Special Theory of Relativity
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Lecture - 4
Look out for a New Transformation

In our last lecture, we had discussed the various postulates of special theory of relativity, there were two postulates, we will sort for revise them. We also discussed about Galilean transformation, we discussed what is a transformation and how the classical Galilean transformation look like, we formalize that transformation.

We discussed some examples, specially with the idea of time in mind, we saw that in the classical mechanics time is always treated to be same in all the frames. We also saw that, if there two events which are appear would be simultaneous in one frame, it means they occur at the same time. Then another frame also they will appear to be simultaneous this is ensured by the fact that time is same in both the frame of reference, so this is what we have discussed in a brief lecture.

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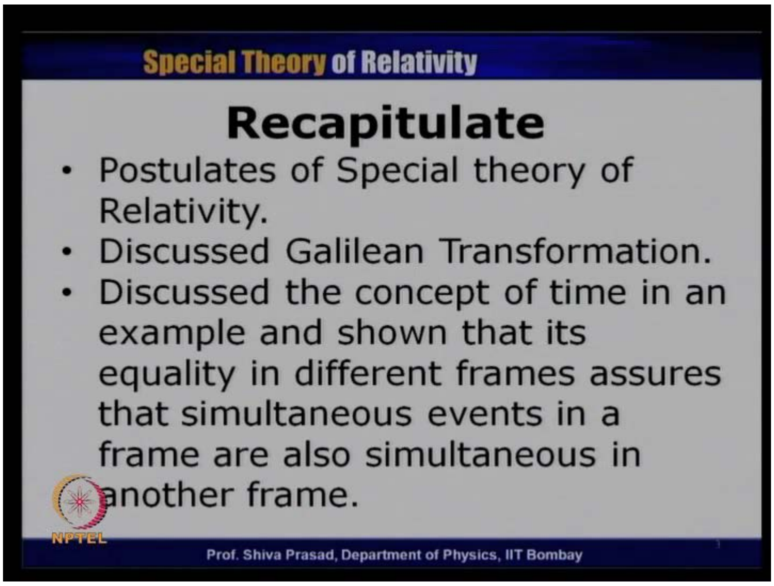

That postulates of special theory of relativity, Galilean transformation and the concept of time, and took some examples to show that it is equality in different frames, assures that simultaneous events in a frame are also simultaneous in another frame. Just to recapitulate, let me just tell again the postulates of special theory of relativity.

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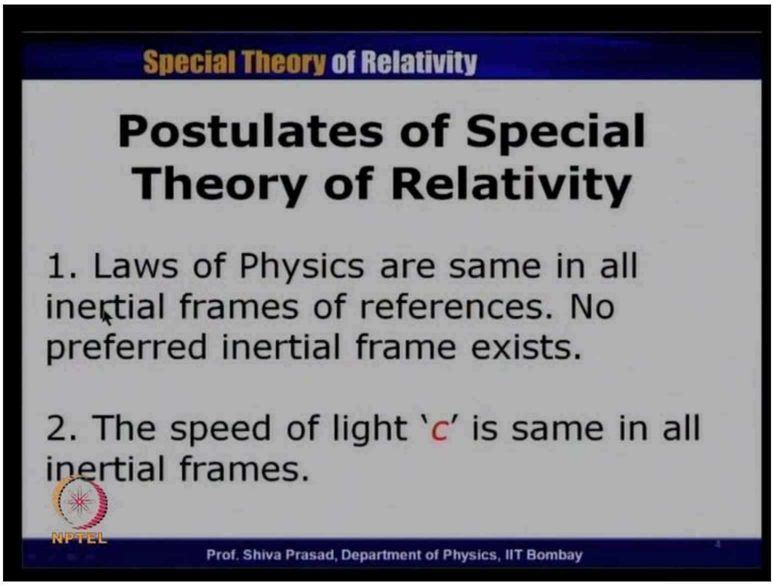

First is the laws of physics are same in all inertial frames of reference, no preferred inertial frame exist, we have always said that all inertial frames the laws of physics should always remain same. You cannot have something like ether, you cannot say something absolute rest, you cannot talk about absolute velocities all you have to talk is only relative in terms of a given frame. You must have something really physical to which you must attach your frame of reference and only with reference to that you can talk about speeds velocities.

Then the second postulate was that the speed of light C is same in all the inertial frame, I must add whenever I am saying speed of light, it means the speed of light in vacuum. Of course, when we talk of medium the light of speed of light may get reduced, but when we are talking of the speed of light C is always the speed of light in vacuum.

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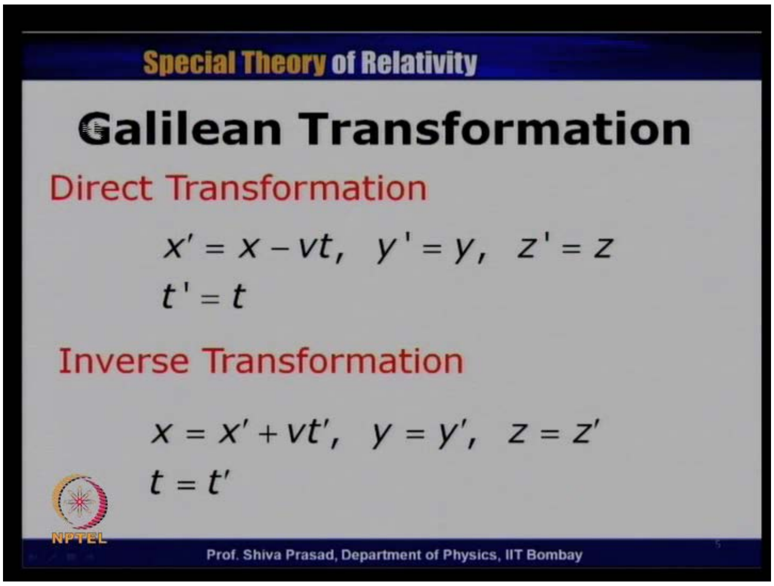

Again, I recapitulate Galilean transformation, this is what we call as a direct transformation, it means if we know the coordinates of an event which happen at X, Y and Z. And it happens at a time t, then the coordinate of the same event as appearing to another frame of reference X prime is given by this equation, X prime is equal to X minus v t. Y prime is equal to Y, Z prime is equal to Z. And of course, we have said that implicitly, we have assumed that t prime is equal to t.

Similarly, we talk of inverse transformation we says that, if we know the coordinates of an event in X prime frame of reference which is X prime, Y prime and Z prime, and it occurs at a time t prime. Then the coordinate of the same event in X frame would appear to be as X is equal to X prime plus v t prime, Y is equal to Y prime, Z is equal to Z prime, t is equal to t prime.

Of course, this transformation required a special set of axis about which we had discussed earlier that we assume that Y and Y prime X is are always parallel, Z and Z prime X is are always parallel, the relative motion is only along the X direction. And the origin of X prime moves along the X axis, time is measured from the time, when the origins of the two frames are constant; this is what we have discussed. Inverse transformation can always be obtained from direct transformation by putting v is equal to minus v.

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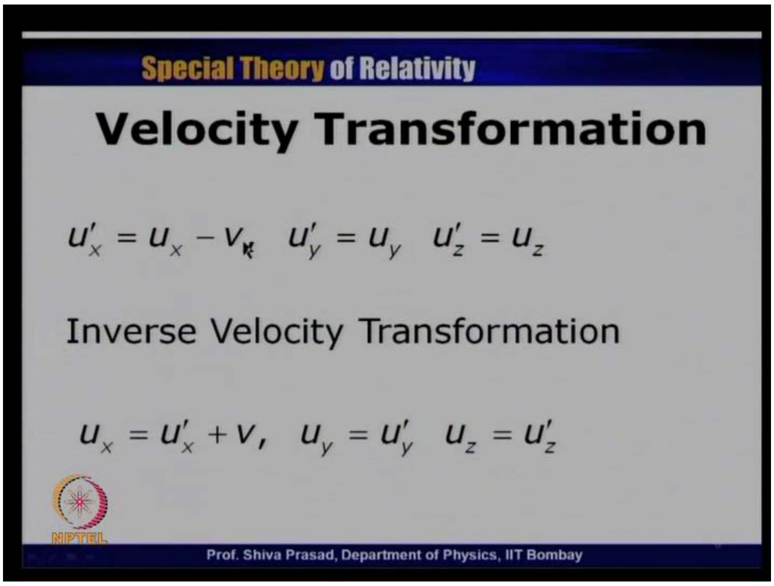

We also discussed the velocity transformation, it means I know the velocity components in X frame, U X, U Y, U Z, we can find out what will be the velocity components in X prime frame of reference, which turns out to be U X prime, U Y prime, U Z prime. And similarly we discussed about the inverse velocity transformation, you also said that this is exactly same as the normal relative velocity concept, with which we are very familiar with classical mechanics.

Today, I will try to give some example, and try to say that how Galilean transformation and second postulates of special theory of relativity do not go along with each other; it means it is not consistent with the second postulate of special theory of relativity. We have mention about this by passing by in our first lecture, when we are trying to do this special theory of relativity, we have measured that from the normal traditional relative velocity, and we will never find this speed of light to be constant.

But, once we have formalize the Galilean transformation, let us try to look with the eyes of Galilean transformation try to see how this particular transformation is inconsistent with second postulate of special theory of relativity. Will also give an example, we are we will see that if second postulates of special theory of relativity is correct, then simultaneity is also relative, it means in a frame if the two events appear to be simultaneous in another frame they may not, these are the two example let us start from them.

Before, we really actually come to the transformation which is consistent with the second postulate of special theory of relativity.

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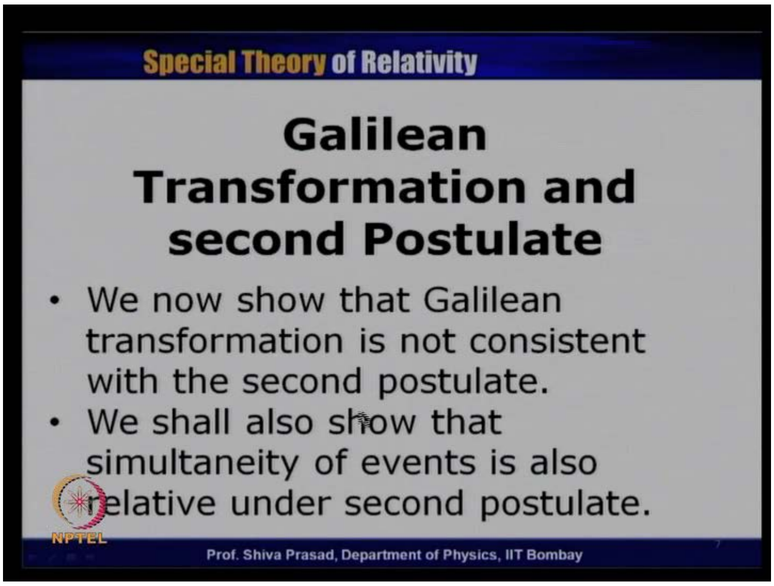

So, this is what I have written here, we now show that the Galilean transformation is not consistent with second postulate; we also show that simultaneity of event is also relative under the second postulates.

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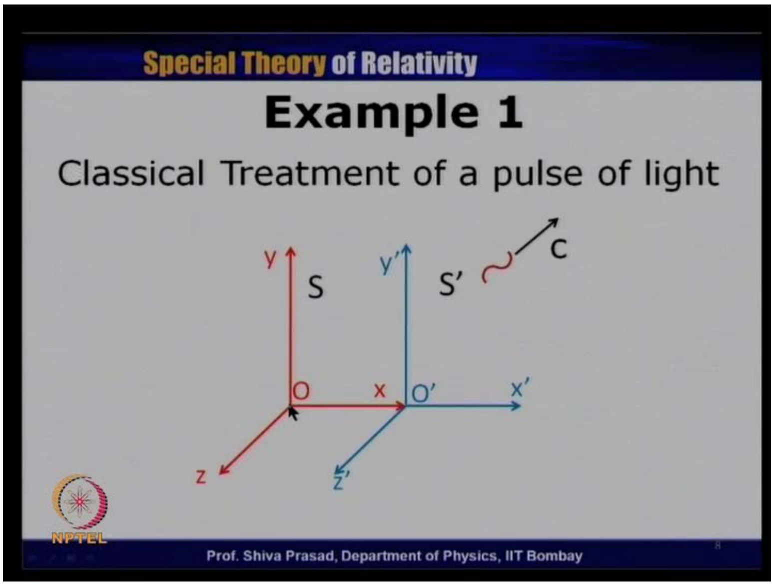

Let us go back to the old example which we have discussed in our last lecture, see what we have done in that particular lecture that we assume that a ball is thrown making a particular angle in the X, Y plane in the X frame. We try to find out the coordinate of that particular ball at time t is equal to 2 seconds in that particular frame, then we found out what are the coordinates of the same event that is finding the ball at t is equal to 2 seconds, in X in X frame, in X prime frame.

And the eventually found out what are the velocity components, so this is other problem which we have discussed. Now, I have modified that particular problem, instead of ball we thrown let image that we throw a pulse of light, most of the things remain same other than we have changed some numbers to make it look really somewhat more realistic.

Otherwise, the problem is more or less identical of what we did for that particular ball bought problem in last lecture. So, here we have a pulse of light, let us assume that this pulse of light is highly localized, so you can determine to a great accuracy, what is the position of the pulse. So, we through a pulse of light from the origin at the same time, when this O prime origin was also constant, which happens to be t is equal to 0, which also gives t prime to be equal to 0, most of the things as I said have remain same.

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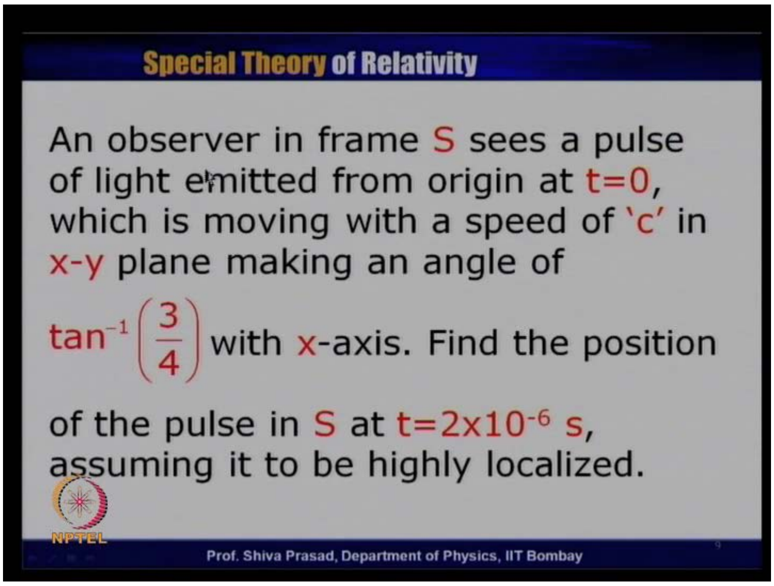

So, we assume that it is thrown in X, Y plane and it makes an angle of tan inverse 3 by 4 with X axis, this angle remains same as we have done in the last problem. What we have said now that find the position of the pulse in S at t is equal to 2 into 10 power minus 6 seconds that is 2 micro second to make number somewhat more realistic, instead of 2 seconds I have made it 2 micro seconds, because the speed of the light is very large. And of course, I assuming that the pulse is highly localized, so I can really determine it is position to reasonable amount of accuracy.

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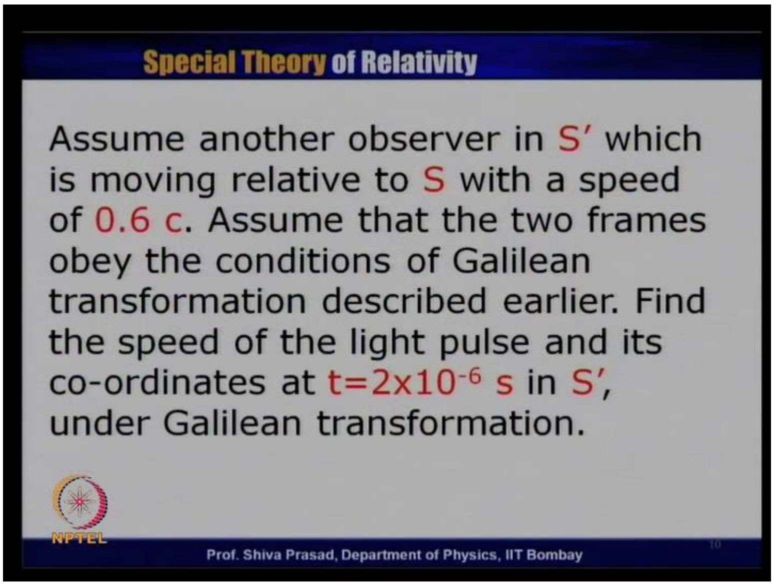

The question is essentially identical, assume that there is another observer in S prime frame which is moving relative to S with speed of 0.6 c, I have changed to the relative speed also to make it somewhat more realistic from the point of special theory of relativity. Assume that the two frames obey the condition of the Galilean transformation, which we have described just now. Again find the speed of the light pulse and its coordinates at t is equal to 2 into 10 power minus 6 seconds in S prime frame of reference under Galilean transformation.

So, we assume that the Galilean transformation is still valid, and try to find out this coordinates and also this speed in S prime of reference, as you can see that this problem is very identical to the problem, that we have just now discussed in our last lecture about the ball.

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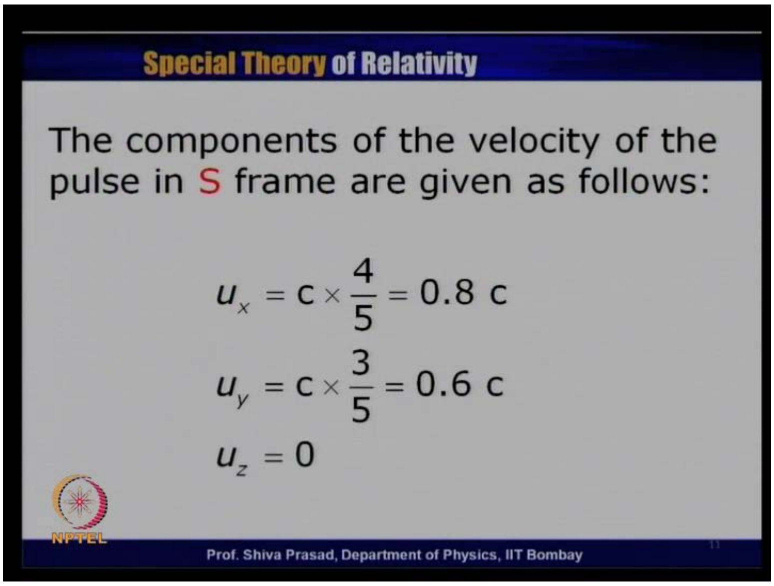

The method that we are going to adopt is essentially identical; we find the X component of the speed and the Y component of the speed. We have discussed last time, that if time theta is 3 by 4 then sin theta will be 3 by 5, and cos theta will be 4 by 5.

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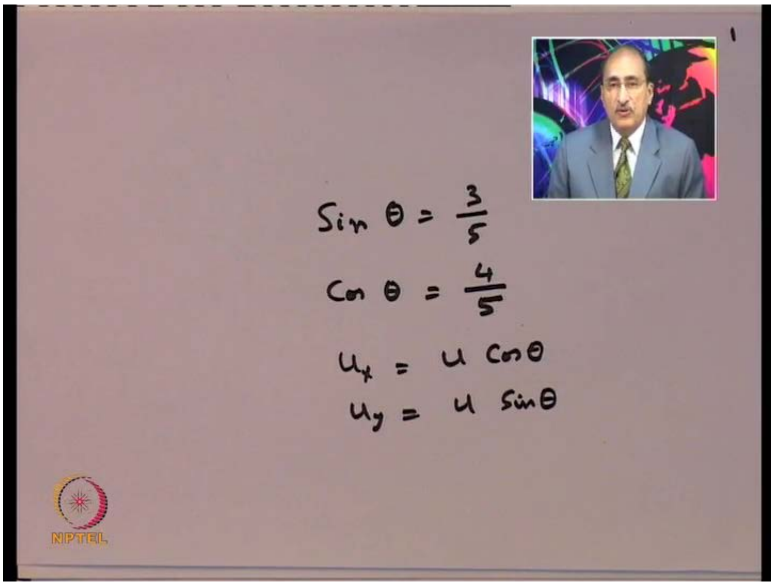

Therefore, we know that U X is equal to U cos theta and U Y is equal to U sin theta standard way of taking resolving a vector long X and Y direction. So, we will get U X to be equal to 0.8 C, U Y is equal to 0.6 C and because the light is through in X Y direction

U Z is equal to 0. So, this remains essentially identical of what we have done earlier, expect that this speed is now C, and not a classical speed.

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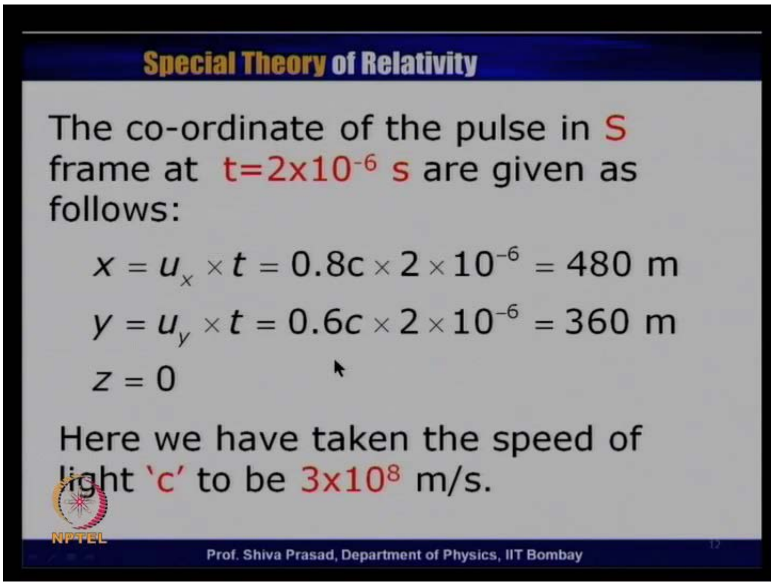

We can find out the coordinates at t is equal to 2 into 10 power minus 6 second that is 2 micro second, again by simple application of formula that the X coordinate will be U X multiplied by t which is 0.8 C is the value of U X time is 2 into 10 power minus 6 second we multiplied by that 2 and we get 480 meters. Similarly, Y will be equal to U Y multiplied by t, evaluated by U Y which turns out to be 0.6 C and t is 2 into 10 power minus 6 seconds, so Y component turns out to be 360 meters; and of course, Z components is 0. Of course, in this calculation I have taken the speed of light as 3 into 10 power 8 meter per second to make our problem simple.

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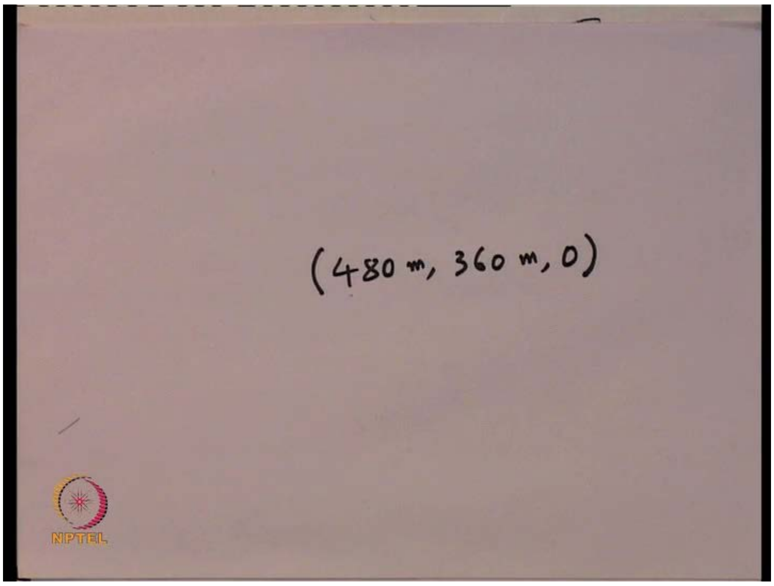

So, we see that the coordinate of this particular light pulse at 2 micro second will be 480 meters, 360 meters and 0. Question is that, what will be the coordinate of this particular light pulse as be viewed in S prime frame of reference assuming Galilean transformation, we apply standard Galilean transformation, this conclusion we have just now given.

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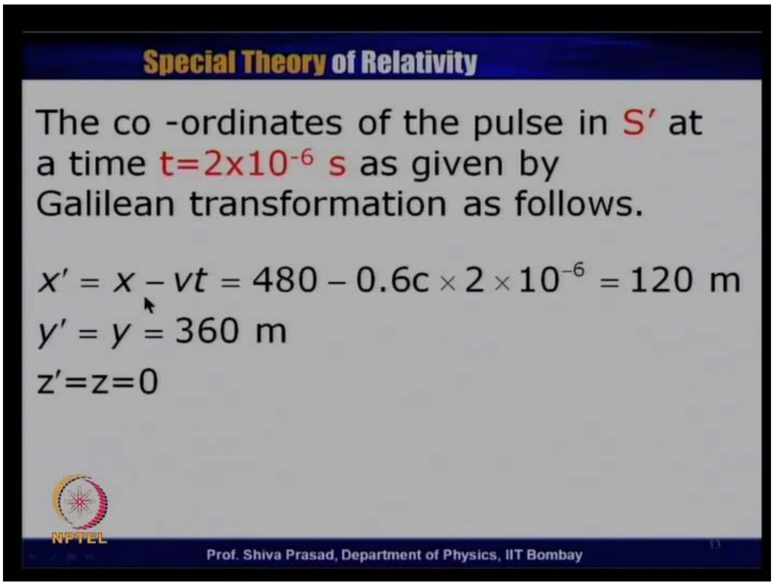

X prime will be equal to X minus v t, X we have now calculated 480, v which is the relative velocity between the frames is 0.6 C, time t is 2 micro seconds we put it here, I will get 120 meters. Y prime remains same which is equal to Y, which is equal to 360 meters.

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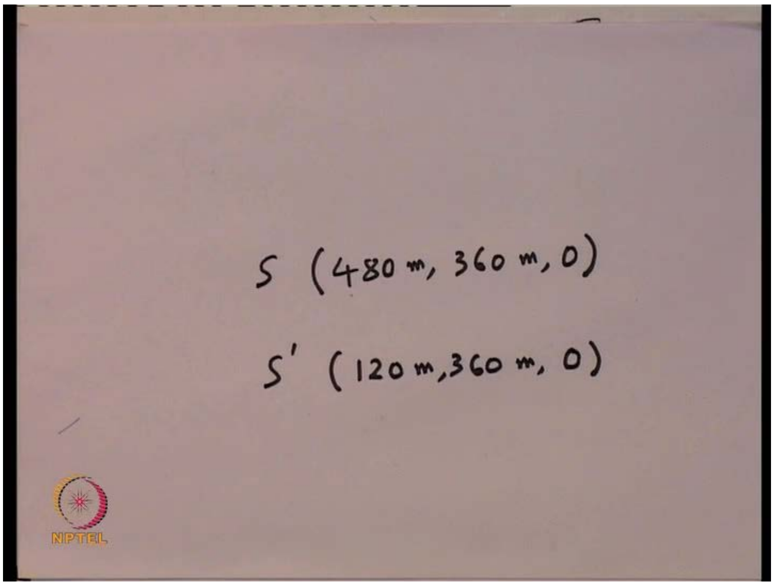

So, according to observer in S prime frame of reference, the coordinate of the same event will 120 meters, 360 meters and 0. So, the two observers will find out that their coordinates are different in their frames, one is 480, 360, 0, under that 120, 360, 0 standard thing expected in classical mechanics. Now, if I want to find out the speed like we did in the case of ball, as seen in S prime of reference this displacement I must divide by time, and the classical mechanics if we remember we always calculated time, assume time to be same.

So, we take 120 divide by 2 micro second which is the same time, as seen in S frame and we will find out U X prime to be 0.6 into 10 to power 8 meter per second, U Y prime will be 360 divided by 2 into 10 power 8 which is 1.8 into 10 power 8 meter per second. As you can see that this U X prime as changed in fact, it has become smaller than U X obviously, you would have expect that the speed of light, in this particular frame will be reduced if Galilean transformation was correct.

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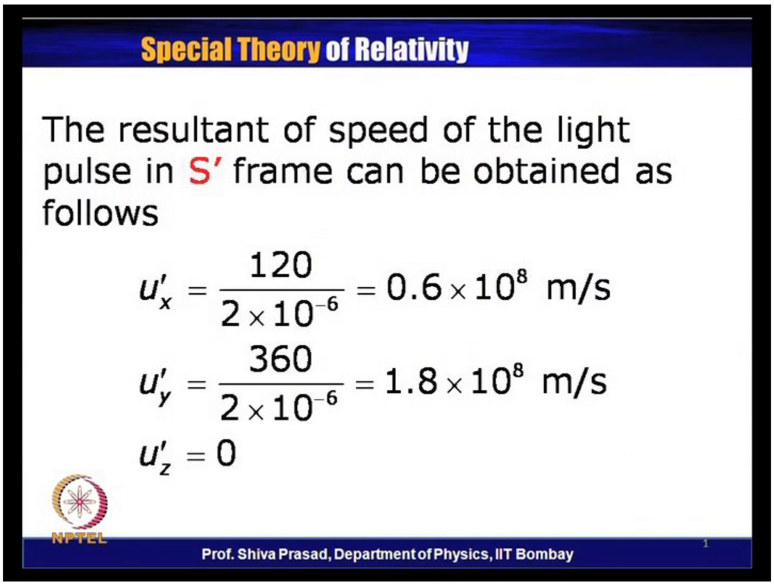

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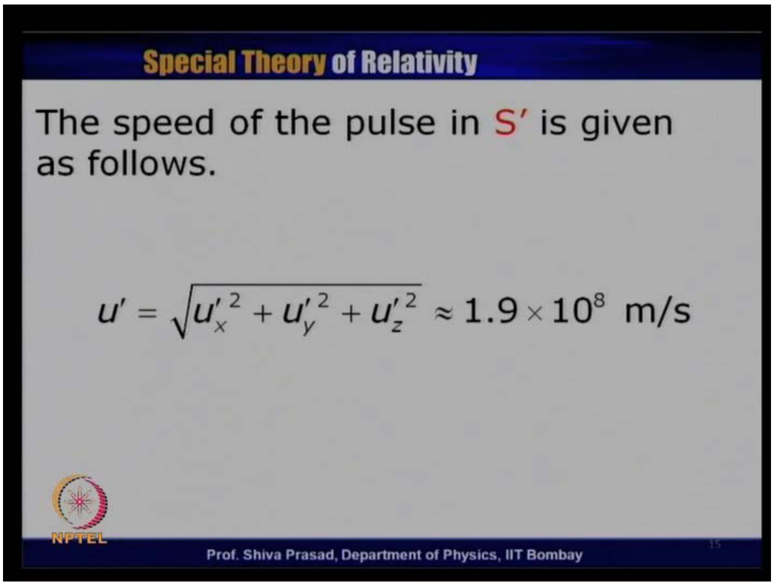

Calculated the speed of pulse in S prime which is given by standard formula of finding out the length of the vector, under root of U X prime square plus U Y prime square plus U Z prime square, this turns out be approximately 1.9 into 10 to the power 8 meter per second. While in S the same speed was 3 into 10 to the power 8 meter per second, so as we can see that if Galilean transformation was correct, then the speed of light will turn out be different in different frame, as was will turn out in different frames. And this is something which was against the second postulates of special theory of relativity.

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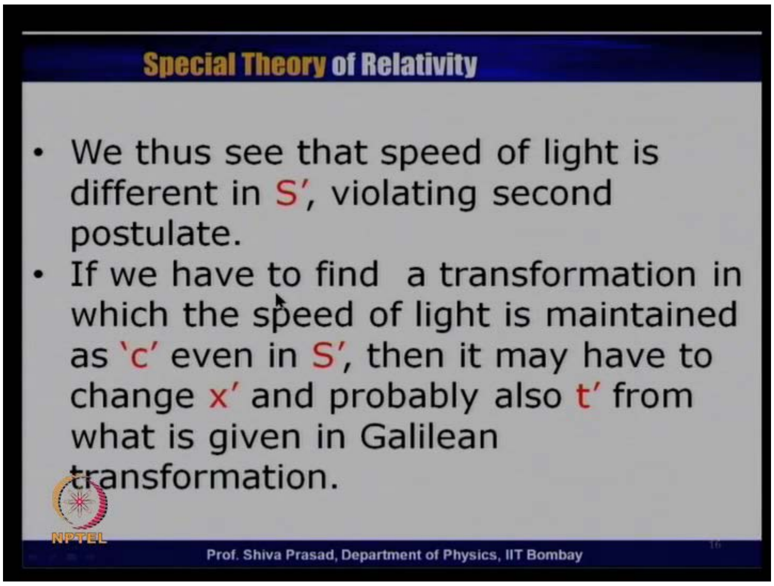

So, this is what I have written, we see the speed of light is different in S prime violating second postulates, if we have to find out the transformation in which the speed of light is maintained as C, even in S prime, S prime of reference. Then, either this X prime or t prime or both X prime and t prime have to be changed, from what has to be obtained from the Galilean transformation. So, we must have a transformation in which still X prime divided by t prime, and corresponding calculating the Y prime U Y prime eventually leads to a velocity which is equal to speed of light.

So, probably we required, we change in X coordinate as well as in Y coordinate. Let us look at the second example which we have done in the last time, which was an example in which a particular observer sitting in a train, of course two box one towards the motion of the train, another against the motion of the train. This particular observer concludes that the two events, event number 1 the ball hitting the front wall, and event number 2 ball hitting the back wall occurred at the same time.

We also concluded that the ground observer would also notice that the times are same, and according to the ground observer also these two events will appear to be simultaneous, it means appearing at the same time. Now, again we replace this particular experiment, this particular example and changed the ball to a light, a pulse of light, most of the problem is this exactly identical.

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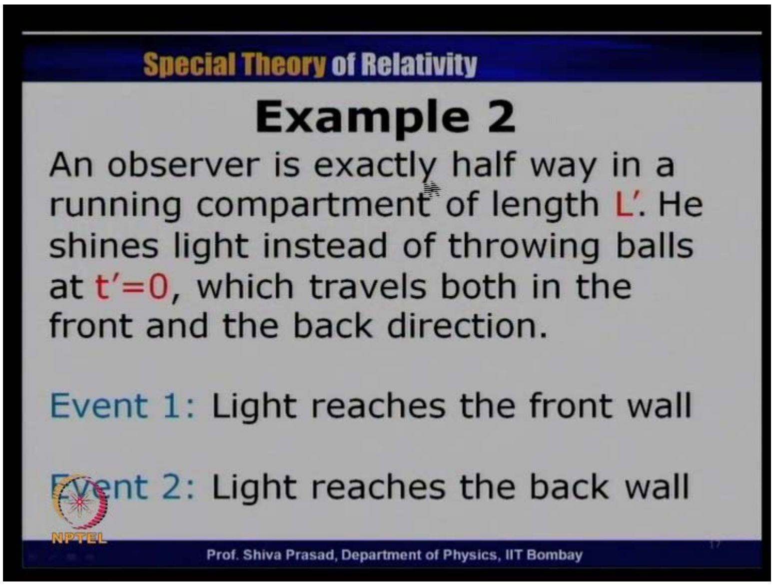

So, I have said that an observer is exactly half way in a running compartment of length L prime, as we eventually be seen that the length have also become frame dependent, so because, this S prime of reference I have decided to call it L prime. So, the length as measured in S prime of reference is L prime and of course, in the classical mechanics Galilean transformation will assume that this length is same, as seen in by the ground observer also.

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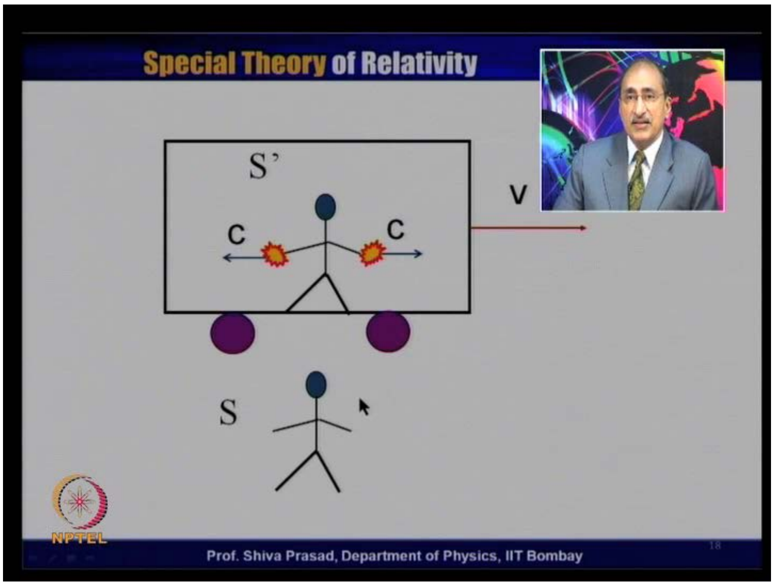

He shines light instead of throwing balls at t prime is equal to 0, which travels both in the front and the back direction, from direction means the direction of the motion of the train, and the back direction is opposite to that. We define events exactly as before, event number 1 light reaches the front wall, event number 2 light reaches the back wall.

I put this (( )) particular figure that this observer is sitting in the train, which is moving with respect to the ground observer with the speed v, this person has one light source here, one light source here which shines light or at the single line that does not make a difference shines light which goes this way which is opposite to that direction of the motion, which I call as back wall and like why going this direction which I am calling it as a front wall. So, light moving to the right, a light moving to the left, this is towards a front wall, this is towards back wall.

And our events are event number 1, this particular light pulse which in here, event number 2 this light pulse which in here at the back wall. Both this events are being observed by another observer as on the ground, which we assume as inertial. Let us look at the motion, both from point of the S prime, and S when we have a light source rather than two balls been thrown.

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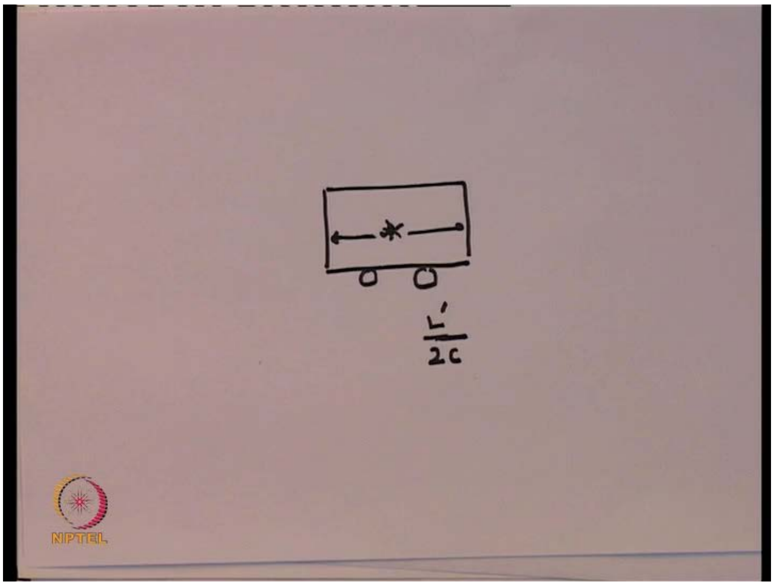

Let us go to the frame of S prime, if we go to the S prime of reference, this was my train light source was let us assume a single light source, this go in this particular direction, this particular light goes in this particular direction. So, each light pulse actually travels a distance of L by 2 according to the observer in S prime, the speed of light is C, so the time taken for each of the event is L prime by 2 C, and then we replace L by L prime as we have just now said. Now, the distance travelled by both the pulses this and this are same which is L prime by 2, both of them travels with the same speed C, so the time taken to be L prime divided by 2 C.

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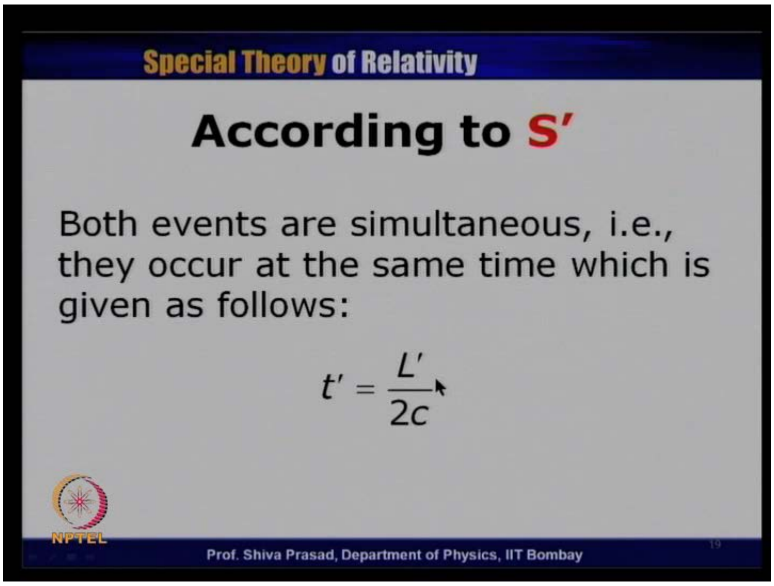

Now, what I have written here both events are simultaneous in S prime that is the occur at the same time, and the time is given by t prime is equal to L prime divided by 2 C. Of course, it assumes that the instant when the light pulses were thrown, on when the light (( )) shine the light at that time, time t prime was equal to 0. So, the observer in S prime frame of reference would feel that these two events are simultaneous, now let us look to with respect to a observer in S frame.

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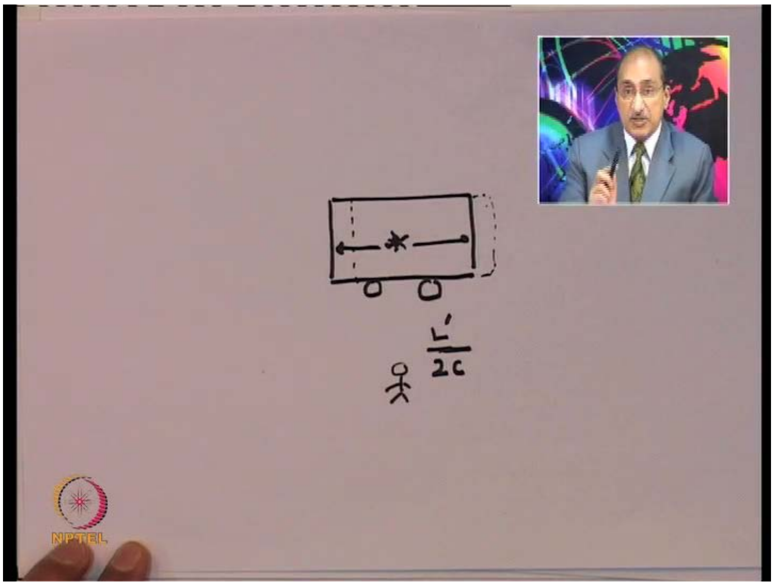

If the observer sitting here in S prime, S frame in the ground and if the second postulates of special theory of light, a special theory of relativity is correct, then according to this observer also this light pulse will be travelling with speed C, this light pulse will also be travelling with speed C. Remember, in the classical mechanics, in the classical velocity transformation the speeds were different, but if the second postulates of special theory is correct, then this is speed must be C, this speed also must be C, because the speed of light is a frame independent quantity, both the observer will feel exactly the speed to be identical.

So, according to the ground observer this pulse which was, which originated from this particular center was actually travelling with the speed C, the one which was going backwards also travel with the same speed with C. But by that time the pulse like, pulse of light moves from this particular position towards this particular wall, this wall has moved ahead; because the train is actually moving. So, it takes a certain amount of time for the light to reach from this particular point to this particular point, and according to the ground observer during this particular time, this train has moved ahead and gone somewhere here.

So, by the time this light will go and hit this particular wall, it has to travel a larger distance on the other hand, this pulse when it was trying to move towards the back wall, this back wall was approaching towards this particular source of light. So, eventually this light has to travel is smaller distance, before hitting this particular wall, so remember the same concept was used also by Galilean observer when we are throwing the balls. The only difference in that particular case was that according to that observer, this ball was thrown with larger speed and this ball was thrown with the smaller speed.

But, now both the light pulses travelling with the same speed, if the second postulate is correct, so what the observer on the ground conclude, the observer on the ground will conclude that this particular light will reach this particular wall later. Then here, because both the light pulses are travelling with the same speed, but this travels the smaller distance to reach the wall, while this travels the larger distance to reach the wall. Obviously, these two events cannot be simultaneous, this event even number 1 will occur later, then the event number 2 that it reach in this particular wall.

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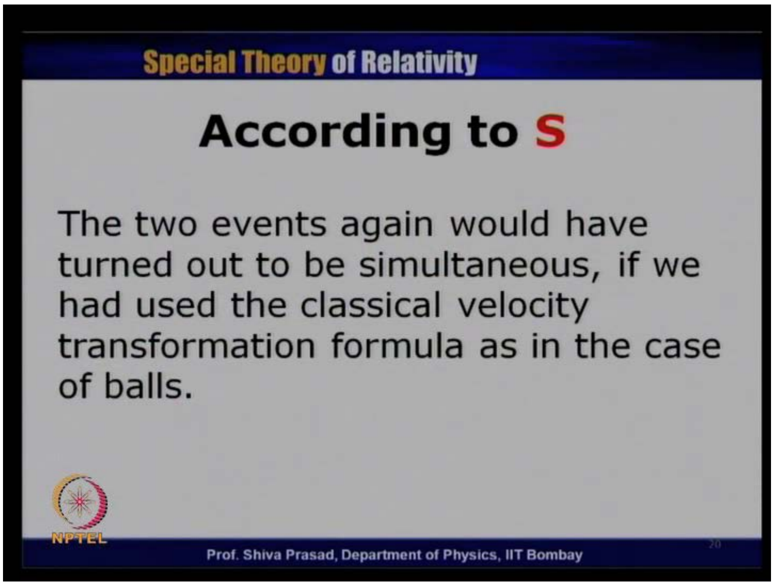

This is what I have written here, the two events again would have to turn out to be simultaneous, if we had used the classical velocity as in the case of balls.

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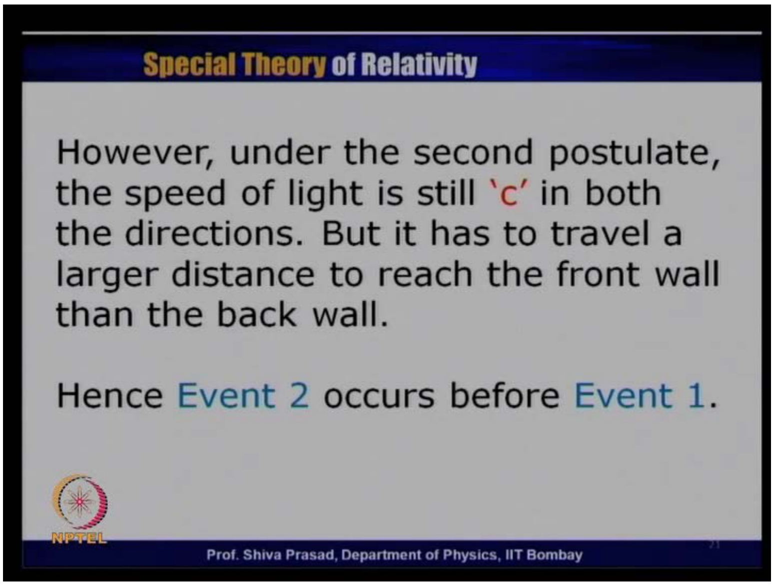

However, under the second postulates, the speed of light is still C in both the directions, but it has to travel in larger distance to reach the front wall than the back wall, as just now I have explained. Hence, event 2 occurs before event 1, so the two events are not simultaneous, so simultaneity is also relative. Now, let us come back out discussion on time, see time is often related to simultaneity of two events, when I say that a particular event happens let us say class starting, and taking another event, always a class starting at 9 o clock. Essentially we mean that, when watch shows 9 o clock and when the class starts, these two events are simultaneous.

Or will we say that a train starts from a given station, at least at 10 o clock it means when watch shows 10 o clocks, and when the train starts these two events are simultaneous. We have just now seen that simultaneity is relative, it means there would be another frame in which these two events may not appear to occurring simultaneous or appearing to be occurred at the same time. It means we probably we have to question the same basic equation which we have written, which we have sort implicitly assumes so far, which is t, which is equal to t prime, so this is what I have written here.

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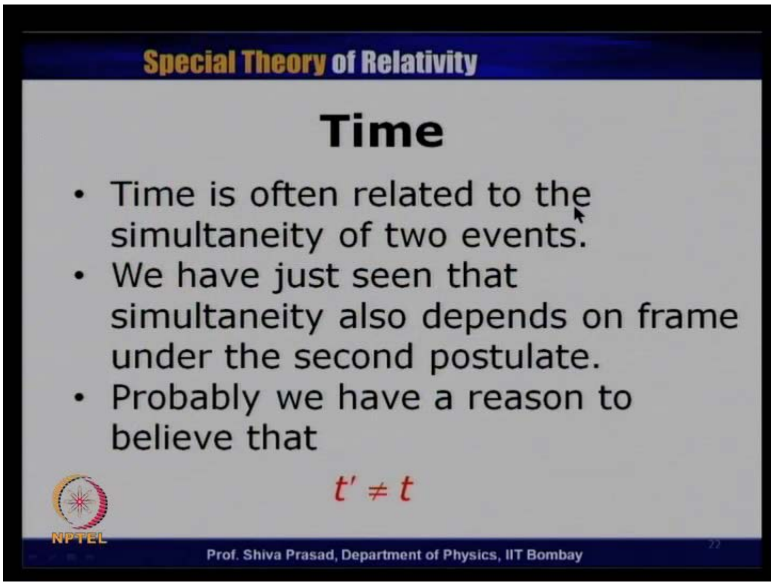

Time is often related to the simultaneity of two events, we have just seen that simultaneity also depends on frame under the second postulate. Therefore, probably we have reason to believe that t prime is not equal to t or t is not equal to t prime; so probably one has to relook at the concept of time, and we cannot use a simple equation like t prime equal to t. So, now we are again back into dark, we do not know where to start, all we have said that probably time cannot be taken to be same in all the frames; so let us start collecting all our evidences, and try to reach a new transformation, this transformation is call Lorentz transformation.

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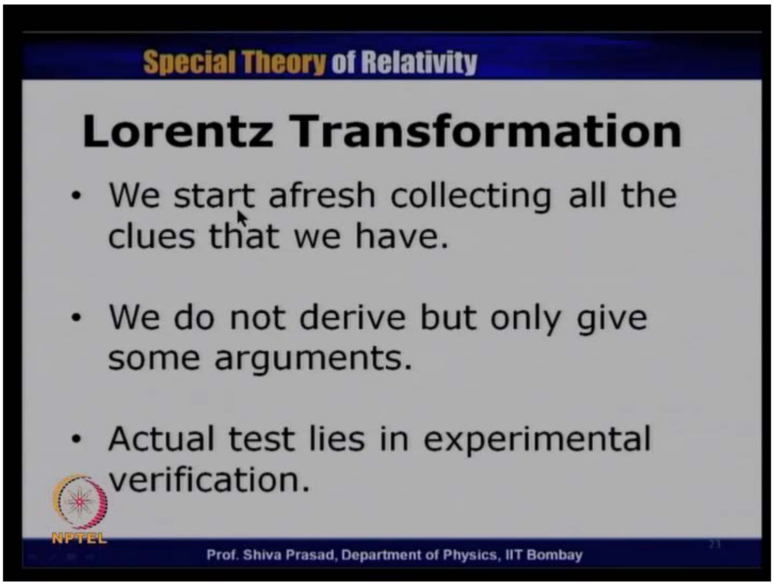

So, let us just before starting (( )) some observation, we are as I said we are essentially as dark, so we have to start collecting all the clues that we can get, I must mention that we are not deriving Lorentz transformation, this things are not derivable, so when we say something to be derive, we can derive it from certain fundamental laws. So, we are only collecting arguments we are not deriving it, these observations whatever we have seen, whatever we feel should to be put into the set of certain equations, but whether these the equations are correct or not depends on whether they can be explained in the experiment.

Like when we say of the Newton's law of motion, Newton's law of motion we do not derive in our high schools we have never pull how to derive the laws of motion, they became a fundamental law. We assume Newton's laws of motion and try to explain various things, and that is a way we have developed our physics. Similarly, Lorentz transformation we are just collecting our evidences, some of these evidences may not look very strong, some of the observations may not very strong, but eventually we believe in them. Because, they are able to explain a large number of experiments which we cannot otherwise explained, that is why we believe in Lorentz transformation.

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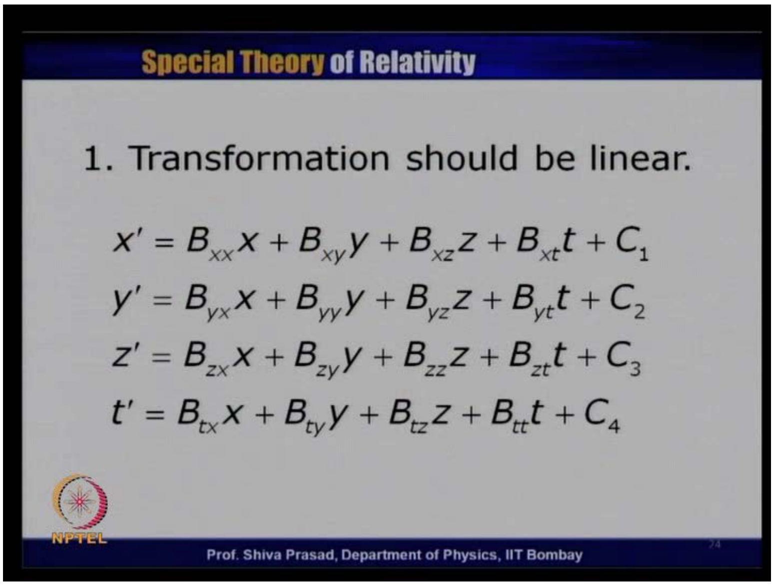

So, the first point about Lorentz transformation is that, the transformation should be linear; I must mention that the thing I am taking this particular treatment is given in Decimix book which I will be the reference will be given in the next. So, what I mean by it linear transformation, if you see this particular equation, we have written X prime equal to one constant which I am calling it as B X X, because this is relating X to X this is B X Y relating X to Y, B X Z, B X t and C 1, we have written as X prime is equal to B X X X plus B X Y Y plus B X Z Z plus B X t t, were all this B is are some constants which we to have determine. By linear mean, we mean that this only one single power of X which is involved, there is single power of Y which is involved, single power of Z which is involved, single power of t involved.

We do not have a turn involving, let us say X square or X cube or Y square are term like X Y or term like X C, we expect the transformation to be linear, I will give you the reason why do we expect like that. You look at Y prime, we have written exactly similar type of equation, expect this concepts are different which in general would be different, this I have written as B Y X, this Y represents this Y, this X represents this X, B Y X X plus B Y Y Y plus B Y Z Z plus B Y t t plus C 2. Similarly, Z prime we have written as B Z X X plus B Z Y Y plus B Z Z Z plus B Z t t plus constant C 3.

Then time we have written, which earlier was always equal to t, t prime is equal to B t X X plus B t Y Y plus B t Z Z plus B t t t plus C 4, let me first given argument why we expect this particular transformation to be linear.

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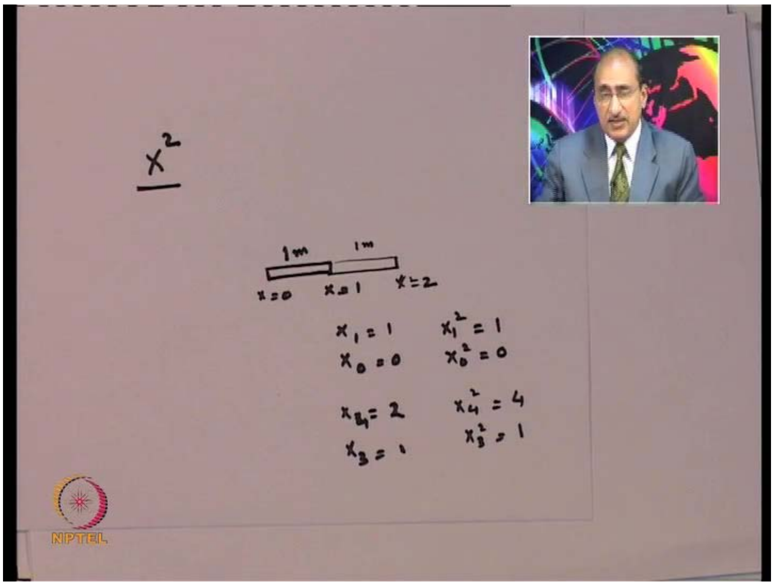

Let us suppose we had a transformation which involve X square term, if we had a transformation which involved X square term and let us suppose, let us just put a rod which is length 1 meter, which is placed between X is equal to 0, and X is equal to 1. If this rod is put between X is equal to 0 and X is equal to 1, then X 1 will be equal to 1 and X 0 will be equal to 0. Now, X 1 minus 0 will be equal to 1, if I take X 1 square this will be 1, X 0 square will equal to 0; now let us suppose we displace this rod, and put this particular rod now between X is equal to 1, and X is equal to 2, same 1 meter rod. Now, let us call this X 3, X 3 will be equal to 1, X 3 let we put X 4 will be equal to 2, X 3 will be equal to 1, because this now at X is equal to 2 meters, and this is at X is equal to X is equal to 1 meter.

If we take X 4 square this would be 4, if we take X 3 square this is equal to 1, so if by transformation term involved X square will find that X square X 1 square minus X 1 square is 1, while X naught X 4 square minus X 3 square is 3. So, this particular length would appear to be 1 meter in X prime frame of reference, if we had a term like this, and this particular length will turn out to be 3 meters. It means if this lens rod was put between X is equal to 0 and 1, its length will turn out to be different, from the 1, from the case from the rod was put between X is equal to 1 and X is equal to 2.

This looks somewhat wizards, this does not look somewhat reasonable that if we just placed our rod, and displaced by 1 meter, the length of the rod in a different frame will appear out to be different; after all whatever we have taken as origin could have been a slightly different origin. We expect this space to be homogeneous, I do not expect the length of a rod to be different in different frames based on where I put the rod, if the length I can expect that you know the length of 1 meter may not appear to be 1 1 meter, it may appear to 1.2 to or 0.8 meter. But if I displace the rod, the same length 1.2 or 0.8 whatever we had observed earlier, same length should be observed in X prime frame of reference, by displacing the rod I should not be able to, I should not measure a different length.

Because, the space is homogeneous I do not expect length to be dependent on the fact where I put this particular rod. Suppose, this is particular pen if I displace it here, it should not appear that in a different frame of reference its length has changed, whatever might be the length larger or smaller, but whatever was the length here, same should be the length observed here also. So, therefore, I avoid all higher order terms and we expect linearity, so with the first clue that we have the transformation must be linear, it must involve only single powers, should not involve power like X square etcetera, etcetera.

So, the simple most transformation in linear is of this particular type which have just now mentioned.

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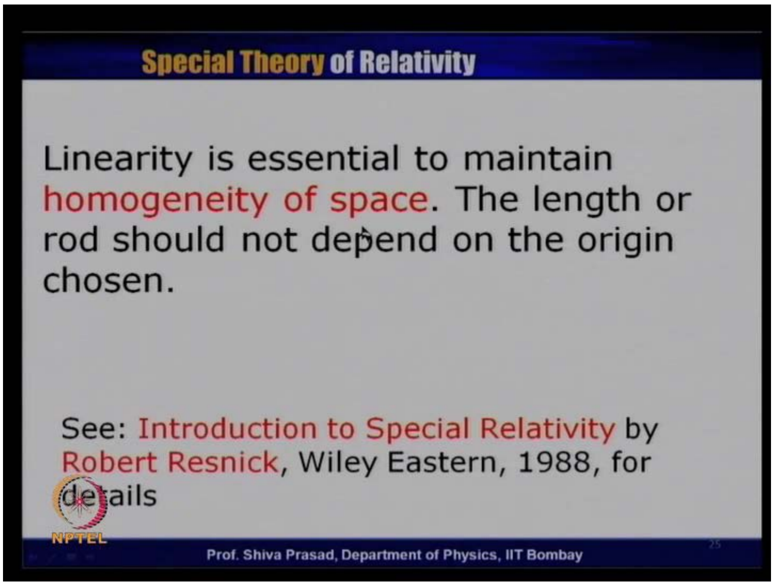

This is what I have said linearity is essential to maintain homogeneity of space, the length of the rod should not depend on the origin chosen. And this is the reference of the book from which we have taken this particular derivation; this is introduction to special relativity by Robert Resnick, by Wiley Eastern 1988.

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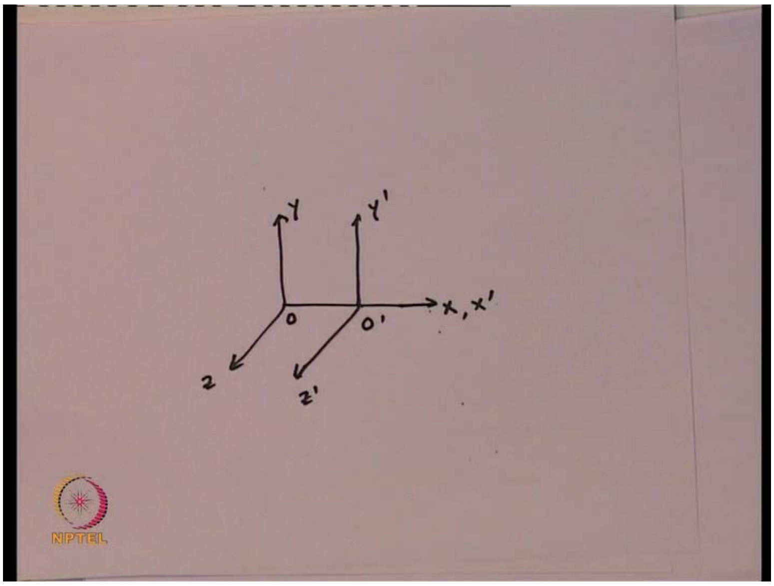

Now, let us collect more evidences, what we will realize see earlier when we do use Galilean transformation, we had arbitrarily put S and S prime frame; and taken a assertions a simply a realistic view of the axes. Now, you realize that this simplistic view help us in getting eliminated a large number of constant and therefore, making transformation equation simple.

I remind again, your two frames S and S prime, this was origin O, this was origin O prime, this was my X axis, this was Y axis, this is Z axis, this is Z prime axis, this is Y prime axis, X and X prime axis are always constant. And the time was time t is equal to 0 was measured, when O prime was constant with O, this are the special set of axes that we have taken for our Galilean transformation. Now, let us see that by checking these set of axes, we are ready to get that we can eliminate a large number of constants.

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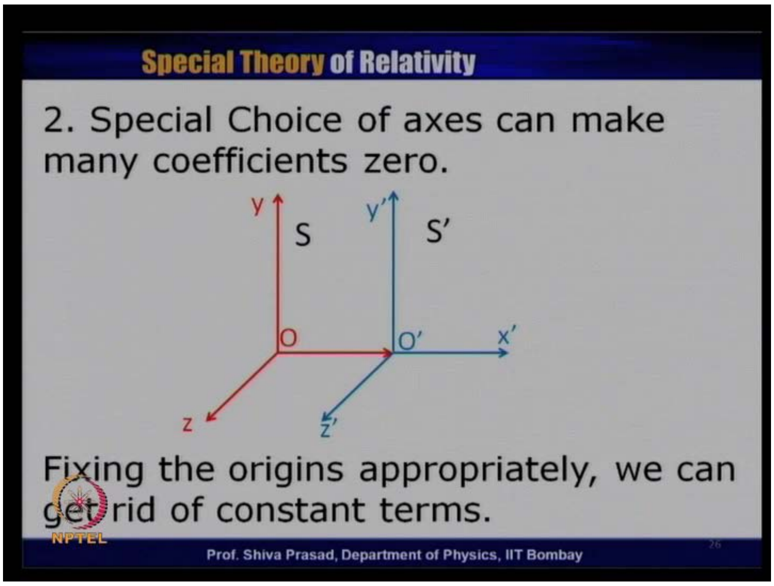

So, this what in have written special choice of axis make many coefficient 0, first making these particular origin choosing these origins appropriately, remember we have chosen these origins that O prime, when it coincides with O. Then and that particular movement the time is measured to be 0, so let us assume that at the time when the origins of the two frames were constant, and event occurred whatever might be the event, it occurred exactly at the origin.

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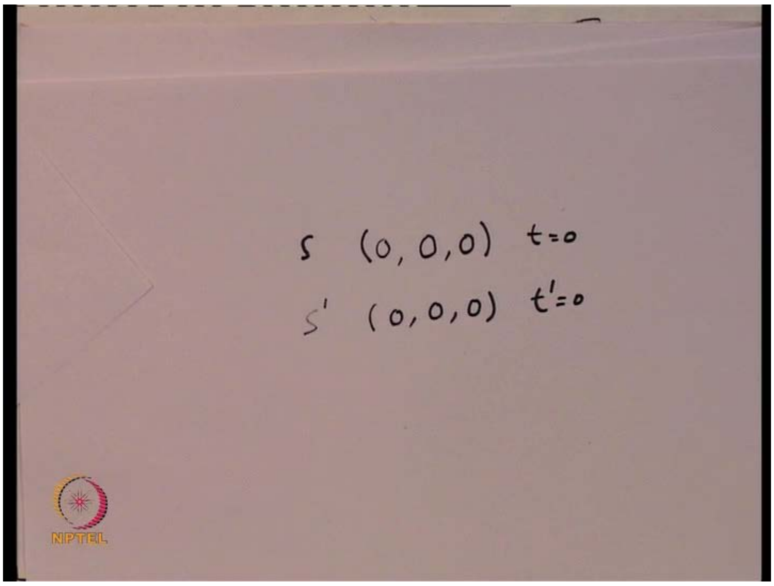

So, it means at that particular when the event occurred, the coordinate of that particular event in S frame was X is equal to 0, Y is equal to 0, Z is equal to 0, because it occurred at origin, and the time limit t is equal to 0. Because, this special choices that we have taken, we expect that this event in X prime of reference would also occurs at the origin, because it was constant at that particular time event, and because time was also measured at that particular instant of time; so time in the particular frame will also turn out to be 0.

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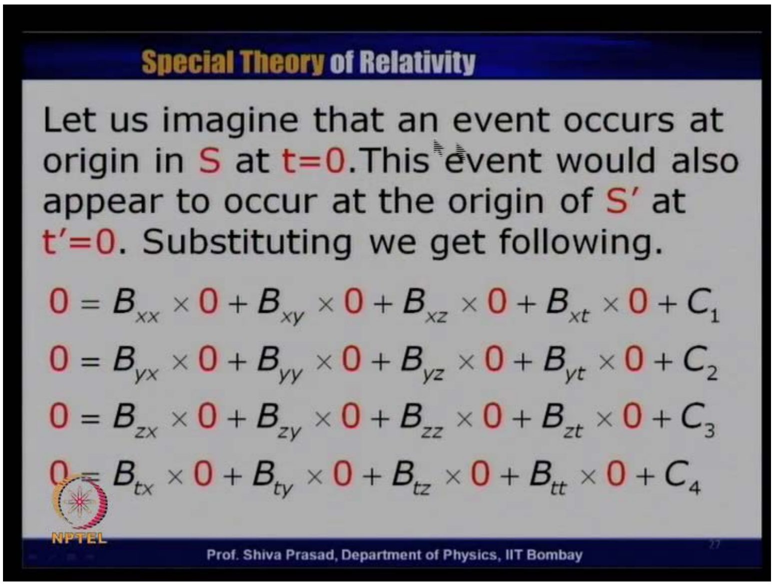

So, I expect that in S prime frame of reference also the event will turn out to be at 0, 0, 0 and at time t prime is equal to 0. Let us put these conditions in this particular transformation equation which we have just now written, and let us see that we can see get read of some constants.

So, I have written let us imagine that an event occurs at the origin in S at t is equal to 0, the event would also appear to occur at the origin of S prime at t prime is equal to 0; it mean if I substitute X is equal to 0, Y is equal to 0, Z is equal to 0, t is equal to 0, then I must X prime equal to 0. So, remember, if you look at this equation this will give you 0 is equal to 0 plus 0 plus 0 plus 0 plus C 1, which means that the C 1 must be 0 that cannot be a constant term. Similarly, we can put these conditions in the Y Z coordinate system also, Y Z transformation equation also, and we will conclude that C 2 is also equal to 0, and C 3 is also equal to 0.

Similarly, for time if the event occurred at the time at t is equal to I put it 0 here, occurs at Y is equal to 0 I put 0 here, occurs at Z is equal to 0 here, it occurred at t is equal to 0 I put it here; then the watch of S prime observer also measured time t prime is equal 0; so if I put this is equal to 0 I will get C 4 also to be 0. So, I conclude that by choosing these special set of axis, this special conditions on the two frames I am able to get read of the constant C 1, C 2, C 3, and C 4.

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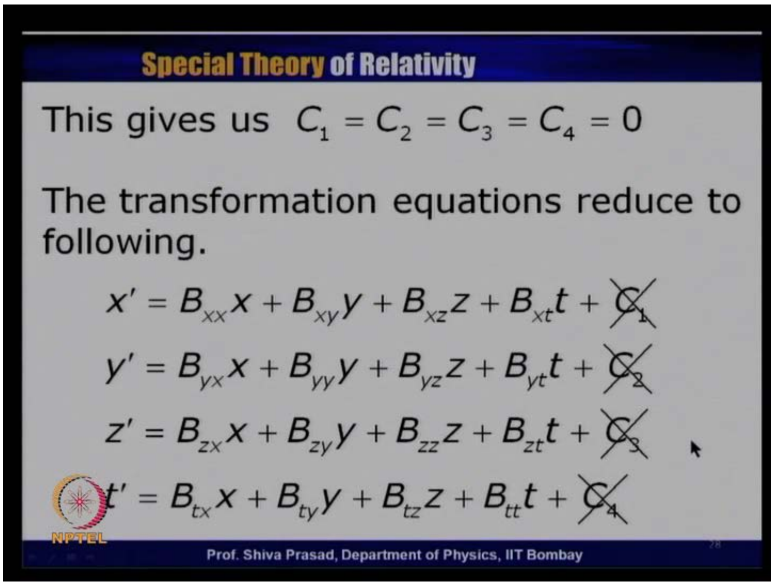

So, this is what I written here, and the transformation equation now reduced to this, so remember, so many constants out of these C 1 count down as become 0, C 2 has been removed is become 0, C 3 has been removed it has become 0, C 4 has been removed it has become 0. So, know we are still left with the 16 constant still very large number of the constant, and let us see if we get read of some other constants. Now, let us start looking again the same axis, and trying to see how this special tries to make that axis can help us of some more constants. So, we try to fix the place appropriately which we have already done by making this special choices which is given here.

Let us now imagine that an events occurred at a arbitrary time in X Y plane, so remember this is your X Y plane, this is your X direction this is your Y direction, so this is X Y plane. Now, we realize that X frame of reference, this X prime X axis is constant for X, Y prime axis is parallel to Y, so we expect that this X prime, Y prime plane is exactly same as X Y plane, we have the same planes. Because, this particular lines moves in that particular plane, the Y prime direction moves in this plane, so X Y plane is same as X prime Y prime plane.

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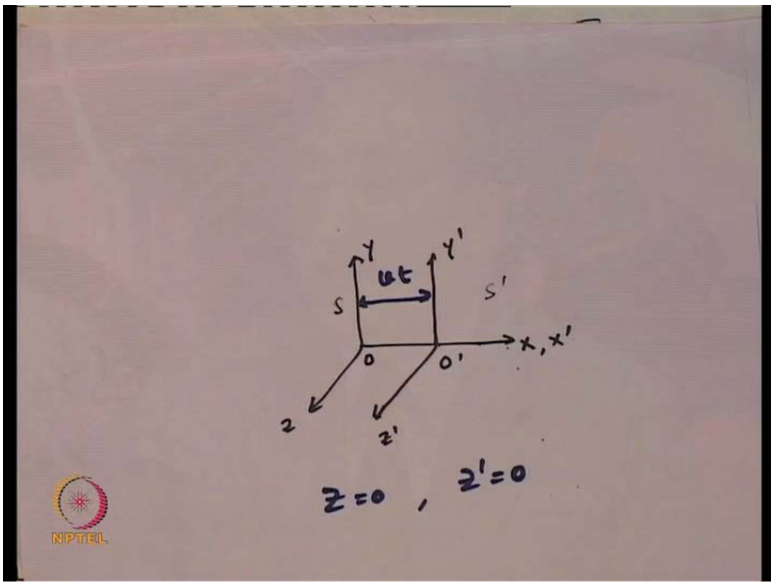

It means that, if an event occurs in X Y plane an observer in X frame of reference will also observe it to be is X prime and Y prime plane, what is X Y plane characteristic of, X Y plane is characteristic by the fact that Z is equal to 0, X Y plane means Z coordinate 0. Now, X prime Y prime plane also means Z prime equal to 0, so what we conclude from this particular discussion that if an event has occurred with Z equal to 0 means at the X Y plane, it would appear to an observer in the X prime also to occur at the X prime in the X X prime Y prime plane, it means according to that observer Z prime will also equal to 0.

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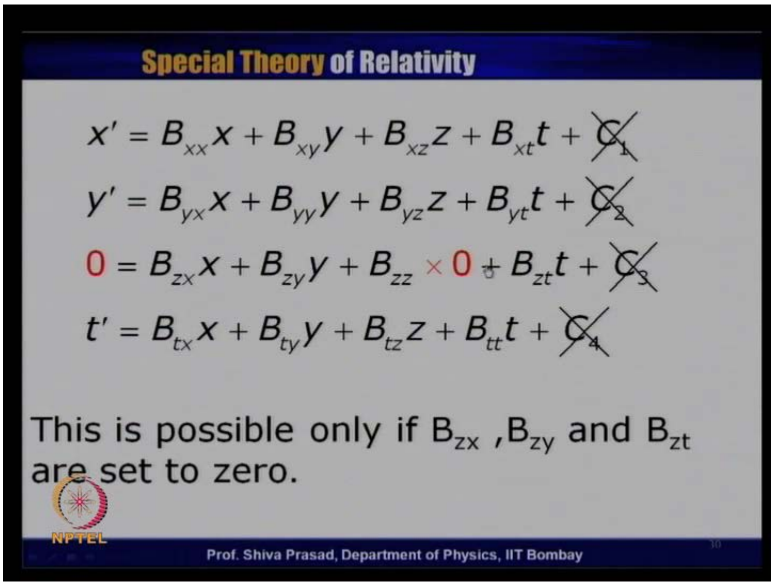

So, let us put this equation at the condition in our equations, and let see what happens, this is the third equation which I have put, I have put Z is equal to 0 here and I expect that if Z is equal to 0 I must also equal to Z prime equal to 0 or this is possible only remember this event occur at any arbitrary value if X, Y and t. But so long it occurs in with coordinate Z is equal to 0, the coordinate of the same event must also have Z prime axis equal to 0. So, irrespective of value of X, Y and t this equation must always be true that is only possible, if this coefficients is 0, this coefficient is 0, this coefficient is 0, this coefficient is 0, so one short we get read of these three constants.

This is what I have written, this is possible only if B Z X, B Z Y, B Z T are all set to 0, then only it is possible that an event which occurs when Z is equal to 0 will also appear to an observer in S prime to occur at the Z prime equal to 0. Similarly, we now look at the Z plane, so let us come back to this particular equation remember at Z plane, the X Z plane, you also realize that X Z plane is same as X prime Z plane, Z prime plane. So, if an event occurs in X Z plane implying Y equal to 0, then the event would also appear in S prime frame of reference occurred in X prime Z prime plane, it must occur with Y prime is equal to 0, exactly the same condition that we have applied for X Y plane, so we put exactly the same thing, the same condition.

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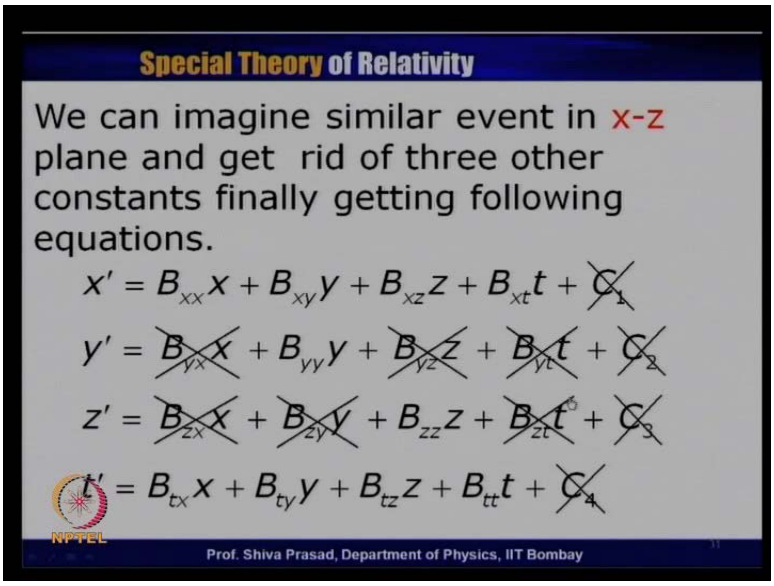

That corresponding to Y equal to 0, you must have Y prime is equal to 0 therefore, you get read off constant this constant, this constant, this exactly the same argument that we have used for X Y plane, so we get read of another three constants. Now, we remember there is one plane which is different and that is Y Z plane, this plane Y Z plane, and Y prime Y prime Z prime plane, these plane is constant only at t is equal to 0, at later time this plane goes there parallel, but they were moving relative to each other. So, it was different for X Z and X Y planes which remains always identical at the frames, but as per the Y Z plane is concerned they were identical exactly at t is equal to 0, but at later time they have moved each other. And how much they have moved, they have moved off by the distance v times t, were v is the relative speed between the frame.

So, an observer in the S frame would find that this particular Y prime Z prime plane has moved with respect to Y Z plane in a time t with a speed v, so the distance between them is v t. So, let me repeat an observer in S frame would find that the Y prime Z prime plane has moved with respect to Y Z plane by a distance of v t during time t, this time t is being measured in S frame, all the observations are being made in the S frame; let us see what does they put, this puts condition on our constants.

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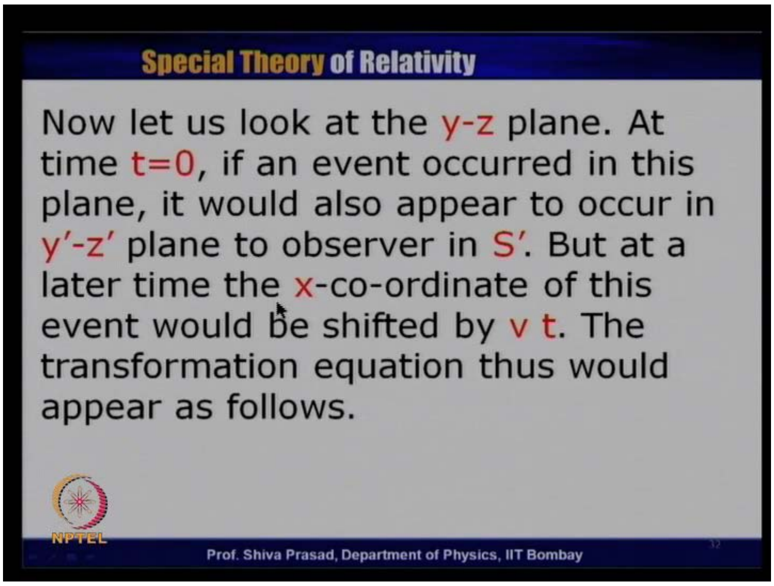

So, this is what we I have written, let us look at the Y Z plane at the time t is equal to 0, if an event occurred in this plane it would also occurred at the Y prime, Z prime plane observer in the S prime. But, at the later time the X coordinate of this particular event will be shifted by v t as observed in the S frame.

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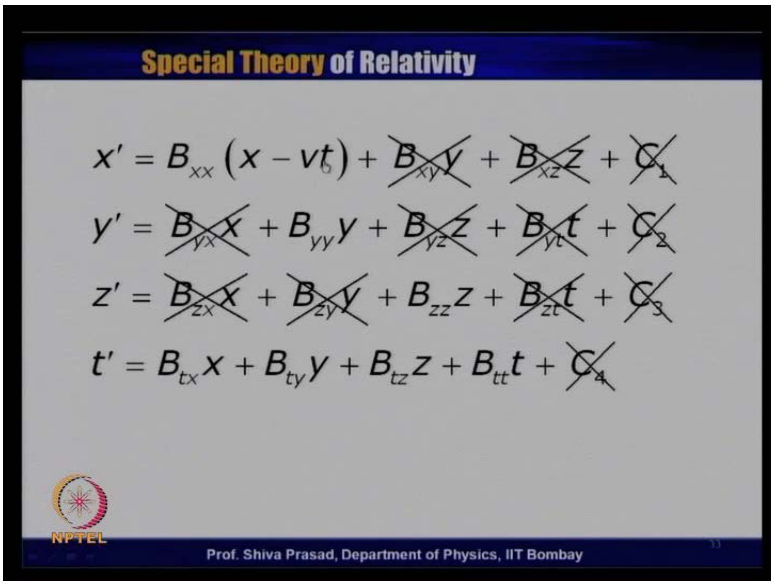

Therefore, the transformation equation must look as follows, taken as quite a bit of jump here which is X prime is equal to B X X into X minus v t, because if I put X is equal to v t then I must get X prime is equal to 0. Remember, this equation if it put X prime is equal to if we put X is equal to v t, then I must get a prime at this particular point Y prime Z prime plane and other constants must turn out to be 0. So, we can find this constants to be 0, we find this constant to be 0, and X prime we expect to be after for B X X multiplied by X minus v t. So, as we can see that this special choice of the axis have let to, let us into a large number into a large number of amount of simplification getting rid of, so many constants.

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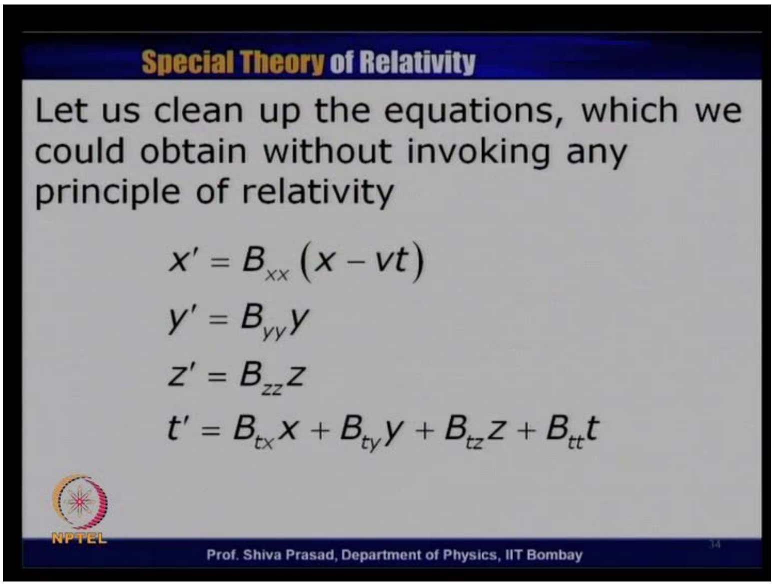

So let us clean up all our equations and we obtain the following equations X prime is equal to B X X into X minus v t, Y prime is equal to B Y Y times Y, Z prime is equal to B Z Z times Z, t prime is equal to B t X multiplied by X plus B t Y multiplied by Y plus B t Z multiplied by Z plus B t t multiplied by t; we still we have 4, 6, 7 constants. Let us see how we are going to evaluate this constants, but I hope you are able to appreciate that the special set of axis that we have chosen (( )) know that they are not really that the special in general enough. But just by choosing this set of axis we are able to simplify in large amount of we are able to simplify our transformation equations otherwise we do not know how to go about it.

Now, let us look at the symmetry arguments, the symmetry arguments are very interesting arguments which often can work out very well in physics. See when I am talking about the X axis, X axis is sort of the well defined axis, because X axis is the direction of the relative speed, see suppose you are just looking at the motion of two frames, and you have to define the relative velocity direction, you have to define the X direction how will proceed, will proceed by taking the direction of the relative motion is the X direction. But once we have chosen X direction, the Y direction whether I took this way, whether I took this way or whether I took another angle, it does not make a difference.

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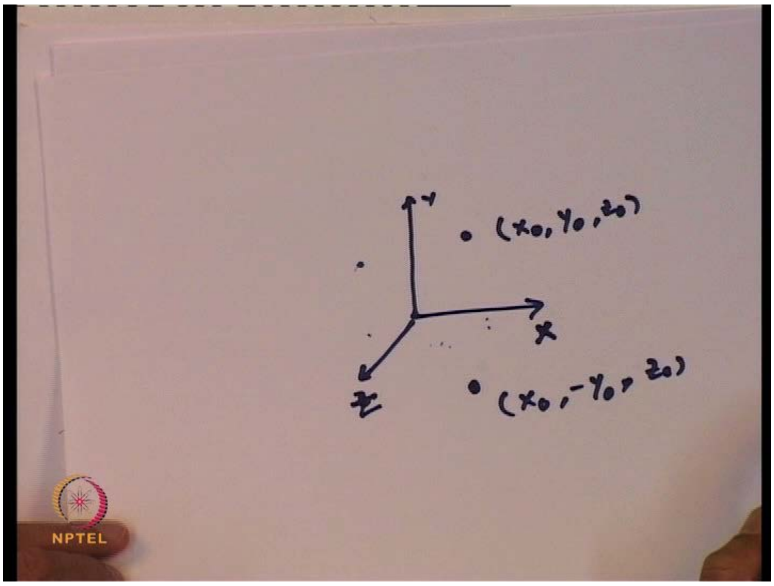

For example, if I take this my X axis, I could have taken this is my Y axis or I could take this is my Y axis making an angle. So, long it is perpendicular take this is as Y axis, Y axis was under my control there is no reason of specifying or choosing a particular specific Y value Y axis, X axis is definitely (( )) X axis is the direction along which the relative velocity occurs. So, this is the particular direction which is the unique direction in along, which sort of symmetry is broken, because that direction is the unique direction defines by the relative velocity directions.

But, the same thing (( )), I can choose Y axis anyway I like of course, perpendicular to X then between X and Y, Z axis has perpendicular to X axis and Y axis, and must follow the written rule, whatever is the standard rule for that transformation we were choosing the axis. But, on the other hand this Y axis could have been any were along this particular plane, so I do not expect my physics laws to change depending on what I choose this is my Y axis. So, there has to symmetric in that particular direction, so let us imagine now, let us imagine that we have chosen some particular Y axis correspondingly we have chosen Z axis, so my minus Y prime axis, minus Z prime axis all this axes are known.

Now, let us assume an event and X naught, Y naught, Z naught and X naught, minus Y naught, minus Z naught, this is your X direction, this is Y direction, this is a direction as I say X direction is a unique direction, which I cannot change, because that depends on the relative velocity. Y direction I could choose anything I have chosen one particular thing which is happening which is around in this particular direction, let us suppose anything that two events which are occurring here and exactly with the negative value Y. So, this occurs X naught, Y naught, Z naught, this occurs at X naught, minus Y naught, and Z naught all, so all that difference between the coordinates of these two events is the Y coordinate, otherwise all other two coordinates are same.

Now, looks back at our time equation which we have written earlier, which is t prime is equal to B t X X plus B t Y Y plus B t Z Z, now if I change Y to minus Y and B t Y is not 0, then t prime will turn out to be different for different frames; this Y change in sign of Y will change the time. But, of course nothing is special note this particular Y, what I have called is this particular Y not positive, I could have chosen my Y axis just an opposite to that, now let my X axis let to change, but this Y axis I now taken as negative of that.

And that event would have now appear to occur at minus Y naught, what event appear be occurring to plus Y naught, may have occurred at minus 1, because this just depends on us how I choose my axis. So, I do not expect that physics will change, because just I have taken different set of Y axis, so the time of event the other person do not know the choose of their axis, it is time note down to be different, depending upon what I have chosen is that X Y axis or what I have chosen my minus Y axis. So, I do not expect that by reversing the coordinate Y naught, I would expect the time to change similarly, if I assume two events to be occurring, at exactly the same value X naught, Y naught and only Z naught being different.

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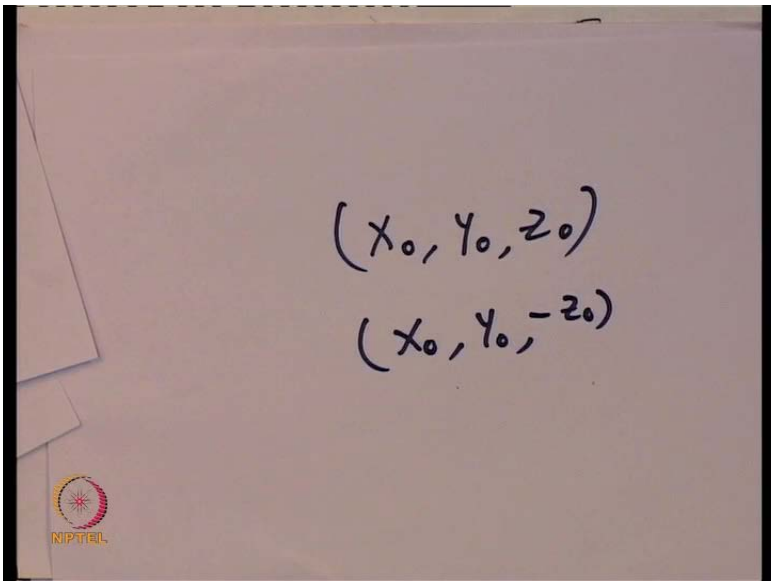

And occurring at X naught, Y naught, minus Z naught I do not expect the time to be different, because these are essential symmetry arguments, the coordinate that I have chosen have just depend upon one particular choice for which I do not have any prior information how to choose the particular, that was totally arbitrary. Therefore, just by taking the choice of the axis time of the event should not change, as we viewed in time frame of reference. Remember, I remain along the X direction is the unique direction, X direction things can change, because there is the special reason to choose the X direction, but not for Y and not for Z. So, I expect that these two coordinates will lead to constants B t Y and B t Z must be equal to 0, in order to maintain symmetry.

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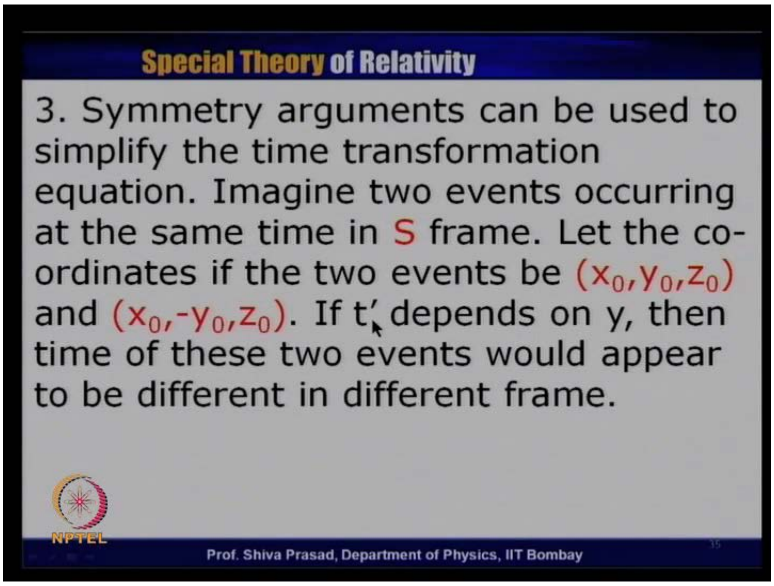

So, this is what I have written, imagine the two events occurring at the same time in S frame, let the coordinate of the two events be X naught, Y naught, Z naught, and X naught, minus Y naught, and Z naught. If t prime depends on Y, then time of these two events would appear to be different in different frame.

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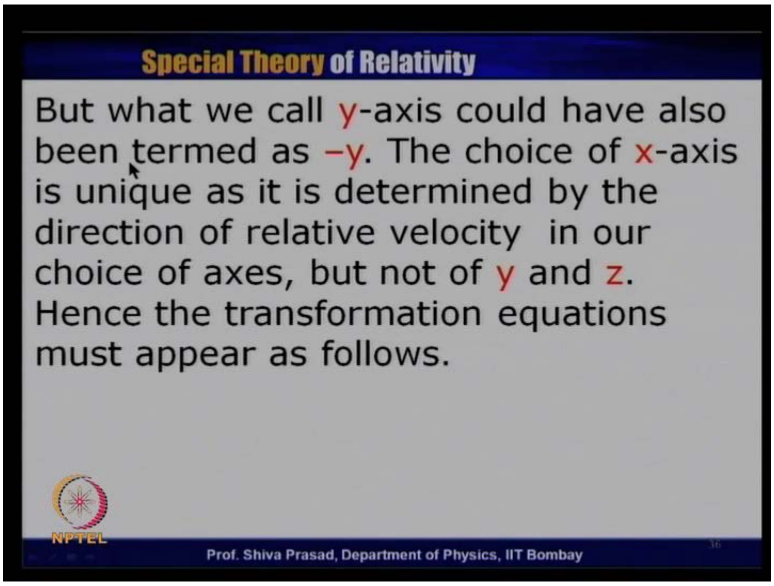

But, what we call Y axis could have also been termed as minus Y, the choice of X axis is unique, as it is determine by the direction of relative velocity in our choice of axis, but not Y and Z hence, the transformation equation must appear as follows.

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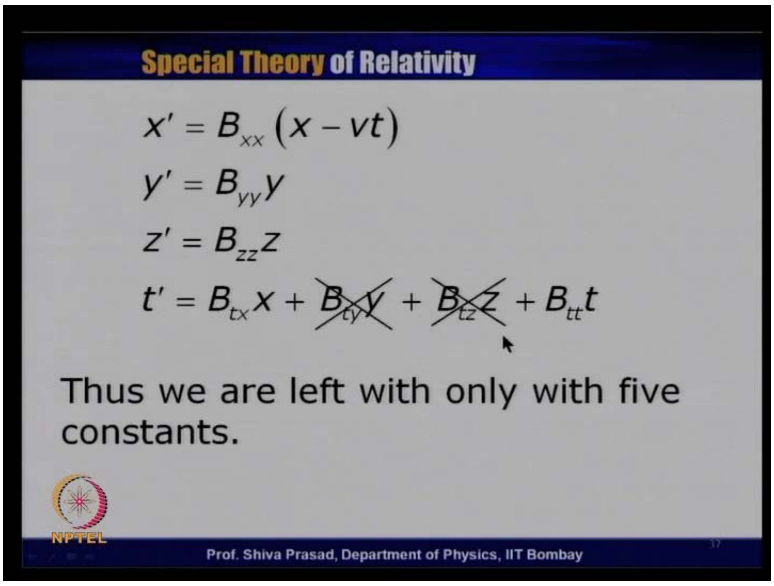

It means, I must have these two terms also becoming 0, therefore I get read of two future constants, so now we are left with only 1, 2, 3, 4, and 5 constants.

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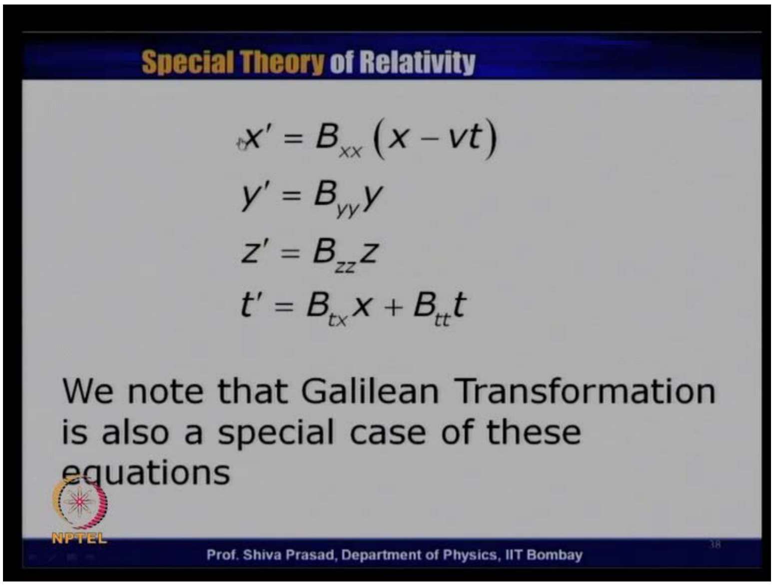

This is what is I am left with, as we can see that simplified reasonable well, purely by physics arguments, and special set of axis transformation equation is simple for. I would just like to remain you that Galilean transformation is also a special case of this equation, remember in Galilean transformation B X X was 1, B Y Y was 1, B Z Z was 1, B t X was 0, B t t was 1. So, Galilean transformation also a special case of this remember, till now we invoke any special or any postulate is the special theory of relativity, this transformation equation whatever we have discussed are general enough, so long we take the set of axis that we chosen. So, this expected to generally to way that the classical rule or under special theory of relativity, we expect the same equations to become.

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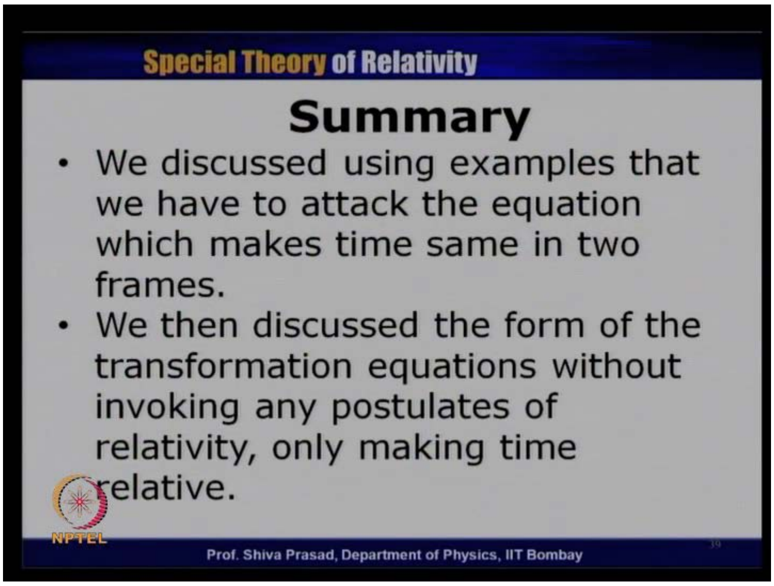

So, let go to the summary of whatever we have discussed today, we discussed using examples that we have to attack the equation, which makes time same in two frames. We then discuss the form of transformation equations without invoking any postulates of relativity only making time relative, only we have taken consider the fact that this time could also be related, we have not invoke any things of special theory of relativity. See in our next lecture, we will invoke now the conditions of special theory of relativity, and we will determine all the constants that we have not so for determine, and we will arrive at what we call today Lorentz transformation.