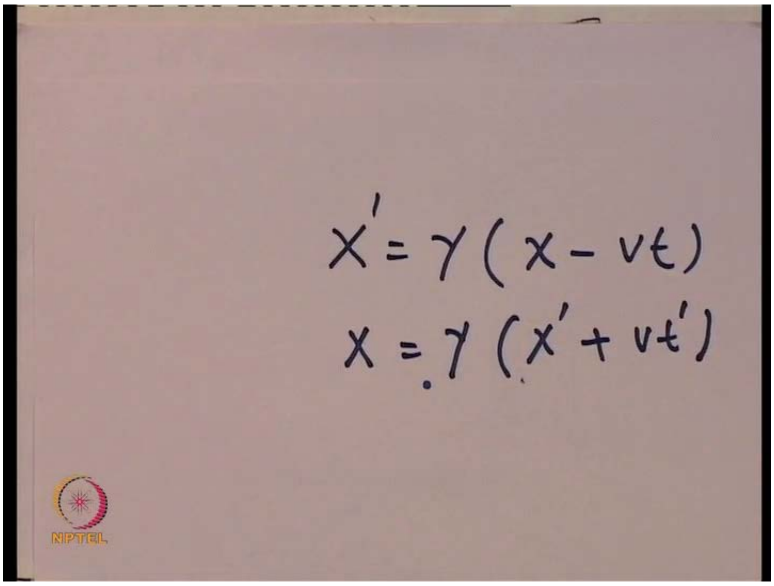

And if you remember the Lorentz transformation gives you X prime is equal to gamma X minus v t. Now the information has been given in X prime frame of reference and I want to find in S frame of reference, therefore, I must use inverse Lorentz transformation, which is given by X is equal to gamma X plus v t prime X prime plus v t. So we said the prescription is change un-primed to prime un-prime to prime and prime to un-prime change the sign from minus v to plus v, change the sign of v.

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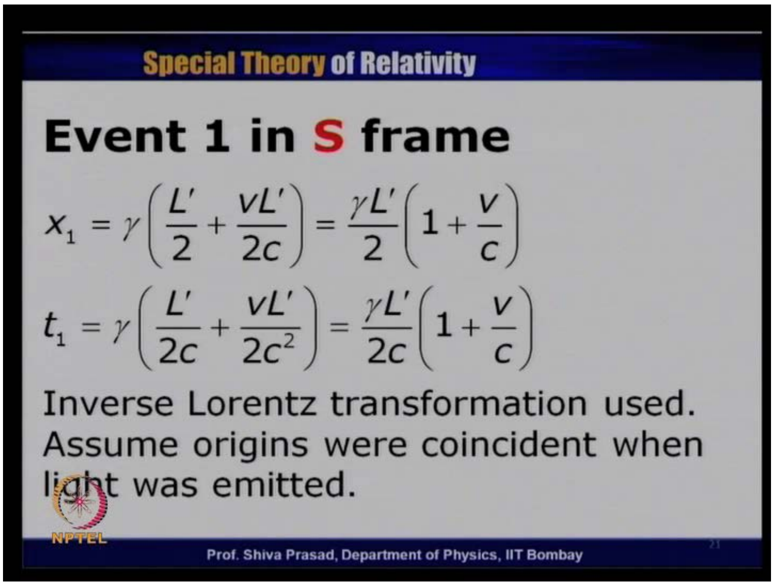

So this is my inverse Lorentz transformation. So I use the inverse Lorentz transformation here to find out X 1 is given by gamma L prime by 2 plus v L prime by 2c. This is my value of X prime, this is the value of my t prime L prime by 2c. So what we have written is X prime plus v t. This what is comes to. So I take gamma L prime by 2 out of this, so this becomes gamma L prime by 2 multiplied by 1 plus v by c or.

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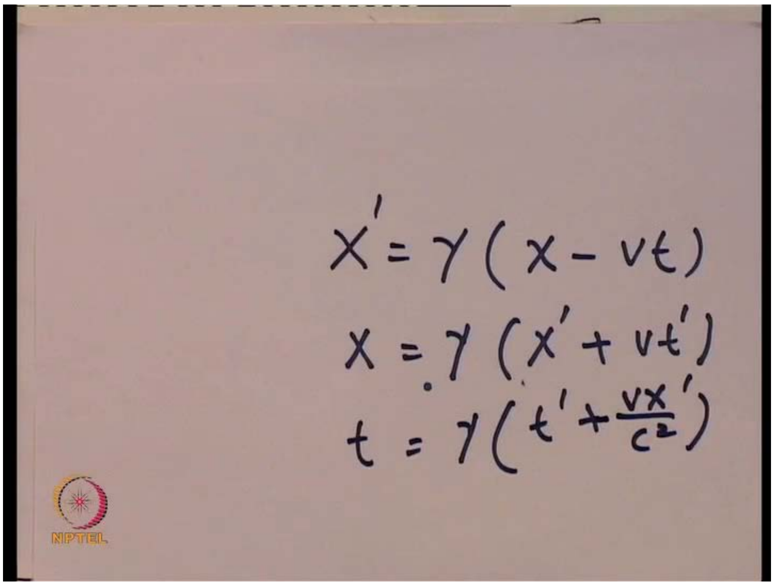

Similarly we write inverse transformation for time. Inverse transformation for time will be given by t is equal to gamma t prime plus v X prime by c square. Again using inverse transformation we go back here to the transparency t1 is equal to gamma times L prime by 2c, this was the time plus, plus because this is a inverse transformation. v relative speed in the frames, x which is the l prime by 2 divided by c square. I take gamma l prime by 2c out of this bracket. This becomes 1 plus v by c. Of course, we have assumed that the origins are coincident with the light where was emitted.

So at that instant of time the observer which is sit standing on the wrong was also at its origin and now just passing by it. Now let us write the coordinate of event number two in S frame of reference. You realize that everything will be same here exactly similar except that that as L prime by 2 we change sign. So this appears here and this appears here this we change sign. So that is what I am writing in the next transparency.

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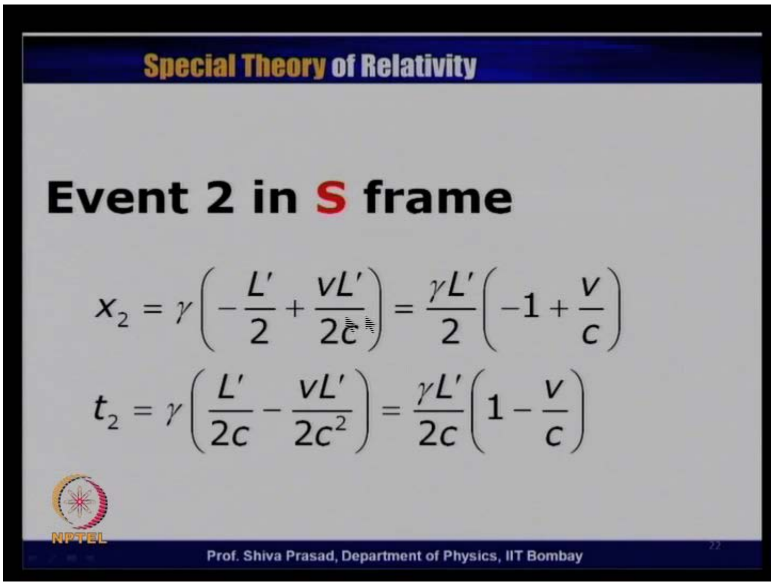

Event number two in S frame, so X 2 that is X coordinate of the second event will occur at gamma minus L prime by 2, that is where it observe with that was X value plus v L prime by 2 c. Remember time is same though event occurred at different location, but, time was same. So this L prime by 2 c, so this remains the same thing. I take gamma L prime by 2, common and you get minus v plus v by c. We take t 2 which is gamma t prime which is same as L prime by 2c minus v into X prime, which is minus L prime by 2, because this event occurred at the back of the train. So this is a minus L prime by 2 divided by c square.. I take gamma L prime by 2 c I out, I get in bracket 1 minus v by c.

So as you can see that this time this time is different from the other time. The in earlier case for the event one there was positive sign here and this time interval was positive. This was one plus v by c. Here it is one minus v by c. So these two intervals are not simultaneous in S frame of reference, which is the ground frame of reference as we had discussed earlier but, now we know what is the time difference. Actually I can calculate t 2 minus t 1 and find out how much difference in the time was observed as according to S observer on the ground observer between these two events.

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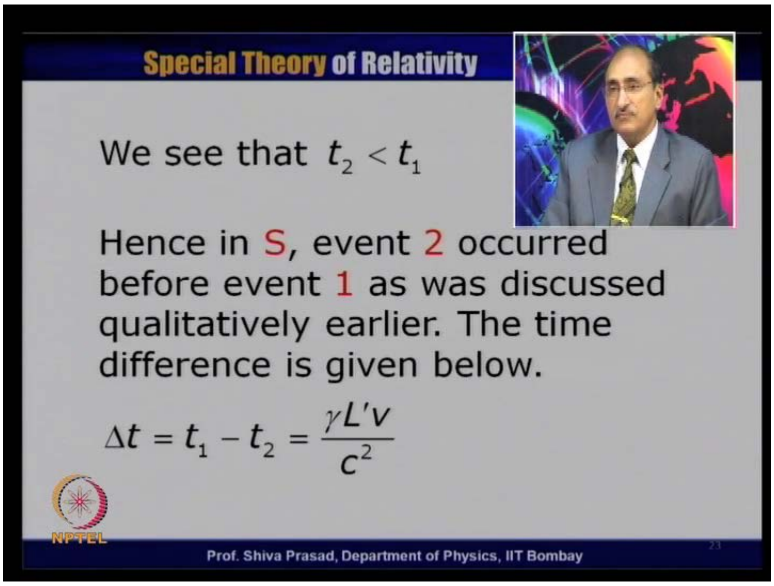

So that is what I have written in this particular transparency, that we see that t 2 is less than t 1. Because there is a negative sign here, one minus v by c. So t 2 is less than t 1. Hence in S event two occurred before event one. This also we had discussed earlier, somewhat qualitatively without Lorentz transformation. We had said that if speed of light should remain same, then actually happen that because the light which travels a backward has to travel a smaller distance.

So if will hit the wall first the light which is emitted in front has to travel larger distance but, with a same speed. Therefore, that event will occur later so that particular time will be larger. So this is what we had qualitatively discussed. Now we know their numerical numbers. So hence in S event two occurred before event one as first qualitatively discussed earlier the time difference you can just calculate because that one factor will cancel out.

So that one only the v by c factor thing has to be taken. And this will turn out to be gamma L prime v by c square. So this is the time interval between the two events as observed in S frame of reference while in S prime frame of reference this time interval was 0. Both these events occur at the same time.

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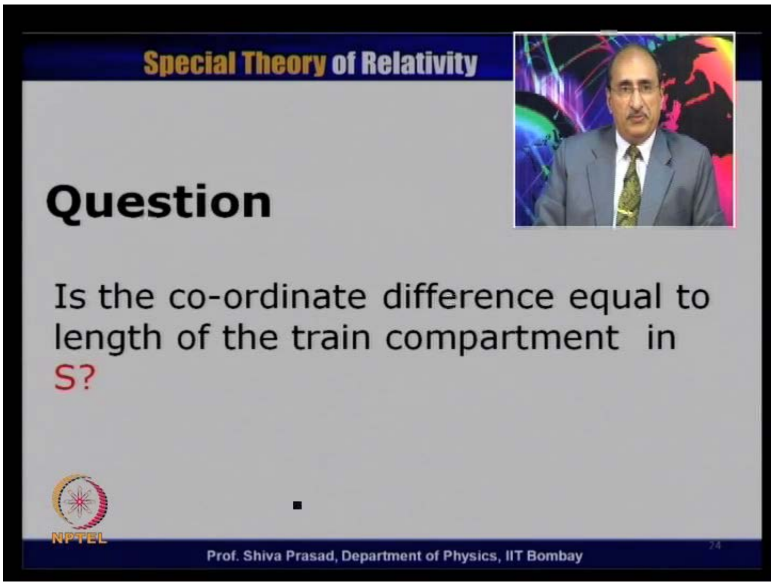

I have another question, is the coordinate difference that we have calculated X 2 minus X 1 will be equal to the length of the compartment in S? I think all should be able to give quick answer that no it cannot be. The reason is that these two events though occurred at two different ends of the compartment, but, that compartment was moving and unless these two events occurred in S in the same time X 2 minus X 1 will not give me correct length. X 2 prime and X 1 prime difference will give me the length. In S prime frame of reference but, remember in that would go would go frame of reference anywhere the compartment was at rest.

So even the time difference tow, in this in the present example in S prime frame of reference these two events occur at the same time but, in that frame of reference the compartment was anywhere at rest even if they would not have occurred at the same time is still X 2 prime minus X 1 prime would have given the correct length but, not in S frame of reference. In S frame of reference these two events occurred at different time therefore, X 2 prime minus X 1 prime is not the correct length. Now can we guess whether we are getting an over estimate or an underestimate of the length? Let us look into the picture.

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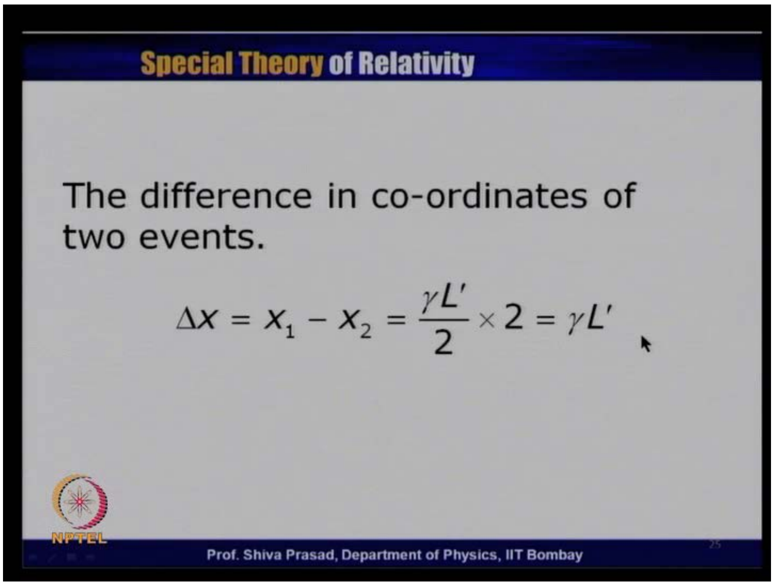

Anyway we let us first I have calculated the difference in the coordinates of the two events. This delta X will be given by X minus X 2, because the event one was on front of the wall. So this turns out to be gamma L prime divided 2 into 2. If we take the X coordinate transformation which we have done just now. The result will turn out to be gamma L prime so according to the S observer these two events occur at a spacing of gamma L prime along the X direction.

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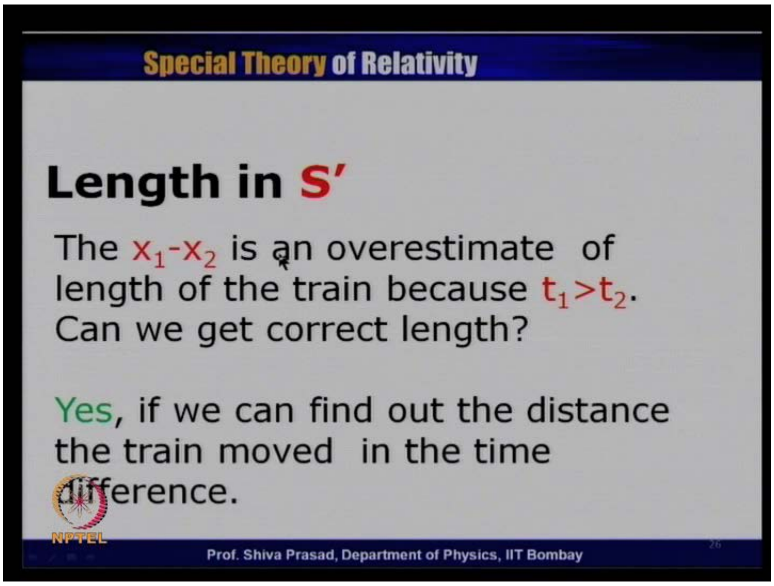

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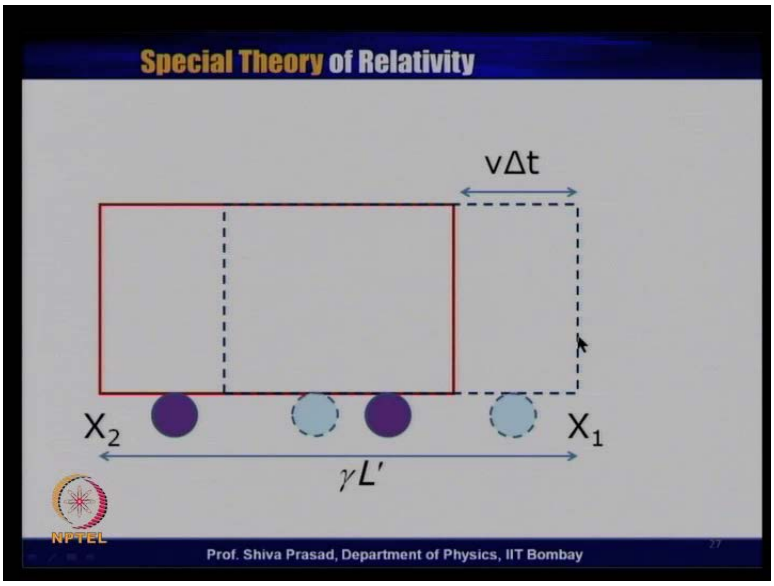

The X 1 minus X 2 is an overestimate of length of the train because t 1 is greater than t 2 and can we correct it. Let me just show you this particular picture. Let us suppose this was the position of the train, when event number two occurred which was in the earlier event, so this was X 2; event number one occurred when the light reached here. This event occurred at a later time, how much was the time difference was delta t which we have just now calculated.

Now if I take the X 1 event according to ground observer this occurred here because the train has been moved and this train has moved by distance of v delta t during the time interval in which the two events occurred according to S frame of reference. According to observer in S frame this event occurred earlier, this event occurred later and during this particular time this train was actually moving. And how much distance you would have move is the speed of the train multiplied by the time difference. This was the additional distance that this particular train moved and therefore, X 1 minus X 2 actually gave me gamma L prime which we have just now obtained. and if I want to determine the correct length what I must do? From this gamma L prime I must subtract v delta t, then I will get the correct length as measured in S frame.

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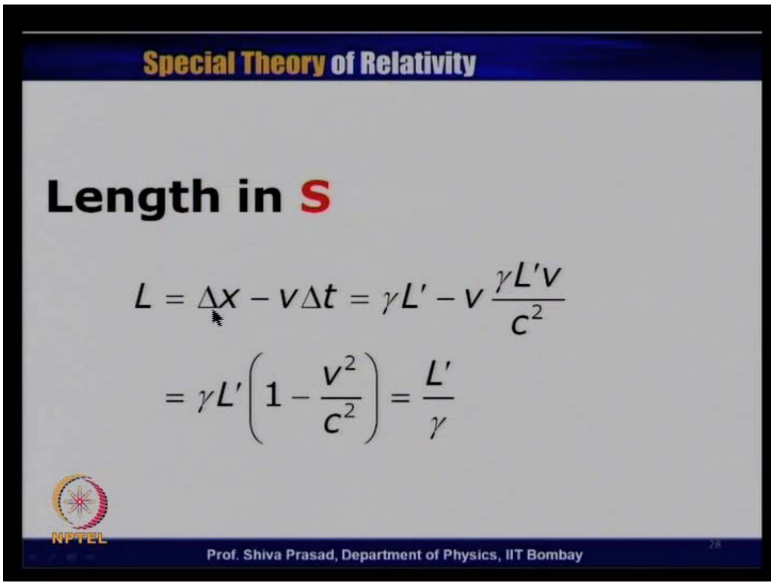

This what I have done in the next transparency. My L is equal to delta X minus v delta t. As we have just now seen delta X is v gamma L prime, v is here delta t is gamma L prime v upon c square. We just redistribute the numbers it becomes gamma L prime to bracket 1 minus v square by c square. If you know gamma is equal to 1 upon under route v square minus c square and there is no under route here. So this whole quantity will become L prime under route one minus v square by c square which is 1 by gamma.

So I get length as L prime by gamma. So as I expected this length turns out to be a contracted length. This is what I had expected that because in S prime frame of reference the length was proper hence it is in this particular frame of reference that the length will turn out to be contracted.

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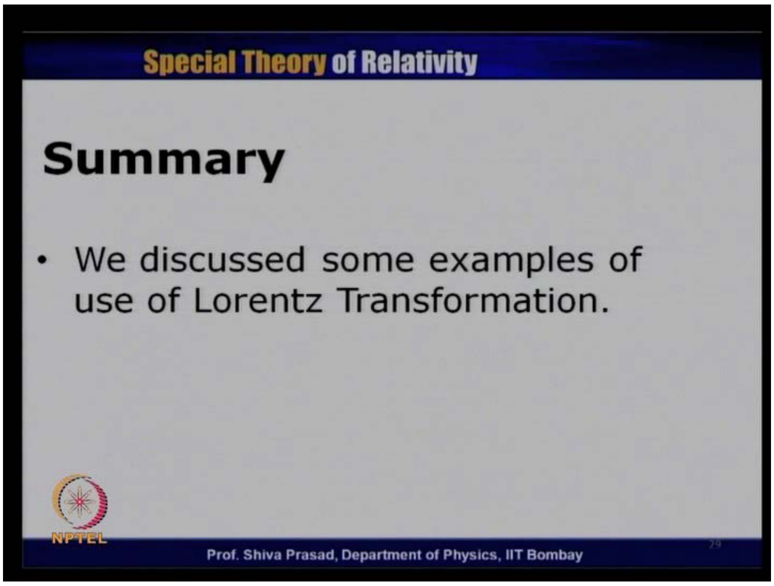

Now let us go to the conclusion, in this particular lecture, we have essentially discussed some examples of Lorentz transformation. In the next lecture, we will obtain the velocity transformation.

Thank you.