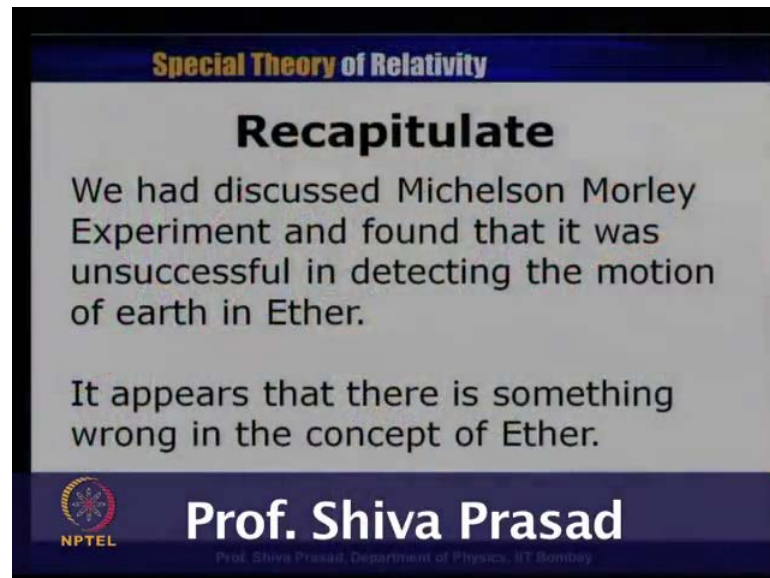


Special Theory of Relativity
Prof. Shiva Prasad
Department of physics
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Lecture - 3
Postulates of Special Theory of Relativity
and Galilean Transformation

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Special Theory of Relativity

Recapitulate

We had discussed Michelson Morley Experiment and found that it was unsuccessful in detecting the motion of earth in Ether.

It appears that there is something wrong in the concept of Ether.

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In our last lecture, we had discussed in detail the Michelson Morley experiment. Michelson Morley experiment was one experiment which was designed to see, if there something like ether in which earth is actually moving the motion of earth in ether was considered to be essential. If we can believe in the classical postulate that there is a special inertial frame of reference or there is something, which we can call as an absolute rest.

When we describe the experiment, we realize that the outcome of there is experiment was negative and it did not show did not seem to show any effect that there is a ether or there is an absolute frame of reference. We also short of describe their some experiments which we are also done with totally different nature in mind looking specially at the electromagnetic theory and they were also unsuccessful. So, it appears that there something wrong with the concept of ether which, we had sort of discussed in our first lecture without going much into detail.

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Special Theory of Relativity

Postulates of Special Theory of Relativity

1. Laws of Physics are same in all inertial frames of references. No preferred inertial frame exists.
2. The speed of light ' c ' is same in all inertial frames.

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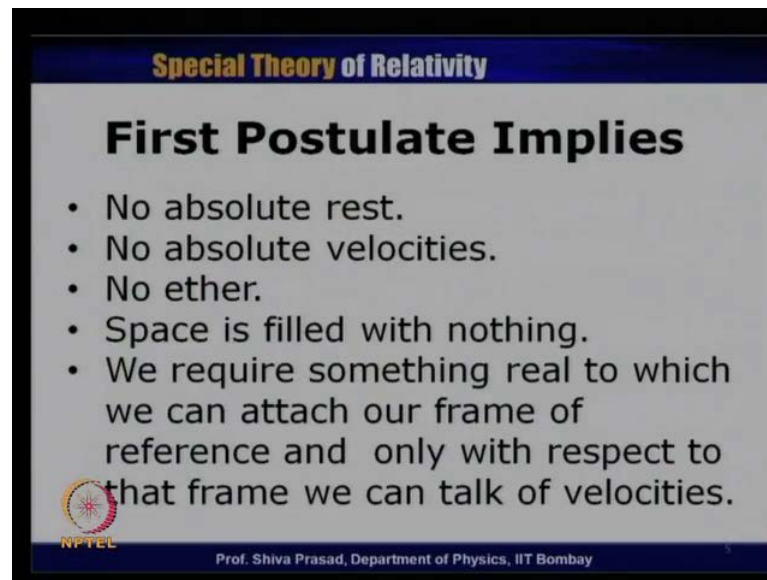
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Now, I come to the Einstein's postulates of special theory of relativity, it is not very sure whether Einstein was aware of the result of the Michelson Morley experiment. He was totally unorthodox thinker, he had far belief on what his perception or his perspective of nature is. Therefore, he made two postulate of special theory of relativity which we will describe, now the first postulate is that laws of physics are same in all the inertial frames of reference more preferential inertial frames exists.

It means there is nothing like ether, there is no absolute rest, there is more inertial frames of reference, all inertial frames of reference are equivalent and law of physics can be applied in one inertial frames as well as any other inertial frames of reference. Now, we have discuss that if we believe Rudely in the classical theory and purely intelligent in transformation. Then, this will laid has to a value which is frames depended and because sea was related to the fundamental constant we have said that this create the problem.

Now, in all to make all the inertial frames of reference equivalent appear display the flight appear to be way of distinguish, but distinguish between different inertial frames. So, here comes the second postulate of Einstein's that the speed of light is same in all inertial frames which is the very bold statement saying that irrespective the frame in which you are, you will always measure the same speed. It is not c minus u or c minus v or whatever it is.

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Special Theory of Relativity

First Postulate Implies

- No absolute rest.
- No absolute velocities.
- No ether.
- Space is filled with nothing.
- We require something real to which we can attach our frame of reference and only with respect to that frame we can talk of velocities.

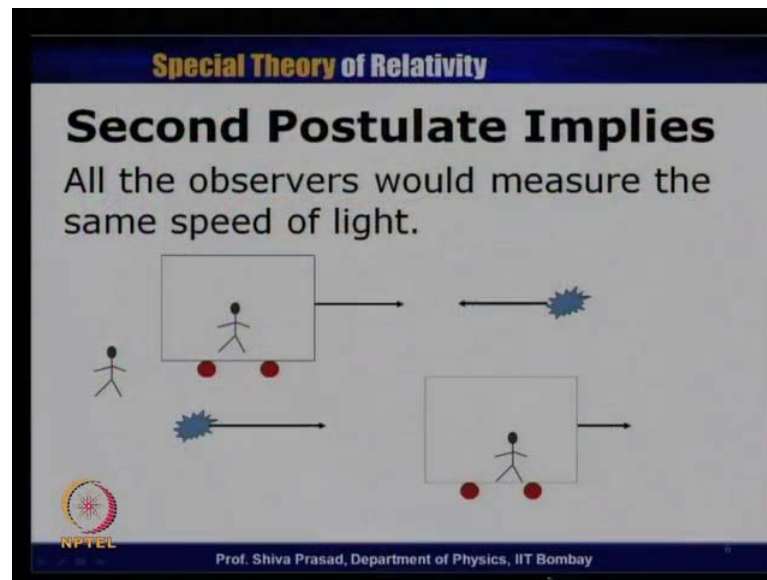
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What does the first postulate implies? First postulate implies that there is no absolute rest, no absolute velocities, we can never give absolute, we can never assign a absolute velocities to any object. This is nothing like ether space is not with you for anything, we required something real to which we can attach our self, and with reference to that frame only we can talk about speeds may be it is dust of particle may be whatever it is.

There has to be something real on which we can elect our frame of reference, we can elect axis and only with respect to that pollute of frame of reference. I can talk velocities, I cannot talk of an absolute velocities, I cannot ask a question, what is the speed of sun, what is the speed of earth? Without specifying which frame of reference, I want to measure this speed, so only speed have meaning when I describe in terms of the frames of reference, this is what the first postulate Einstein's will be.

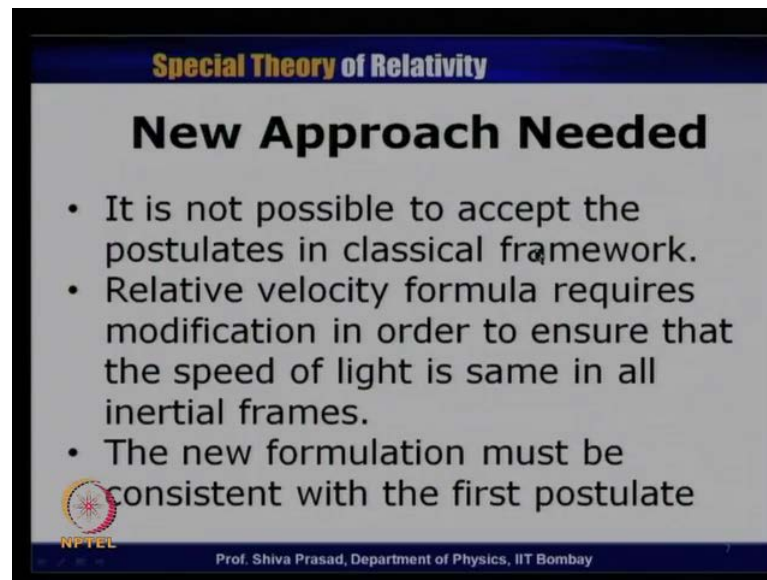
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Second postulate could mean that irrespective of whether by observer here, we had said that, suppose this is a source of light and this is stationary in the frame of reference of this particular observer he measure the speed of light. There is one particular train compartment which is moving towards of the light this particular person would also measure the same speed c . There is one particular person which in this compartment which is moving in the same direction in the speed of light fuel, also measures the speed of light to be seen.

So, there is no c minus u , there is no c minus c plus u , all this three observers will measure the same speed of light obviously it tells the normal velocities addition formula or relativity velocity formula. Therefore, if we believe in the postulate of Einstein or postulate special theory of relativity we much change our ideas we much change our velocity addition formula. So, we required totally new approach very essential, now in dark we have to really look how to go head and try to solve this problem.

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Special Theory of Relativity

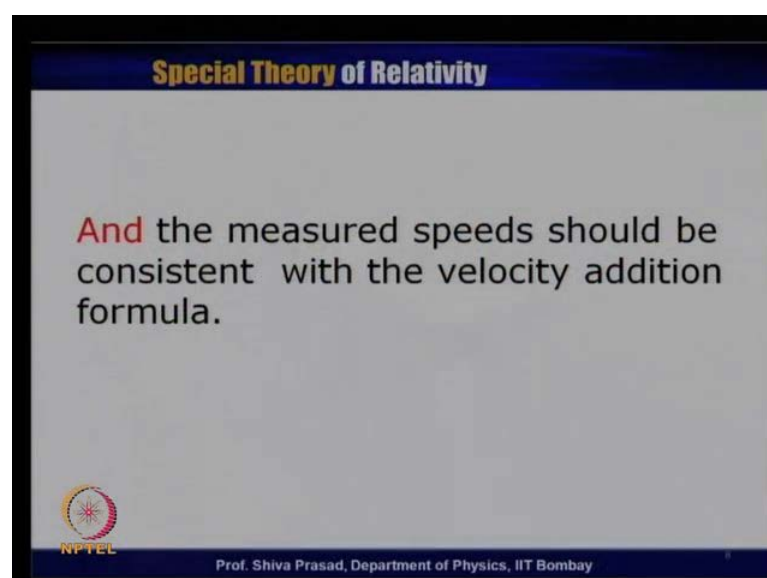
New Approach Needed

- It is not possible to accept the postulates in classical framework.
- Relative velocity formula requires modification in order to ensure that the speed of light is same in all inertial frames.
- The new formulation must be consistent with the first postulate

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We have just now said that not possible to accept the postulates of special theory of relativity in classical framework because that could give a speed of light that will make speed of light frame independent. Relative velocity formula required modification in order that the show that the speed of light is same in all the frames. So, we much change our relative velocity formula and in addition to that whatever did light as the new velocity formula that much be consistent. First postulate means that much may all the inertial frames of reference equivalent if we believe in whatever we are saying about special theory of relativity.

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Special Theory of Relativity

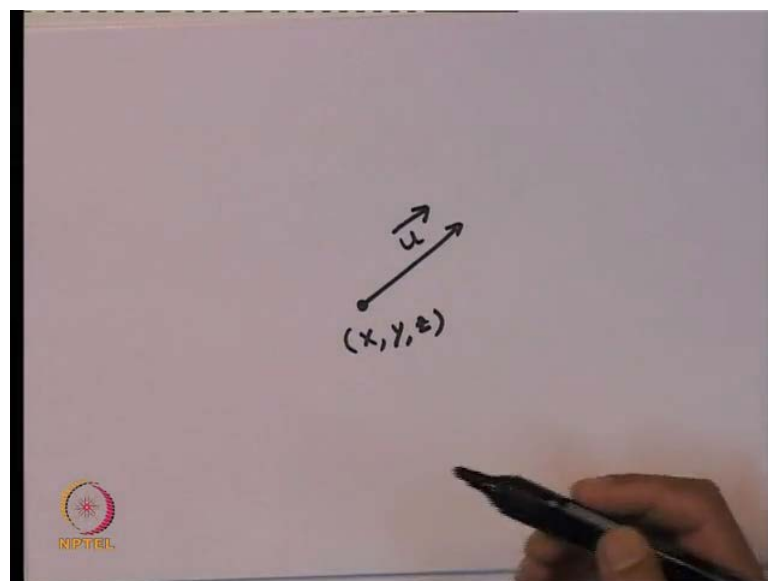
And the measured speeds should be consistent with the velocity addition formula.

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I would like to add that the measure speed must be consistent with velocity addition formula because eventually the test of any theory module lies in where it can explain experiments. So, whatever we are saying must be experimentally verifiable, then only to believe in whatever we are saying as a postulate. Now, let me introduce a concept which is probably very simple concept and initially I will discuss a concept which is purely in the classical way. Probably, everyone should be aware of this particular concept, only thing because I am going to use some formal terms.

Therefore, I am describing this thing a little bit more detail than probably necessary, but I think it is essential to get our ideas, I introduce the concept of what we called transformation. Transformation is a very simple thing and I will give an example what actually transformation is, it effectively means that suppose there is the particle which is moving.

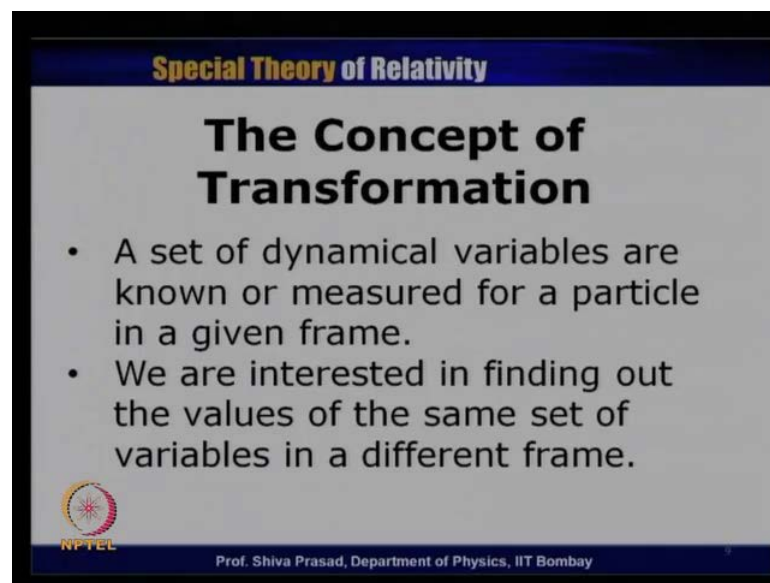
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Let us imagine that this particle is moving in this particular direction, there is one particular coordinate to be different. Now, if there is an equation or there is a set of equations which are related to these coordinates and these velocities as observers in one frame that will be an observer in any other frame. These equations, these set of equations are called transformations, so there can be a coordinate transformation where I know the coordinate, the particular particle.

This coordinate of this same particle in different frame of reference can be obtained by using this transformation equation or it could be a velocity transformation where I know the components velocity u_x u_y use it. Then, I want to find out what are the components of the velocity as observed in different frames of reference specify. So, this is what is essentially called transformation as I said it a very simple concept, only thing the name now we start putting a name to formalise it.

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Special Theory of Relativity

The Concept of Transformation

- A set of dynamical variables are known or measured for a particle in a given frame.
- We are interested in finding out the values of the same set of variables in a different frame.

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So, this is what I have written here, the concept of transformation is a set of dynamical variables are known or are measured. A particle in a given frame like for example, in this case we talk about the coordinates, we talk about the velocity. We are interested in finding about the values of the same set of variables, it means if you coordinates we are interested in finding out the values of the coordinate. If these are the velocity components, then we are interested in finding out the velocity components in the different frame.

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Special Theory of Relativity

- The equations that we shall use are called transformation.
- **Example:** Co-ordinates or velocity transformations.
- We shall first formalize classical transformation.

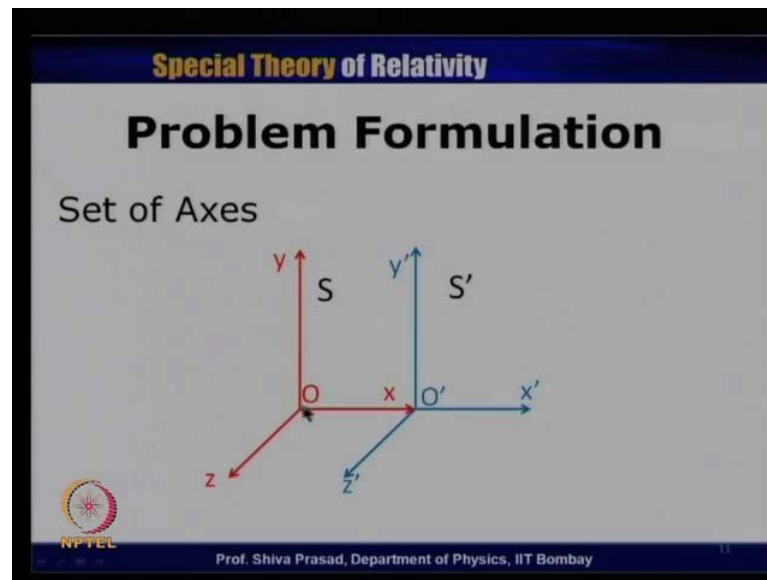
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The equation that we change these variables for one frame to another frame, these are called transformation and I had just given an example this example is could be a coordinate transformation or a velocity transformation. So, lets us first start with the simplest transformation which is a classical transformation which we call as a Galilean transformation before we talk about the particular thing lets define our problem.

We have seen the special theory theory of relativity, it can be complex in terms of equations and we do not want to start with most difficult problem right in the beginning. So, without losing generality, we can always define a set of axes which are convenient, so that eventually equation that we are going to deal are going to be simple, so we assume that we have two set of frames S and S' .

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This is what I have written here, one is the red frame which is called s , it has x axes, y axes and z axes. To remember in physics, we are free to choose the set of axes that we want to describe physics is not going to change if I chose a different set of axes. There is another frame of reference here which I am putting blue in colour which I am calling s prime for its reference which has also x prime, y prime and z prime axes. Now, I make certain conditions on this particular set of axes as I said to make our simple as for the future equation.

So, if I do the rate of velocity direction between this two frames, I decide to choose my x axes along that particular direction. It may mean I assume that x prime frame of reference is moving in such a way that its origin is always along the x axes of s frame. So, it always moves along this particular line, so x and x prime are always coincident with each other and origin is moving just along the x axes. Similarly, we can of course say that the origin O is moving along the minus x direction of s prime frame of reference because if there is an observer in a prime frame of reference.

He would feel as if this particular frame of reference is going behind him because observer's setting is his own frame does not see the motion of its own frame like when we are sitting on earth. We do not notice the motion of earth we feel the earth is at rest and we accordingly change our idea of direction of motions of let say sun or moon. Similarly, if a person was

sitting on a prime, he would notice that this o is going behind him along minus x prime directions.

So, this is the direction of the relative velocity between the frame, I assume that y prime axes is always parallel to y axes, I assume that z prime axes are always parallel to z axes. So, this is the special set of axes that we have chosen to describe special theory of relativity it actually does not lose generality, but because we can always we can always know the relative velocity direction. Along that particular direction I can choose my x axes as I have said physics is not going to change if it chose a different set of axes. So, this is a special set of axes which we have chosen so this is what I am describing in my next transparency.

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Special Theory of Relativity

Conditions

- Frame S' moves with respect to S along x -axis with a velocity $\vec{v}$.
- Clock of each observer is set to zero when their origins coincide.

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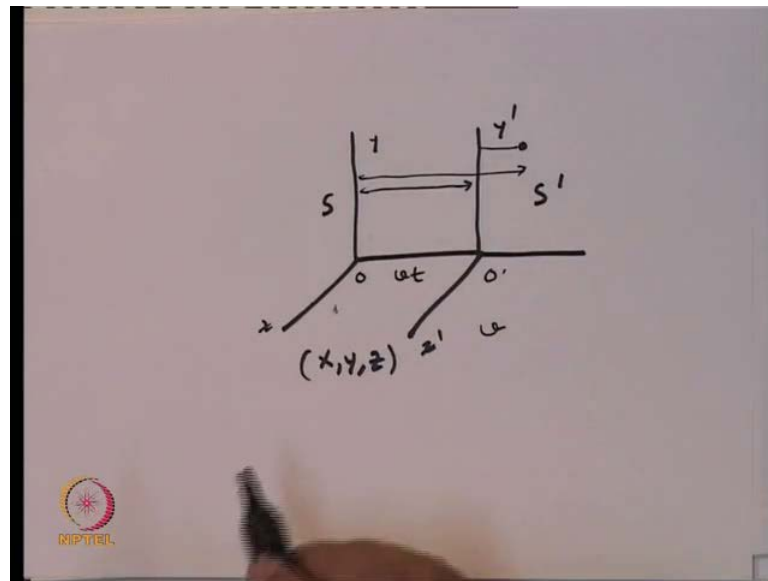
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Then, these are the conditions that I have put on the two frames the frame S' moves with respect to S along x axes with a velocity v . Let me just say that velocity symbol v is reserved for their relative velocity between the frames then the additional condition we are putting that clock. There is relative motion and O' is moving along the x axes of O , there will be some time when O and O' will be coincident. Let me go back to my old transparency, here look of there is relative motion there will be a time when O' will be always moving will always be coincident, sorry.

When O and O' will always be coincident we assume that that is a time when the watches of S and S' both are set equal to 0. So, at that time S also makes the time t

prime equal to 0 as time observer also make time t prime equal to 0 in history. Let me also relocating one of the things because the frame of reference that I am going to talk is initial, so this velocity v that I am noticing here and writing here is constant. It does not change because if we was changing function of time then at least one of the frames would be non induction, so it means essentially that we has to be constant because that two frames s and s prime that I am talking are both inertial.

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Now, let me come to Galilean transformation has I say it completely simple transformation and purely classical transformation. So, this was my s frame this was my prime frame let us assume that we are talking of one particular here and the observer. Here, s measures a coordinate of this particular particle in his frame and finds out to be x, y, z , my question is that what to be the coordinate of the same particle s seen in s prime frame of reference. So, observer is sitting in s prime what will be the coordinate of this particular particle as measured by him as we can see because this was your y axes this was your z axes because y and y prime axes are always parallel to each other.

So, this particular component y component will always be seen this y component will always be same where s you are talking about s prime frame of reference. Similarly, z component will also be same so that coordinate y and z will be same as measured in s and s prime. Its x coordinates which will be different because according to the observer s

prime he would measure this has x coordinate while according to observer in s the x coordinate will be this 2 x coordinate will be disfiguring by this much amount.

This is essential, the separation of origins between o and o prime, so this coordinate x has measured in s will be larger from what was measured by s prime by this much amount. This amount this distance o and o prime would depend upon how much time passes because that relative velocity is v. So, if this would can measurement was done at time t then this particular o origin would have displaced by a distance v t remember time t was 0 when the two origin were coincident, so this distance will be v t.

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Special Theory of Relativity

Galilean Transformation

Direct Transformation

$$x' = x - vt, \quad y' = y, \quad z' = z$$

$$t' = t$$

Inverse Transformation

$$x = x' + vt', \quad y = y', \quad z = z'$$

$$t = t'$$

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Hence, I accept that x prime would be equal to x minus v t y prime would be equal to y and z prime will be equal to z this is what is called the Galilean transformation, I will take minute of time to explain this particular thing. We have also written clearly in this equation that t prime is equal to t time is something which normally we have not been talking in classical mechanics. We have just been in sided we just sort of ignore this assume that there is uniform flow of time in all the frame of references so was they two observers in s and s prime have measured there time.

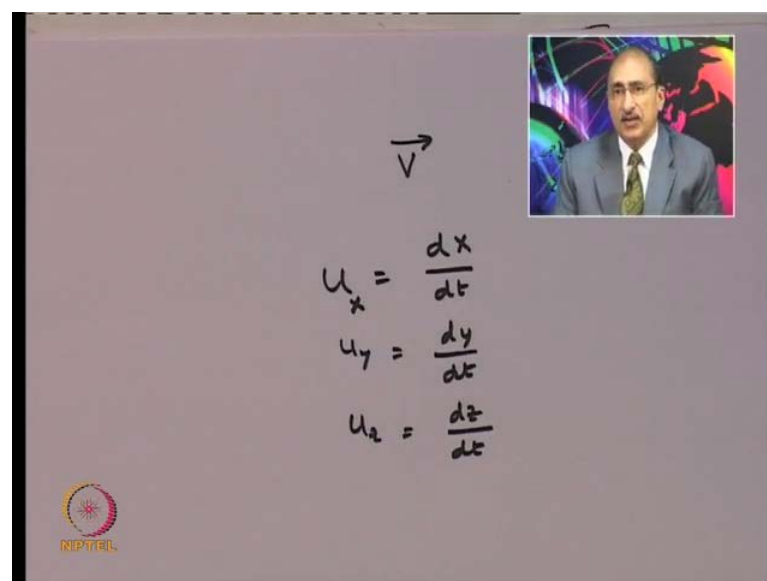
Wherever a measurement of the coordinates of the particles is done, whatever was time in s watch in s watch same will be the time s times watch. So, we assume that t and t prime is equal to same even though the classical mechanics we have not discussed it, we

have never talks specifically about the time, but it is a sort of implicity summation that the two observers was there shrunk the about this.

Then, there times will always be same were ever we make a measurement or whenever the event occurs so we assume specifically that t prime is always equal to t . Let me call a transformation as inverse transformation, actually inverse transformation is nothing but same as direct transformation except that we are changing the frame for x to x prime. It means information is available or the coordinate are available as prime frame of reference and I won't to find out the coordinates as frame of reference. If it is I know what x prime y prime z prime and I want to find out what is x finds z if you just inversing because they has to be some sort of cemetery.

So, essential mean that all your have do is to change v to minus v because the equation have to be same, so what we called as inverse transformation if I know x prime. Then, I find out x as x prime plus v t prime y is equal to y prime as we have discussed z is equal z prime and off course assume it implicitly that t is equal to t prime the two time a same. So, this is the Galilean transformation which involves the transformation of the coordinates of a particle. Now, let us look at the velocity transformation as we know where if form the standard traditional classical mechanics.

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$$\vec{V}$$

$$u_x = \frac{dx}{dt}$$

$$u_y = \frac{dy}{dt}$$

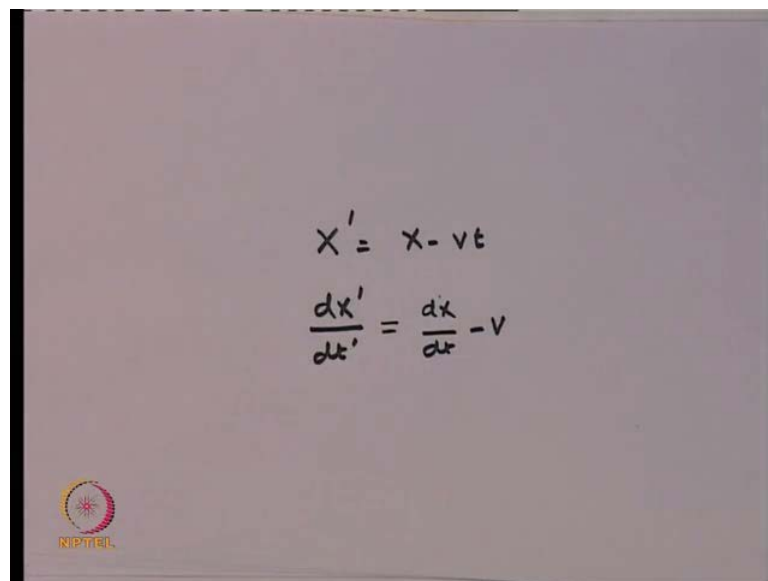
$$u_z = \frac{dz}{dt}$$

The speed of the particle is given as dx/dt i much write u_x because I am differencing only x , similarly u_y will be given as dy/dt u_z will be given as dz/dt . Let me just spend

couple of minutes on my notations, I am using the symbol u here to define the particle velocity. Earlier, remember we had used the symbol v was a relative velocity between the frames the symbol u have reserved or it is traditional reserved for that particle velocity. Remember the particle need not go moving with constant velocity because particle could be a only v is just constant of function of time.

This is a relative velocity between the frames, but would not be observers s and s' could be observing what some other particle and could be a particle could be executing could be stationary could be moving there any velocity. So, what we are talking is the instantaneous velocity of the given particles so the instantaneous velocity of the given particle is given in terms of the derivatives $\frac{dx}{dt}$ $\frac{dy}{dt}$ $\frac{dz}{dt}$. Using Galilean transformation which we have just used, we say that x' is equal to x minus v t .

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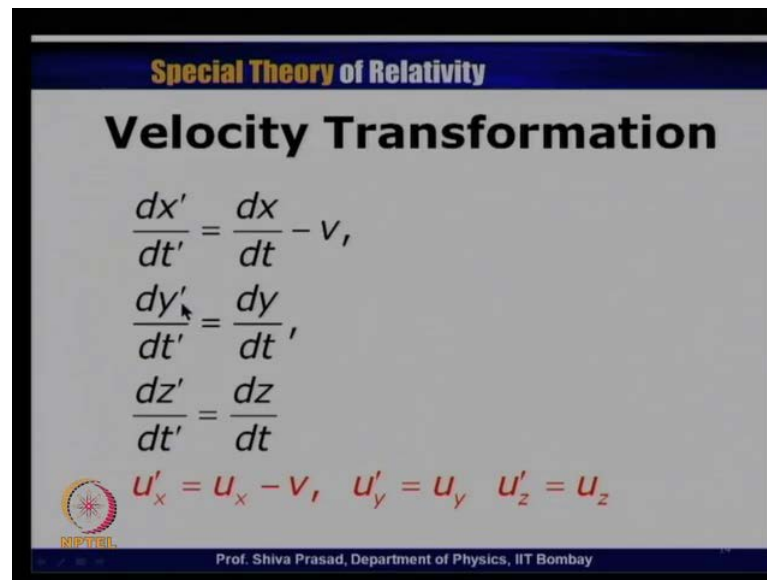


$$x' = x - vt$$

$$\frac{dx'}{dt'} = \frac{dx}{dt} - v$$

We can were easily write we had said x' is equal to x minus v t in we differentiate with respect to time the speed has measured in prime frame of reference could be given by $\frac{dx'}{dt'}$ divided x by $\frac{dx}{dt}$. Remember, I must differentiate with the respect to the time measured in the observer home frame of reference, but at this movement we had said t is equal to t' , so we need not bother about it we depreciate this with time I get $\frac{dx}{dt}$ minus v .

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The slide displays the following equations for velocity transformation:

$$\frac{dx'}{dt'} = \frac{dx}{dt} - v,$$
$$\frac{dy'}{dt'} = \frac{dy}{dt},$$
$$\frac{dz'}{dt'} = \frac{dz}{dt}$$

Below these, the velocity components are given as:

$$u'_x = u_x - v, \quad u'_y = u_y, \quad u'_z = u_z$$

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This is what we have written this particular transform velocity $\frac{dx'}{dt'}$ equal to $\frac{dx}{dt}$ minus v . Similarly, because in Galilean transformation y' was equal to y there for $\frac{dy'}{dt'}$ is equal to $\frac{dy}{dt}$, similarly $\frac{dz'}{dt'}$ is equal to $\frac{dz}{dt}$ because this is the velocity x component of the velocity measured in S' frame of references. This is the x component of the velocity is measured in S frame of references I can write the equation as u'_x is equals to u_x minus v .

Similarly, this equation can be written as u'_y is equal to u_y , this equation can be written as u'_z is equal to u_z . So, this gives me my velocity transformation equation which means that if I know u_x u_y u_z , I know how to find out u'_x u'_y u'_z actually this transformation is as simple as the relative velocity standard relative velocity.

The relative velocity has you know is given by this same expression, so velocity transformation is same as the relative velocity expression which with which we are quite used to it similarly, I can write an inverse velocity transformation. It means if the velocity components are known in S' frame of reference, how do I find out the velocity components in S frame of references and as we have discussed the prescription is very easy just change v to minus v .

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Special Theory of Relativity

Inverse Velocity Transformation

$$u_x = u'_x + v, \quad u_y = u'_y, \quad u_z = u'_z$$

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So, it becomes u_x prime plus v u_y becomes equal to u_y prime u_z becomes equal to u_z prime, this is what is called inverse velocity transformation.

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Special Theory of Relativity

Example 1

A particle moving with constant velocity

Diagram showing two frames S and S' with axes x, y, z and x', y', z'. A particle is shown moving with velocity $\vec{u}$ in frame S'.

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Now, let us take few examples of Galilean transformation to illustrate my point specifically to initiate little bit of discussion on the concept of t is equal to t prime. So, let us assume that this one particular particle which is moving with a speed u of with a velocity u which is going in this particular direction. Some arbitrary direction which is being observed by a frame S because I have not put a prime u on u . It means this is the

velocity s has been observed in s frame of references, now my question about this particular problem is given in the next transparency.

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Special Theory of Relativity

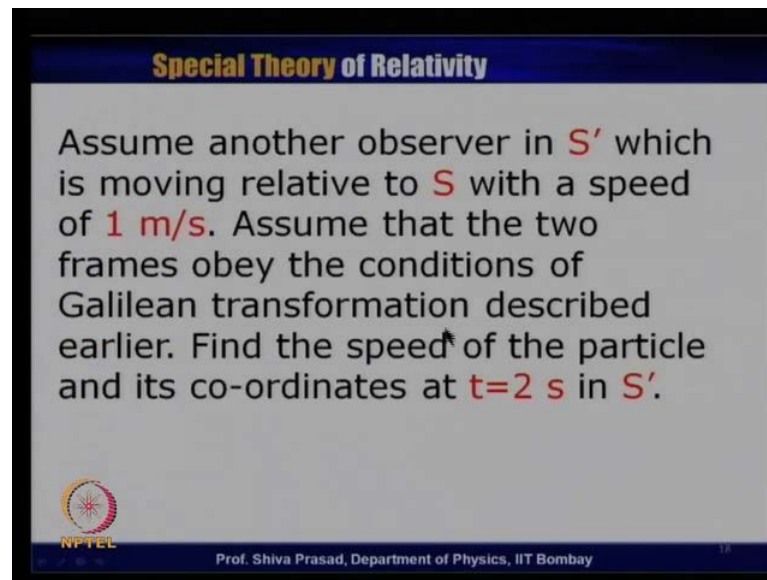
An observer in frame S locates a particle at origin at $t=0$, which is moving with a constant velocity of 5 m/s making an angle of $\tan^{-1}\left(\frac{3}{4}\right)$ with x -axis. Find the position of the particle in S at $t=2 \text{ s}$.

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This says that an observer in frame s locates a particle at origin at t is equal to 0, so at t is equal to 0. This particular observer feels that this particular particle was there at origin and this is moving with a constant velocity 5 meters per second making an angle of $\tan$ inverse 3 by 4 with x axis. I have taken some arbitrary velocity, I have taken some arbitrary angle just to illustrate my point. So, this particular particle which was moving here its speed is measured by observer s and is found to be 5 meter per second.

The angle that this makes this speed makes with s is given by $\tan$ inverse 3 by 4, this is what I have just assumed. Question is that find the position of the particle in s at t is equal to 2 seconds a very simple problem a typical standard classical mechanism problem what is the position of the particle in s frame of reference at t is equal to 2 seconds.

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Special Theory of Relativity

Assume another observer in S' which is moving relative to S with a speed of 1 m/s . Assume that the two frames obey the conditions of Galilean transformation described earlier. Find the speed of the particle and its co-ordinates at $t=2 \text{ s}$ in S' .

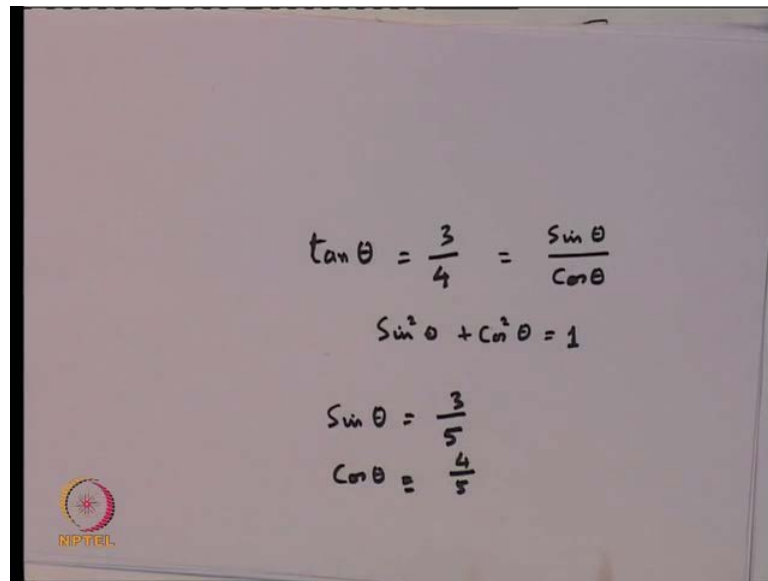
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Then, you assume that there is another frame s prime the one which I have drawn as a blue lines in my earlier figure. This frame I assume that this observe this frame s prime is moving with a speed of 1 meter per second, it means some arbitrary value I have taken assume that the two frames obey the condition of Galilean transformation described earlier. It means I assume that y axes is parallel to y prime axes z axes is parallel to z prime axes relative motion is only along the x direction and the origin of s prime always moves along the x axes of the origin of o of s . The two times are measured when the two frames the two origins were consulate so these were the conditions under which we have derived Galilean transformation.

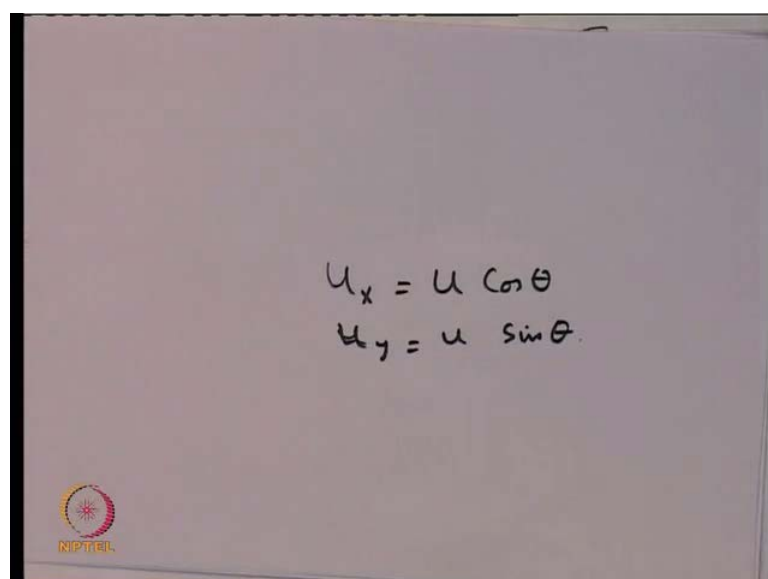
So, we assume that these conditions are also true for these two frames s and s prime, now the question is that what is the speed of the particle and its coordinate at t is equal to 2 seconds also in s prime frame of reference. So, we have to not only find the coordinates in s frame we have also to find out the coordinates in s prime frame. So, this is what is my question first we find out the coordinates of this particular particle in s frame which is very simple because I know the velocity, so I must be able to find out what is x component of the velocity and y component of the velocity.

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$$\tan \theta = \frac{3}{4} = \frac{\sin \theta}{\cos \theta}$$
$$\sin^2 \theta + \cos^2 \theta = 1$$
$$\sin \theta = \frac{3}{5}$$
$$\cos \theta = \frac{4}{5}$$

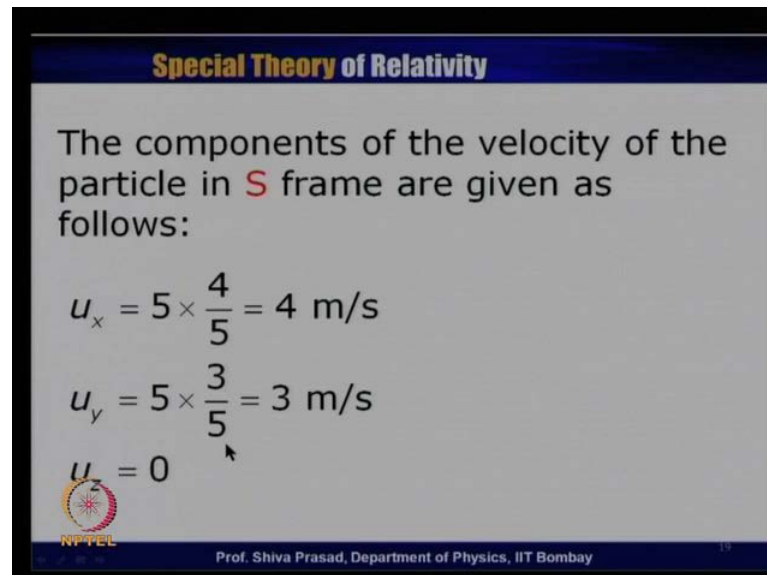
We have said that tan theta is equal to 3 by 4 and we know that tan theta can be written as sin theta divided by cos theta. Also, we know that sin squared theta plus cos squared theta must be equal to 1, if you solve these equations you will get that sin theta will be equal to 3 by 5 and cos theta will be equal to 4 by 5. As you can see tan theta is sin theta divided by cos theta, so if I divide this 2, I get 3 by 4. If you square and square this and add this will give you 5 by 5 which is equal to 1 and 5 square divided by 5 square which is eventually give you 1. So, sin squared theta plus cos squared theta will actually turn out to be equal to 1 so the value of sin theta is 3 by 5 and cos theta is 4 by 5.

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$$u_x = u \cos \theta$$
$$u_y = u \sin \theta$$

So, when I write u_x I know that this is given by $u \cos \theta$, similarly u_y is given by $u \sin \theta$.

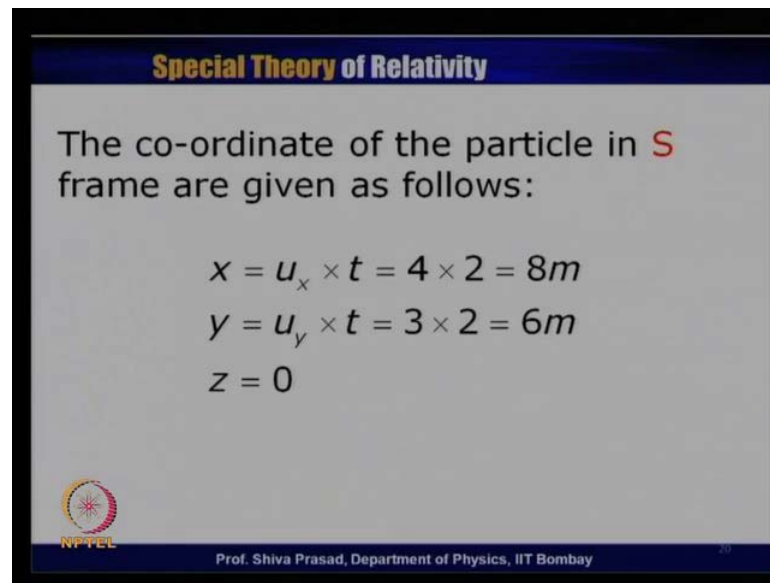
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The slide is titled "Special Theory of Relativity" in a blue header. The main text states: "The components of the velocity of the particle in **S** frame are given as follows:". Below this, three equations are listed: $u_x = 5 \times \frac{4}{5} = 4 \text{ m/s}$, $u_y = 5 \times \frac{3}{5} = 3 \text{ m/s}$, and $u_z = 0$. A small circular logo with a star is positioned to the left of the $u_z = 0$ equation. At the bottom of the slide, there is a footer with the text "Prof. Shiva Prasad, Department of Physics, IIT Bombay" and a small "NPTEL" logo on the left.

We have just now said $\sin \theta = \frac{3}{5}$, so u_y will be equal to 5 multiplied by $\frac{3}{5}$, we have said $\cos \theta = \frac{4}{5}$. So, u_x be given by 5 multiplied by $\frac{4}{5}$, therefore the x component of the velocity of the particle is 4 meters per second while the y component of the velocity of this particular particle is 3 meters per second. Of course there is no speed in the z direction under the examine under the what has given the problem, so this is speed is 0.

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Special Theory of Relativity

The co-ordinate of the particle in **S** frame are given as follows:

$$x = u_x \times t = 4 \times 2 = 8m$$
$$y = u_y \times t = 3 \times 2 = 6m$$
$$z = 0$$

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If you have to find out the coordinates all we have to do is to use simple kinematical equation, I have assumed that particle is not accelerating, so acceleration is 0. So, the displacement is just given by $u \times t$ or u_i multiplied by t as the case may be, so along the x direction the x component is given by u_x multiplied by t u_x is 4 time which has been given is 2. Second, this term is to be equal to 8 meters, similarly y will be u_y multiplied by t u_y , we have just now calculated is 3 multiplied by 2, 6 meters.

So, eventually the coordinate of the particle at t is equal to 2 second in S frame will be given by 8 meters and 6 meters, x component is 8 meter y component is 6 meter. Now, if I want to find out the coordinate of this particular particle and the same time in x' prime of reference, all I have to do is to use a Galilean transformation this is what I have used for this particular transparency.

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Special Theory of Relativity

The co -ordinates of the particle in S' at a time $t=2$ s is given by Galilean transformation as follows.

$$x' = x - vt = 8 - 1 \times 2 = 6 \text{ m}$$
$$y' = y = 6 \text{ m}$$
$$z' = z = 0$$

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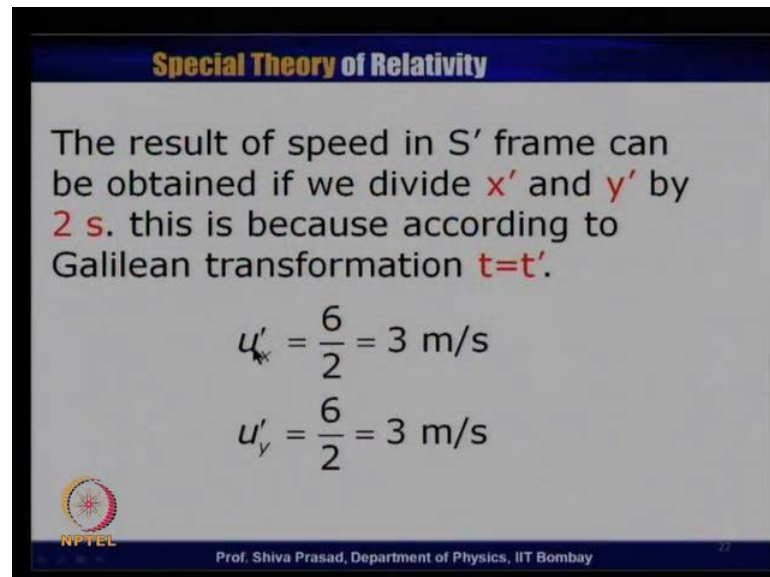
So, x' is given x minus $v t$ this is what we have just now calculated, x was equal to 8 which we have just now valued, v is the ratio velocity between the frame which has beginning the given has 1 meter for second, 2 is the time. So, we get x' is equal to 8 minus 1 multiplied by 2, this be equal to 2 x' is equal to 6 meter. According to Galilean transformation y coordinator not change has we change s to s' the frame of reference under the condition that had been described.

Therefore, five coordinate reverses exactly same which is 6 meter of course set coordinate also remain same z coordinate initial was 0, it remains 0 it continuous remains 0 in s' frame. Now, if I want to find out velocity now the same particle of prime frame of reference i have two methods one is that I realise that the time interval measured between 0 and t particle started let say t is equal to 0. Eventually, the next measurement of the particles position has done at between t is equal to 2 seconds, so this time interval of 2 second remains identical s and s' frame of reference.

It means when the observer s measured the particle coordinate and he found time and that particular time in his watch as 2 second. According to the observer as prime frame of reference his watch also showed a time t is equal to 2 seconds. Therefore, whatever coordinate off course coordinate are changed whatever coordinate his measure according to him the particle remember a t is equal to 0 the origins of both the frame s and s' which called the coincident.

So, according to an observer s prime this particular particle has move along x direction not by 8 meters, but by 6 meters because it started at x prime is equal to 0, but only landed up add x is equal to prime is equal to 6 meters. That particle took the same amount of time as observer in s frame has evaluated within 2 seconds.

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Special Theory of Relativity

The result of speed in S' frame can be obtained if we divide x' and y' by 2 s . this is because according to Galilean transformation $t=t'$.

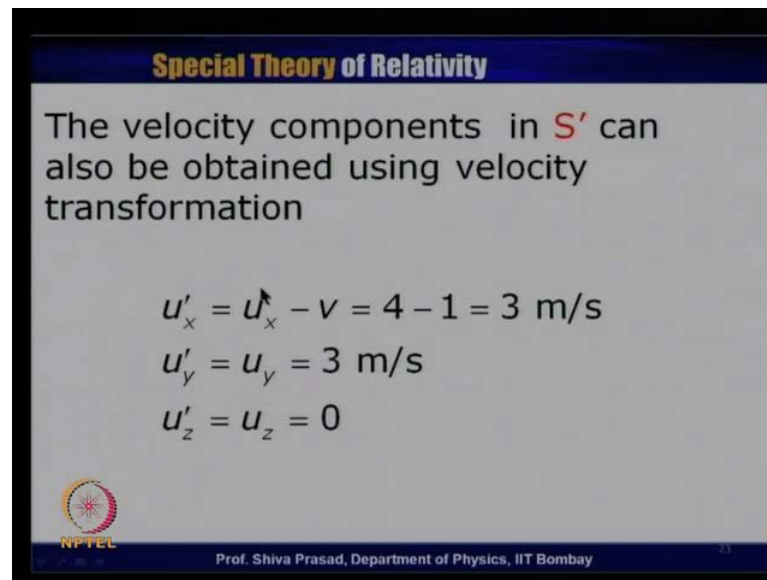
$$u'_x = \frac{6}{2} = 3 \text{ m/s}$$

$$u'_y = \frac{6}{2} = 3 \text{ m/s}$$

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So, you would calculate u_x prime and find it out to be 6 divided by 2 which is 3 meters and u_y prime off course will be y coordinate as measured by his frame which is the same as y coordinate measured in x frame which is 6 meters divided by time. Measure in this frame which is same has the time measured in x frame, therefore u_y prime is equal 6 divided by 2 which is 3 meters per second, this is has very simple ideas, but again I am less sort of impressing on these thing. So, the comparison with the relativity become easier has I have said that we could also use the velocity of a transformation formula without using the time concept.

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Special Theory of Relativity

The velocity components in S' can also be obtained using velocity transformation

$$u'_x = u_x - v = 4 - 1 = 3 \text{ m/s}$$
$$u'_y = u_y = 3 \text{ m/s}$$
$$u'_z = u_z = 0$$

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We would get u'_x is equal to u_x minus v . u_x we have just now calculated is 4 meters per second. v is 1 meter per second. So, u'_x will be 3 meter per second. u'_y is going to be same as u_y which is 3 meters per second, so we get the same answer from the two approaches for the x component. The velocity as measured in the x prime frame of reference will be 3 meter per second. The y component of velocity as measured in the x prime frame of reference will be 3 meters per second.

Obviously, the components have become different. More specifically, the x component of the velocity has changed as I have changed the frame of reference; this is something which is very normal which we have not been discussing. From our first lecture, the speed is a frame independent quantity. If I am interested in finding out the speed of a particle in a x prime frame of reference.

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Special Theory of Relativity

The speed of the particle in S' is given as follows.

$$u' = \sqrt{u_x'^2 + u_y'^2 + u_z'^2} = 3\sqrt{2} \text{ m/s}$$

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I have to take overall all the mode of the velocities which means the velocity would be given by u' is equal to under $u_x'^2 + u_y'^2 + u_z'^2$ square. We have to calculate this three, this we have calculate has 3 meters per second, so this becomes 9 plus 9, eventually if you relative prime this speed will be three multiplied by root 2 meters per second. So, this speed which was observer in S frame was 5 meters per second the speed of the same particle as measured in S' prime frame of reference turn out to be $3\sqrt{2}$ meter per second.

Now, let me take another example which is a more interesting example and it discusses a concept which again we will be discussing quite bit in relativity which is the concept of simultaneity. We called two events to the simultaneous, let me as say at this movement the relativity many times we talk about events. For example, in the example which we have given earlier when observer S measured coordinate of the particle whatever met it be the coordinates whatever met it be the time we called that has an event. So, a particular took of observer measure the position of the particle that is and event this same event the measurement of the coordinate of the particle is been observed by another frame of reference another observer in a different frame of reference.

So, an event is an event this observed, all the frame of reference only thing which could be different are there dynamically variables. There coordinates could be different, there velocity components could be different, but what we are measuring are there coordinates

or dynamical variables corresponding to a given event. So, if this two or either any two events which occurs at the same time we call them a simultaneous events, if we are seeing an observer and finding out two events.

For the example we are sitting here on the earth and we find that it a plain taking of and let us say train departing. We find that both this event occur that that same time, then we occur the position the plain may take off from airport while train may depart form particular railway station. If they happen according to me the same time I will call that this two event, there simultaneous, now in classical mechanic if two event are simultaneous in a given frame they remains simultaneous all other initial frames.

So, this is an example which is I am giving again use in Galilean transformation describing the fact that two events. If they happen to the simultaneous in one frame will turn out to the simultaneous in other frame put this is has specific example, but this is a general statement.

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Special Theory of Relativity

Example 2

An observer is exactly half way in a running compartment of length L . He throws two balls at the same time ($t'=0$) with a speed u' as measured by him. One towards the front wall and other toward back wall.

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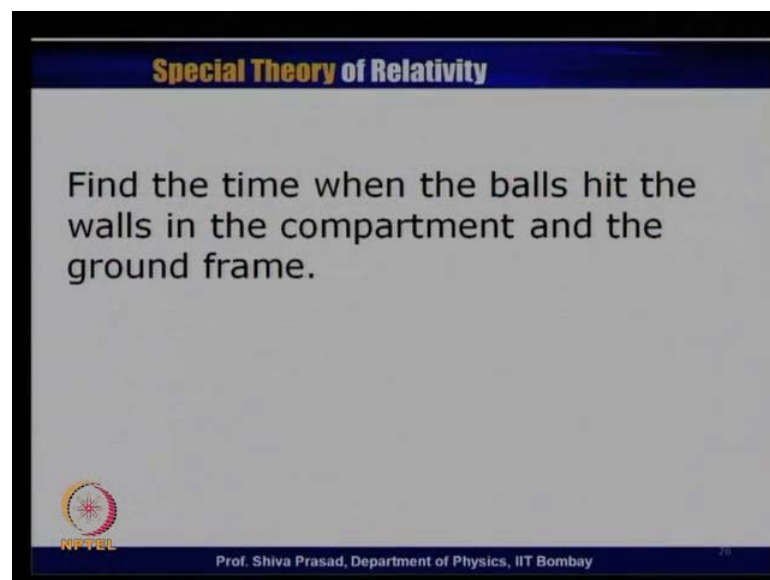
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So, my example as follows an observer is sitting or standing whatever want you called it exactly off way in a running compartment of length l . So, there is a train compartmenting which as a total length of l and he is sitting or standing just exactly half the way of the compartment. He throws two balls in the same time which I am assuming at the time t prime as a 0 with a speed u prime as measured by again, I remind you I am using symbol u because this an object who velocity is been measured in s and s prime. These two

different frames of in this one frame is the component frame, so in the compartment frame it is found that this particular ball is thrown with the with speed u prime has measured by him off course one towards the front ball.

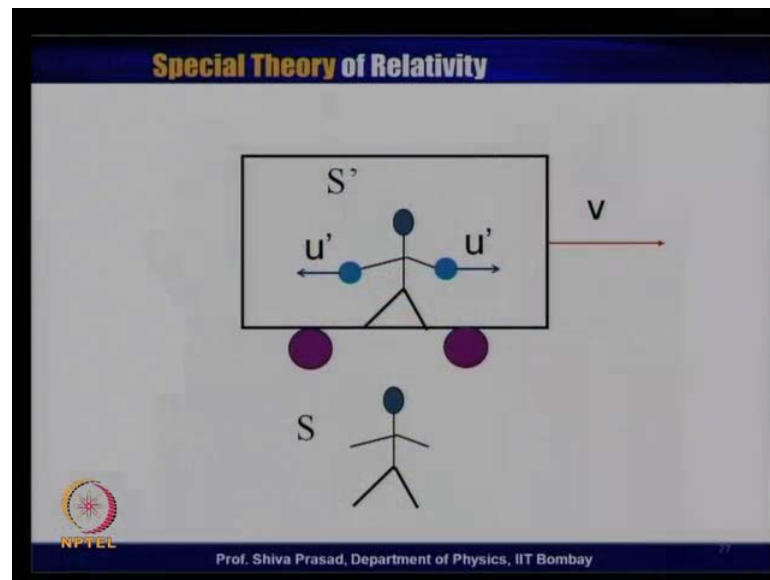
The second one towards the back word see the compartment is moving there is a wall which is moving along the direction of the speed which is front, so let suppose he is standing in between. So, that his left hand is towards the motion and right hand the again the motion then he takes ball in left hand and throws the particular direction towards the motion and other ball throws in a direction opposite in the motion.

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Find the time when the ball hit the wall in the compartment and the ground frame, so these balls go and hits the wall of the compartment. One along the direction another opposite to the direction of the velocity of the compartment and what I have to find are the time when these balls hit the walls.

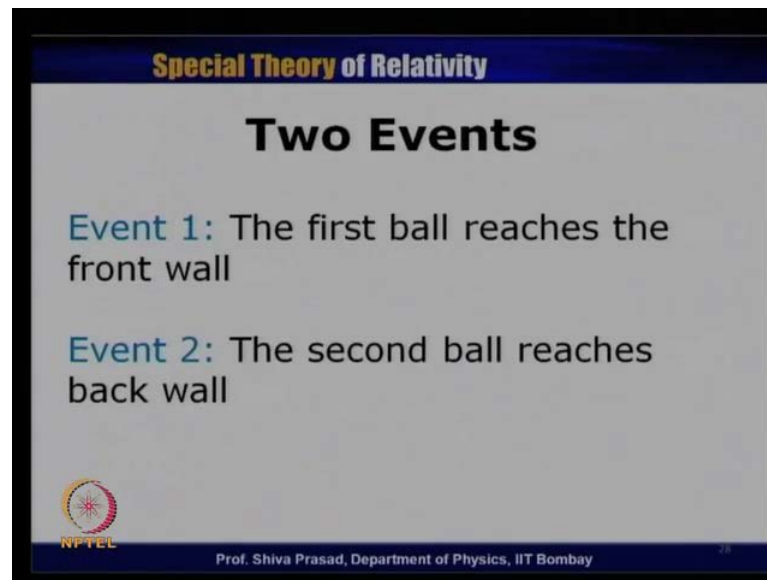
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This has been picturized somewhat in this particular cartoon at this is an a compartment which I assume is moving with the velocity v relative to observer on a ground. I assume that ground frame is an initial frame this person is sitting in half way exactly in this particular come compartment his one ball is pointing towards the direction of the motion. There is another ball which is pointing in a direction opposite to a motion he throws one ball in this direction he throws another ball in this direction this ball goes and hit this wall this ball goes and hits this wall.

I find out the time then this happens as for this observer constant assuming that these balls are thrown at time $t' = 0$. Then, I would find the same motion I describe the same motion with refer reference to the observer as prime which is standing on the ground. From now onwards, we will try to define the event to make thing corporately simple.

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Special Theory of Relativity

Two Events

Event 1: The first ball reaches the front wall

Event 2: The second ball reaches back wall

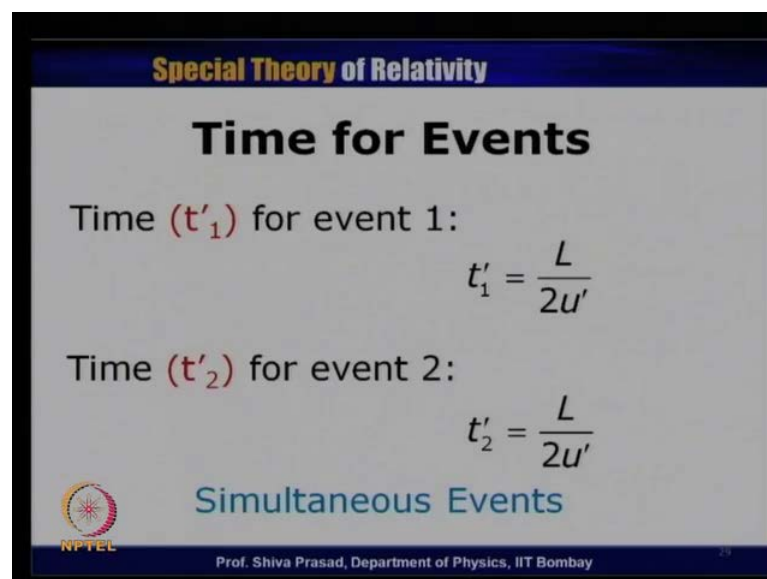
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So, I describe two events the first event is the first ball reaching the front ball it hits and if the front ball the ball which is towards the direction of the motion, so this is the ball which comes and hits here this my event number one. This ball goes and hit the back ball this is my event number two, so event number one, so this ball coming and hitting here this event number two this ball coming and hitting there. These are the two event that I am describing event one the first ball reaches the front ball event number two the second ball reaches the back ball we are doing all this calculation.

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Special Theory of Relativity

Time for Events

Time (t'_1) for event 1:

$$t'_1 = \frac{L}{2u'}$$

Time (t'_2) for event 2:

$$t'_2 = \frac{L}{2u'}$$

Simultaneous Events

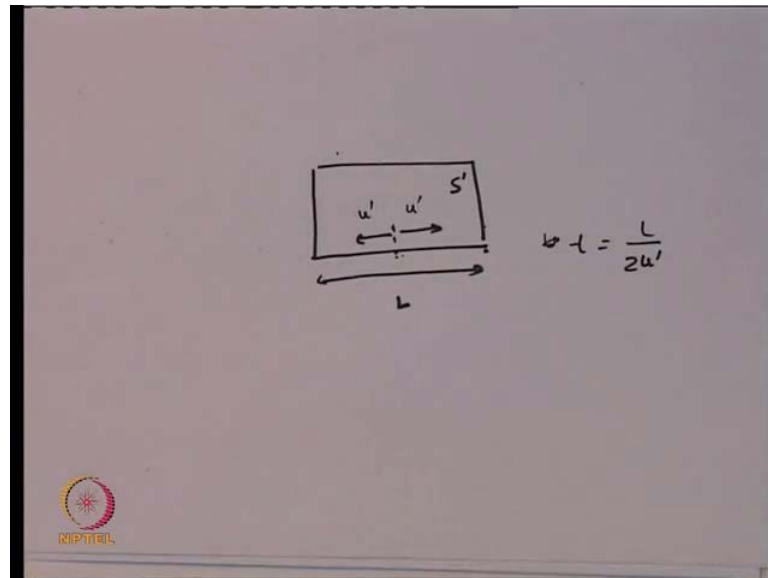
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In s prime frame of reference we have not that is compartment frame of reference I have not really talk about the ground frame of reference. Now, if I find out what is the time of event one it very simple these ball as to travel horizontal distance of l by 2 because the length of the compartment was l .

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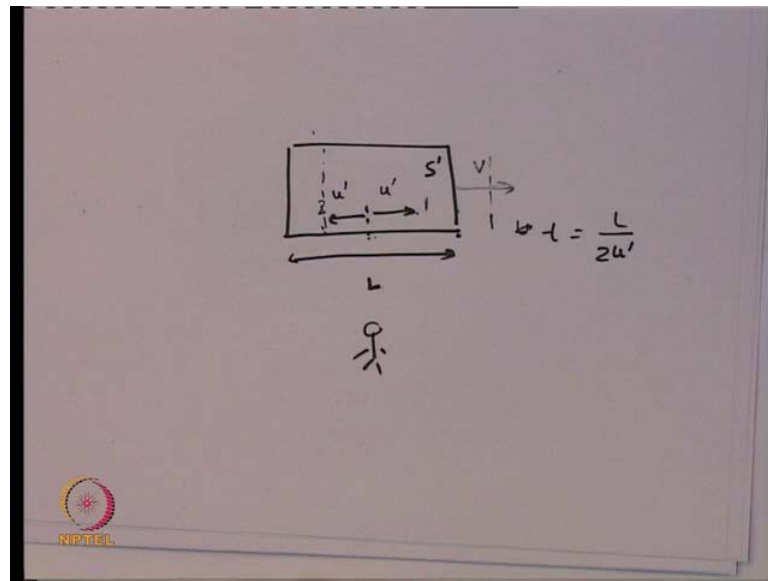


This is my compartment this length was l ball is gone exactly at half way, so one ball goes like this another ball goes like this then distance. This ball the horizontal distance that this ball travel will be l by 2 would may it had slight down wards displacements because of gravity that is not bother that particular time that does not change this time. Similarly, this ball also have to travel the distance of l by 2 , I remember I am talking everything in terms of this frame of reference which is the compartment frame of reference.

So, this distance is travel by 2 this distance is travel l , l by 2 we have also assume that this two speed are same this is speed is u prime this is speed is also the u prime. So, obviously the time that this particular ball take to hit here looking only at x component use this time t will equal to l by two u prime. So, this is what I have written here this particular transparency that time t_1 prime this one refers to an event one. This time refers to the fact that this particular time is been measured in s prime frame of reference, so t_1 prime is equal to l divide by 2 which is length divided by the speed of the ball.

This is u' , so this is the time t_1' , similarly time t_2' referring the event to is given by $t_2' = l / 2u'$. So, these events are simultaneous because $t_1' = t_2'$, so the observer in S' frame of reference will conclude that these two events, the ball hitting the front wall and the ball hitting the back wall, are occurring at the same time. Therefore, they are simultaneous events; now let us look at the same situation as observed with respect to the ground frame.

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So, there is an observer sitting here at the ground. Let us assume that at the moment the balls are thrown, the origin of the particular person was also coincident with the origin of S' . It means these two observers are just sitting side by side; therefore, when S' watch shows $t' = 0$, the watch of this particular observer also showed time $t = 0$. However, because this particular train is moving with speed v in the frame of the ground observer, his perception about speeds and distances would be very different.

Why it could be different, because this ball which has been thrown in the particular direction according to this observer. He would find that the speed of the ball is different using standard relative velocity or velocity transformation formula. So, this speed of the train gets added up, so in fact what he would find is that the speed of this particular ball is larger than u' . Similarly, this particular ball which has been thrown backward

according to this observer this ball would not actually be thrown with this prime because this particular train from which this particular ball was thrown was itself going.

So, he would find that this speed is smaller than this speed of the front ball, if we call this ball 1 and we call this ball 2, according to the observer ground ball 1 moves with a larger speed than ball 2, but as the ball keeps on moving on this particular direction. It tries to approach this particular ball this ball according to him also is moving towards the right. So, according to this observer this ball as to travel a larger distance because by the time ball starts from here and reaches the wall this wall has already moved ahead because during this particular time there is been motion so this wall may be somewhere here.

So, this particular ball as to move a larger distance on the other hand this particular wall because it's moving towards this particular in this particular direction. So, by the time it hits a wall this particular ball would have moved towards it so this ball has to travel a smaller distance to hit the back wall. Then, this ball number 1 which has to travel a much larger distance to hit the wall, so the perception of ground observer will be quite different according to him.

The ball number 1 will be thrown with the larger speed, but it has to travel a larger distance to reach the wall while the second ball has been thrown with a smaller speed, but also it has to travel a smaller distance to reach this particular wall. What we are going to show next now is that because this ball has to travel larger distance and larger speed by making calculation we will show that still the times measured by this particular observer.

For the two events will turn out to be same exactly as what has been observed by S prime observer, so let us try to look at the particular calculation it's simple calculation. So, we look at the same two events in x prime frame S frame sorry in S prime of reference which is the ground frame of reference, so I calculate the speed of the two balls in the ground frame of reference using velocity transformation.

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Special Theory of Relativity

Same Events in S'

The speed of the first ball using inverse velocity transformation.

$$u_x = u'_x + v = u + v$$

Similarly the speed of the second ball

$$u_x = u'_x + v = -u + v$$

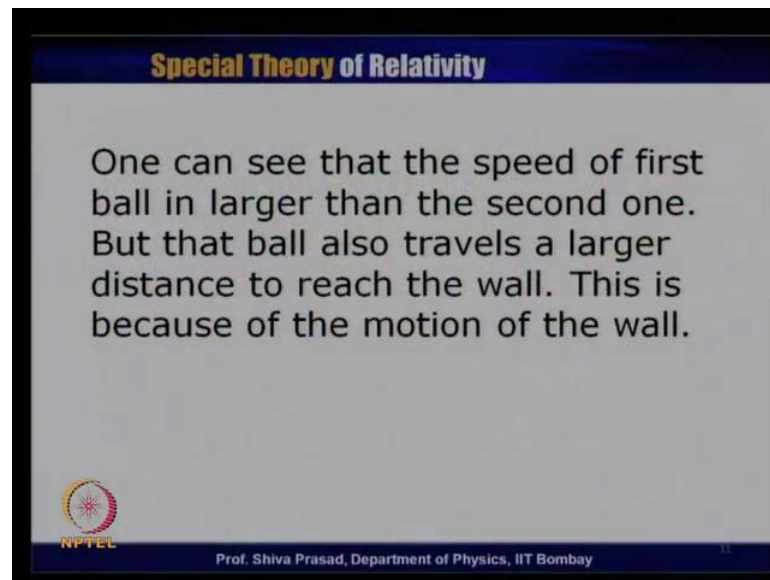
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So, the speed of the first ball using inverse velocity transformation is in fact what I have used is inverse velocity transformation because the speed had been given in S' frame of reference. I want to be speed in S frame of reference, so remember in inverse velocity transformation it was $u_x = u'_x + v$ $u'_x = u - v$ for the first ball and v is relative velocity. It means it is the velocity of the train as measured by the ground observer so the x component of the velocity of the ball as measured the ground observer will be u plus v .

Similarly, the speed of the second ball we apply the same velocity transformation inverse velocity transformation will be given by same expression $u_x = u'_x + v$. For the second ball because it moves in minus in direction this will be minus u plus v , so as we had said that this velocity this x component velocity is going to be a smaller than this component will find that u_x for second ball will be smaller than u_x for first ball.

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Special Theory of Relativity

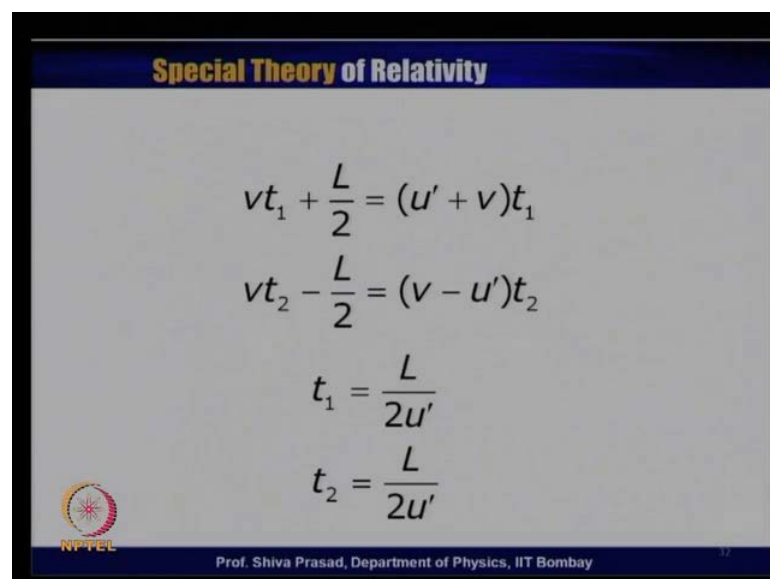
One can see that the speed of first ball is larger than the second one. But that ball also travels a larger distance to reach the wall. This is because of the motion of the wall.

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So, this is what I have written here in this transparency that one can see that the speed of the first ball is larger than the second one, but that ball also travels in larger distance to reach the wall this is because of the motion of the wall.

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Special Theory of Relativity

$$vt_1 + \frac{L}{2} = (u' + v)t_1$$
$$vt_2 - \frac{L}{2} = (v - u')t_2$$
$$t_1 = \frac{L}{2u'}$$
$$t_2 = \frac{L}{2u'}$$

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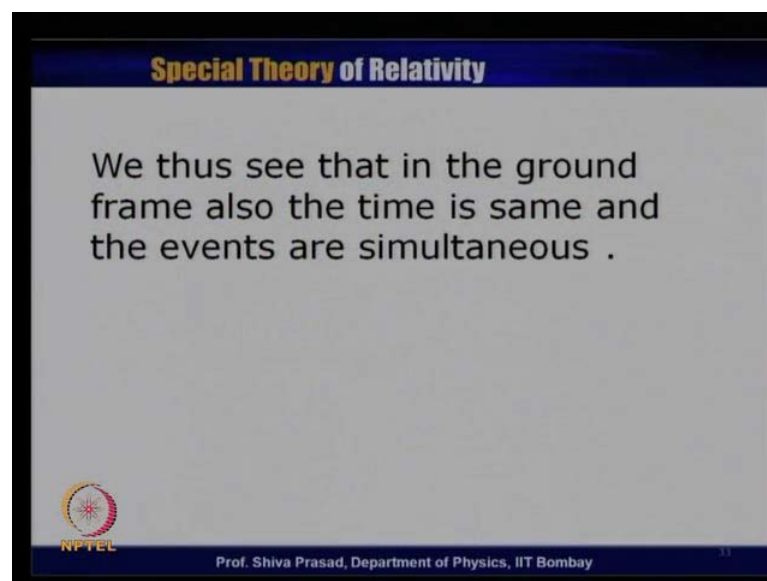
Now, what will be the observation of the observer in S frame of reference according to the observer? Let us look at the first ball which was the ball which we have thrown in this particular direction as we have mentioned that this particular ball has to travel a larger distance not only $\frac{L}{2}$, but also the additional distance this additional distance is the

distance move by the train during. The time that this particular time wall took to hit the wall that particular time it happens to t_1 . Then, the total distance this distance will be v multiplied by t_1 .

So, the total distance that the wall has to travel is what is written in this particular transparency is $v t_1$ plus l by two this particular distance is covered by the first ball with a speed of $u' + v$. So, u' multiplied by t_1 is given by $v t_1$ plus l by 2 if we look at second ball the distance travel, now gets reduce because by in amount of v multiplied by t_2 if the t_2 is the time when the second events occurs. This will be given by $v t_2$ minus l by 2 because numbers there is negative sign the motion of the ball is minus x direction and it travel with the speed of $v - u'$ into t_2 .

So, this travels is small t_2 , so this travel is smaller distance with a smaller velocity if you just look at this two equation here $v t_1$ will cancel of the $v t_1$. Here, $v t_2$ will cancel I hope with this $v t_1$ will get the same expression t_1 equal to l divided u' t_2 equal to $2 l$ divided $2 u'$ prime the same expression we have obtain from the observation of the observer in s prime. So, observer in s also finds that the two times are same and he will also notice that this two events are simultaneous this is what a point was trying to bring home.

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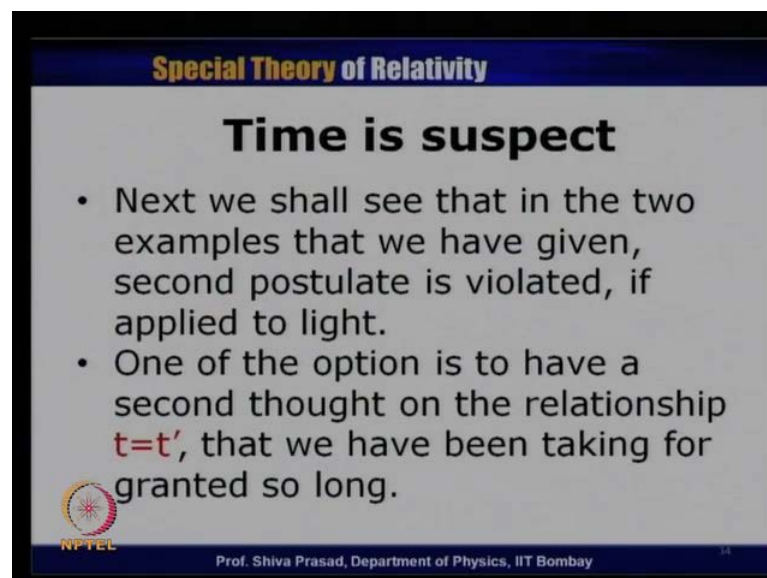


Saying that the two events which are simultaneous in s frame also appears simultaneous s prime frame of reference this what I have written we thus see that the ground frame

also the time is same and the event are simultaneous to you to earth to statement. The two times are same and the event is also simultaneous, now before you go to the next lecture let me just pose one question just pose the particular issue.

Eventually Einstein realise that it is this particular time which we are assuming to be same in the all the frames this particular simultaneity which we are assuming to be same in all the frame is something be suspected. If we are looking into a different set of transformation which is consistent with the postulate of special theory, probably we have to relook at our concept of time, so this is what I have written.

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Special Theory of Relativity

Time is suspect

- Next we shall see that in the two examples that we have given, second postulate is violated, if applied to light.
- One of the option is to have a second thought on the relationship $t=t'$, that we have been taking for granted so long.

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Next, we shall see that the two a will effect will work out the same examples and will see that if we assume time to be same then the postulate of special theory of relativity or start of valuated in order that we have to maintain the postulate of special theory relativity. Probably, we have to have to re look the time t something which was taking for granted so long.

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Special Theory of Relativity

Summary

- We described the postulates of special theory of relativity.
- We described the concept of transformation.
- Discussed the classical Galilean Transformation and some examples based on that.
- Finally we raised an issue over time.

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I just give a summary whatever we have discuss today we described postulate of special theory of relativity we described concept on transformation we described the Galilean transformation. The velocity transformation gives some examples for that and finally we raised an issue over time see that may be is a time which we much suspect.

Thank you.