Special Theory of Relativity
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Lecture - 5
Lorent Transformation

Hello. So, let us recapitulate of what we did last time. We discussed the simultaneity of two events; that was the first thing that we discussed. We try to see that if there are two events which occur at the same time in a given frame of reference which we normally call as simultaneous events may not appear to be simultaneous in a different frame or in general, they would not appear to be simultaneous in another frame. So, this is what we say that simultaneity is relative. So, this will be true if second postulate is correct.

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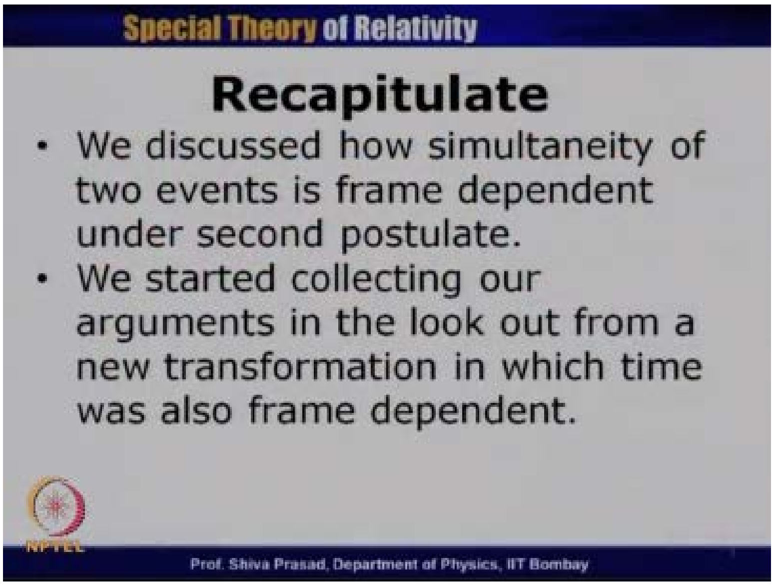

Then we also started collecting our arguments to look for a new transformation, because we realize that Galilean transformation would not be consistent with the second postulate. So, we started collecting all our arguments in the lookout for a new transformation in which time has also to be made frame dependent; in fact we gave reasoning's that why we expect time also to be frame dependent. After that we started looking for a new transformation without invoking any of the postulates of special theory of relativity. Just purely on physics grounds we try to look for a transformation which we try to look at a form of the transformation; the way it should look like and we also realize that Galilean transformation is also a special case of that.

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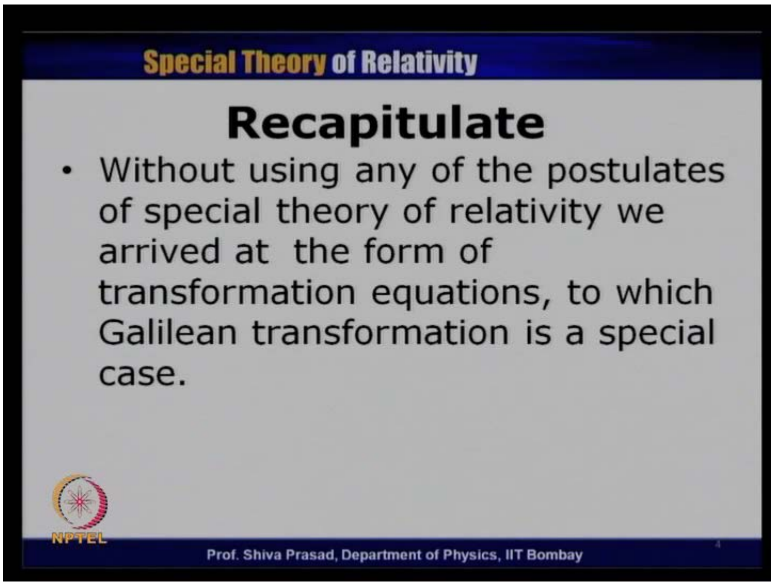

So, without using any of the postulates of special theory of relativity we arrived at the form of these equations to which Galilean transformation is a special case.

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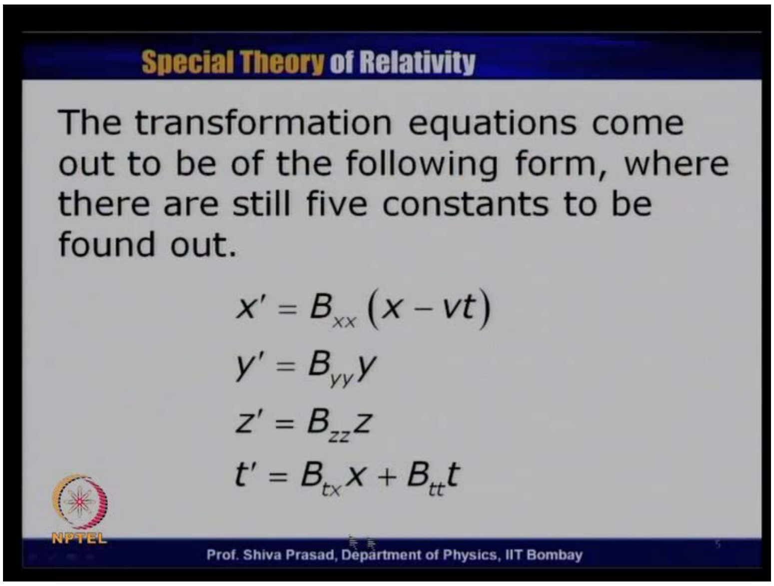

The transformation equations eventually turned out to be of the following form which is written here. So, x prime is equal to B xx multiplied by x minus vt where B xx is a constant which is yet to be evaluated. y prime is equal to B yy multiplied by Y; again B yy is a constant to be evaluated. z prime is equal to B zz times Z. t prime is B tx X plus B tt t. We realize that this transformation similar to the Galilean transformation in which B xx was one, B yy was one, B zz was one, B tx was zero and B tt was one. So, by taking special values of these constants Galilean transformation can be arrived at.

Now we have to evaluate these constants by invoking the postulates of special theory of relativity; that is what we are going to do just now. So, first postulates that we try to invoke here in order to evaluate these constants are the fact that we expect all inertial frames of references to be equivalent. We do not expect any preferential inertial frame of reference. Now we realize that if we go back to our equations and we take y prime is equal to B yy multiplied by Y as this equation suggest. Then let us suppose we put a rod in s frame which is zero and one. So, we have a rod which is we are putting and here this y is equal to zero; here y is equal to one as measured in s frame.

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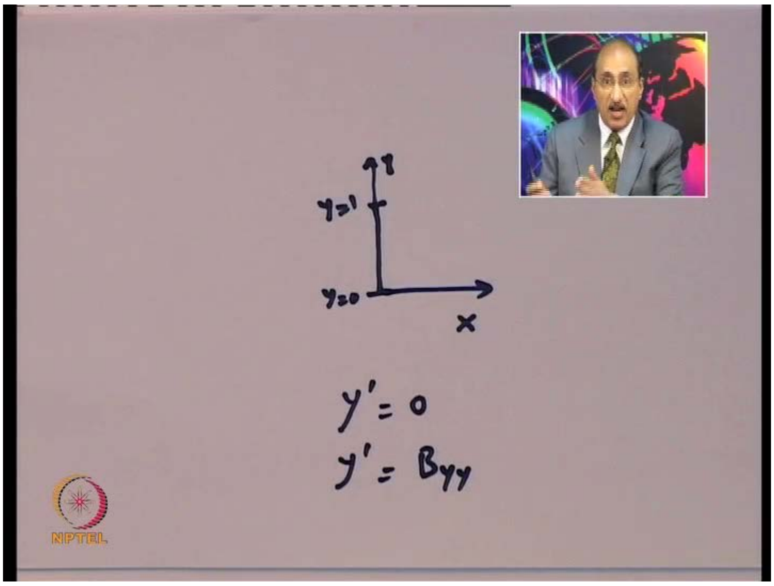

I am drawing the same rod here y is equal to zero and y is equal to one. This is my x-axis, this is my y-axis and let us assume that this is put along the y-axis. Now according to this particular transformation law which is y prime is equal to B yy times Y; Y is equal to zero would give me y prime is equal to zero, Y is equal to one will give me y prime is equal to B yy. So, from this an observer at s prime would conclude because this is transformation equations which are supposed to translate in formations from one frame to another frame.

So, according to these transformation equations if they are correct, the length of that particular rod would be found out to be B yy by an observer in s prime because according to him the rod is put between y prime is equal to zero and y prime is equal to B yy. Now let as imagine in a different situation, an inverted situation, in which the rod was actually put in y s prime frame of reference between y prime is equal to zero and y prime is equal to one.

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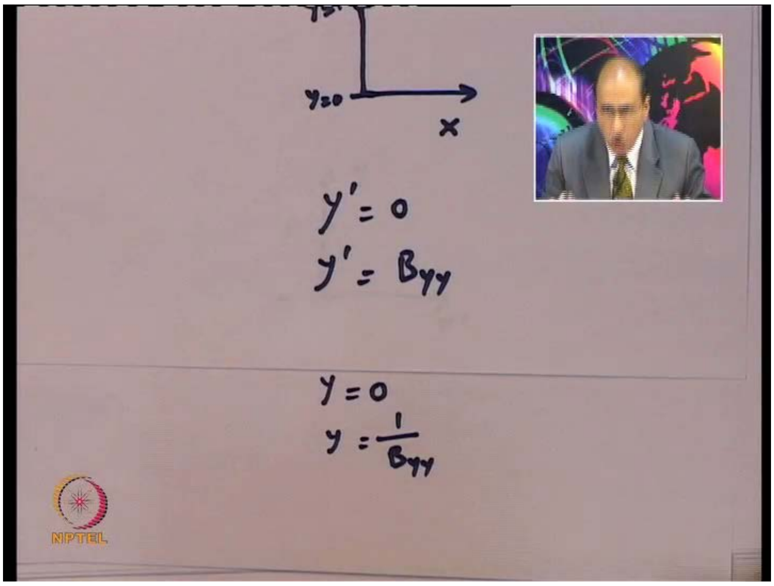

Exactly the same thing except that now the rod is put in x prime in s prime frame of reference and is put between y prime is equal to zero and y prime is equal to one. So, this is my rod or if this transformation equations are correct, the one which we have mentioned, we apply the same transformation here. So, y prime is equal to zero would give me y is equal to zero, no problem; y prime is equal to one will give me y is equal to one divided by B yy. Now if B yy happens to be anything different from one, then in that particular case, this observer here would say let us assume that B yy is, say, two. So, this observer here would assume would find out that the length of the rod is 2 meters.

But if the same rod was put in y prime frame of reference, the observer in s frame will find out that the length of the rod is only half meter, one divided by two. It means one frame of reference is magnifying the length of the rod; another frame of reference is reducing the length of the rod. This we do not expect. See I do not mind that in y frame if I put a rod along y direction or any direction, its length turns out to be reduced in y prime frame of reference or s prime frame of reference but they inversely should also be true. It means if exactly in a similar location in s prime of reference, we put a rod in s prime also it should turn out to be reduced; that is what is meant by the equivalence of the two frames.

It cannot happen that in one frame all the rods are to be elongated in comparison to another and when we inverse the frame, in the other frame, they will turn out to be all reduced. It means they are not equivalent; I can distinguish by knowing the length of the rod whether this frame is different from the other which is not allowed by the first postulates of special theory, first postulates of special theory of relativity. Hence I expect that B yy must be equal to one because unless is equal to one I will be able to distinguish and evolve a method of distinguishing between two frames, one as magnifying frame and other is reducing frame. Hence I expect B yy to be equal to one. So, this is what I have written in the next transparency.

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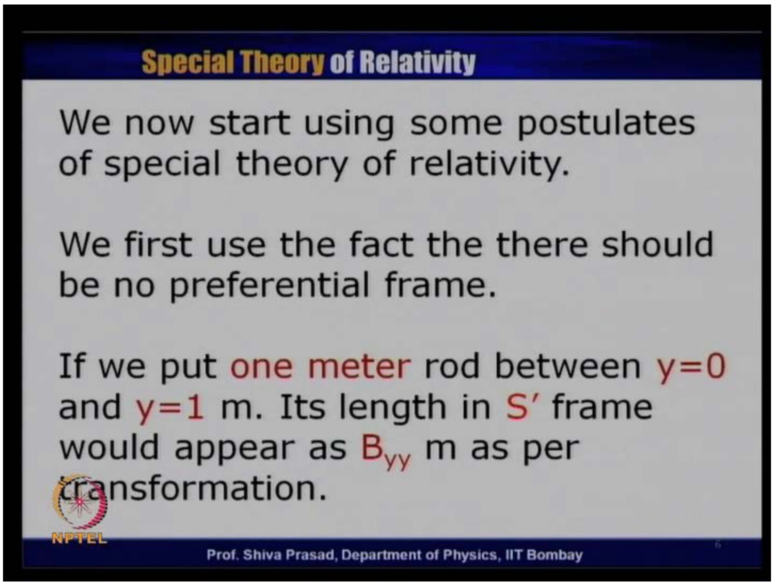

We first use the fact that there should be no preferential frame. If we put one meter rod between y is equal to zero and y is equal to one meter, its length in s prime would appear to be B yy as per transformation.

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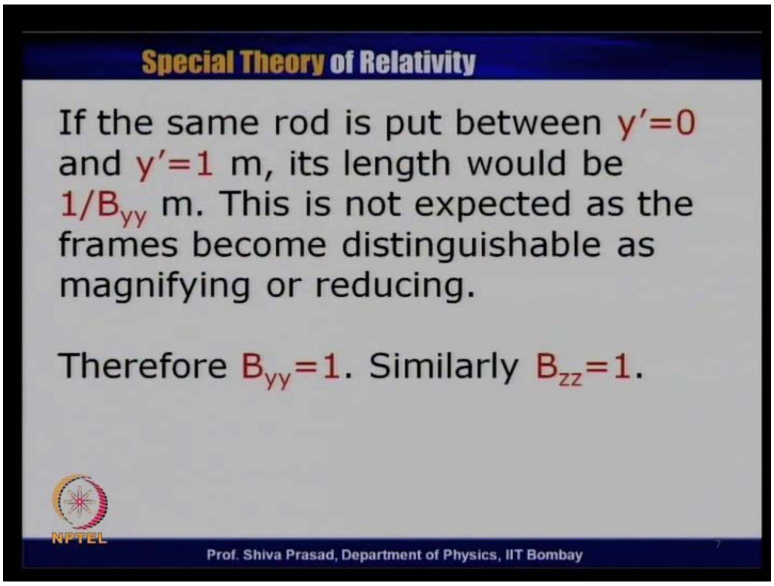

If the same rod is put between y prime is equal to 0, and y prime is equal to one meter, its length would be one divided by B yy meter just now the way I have discussed. This is not expected as the frames become distinguishable as magnifying or reducing. Therefore, we expect B yy to be equal to 1. Now exactly in the similar way, I can put the rods along the z direction and using exactly the same argument I can put B zz equal to 1. So therefore, I must have been B yy is equal to one and B zz is equal to 1.

So, two unknown constants have now disappeared; we have now left with three constants. Remember same arguments I cannot apply along the x direction because in x direction there is time dependence as well x dependence. So, things are little more difficult as far as the x direction is concerned. Remember x direction is somewhat unique, because this is determined by the direction of the relativity velocity while as I have discussed in last lecture, y direction could have been rotated with respect to x direction and things would not have changed.

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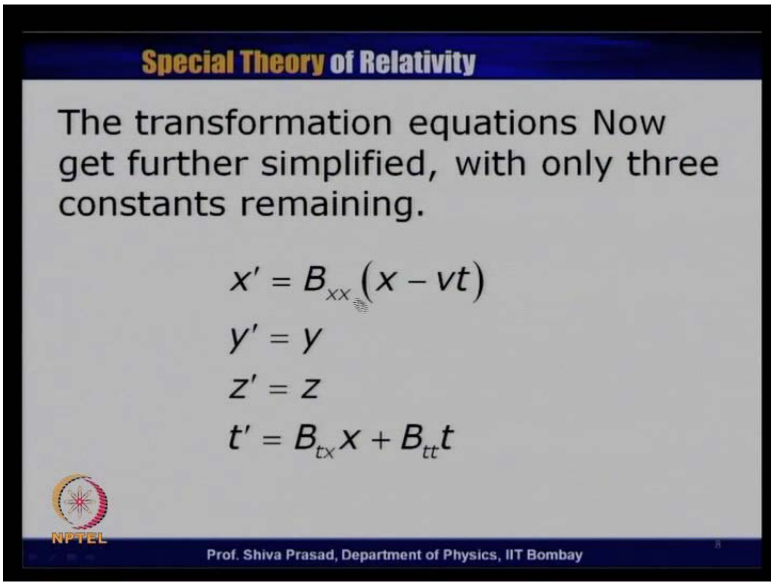

So, now my questions become comparatively simpler with only three unknowns which is x prime is equal to B xx multiplied by x minus vt; y prime now becomes equal to y; z prime is equal to z and t prime is equal to B tx X plus B tt t. These are the four equations in which there are three unknowns B xx and B tx and B tt which have to be still determined using the postulates of special theory of relativity. Now this is the time I will involve the second postulate or special theory of relativity to determine the three constants.

Let us assume that at t is equal to zero, when the origins of the two frames are coincident, a particular source of light emits a light from origin; we can always imagine like that. Had it been classical mechanics we would always asked the question; suppose we are asking a question, a ball is been thrown from the origins; we will always ask the question, whether this person who was throwing a ball was he stationery in s frame or was his stationary in s prime frame or was his stationary in any other frame. But for light we did not answer this question because whatever might be the frame and according to the second postulate, light, speed will always be same. So, it makes no difference whether the source which was emitting light was stationery in s or at s prime or in any other frame.

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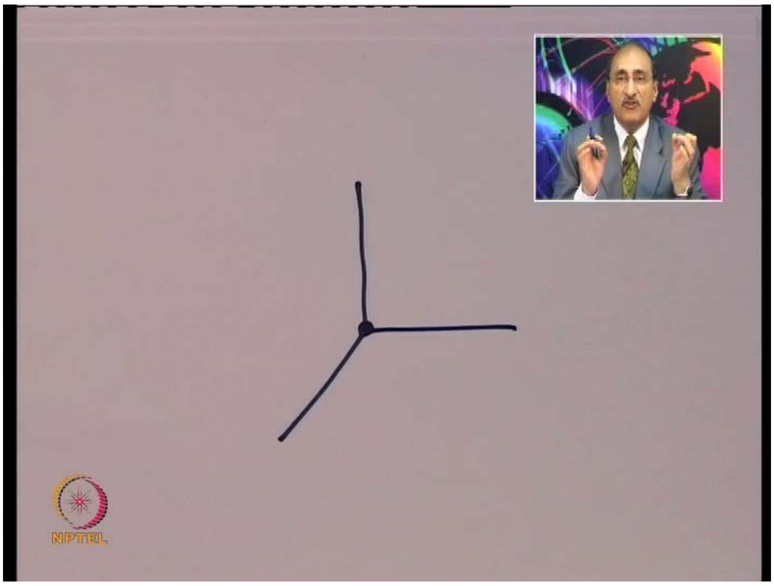

All I am saying that the incident when the two frames were coincident at that incident which is time t is equal to zero and t prime is equal to zero, there was a source of light at the origin which emits light. Now this light let us assume an observer in s frame, according to that particular person, this light will emerge in a spherical wave front which will appear as a sphere with the as the origin as a center of the sphere. In all the direction it has to travel with the speed of light c as observed in the s frames of reference. In s prime frame of reference exactly the same thing would happen.

That person would also feel that this light was actually originated from the origin of his frame and in all the directions this spherical wave front is emerging with his origin as a center; that is what the observer at s prime frame will also feel. So he will also feel, remember at given literal time, the origins of the two frames of reference are no longer coincident, but still both of them will be feeling as if the spherical wave front is emerging with their respective origins at the center of the sphere; this is what I have showed in this particular transparency, it is something like this.

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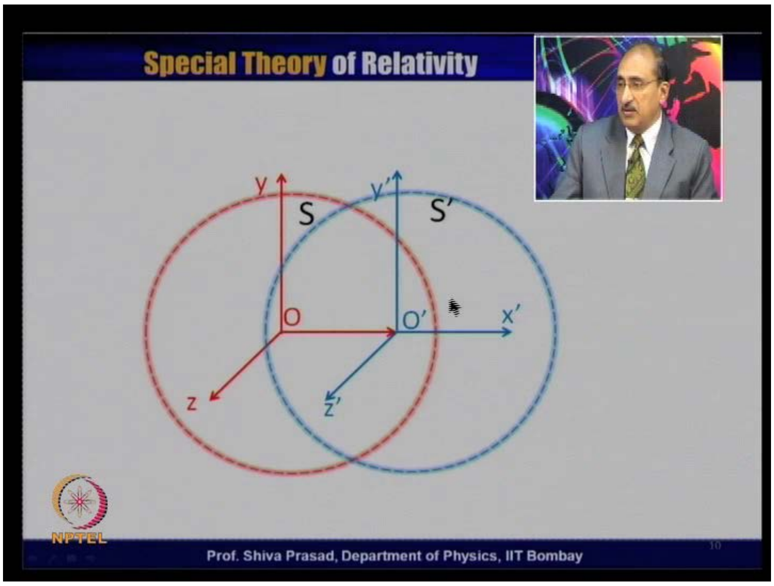

This is sphere, this is another sphere. So, it appears to be emerging according to the observer in this red frame of reference in s frame of reference; this wave front appears to be spherical with its center at the origin. According to this observer also this particular wave front would appeared to be spherical with this o prime as the origin. Remember at a given time o and o prime are not coincident; still both of them will feel as if it has been emerging from their own centers.

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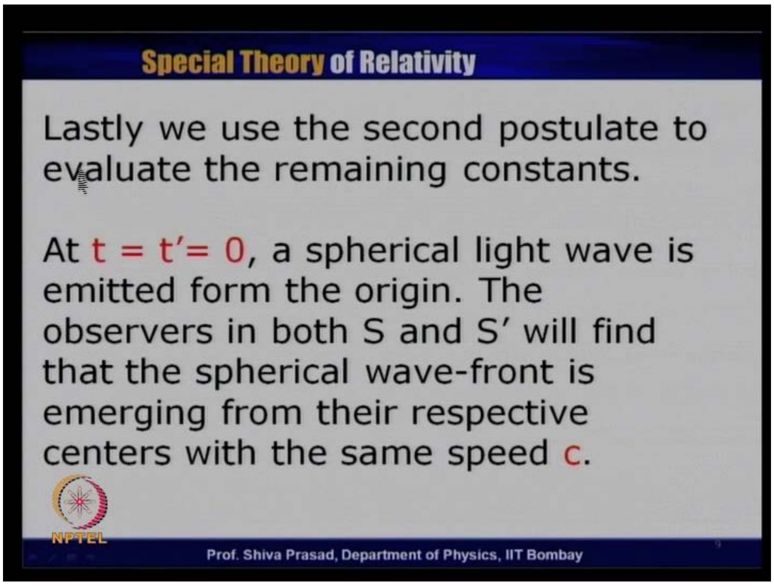

So, this is what I have written here; at t is equal to t prime is equal to zero, a spherical light wave is emitted from the origin which is going in all the directions; that is what we mean by a spherical light wave. The observer in both s and s prime will find that the spherical wave front is emerging from their respective centers with the same speed c in all the directions. Therefore, if I have to write the equation then observer in an equation of the spherical wave front, according to the observer in s prime this equation will be an equation of a sphere, the radius of which is changing as a function of time and at a given time, the radius is given by c multiplied by d where c is the speed of light. So, it will appear to be x square plus y square plus z square is equal to c square t square; that is the equation of spherical wave front which observer in s prime will right. On the other hand observer in s prime, remember he has to be consistent in his frame.

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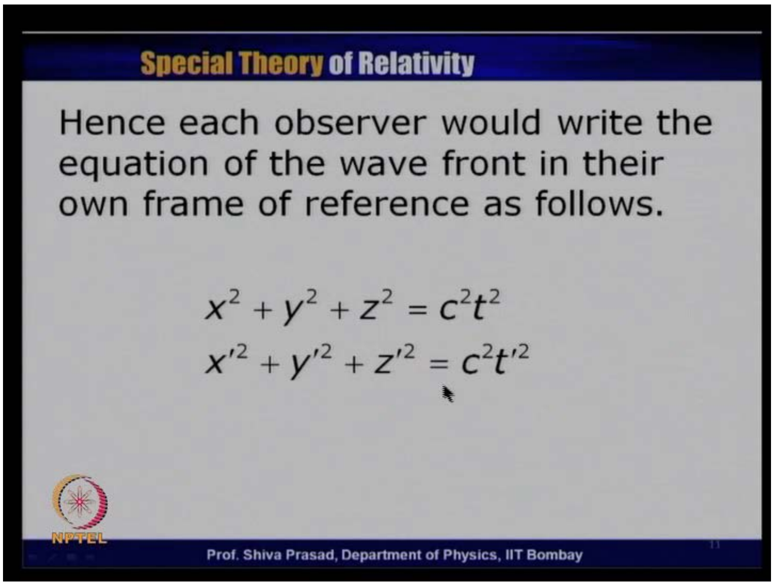

We will write an equation which is x prime square plus y prime square plus z prime square is equal to c square t prime square because according to that particular observer, the radius of the sphere is given by c multiplied by t prime where t prime is the time in his frame; x prime y prime z prime are the coordinates measured in his frame. So according to s, this equation must be correct as the equation of the spherical wave front according to s prime equation the observer, this should be the equation of spherical wave front.

So whatever transformation equations that we have written, if I substitute those transformation equations here in this particular frame because transformation equations are supposed to translate information from one frame to another frame; so if these transformation equations are correct, if I substitute this transformation for x prime y prime z prime in these equations and of course t prime, then I must get back this second equation. If that is correct, then I have to found out a correct transformation equation. So, remember we had already looked out our transformation equation; let us substitute it back in this particular thing and try to see whether we can evaluate the other three remaining constants which we have not done so far.

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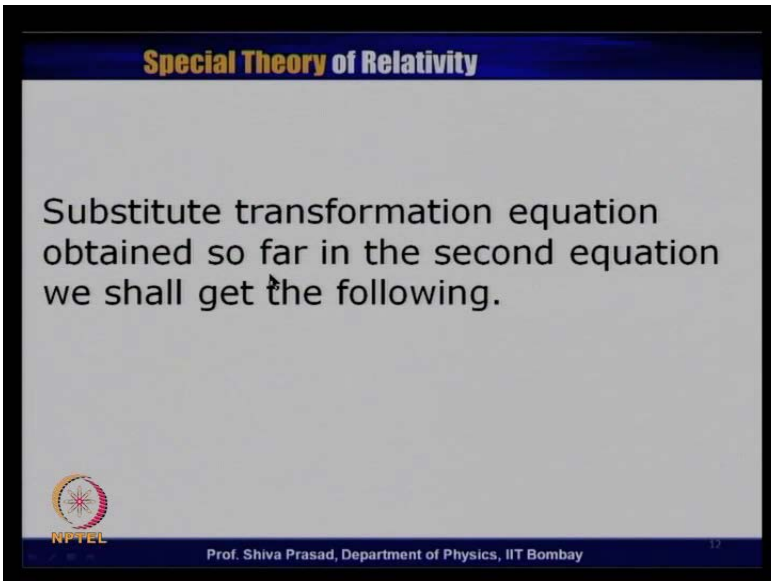

So, this is what I have written; substitute transformation equations obtained so far in the second equation and if we substitute we will get the following.

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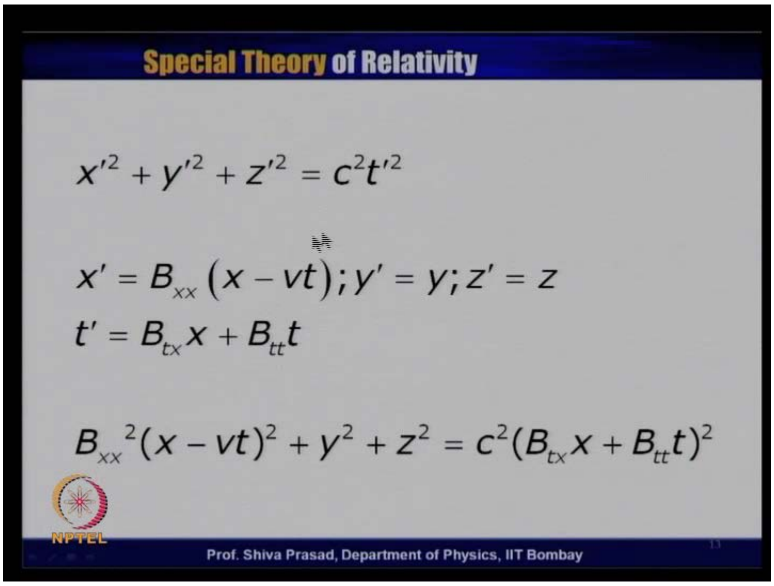

This is the equation which an observer in s prime root x prime square plus y prime square plus z prime square is equal to c square t prime square. If you remember we have reached up to this particular point as far as transformation equations are correct; x prime is equal to B xx multiplied by x minus vt, y prime is equal to y, z prime equal to z, t prime is equal to B tx x plus B tt t. So, I substitute back in this equation. So for x prime, I write this thing; this is what I have written. So, once I substitute this thing here in this equation, this whole thing will get squared. So, this becomes B xx prime squared square of this x minus vt; so square of x minus vt. So, I have just put x prime square. About y prime there is no problem; about z prime there is no problem because y prime happens to be equal to y, z prime happens to be equal to z.

So if I substitute it back here. I just get y square and z square. Then on the right hand side, I have c square which anyway does not change with frame; we have t prime. So, for t prime I have put this equation which is B tx X plus B tt t. So, on the right hand side I will write c square which I have put it here c square t prime square; it means the square of this particular quantity. So, i have put B tx X plus B tt t whole square. So, all I have done is this trial transformation equations have been put in x prime y prime z prime equation and I want to see arrive that I arrive back at my first equation which is x square plus y square plus z square is equal to c square t square. If that happens to be true, then only my transformation equations are correct. So, let us go to the next transparency and try to expand this.

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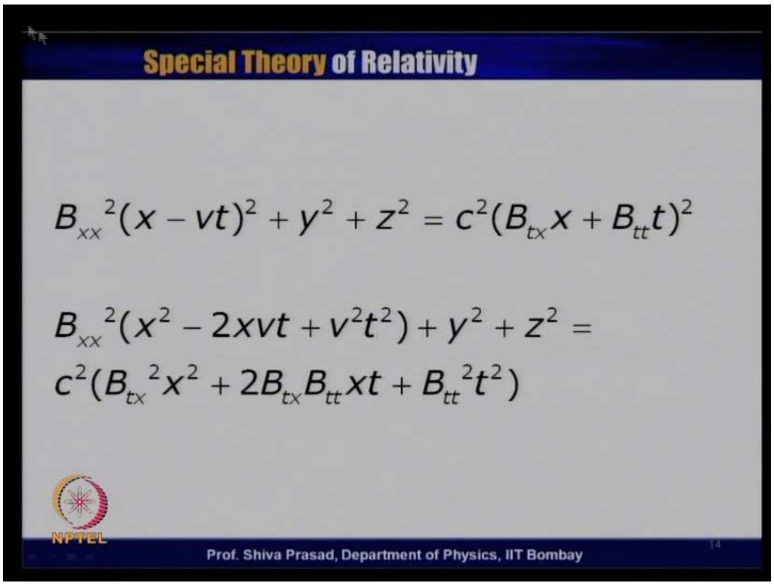

This is the equation which I had just now written in my earlier transparency; I am expanding this squares. So, this is B xx square which remains x minus vt I expand this equation. You know the standard equation for a plus b whole square is equal to a square plus 2 ab plus b square; I use exactly the same thing here. So, my a is x and my b is vt; so a square which is x square minus two ab. So, there is a two multiplied by x multiplied by vt plus b square. So, v square t square plus y square plus z square is equal to c square which is here; then I explained this square. So, this becomes B tx X whole square; that is a square.

So, this is B tx square X square two ab. So two, this multiplied by this, two btx I have taken this B tt first. So, this becomes two B tx multiplied by B tt into x multiplied by t; I have slightly reorganized this term. Then b square which is the square of this particular term. So, this becomes B tt square plus t square. So, all I have done is expanded this square quantities to write this equation and I want to now pick up x square y square z square and try to see that it matches with the original equation x square plus y square plus z square is equal to c square t square. So, let us do that in the next transparency.

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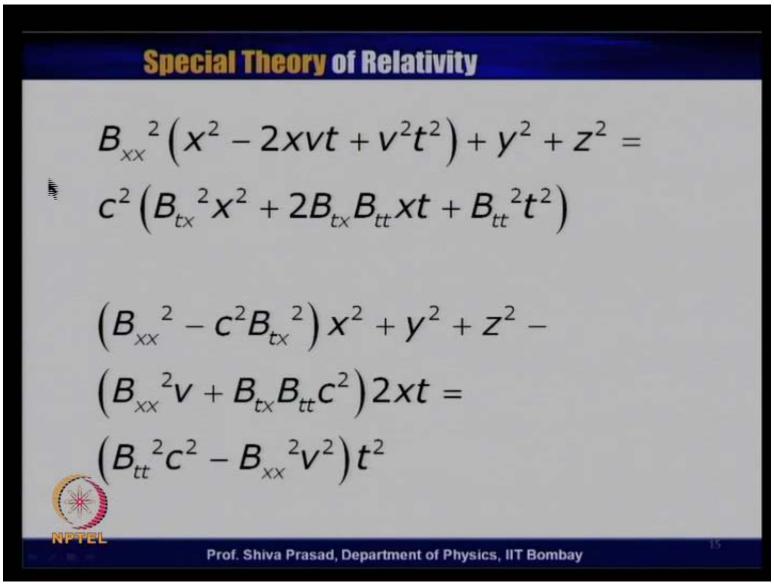

This is the equation which I had written in the last transparency; that is the beginning equation. Now I have started picking up terms involving x square. So, this is x square. So, this terms contains x square; so this is B xx square. Let us see which other term contains x square; this does not contain x square, this does not contain x square. Only other term which contains x square is this which is on the right hand side of equal sign. So, I take it on the left hand side. So, this equation becomes negative. So, it becomes minus c square multiplied by B tx square multiplied by X square. So, this is what I have written here. So, this is the second term which contain x square.

So, the coefficient of x square becomes B xx square minus c square B tx square. No other term contain x square; y square is straight forward y square only; z square there is no problem, it is z square only. Then I am trying to collect 2 x t term; there is x and t here. So, I am trying to collect 2 x t term which is here. I am taking negative sign out here. So, this term becomes positive. The coefficient of 2 x t will be v multiplied by B xx square. So, this I have written here; Bxx square multiplied by v, this is this term. There is another term which contains 2 x t term which is on the right hand side of the equal sign. So once I bring it back, it will become negative; a negative has already been taken out. So, this term becomes positive. So, remember we are looking at the coefficients of 2 x t.

So, it becomes B tx B tt multiplied by c square; this is what is here. No other terms contain x and t terms together. Now let us look at the t square term; t square term, there is one t square term here. The coefficient of t square term is B xx square plus v square, but I want to take this on the right side of the equal to sign because it is equal. So, I am putting this on the right hand side. So, this term will now become negative. So, B xx square multiplied by v square will become negative. So, that is what I have written here, bxx square v square as negative. There is already one term of t square on the right hand side which is B tt square multiplied by c square. So, this remains positive B tt square c square minus B xx square v square. So, this is simply collection of all the second order terms involving x square y square z square xt and t square.

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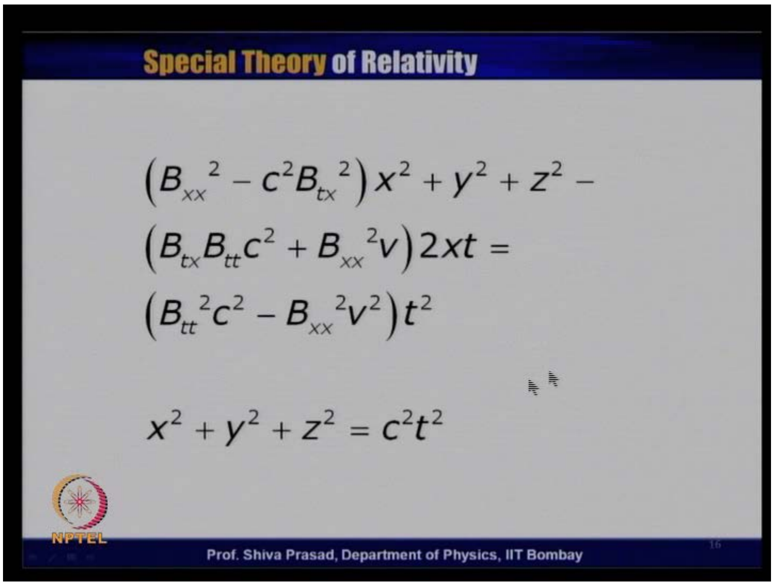

I have put the equation back here except to this equation which I have put in the last transparency. Remember if my transformation equations are correct, then this equation should exactly look like x square plus y square plus z square is equal to c square t square; that is what we have been discussing just now. If these equations have to match, it means this coefficient must equal to one; y square z square anywhere there is one notion, 2 x t there is no term of 2 x t there. So, I cannot allow a 2 x t term here. It means this coefficient must be zero, then only there will no 2 x t term.

This t square coefficient in this particular equation is c square; therefore, this coefficient this particular term must be equal to c square. So, I get three equations out of this; one is this whole term should be equal to one, this whole term should be equal to zero, this whole term should be equal to c square. So, whatever we have said is correct; then these three equations must be valid and remember we had already three unknown constants and if we have three equations, we can determine these coefficients and my transformation equation becomes known. This is what I have written in the next transparency.

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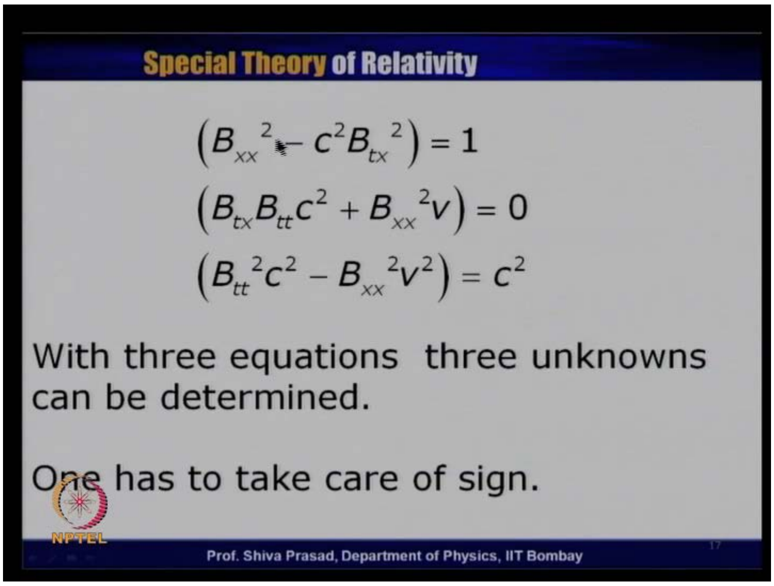

So, we have B xx square minus c square B tx square is equal to 1; B tx B tt c square plus B xx square v is 0; B tt square c square minus B xx square v square should be equal to c square. With these three equations, three unknowns can be determined. Only one thing one has to take care, because there are so many quadratics involved here. Once you try to solve you will find out that there may be two roots possible for some of these terms. So, you have to properly take care of sign; how to take care of a sign I will just explain in a minute.

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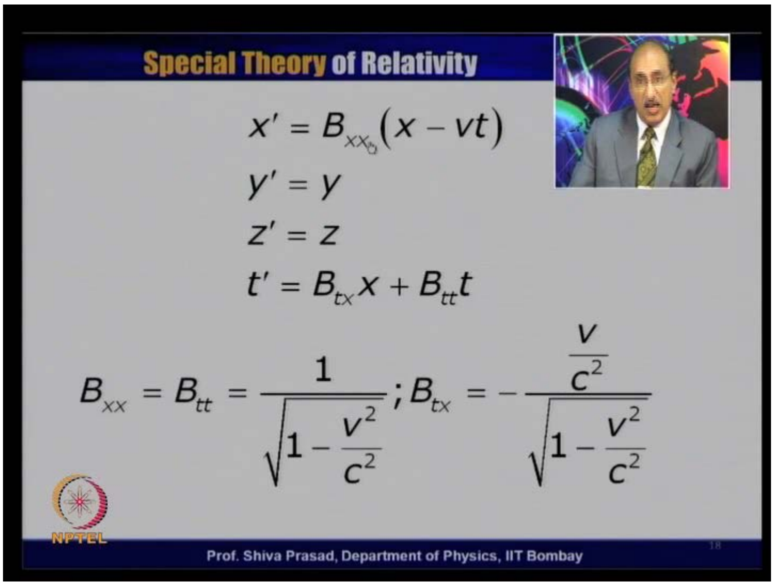

Let us go back to the equations which we have just now written. In fact, these are the sort of solutions but before I discuss that, let me just discuss this particular point here. I expect this coefficient to be positive. Even though there may be a negative root which may be allowed as a solution, this B xx has to be positive. The reason is that at time t is equal to zero, if an even occurs at a positive value of x in s frame, I expect it also to occur at a positive value of x prime in s prime of reference. If B xx was negative, the axis will change sign at t is equal to zero which we not expect because it should not happen in one frame; even I do not physically expect that something which is happening at positive values of x will happen at a negative values of x in a different frame. It is not of a reflection we do not expect. Therefore, I expect B xx must be equal to positive.

Exactly in a similar condition we must also expect that the coefficient of B tt must also be positive because at x is equal to zero whatever is the time sequence, if it is positive time, it should also appear to be positive in s prime frame of reference and if you look back at this particular equation if this has to be positive, this has to be positive. This B tx must be taken negative, so that this equation must be equal to zero. So, this are some of the sign care one has to take in order to arrive finally at the solutions and these are the solutions that we get; B xx is equal to B tt is equal to 1 divided by under root 1 minus v square divided by c square. For B tx I will get negative as I had said; that should be negative minus v divided by c square divided by under root 1 minus v square by c square. So, I have found out all the three coefficients.

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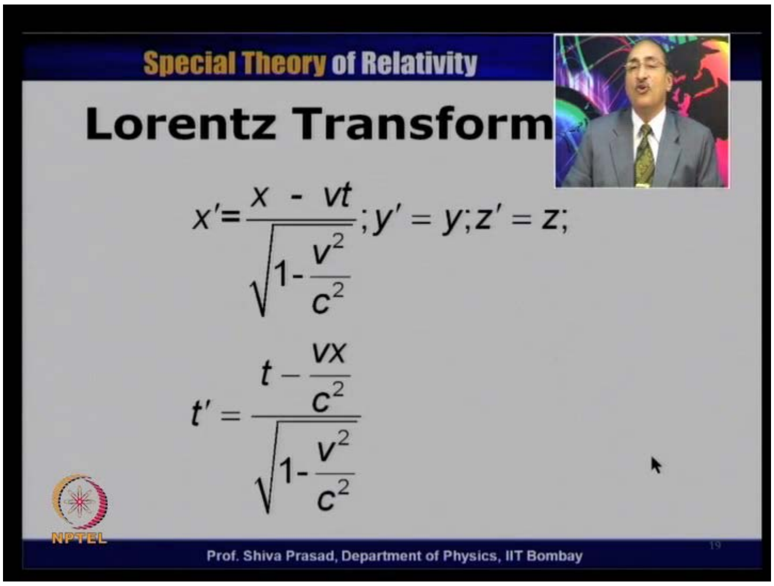

And hence I have found out the transformation which normally we call as Lorentz transformation; this is called Lorentz transformation. This particular transformation it appears was derived by Lorentz with slightly different meaning in a different context; that is why this particular transformation is named as Lorentz transformation. So, these are my Lorentz transformation equations where I have put all these constants back. So, x prime is equal to gamma minus vt divided by under root 1 minus v square by c square. These two are simple equations; t prime is equal to t minus vx upon c square divided by under root 1 minus v square by c square. Normally this equation tends to become somewhat clumpsy.

So, we use some notations which are generally very standardly used in special theory of relativity. v upon c we call as beta. So, beta is equal to v divided by c and gamma we define as 1 divided by under root 1 minus v square by c square. We have already defined beta as v by c. So, this becomes 1 minus beta square. So, I normally use gamma as 1 divided by under root 1 minus beta square. Looking at this and using this particular abbreviations, the equations appear somewhat simple to write x prime is equal to gamma times x minus vt; y prime is equal to y; z prime is equal to z and t prime is equal to gamma multiplied by t minus vx upon c square. So, these are my Lorentz transformation equation.

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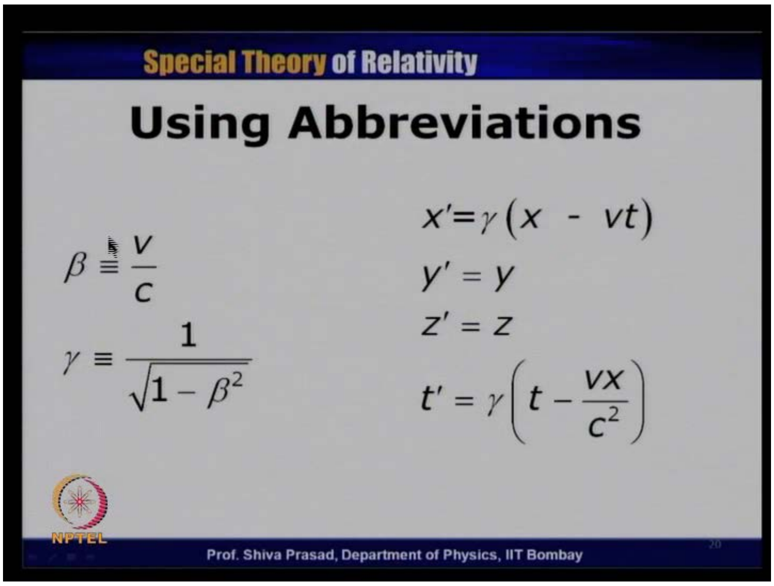

Before we start discussing these equations, let we make some observations on Lorentz transformation which are fairly interesting and probably try to bring out some similarities or some dissimilarity with the traditional or classical Galilean transformation. First thing that we observe that if the relative speed v between the frame is comparatively very small, this particular transformation will reduce to Galilean transformation. It is easy to see because if v is very small in comparison to c; it means beta is negligible.

If beta is negligible in comparison to one, then gamma is essentially very close to one; and if gamma is very close to one, then this becomes x prime is equal to x minus vt which is the x coordinate transformation equation of Galilean transformation. If I look at this particular term, gamma is one; v being very small in comparison to c, this term can also be neglected and this equation also becomes t prime is equal to t which is the traditional Galilean transformation equation for time which assume that time is same in all the frames t prime is equal to t.

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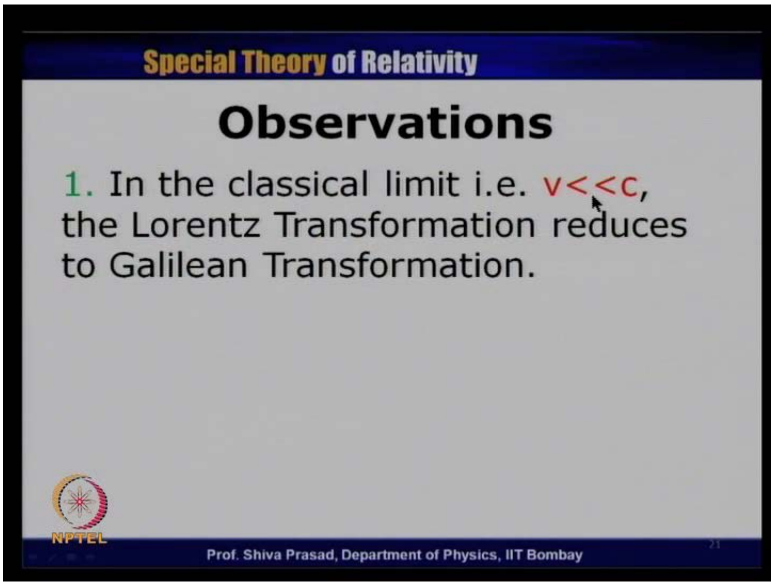

So, we realize that in the classical limit which is v much smaller in comparison to c, the Lorentz transformation is reducing to Galilean transformation. This is something which we expected because in normal life, for example if a car is moving or a train is moving or a plane is moving, we never see relativistic effects; the effects that we are going to describe later. So, we definitely expect that the speeds that we are talking, if they are not really that high as comparison to speed of light; we mean we should not expect relativistic effects to be observed and that is what we normally do. So, we do find that in the classical limit what we call as a classical limit that v much smaller than c, you will always reduce back to Galilean transformation. So if we are not talking of large speeds, Galilean transformation is alright which is simple. Second thing is probably an interesting comparison in the Galilean transformation and the Lorentz transformation. So, let as imagine a situation when an event occurs at an origin in s.

Let suppose this is an observer s and some event occurs at the origin. So, an observer is sitting at the origin some event, let us assume that a train is just passing by the side or anything. So, a particular event occurs just at the origin. Now my question is that will to an observer in s prime frame of reference would it also appear to occur at origin; classically you can ask this particular question, it may not be; because it depends where the origin of that particular observer in s prime or origin of s prime frame where is it at particular time.

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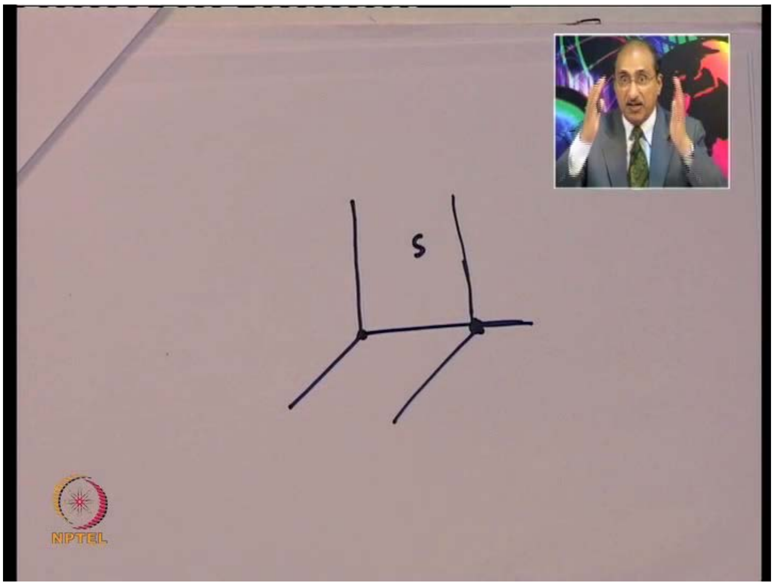

So, if the origin was here; the s prime frame of reference as origin as here, then to this particular observer, this event would be appearing to occur at different value of x unless the time for this event was also zero. If the time of this particular event was zero, in that particular case this origin was constant at this origin. Remember our special condition on the axis and at that time the observer in s prime frame of reference would also find that this particular event occurred at the origin. Let me just write the Lorentz transformation equation again.

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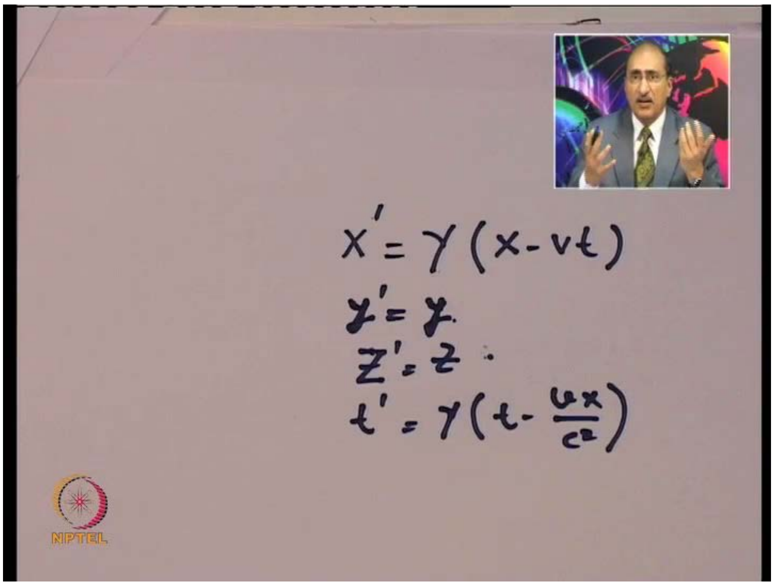

We have to keep on referring it quite often; t prime is equal to gamma t minus vx upon c square. Remember gamma let us not be confused with, this is gamma, this is y, this is y prime is equal to gamma. This gamma is also dependent on the speed of light. These are my Lorentz transformation. Now same thing is seen in the Lorentz transformation also. If the event occurs at x is equal to zero, the same event would appear to occur at x prime is equal to zero only if t is equal to zero. If t is not equal to zero, this x prime may occur may have a different value from zero. This is something which is classical and it is generally true; we all know there is nothing surprising it, Lorentz transformation also gives the same thing. Why I am telling something so obvious is only because I want to compare with the second statement which happened just now. So, let me just read whatever I have written here.

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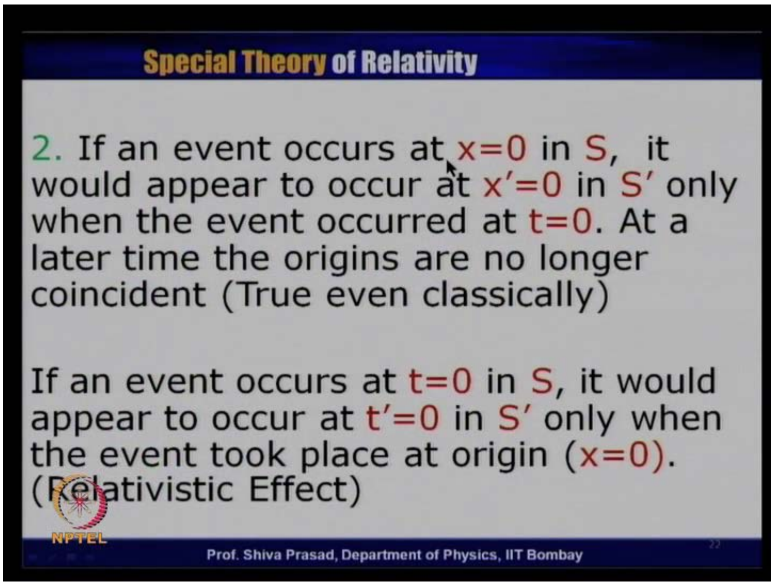

If an event occur at x is equal to 0 in s, it would appear to occur at x prime is equal to 0 in s prime only when the event occur at t is equal to 0. At a later time the origins are no longer coincident; therefore it may not appear to be occurring at x prime is equal to 0; this is told even classically. But let us look at this time equation. If an event occurs at t is equal to 0 in s frame, same event for an example, let us assume a train is passing with a very high speed with relativistic speed; very close to the speed of light. It is passing by the origin at t is equal to 0 it appears to be to passing. Now at t prime is equal to the observer in s prime would also find it out that this is at same time t prime is equal to zero if this event occurs only at x is equal to zero.

If for example this train was passing at a distance of 20 kilometers away from the origin in s frame at time t is equal to 0, then s prime observer would not find this particular event of train passing happening at t prime equal to zero or let us assume that there is a lightening which happens at 20 kilometers away from the origin at time t is equal to zero. The observer in s prime frame of reference would not find that this particular lightening occurred at the same time; look at this particular equation t prime is equal to gamma t minus vx upon c square. If t equal to zero that does not guarantee t prime is equal to zero unless x is also equal to zero. So, if x is not equal to zero t prime is different from t. So, is an event occurs at s at zero time, then that event may not appear to be occurring at same zero time in s prime frame of reference unless that event occurs at origin.

Simultaneity is relative; that is what we have been discussing. So, this is one of the very interesting consequences of Lorentz transformation that one notices. The third thing is about the inverse transformation; we have often talked about the inverse transformation. It means the information is given or the coordinates of an event is given in s prime of reference and I want to find out this transformation or these particular coordinates in s frame of reference and we have said that there is a very simple prescription of that; replace v by minus v, change prime to one prime and another prime to prime, you will get inverse transformation.

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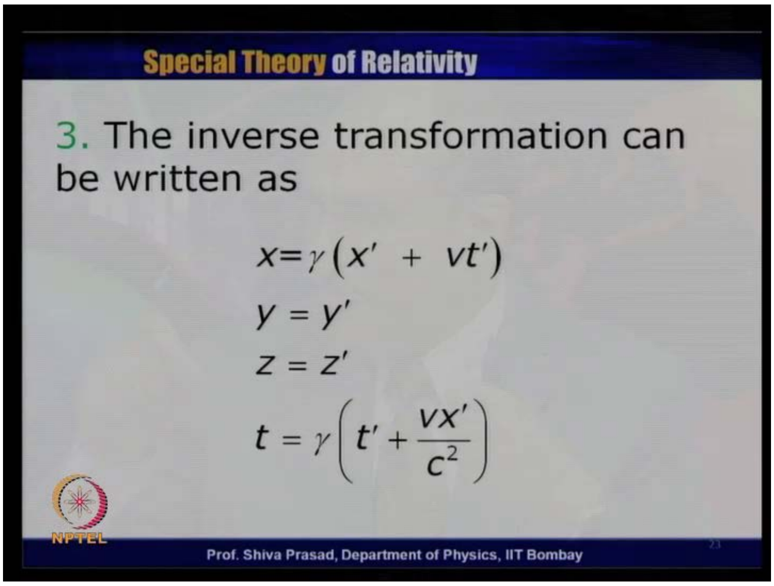

So, this is what we have written as inverse transformation. Again we sort of insist on these things which are very obvious because as we will realize that the relativity tends to become little more complicated. So, it becomes somewhat easier to think that when I am applying direct transformation when I am applying inverse transformation that is all. So, this is my inverse transformation which is x is equal to gamma x prime. So, remember I have changed the primes and instead of v, I have put minus v. So, this becomes plus.

Earlier there was a negative sign here plus v t prime; y is equal to y prime z is equal to z prime t is equal to gamma t prime plus vx prime divided by c square. So, the sign of this particular has also changed because v has been replaced by minus v. Now let us come back to our old example that we have done in our last lecture. In fact in some form or the other, this particular example we have been discussing various times. Now once we have obtained Lorentz transformation, let us look at this particular transparency and this particular example once more.

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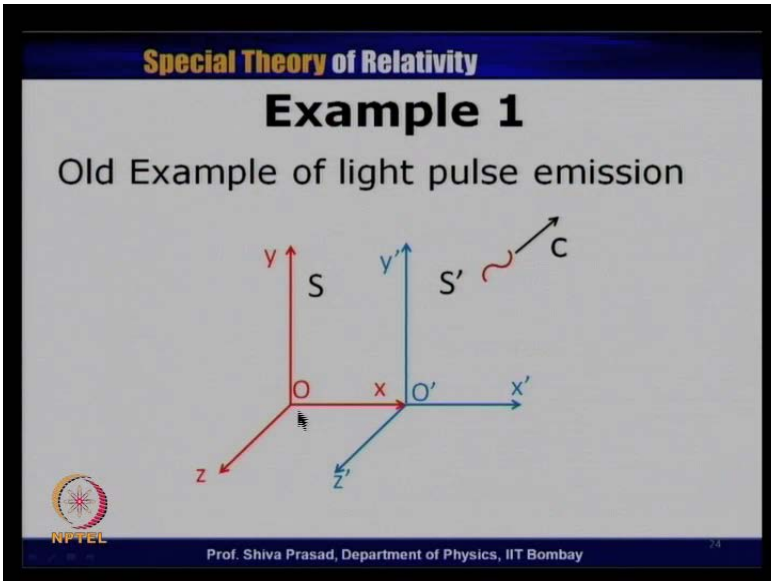

So, let me remind you by old example. This was s frame; this was s prime frame of reference. We have talked number of times about this s and s prime frame of reference, what are the specialties about them; y being parallel to y prime, z being parallel to z prime, relative motion along x direction, origin o prime moving along x direction. A light was emitted from the origin at time t is equal to zero; at the time of course the origin of o prime was also coincident and this is moving in a particular direction. This is small pulse assuming it to be highly localized.

So, I can determine what is the location of this particular pulse at a given time and at a time two microsecond, we had evaluated what would be the position of this particular pulse in s frame. Then in our last lecture using Galilean transformation we had found out the coordinate in s prime of reference, found out the time that eventually evaluated the speed in s prime frame of reference and we found that this does not allow to be seen because Galilean transformation is not consistent with second postulates of special theory of relativity. Now I am going to show that with this example with Lorentz transformations the things would change and now observer in s prime would also notice the speed of that particular pulse to be same. So, lets us just quickly go through the problem once more.

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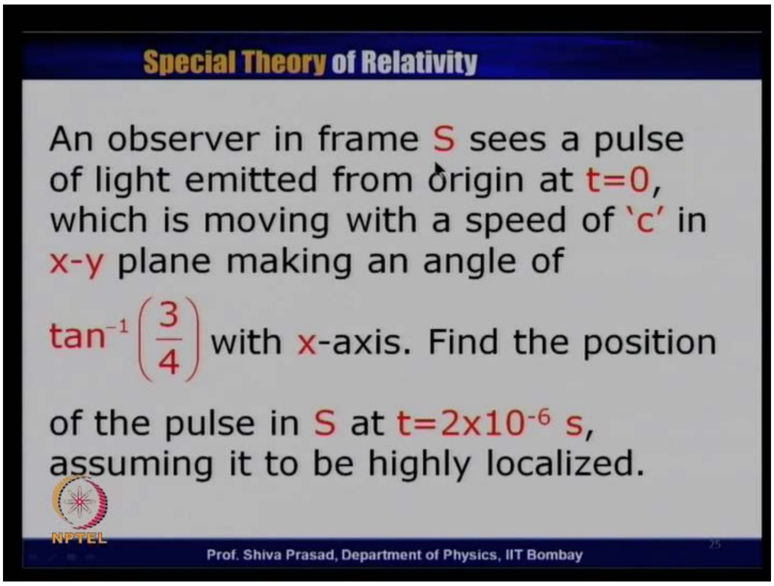

An observer in frame s sees a pulse of light emitted from origin at t is equal to 0 which is moving with a speed of c in x-y plane making an angle of tan inverse three by four with x axis. Find the position of the pulse in s at t is equal to 2 into 10 power minus 6 second which we call it as 2 microsecond assuming it to be highly localized. This part of the problem there has not been any change. This transparency also there has not been any change because as for as the formation in s is concerned, it is all given there; i do not need any transformations equation. So, long all the transformations is given in my own frame; I do not require Lorentz transformation, I do not require Galilean transformation. It is a pure simple classical kinematics; that is what we are doing here.

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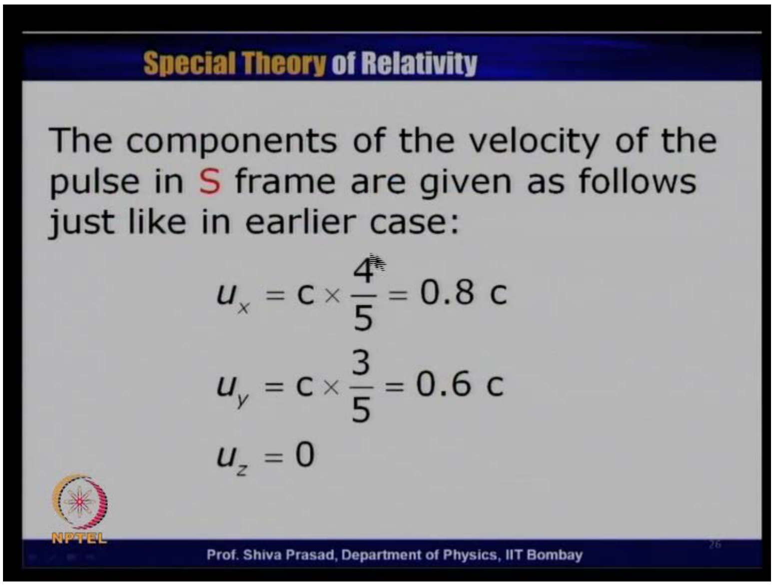

So, we have set that this will be the value of cos theta; this will be the value of sin theta. So, U x will be equal to 0.8 c, U y will be equal to 0.6 c, U z will be equal to 0; exactly the same transparency which we have shown in our last lecture, there is no change here. The change would occur where I go back to s prime of reference. When I want to calculate this particular coordinates in s prime frame of references. Let us first calculate the coordinates in s frame itself. No transformation required; no Lorentz transformation required.

Just take simple kinematic equation x equal to U x multiplied by t which is also exactly the same as my last transparency. My transparency in the last lecture x is equal to U x multiplied by t which is 0.8 multiplied by this term which gives you 480 meters, y U y multiplied by t which gives me 360 meter, z is equal to 0 taking speed of light c as 3into 10 to power 8 meters per second. So, as we had discussed last time according to an observer in s frame, the coordinates of this particular light pulse assuming it to be highly localized, at two microsecond will be 480 meters, 360 meters, zero; x is equal to 480, y is equal to 360, z is equal to 0. Now I want to find out the coordinates of the same light pulse in s prime frame of reference.

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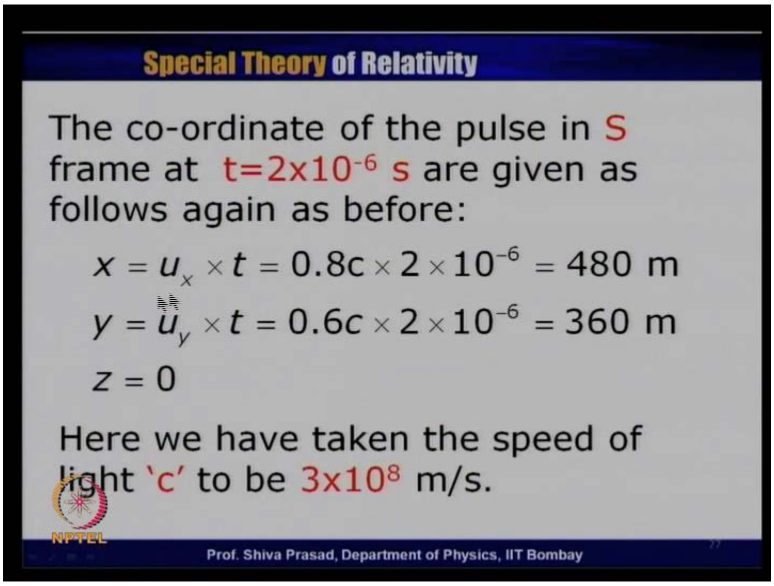

I would need Lorentz transformation. So, I would need x prime is equal to gamma multiplied by x minus vt; remember earlier this gamma was one in Galilean transformation. So, x prime would turn out to be different from x; x prime will not equal to x, it will turn out to be different from whatever we have obtained last time. Last time we have evaluated this x prime as by using the equation equal to x minus vt; y prime is anyway equal to y; z prime is equal to z as per Lorentz transformation. So, first I need to calculate gamma which I am calculating in the next transparency.

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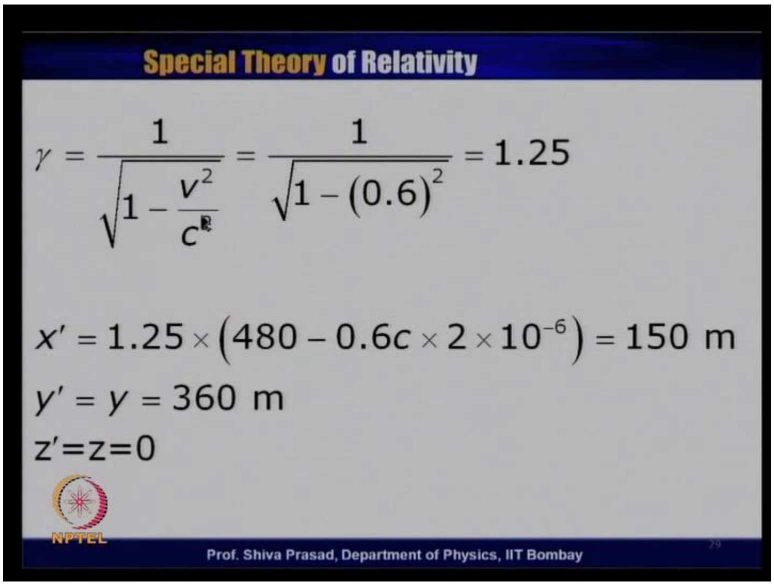

If you remember gamma has been defined as 1 upon under root 1 minus v square by c square. At a little later time, we will discuss that normally relativity we do not expect the values of v to be larger than speed of light though this is generally commonly accepted fact by now. Therefore, the value of gamma that I expect will always be larger than one because this I am reducing something from one. So, if I take under root of this particular quantity, this will be smaller than one and one divided by something which is smaller than one will always be larger than one. This will be equal to one upon under root v was equal to 0.6 c. When I was going to s frame to s prime frame, the relative speed was 0.6 times the speed of light. So, this becomes 1 minus 0.6 square of we calculate point six 0.6 square. This is 0.36 where 0.6 is a very nice number. It gives you a very nice value of gamma.

So, many examples we use this particular number. So, this is 0.36; if you subtract 0.36 from 1, you will get 0.64 and if you take under root of 0.64, this becomes 0.8. So, 1 divided by 0.8 becomes 1.25. So, x prime now becomes 1.25 multiplied by 480 minus 0.6 c multiplied by 2 microsecond. In Galilean transformation this quantity was one. Of this distance x prime turns out to be 150 meters; y prime there is no change, it is 360 meters; z prime was equal to zero, it is anyway equal to zero. So now, according to Lorentz transformation the coordinate of this particular light pulse will be x is equal to 150 meters, y is equal to 360 meters, z is equal to 0 which turns out to be different from what we have evaluated using Galilean transformation in our last lecture. Now not only these coordinates become different.

Using Lorentz transformation even the time becomes different. So, that also has to be looked into. So, I must also find out what is the time according to s prime observer. So, let us assume that this observer in s frame made a measurement at time t is equal to 2 microsecond to find out where is the pulse of light. Let me call that as an event. Now according to an observer in s prime, this event would occur at a different time and not at 2 microsecond. As far as emission of light pulse is concerned because this occurred at t is equal to zero, so t prime was also equal to zero; there was no issue. But now this event is no longer at the same t value as observed in s frame. So, what will be the time of this event as seen in s prime frame of reference; how do we find out, use the fourth equation of Lorentz transformation .

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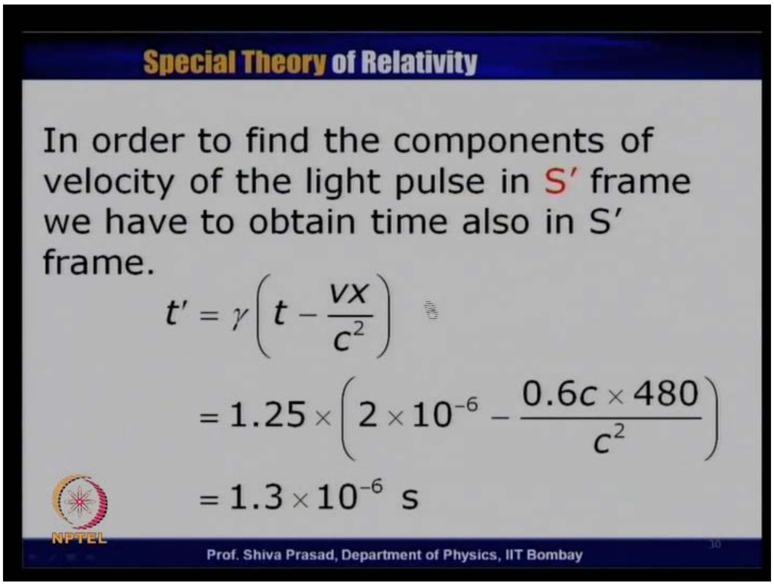

So, this is what is the fourth equation of the Lorentz transformation. This t prime is equal to gamma t minus vx divided by c square. Gamma we had already calculated; this is 1.25, so this was 1.25. What was the t measured by s observer which is 2 micro seconds. So, this is 2 into 10 to the power of minus 6second. v is the relative velocity between the two frames which is 0.6 c. So, this becomes 0.6 c. x is the x coordinate of the event as seen in s frame which happens to 480 meters. So, this x must be substituted. We must substitute for x as 480 meters divided by c square taking c as 3 into 10 power 8 meters per second as we have normally been taking.

To make our numerical calculations easier, we must subtract this number, multiply by 1.25; this particular event according to s prime observer is observed at 1.3 into 10 to the power of minus 6 seconds. So, what an observer in s prime frame would conclude? He would conclude that in a light pulse has been emitted from his origin; remember as per his observation is concerned, it has also come for his origin because this event occurred at t is equal to zero is equal to t prime is equal to zero at the origins, but now the light has travelled only for 1.3 into 10 to the power minus 6 second at a time when observer in s measure the position of the light pulse. So, they will differ in their times and according to the observer in s prime, the x coordinates will be 150 meter and y coordinate will be 360 meters.

So, according to him the light has travelled for a time 1.3 into 10 to the power minus 6 second, not 2 into 10 to the power of minus 6 second. And have gone in x direction, a distance of 150 meters and in y direction, a distance of 360 meters. So, I can calculate what will be the speed of the light as will be determined by an observer in s prime. All you have to do is to divide x and y by their times. Remember we have to consistent in the frame. This information must be in s prime in own frame. x prime is also in s frame and y prime is also in s frame t prime has to be in s frame.

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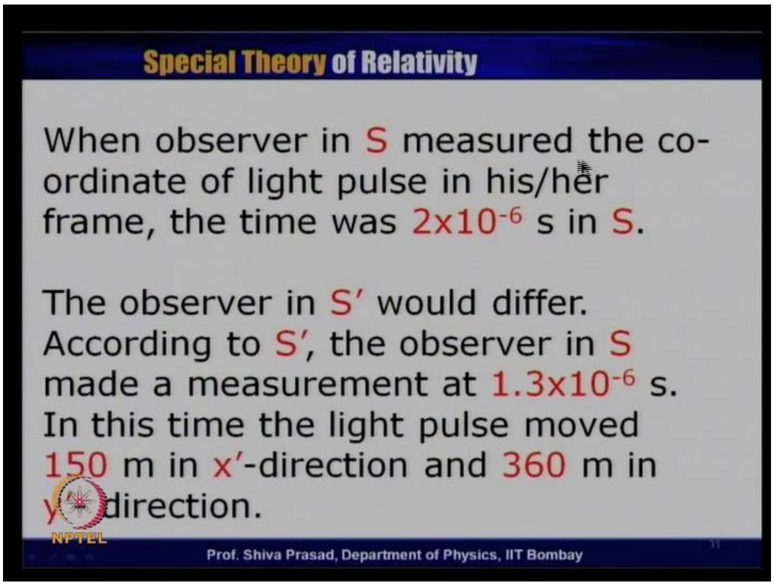

Let me just read what I have written. When observer in s measured the coordinate of the light pulse in his or her frame of reference, the time was 2 into 10 to the power of minus 6 second in s. The observer in s prime would differ. According to s prime, the observer in s made a measurement not at 2 into 10 to the power minus 6 seconds, but at 1.3 into 10 power minus 6 second. In this frame in this particular time, the light pulse actually moved a distance of 150 meters in x prime direction and 360 meters in y prime direction. This is the statement which I was trying to make saying that distances have become different; the time also has become different.

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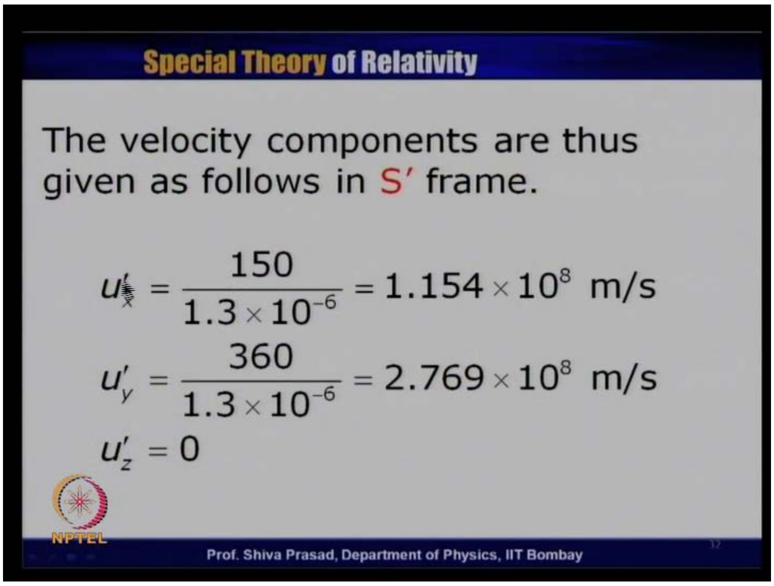

Now if the observer tries to calculate the speed of light, different components of the speed of light, according to him I have put a prime here because this is an observation which has been made according to observer in s prime frame of reference. So, this is U x prime would be the x distance travelled by the light which is 150 meters in a time of 1.3 into 10 to the power of minus 6 second. This would give the x component of the speed of light or speed of that particular pulse which now turns out to be 1.154 into 10 to the power of 8 meters per seconds. U y prime is 360 divided by the time. So, it has travelled 360 meters of distance along the y direction in a time of 1.3 into 10 power minus 6 second.

So according to him, the speed will be 360 divided by 1.3 into ten into 10 to the power of minus 6. This is 2.769 into 10 power 8 meters per second approximately. Of course U z prime is equal to 0. Now if everything what I am saying is consistent, then if I take this square plus this square plus this square and take under root of that, I must get the speed of light. Because according to observer in s prime frame also, the dislike pulse must travel with the speed of light; that is the postulate with which we have started; that is the postulate with which we have evolved Lorentz transformation. So, we must get back the speed of light.

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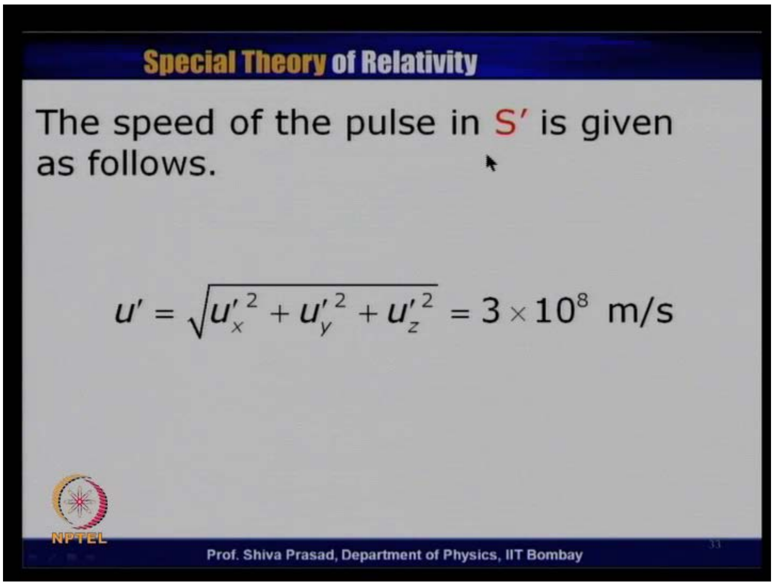

This is what I have done here. The speed of the pulse in s prime now will be given as u prime is equal to under root U x prime square plus U y prime square plus U z prime square and if you just take the exact numbers, you will get it equal to 3 into 10 power 8 meters per second as expected that turns out to be the speed of light. Now you remember, see last time what was the difference when we had evolved everything as per Galilean transformation. The coordinates had changed last time, but this time was same. So, this U x prime was turning out to be smaller than U x while U y prime was same as U y.

So, obviously when I was taking square and adding to evaluate the magnitude of the speeds this was turn out to be different. In fact, at that time I had even commented that if I want to make the speed of the light same in the two frames, probably x prime has to be different from what we have evaluated and also probably t prime has to be different, Now we know if we believe in Lorentz transformation which we today believed that the x prime is also changed from Galilean value; t prime also has changed from Galilean value and thus making eventually the speed of light same in both the frames.

So, let me read here again. We thus see that as per Lorentz transformation both x prime and t prime have changed in s prime from what we had obtained earlier classically. x prime has gone up while t prime has gone down making both the velocity components increase from classical values. Even U x prime has changed increased. U y prime has also increased thus maintaining the speed of light c to be same in both s and s prime frame of reference.

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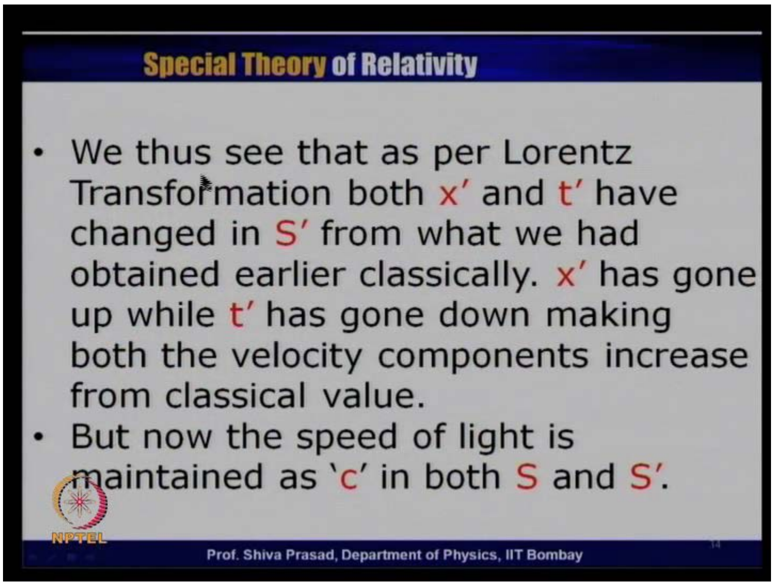

Let me do a summary of whatever we have discussed today. First thing that we did is we arrived at Lorentz transformation; the well-known Lorentz transformation which is consistent with the second postulate. So, that is my first thing.

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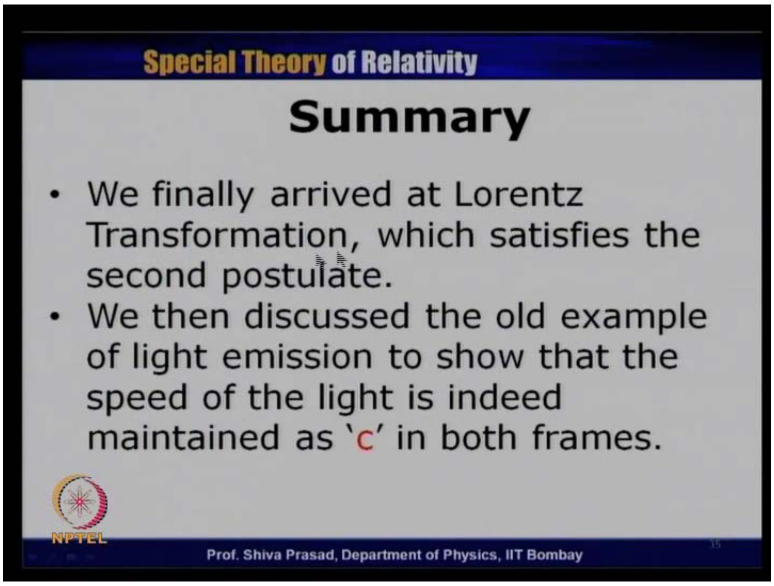

We finally arrived at Lorentz transformation which satisfies the second postulate. Then we discussed the second old example of light emission to show that indeed the speed of light is maintained in both the frames.

Thank you.