Special Theory of Relativity

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Lecture - 24
Current Density Four Vector and Maxwell Equation

Hello, so we have now come to the end of the course. This is going to be my last lecture; trying to wind up whatever we had started about the electric field and magnetic field. In our last lecture, we had taken one example and discussed the electric field and magnetic field transformation. We had said that, what appears to be a purely a magnetic field or an electric field in the frame may appear to be combination of electric field and magnetic field in a different frame.

So, whether you term a field as electric or magnetic or partly electric or partly magnetic, it could depend on the frame of reference from which you are observing. Today what we will try to do is try to analyze the situation little bit more and try to see how we can think about the presence of these fields or change these fields, by taking one simple example of a current carrying wire or current carrying conductor. Then eventually will show, I will not really be able to show because you know this is actually beyond the scope of this particular lecture series, that Maxwell's equations which are supposed to be the basic equations in electrodynamics, they remain unaltered even after special theory of relativity.

So, there those equations do not change once that special theory of relativities also introduced. So, they are invariant under special theory of relativity. So as I said this particular aspect will not be able to be prove, as it is beyond scope of this particular course, but we will just take one equation and try to sort of convince I would say, try to convince that for one equation will sort of say that you know, you can sort of see that Maxwell's equation is expected to be obeyed. So, this is what we are going to do in our last lecture today.

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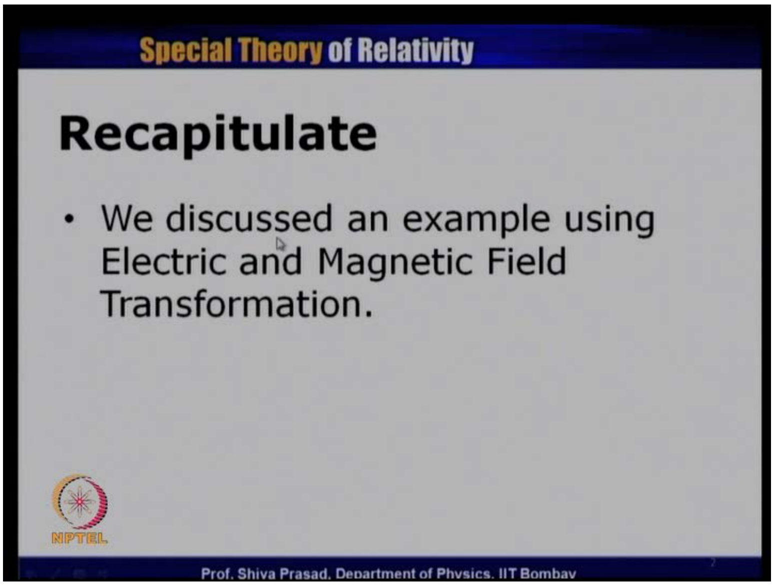

So, this is what we have said we recapitulate; we discussed an example using electric and magnetic field transformation in our last lecture.

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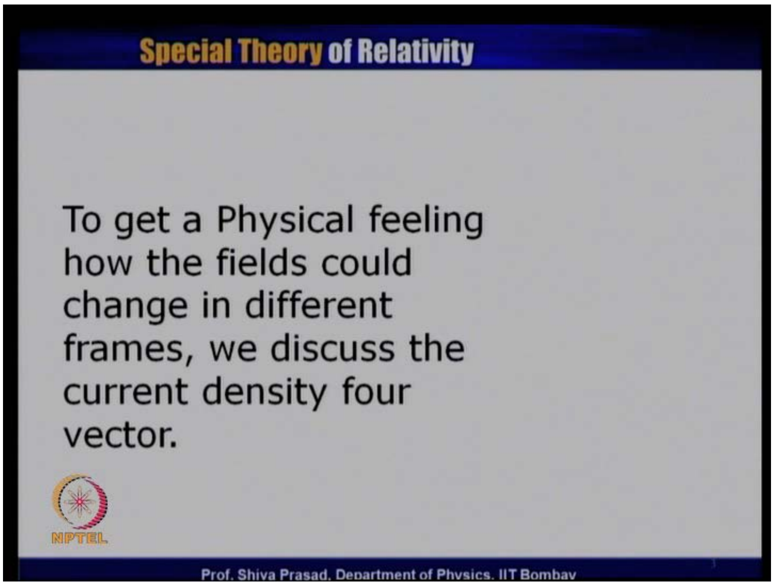

To get a physical feeling how the fields could change in different frames, we discuss a new concept today which you call as a current density four-vector. As I said, we want to discuss little bit more physically by taking current carrying conductor but before we come to that situation, we will define what we call as a current density four-vector.

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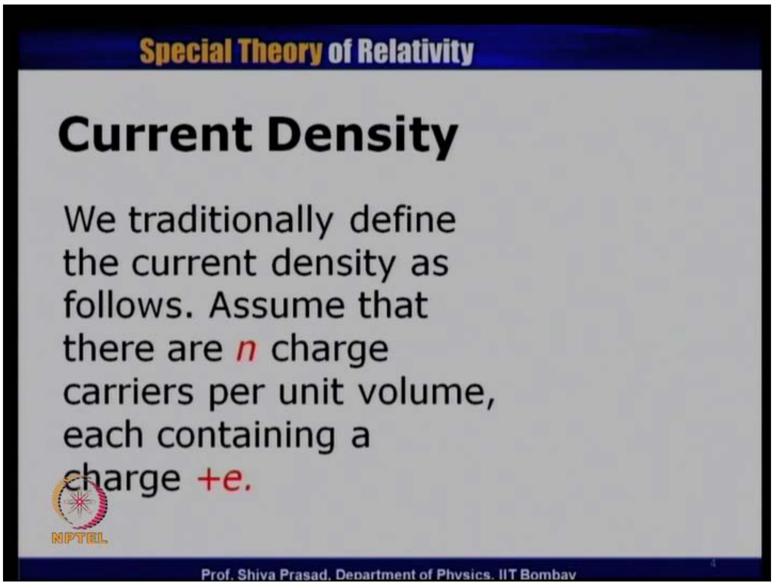

Before we do that, let us first take traditionally what we mean by a current density. In physics generally, current density is considered a little more fundamental than the current and current density is always a vector quantity. This concept of current density emerges more from the fundamental aspects of how the motion of charge carriers takes place in a particular wire or particular conductor. So, let us assume that we have n charge carriers per unit volume in a given conductor or given wire which is carrying current. Now of course, we know that is actually the electrons which carry the current which are negatively charged particle.

But somehow traditionally the direction of the current has been always defined to be opposite direction of the flow of the electron. So, if the electrons sort of flow in the negative x-direction, we define the standard traditional current in the positive x-direction. As we said let us assume that there are n charge carriers per unit volume and each one of them is carrying a charge of plus e. So, I am just assuming that the charge carriers are positive, though as I have said that actually the charge carriers in a real conductor is posed to be electrons which are negative charge carriers.

But just for defining the concept of current density, I take this as positive charge carriers. Now we evolve the concept of what we call is the drift velocity. Actually once we apply electric field, these charge carriers start accelerating under the influence of the force which is created by this particular electric field, but because of the scattering that these electrons or the charge carriers, whatever they are, freeze within a metal or within a conductor.

They are not able to move too much ahead, they go little bit ahead and then they get scattered; then they again accelerate and then they get scattered and if you remember, what we can define that under the influence of electric field, these electrons will eventually develop what we call as the drift velocity. It is something like the motion of air, breeze, the molecules in air may be moving with very large speeds, but we do not feel it.

Only when the overall air drifts, which may be drifting with very small speed in comparison to the actual speed of the molecules; that we feel that there is a breeze. So, that is what is the drift of this whole air. So, similarly is the concept of electrons; the electrons could be moving in very large speed inside a conductor, but if there is one electron is way, another electron is moving this way. So, eventually you do not see any drift; only when there is net flow of electron in a particular direction, then only we see the current and that happens when we apply an electric field and then we define what we call as a drift velocity.

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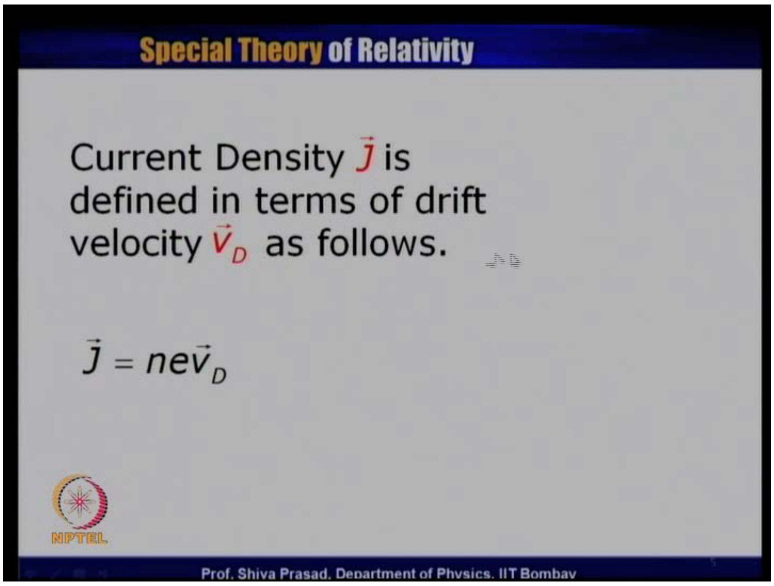

Now current density vector J is defined in terms of this drift velocity as follows. J is equal to ne times drift velocity, where as I have said n is the total number of charge carriers per unit volume, E is the charge of each of the carrier which I am assuming at the moment to be positive; actually it makes no difference if we take for an electron. We have to take care of science properly; otherwise, this equation which defines the current density inside the metal or inside the conductor.

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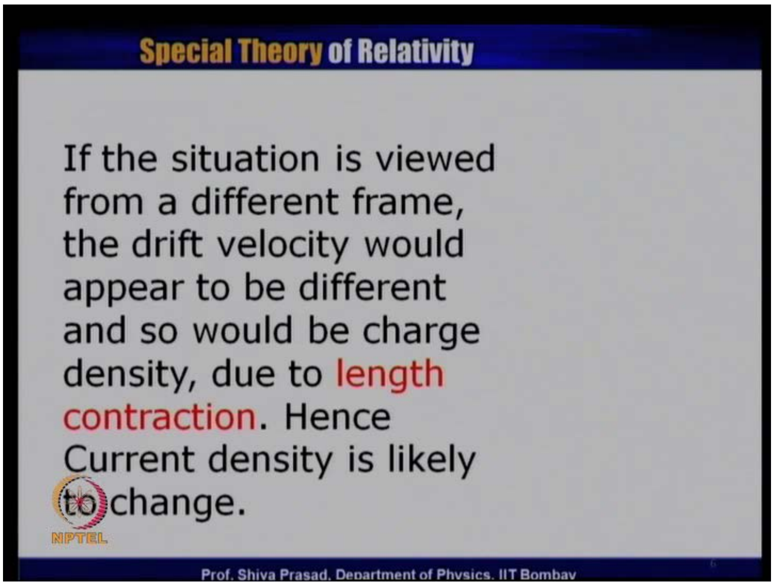

Now if the situation is viewed from a different frame, then obviously we expect that the velocity of the electron would not be same, because we have discussed number of times that the velocity is actually frame dependent quantity. So if I go to a different frame, the drift velocity that you are seeing in a given frame may turn out to be different; it is very well expected. Also I would like to emphasize that even the current density or charge density, not sorry current density, the charge density would be different and that is mainly because of the length contraction.

Because once we have come to the case of relativity theory, we will find that there is a contraction of length; and that itself would change the charge density, and if charge density change, the drift velocity change. So when I look into a different frame of reference, the current density is also expected to change. Therefore, I must look on in relativity, how this current density will change if I change my frame of reference; that is what is the question that we are going to answer now.

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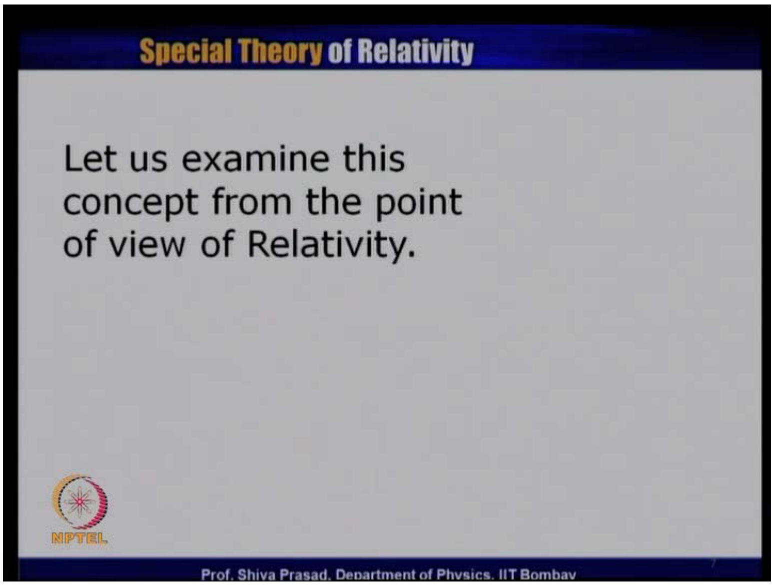

That is what I said; let us examine this concept from the point of view of relativity. For that particular thing, let us assume comparatively simpler picture. Let us not bother about the drift velocities and other things. We will just take comparative, very very simple picture and try to see how this current densities will transform if I go from one particular frame of reference to another frame of reference.

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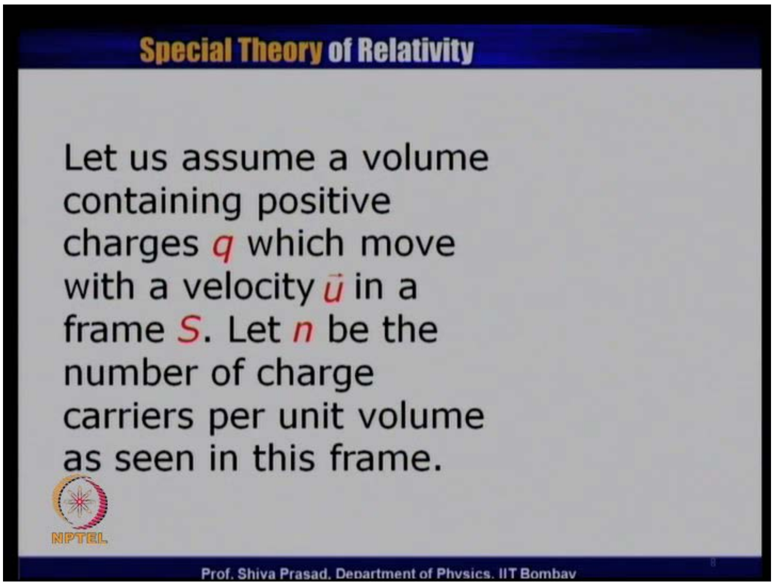

So, let us assume that we have a volume which contains all the positive charges; each one of which has a value of q which moves with a velocity u. So, let us assume that all of them move with the same velocity because there is no question of drift here; I do not want to confuse, will about drift little later. So, let us assume that they all of them moving with the same velocity u as seen in the given frame S; obviously, when these charges move in a particular direction, this will constitute current or eventually current density.

Let n be the number of charge carriers per unit volume as seen in this frame. We have just now said that this n is also likely to be frame dependent; I would discuss this point little more in detail little later. So, we have a frame; as we are always telling, we have to be consistent in our frame. So, there is an observer sitting in s frame, which absorbs the velocity of the charge carrier to be u and first he calculates the total charged number density or charge density, if you call it as number density; then that is the number of charge carriers per unit volume. He absorbs that number to be equal to n.

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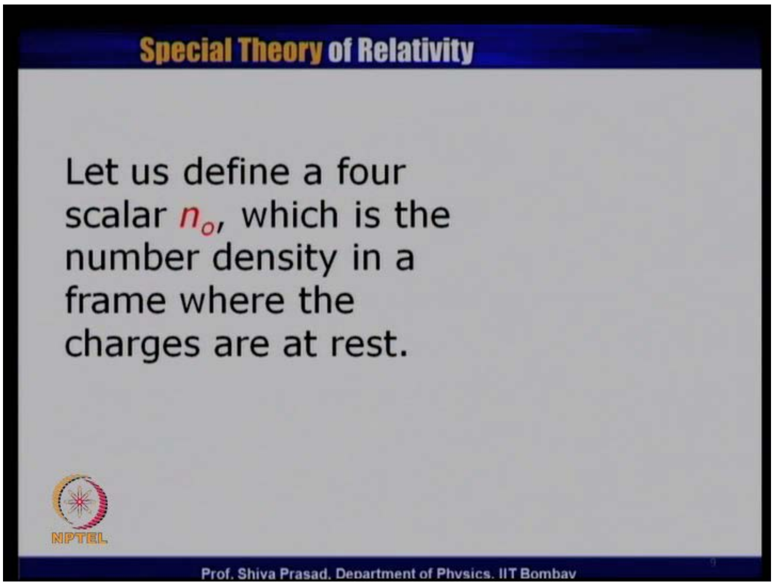

Now let us define a four scalar; we have used the concept of four scalar earlier. This is the quantity which does not change, when we change our frame of reference. See like we have defined, when from velocity to momentum; at the time, we multiplied by four scalar which we called as the rest mass of the particle. Similarly here, we define a four scalar n naught which does not change on frame of reference. This of course is going to be different from n and this is the number density; that is the number of charge carriers per unit volume in a frame of reference in which these charges are at rest.

See like rest mass, we define that this is the mass what we call in a frame of reference. When the particular mass is at rest, take analogy from the particular concept. Similarly we define a number density, total number of charge carriers per unit volume as evaluated in a frame of reference in which these charges are at rest; this n that a person is going to find out in S is going to be different from n naught as we will see just now.

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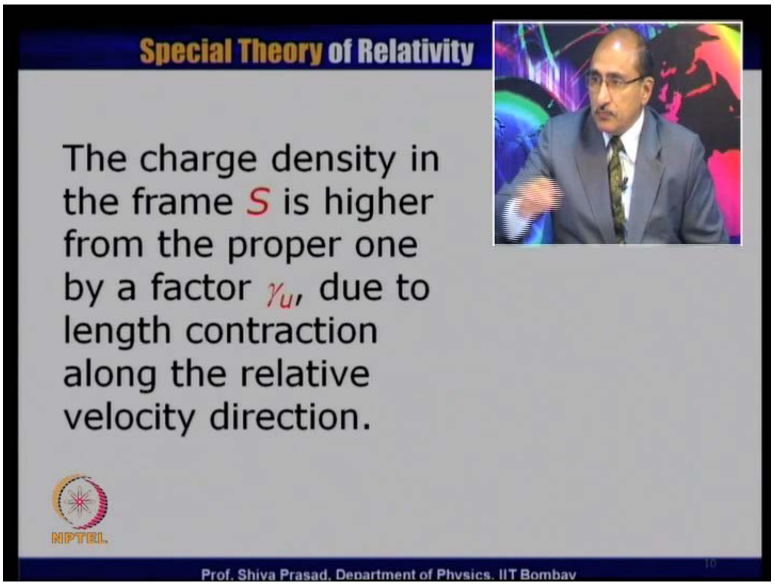

Now the charge density in the frame S is higher from the proper one; this number n, I can call as a proper number density, n naught sorry, this n naught we can call as a proper number density because this is evaluated in a frame of reference in which these charges are at rest. Now what I insist that, the n that I am evaluating in S; S is the frame in which the charges are actually moving. In this frame, the number density would be different from its proper density n naught mainly because of this length contraction; and this length contraction always occurs along the relative velocity direction. So, let us look in to this particular picture.

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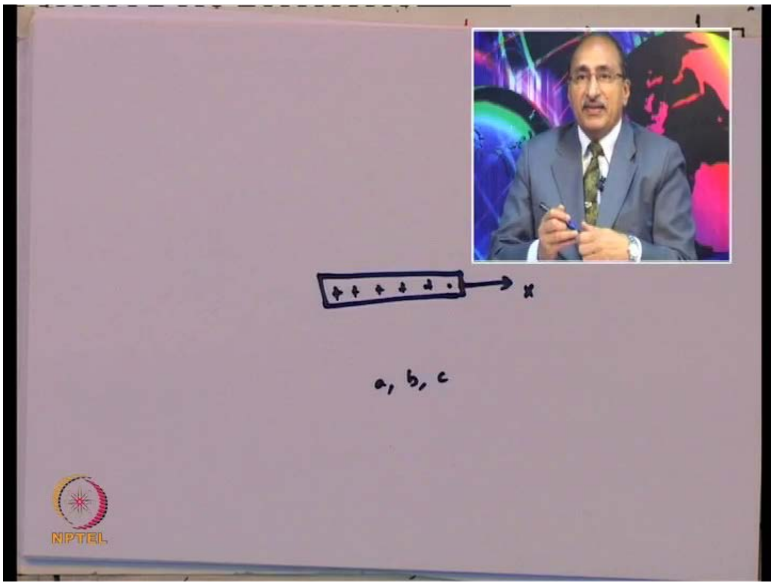

Let suppose we have a wire or a conductor like this and let us assume that this is in the x-direction. Now they are charge carriers here; that is the negative charge carriers whatever are. Here we have assumed positive, so let us assume positive. Let us assume that they are moving in a particular direction x. Now if I view this particular thing from a different frame of reference, then in that particular case and let us assume that this particular frame of reference; let us say as prime or whatever is that particular frame of reference. A person sitting in that frame views these charge carriers, then that particular person would find that this length is contracted. I am assuming that this particular frame is actually moving in the same direction as x.

So, we know that standard length contraction formula. So according to this person, the length will turn out to be smaller and the dimensions would not change. And this length will become smaller by a factor of gamma u; new length or the length as seen in different frame of reference will be gamma u times the original or the proper length. Therefore, the volume of this particular material will also go down by gamma u, because no other dimensions change. So, if you have let us say now the dimensions are a, b, c, then only one of them becomes gamma a and b and c remains same.

So when I take the volume, volume goes down by a factor of gamma u and therefore charge density, the total number of charge carriers per unit volume go up by a factor of gamma u. So, that is what I say that if I have come to this particular frame of reference S in which I find that these charges are moving, then this number density that will be evaluated by the frame will be different from its proper number density because of the length contraction and therefore, this particular factor will get modified or get multiplied by a factor of gamma u.

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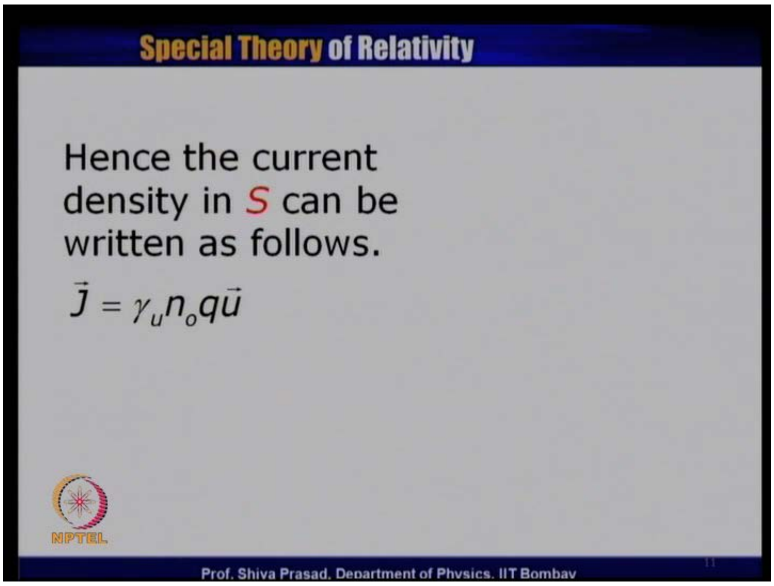

Hence the current density if I have to write in S frame, I have to write the total number of charge carriers per unit volume which I know; I have earlier written as n, will actually be gamma u times and not where n naught is actually four-scalar. This factor is n and this n is gamma u times n naught. Using the definition of charge density that we had used earlier, this equation we can write that current density is total number of charge carriers per unit volume, which is gamma u times n naught multiplied by the q multiplied by the velocity; I am assuming all of them are moving in the same velocity.

So, this will be the current density as seen in this particular frame of reference. So, the only reason why I am writing this equation in this particular form is because I am using a four scalar here. If I would have written just n, this n would have changed if I would have want different frame of reference, while this n naught is going to be the same in all the frames. Now instead of number density sometimes we define charge density. This charge density is very simple; just multiply whatever is the n naught multiply by the charge which is q.

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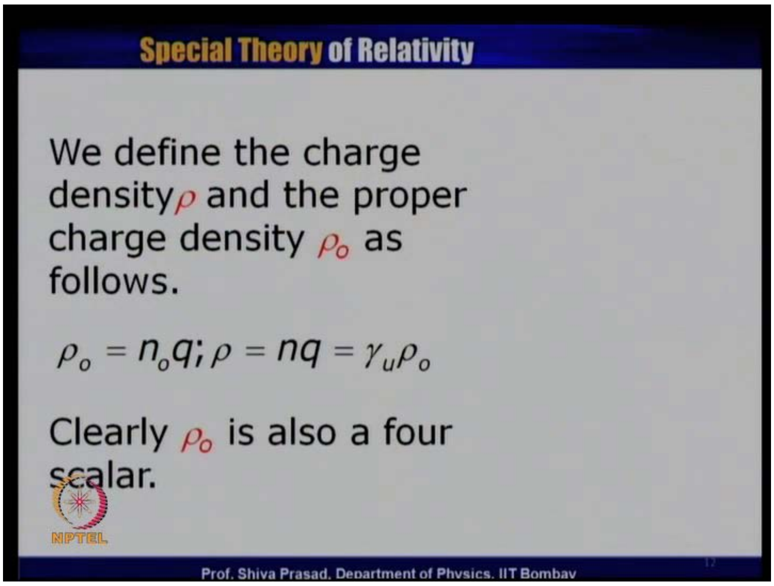

So, this is a very simple definition; most of the time we talk in terms of charge density rather than number density. So we define the charge density rho and like we have defined a proper number density, we can define a proper charge density rho naught as follows. So, rho naught whatever was the number density multiplied by charge; it is very simple it is the charge density. The total number multiplied by charge. Similarly, the charge density in a particular frame is the total number density as seen in that frame multiplied by q. So, this rho is equal to n q and rho naught which is the proper charge density is n naught q and because as we have discussed n is equal to gamma u n naught.

So, this particular factor rho naught and this equation rho can be written as gamma u rho naught, because this is n naught q. This n I can write this as gamma u n naught, n naught if I observe here this becomes rho naught. So, this becomes gamma u rho naught. So, rho charge density in a particular frame of reference can be written as gamma u rho naught and only thing that I insist, that this rho naught is also a four scalar because n naught is a four scalar. And q the charge in relativity we do not expect this to change, first we change the frame of reference. This point we have not specifically mentioned earlier, but let us emphasize it now. The charges do not change once we change the frame of reference. Therefore, this quantity rho naught is also a four scalar.

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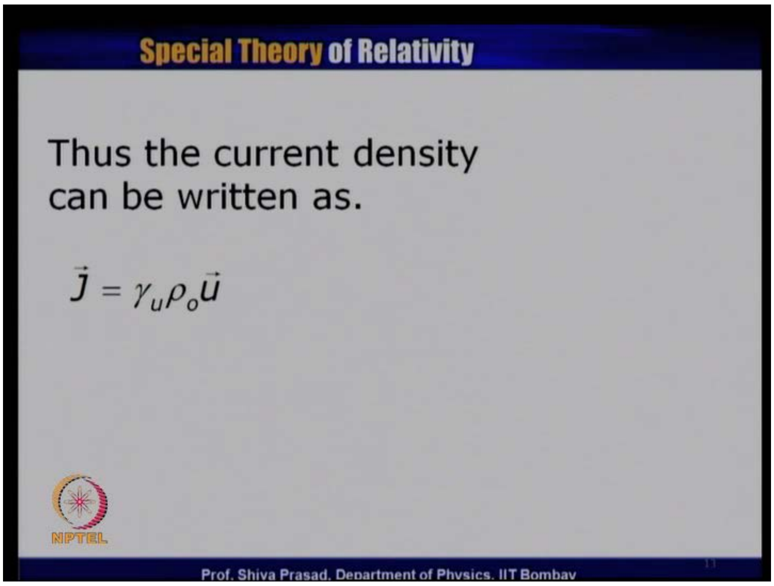

So, we can write the current density now in terms of this equation J is gamma u rho naught u is what we have written here earlier; except this gamma u n naught. This n naught, I am sorry, have been written in terms of rho naught; that is all have done in this equation. So, this is J is equal to gamma u rho naught u.

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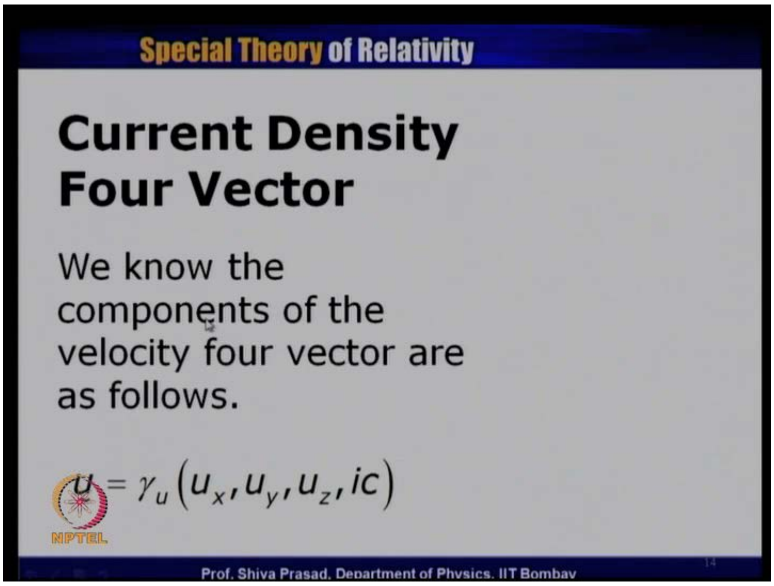

Now I am in a position to define current density four-vector. If you remember in one of our earlier lectures, we have talked about the velocity four-vector. We have said that U x, U y, U z, they are not the components of the four-vector or the velocity four-vector. See remember P x, P y, P z are the first three components of the momentum four-vector, but U x, U y, U z are not the component or not the first three components of velocity four-vector. In fact, what are actually the first three components? They are gamma u U x, gamma u U y gamma u U z. This aspect we have discussed much more in detail when we were discussing the velocity four-vector.

So, this velocity four-vector, the components are given gamma u times U x; this is the first component. Second component is U y, third component is U z. This is iC; all have to be multiplied by gamma u. So, strictly that mean that the first component of velocity four-vector will be gamma u U x; second will be gamma u U y; third will be gamma u U z; and the fourth component will be I gamma u C. This is what we have defined earlier. Now I can use this particular thing to define charge density four-vector; all I have to do is to multiply by a four scalar which I can do by multiplying by rho naught, because we have earlier said that 0rho naught is actually a four scalar.

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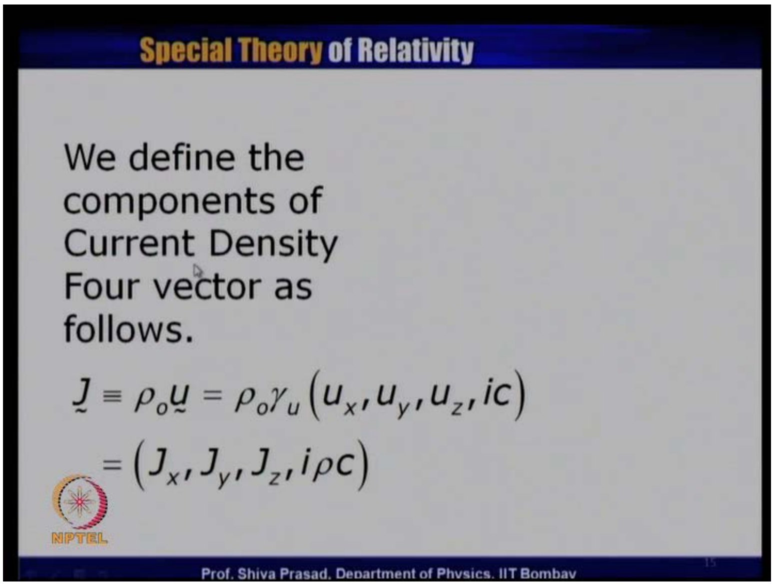

So, we define the components of current density four-vector by multiplying this u four-vector by a four scalar which is rho naught. This has been multiplied by rho naught. All other components are exactly same; only thing this has been multiplied by rho naught. Now we realize, if you look back at the equation of current density this I can write component wise; I if write component wise, it will become J x or the x component become J x gamma u rho naught U x. The y component of this equation becomes J y is equal to gamma u rho naught U y and things like that.

So, once I write here rho naught gamma u U x, this will become J x. Once I say rho naught gamma u U y, this will comes J y; rho naught gamma u U z, this will comes J z and I am retaining this particular thing writing this rho naught gamma u as rho which is the charge density. This becomes i times rho multiplied by C. So what we see, that the first three components of current density four-vector are actually the real current density as seen in that particular frame; it is like momentum, momentum four-vector. The first three components are the momentum as absorbed in a particular frame of reference.

Similarly current density components are the first three components of the current density four-vector and the fourth component, like in the case of momentum four-vector depended on the energy of the system. Here the fourth component of the current density four-vector depends on the charge density. So, the fourth component is rho. See like in that particular case of momentum, it was I E upon C; the fourth component dependent on the energy. Here the fourth component depends on the charge density.

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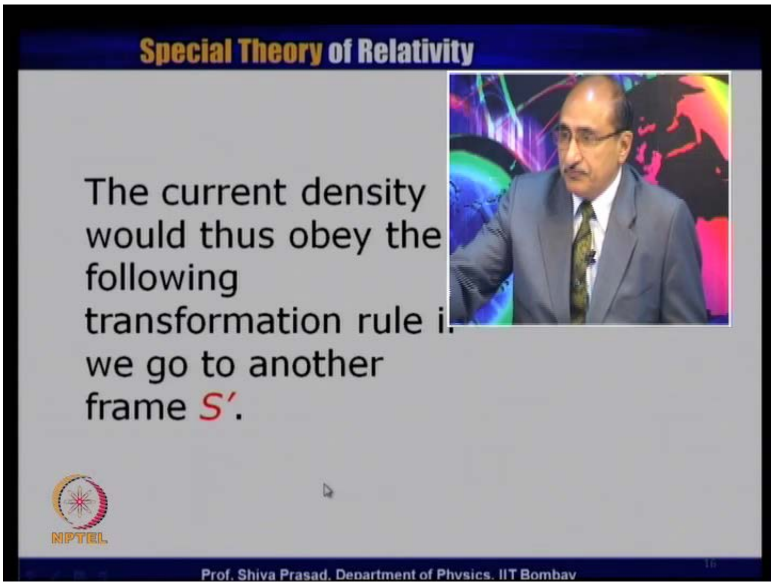

Now once we have found this particular four-vector, we know how to transform it. If I go for particular frame of S to any other frame S prime and if I know the relative velocity between S and S prime to be V, I can always transform this particular four-vector components into any other frame by using this standard matrix equation which we have been doing earlier number of times. So, the current density will obey the following transformation rule if we go to another frame S prime.

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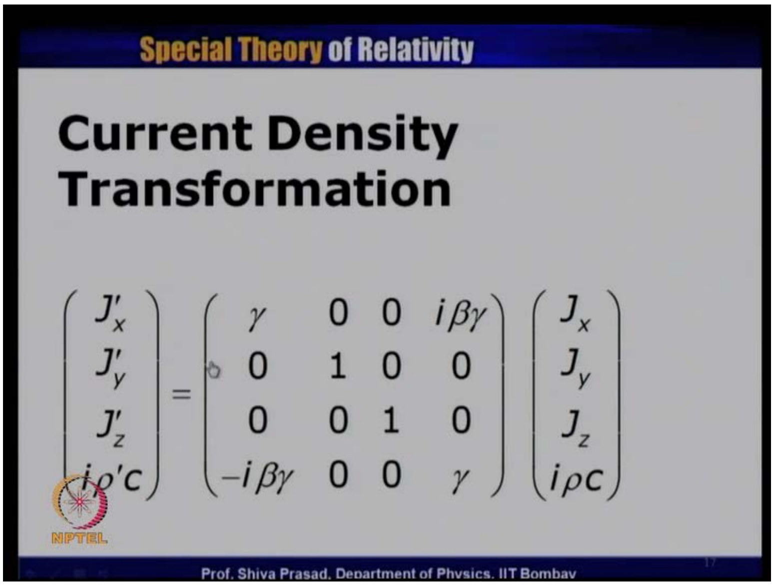

This is the current density transformation. So we have in S frame, these are the components. The current density is J x, J y, J z; these are the x, y and z components of the current density. The fourth component is I rho upon C where rho is the charge density as seen in the particular frame of reference. I want to find out the current density and the charge density in a different frame as prime frame of reference. And let those current densities be give or the components of the current densities in as prime be given by J x, prime, J y prime, J z prime, and let the charge density be given by rho prime.

Then these J x prime, J y prime, J z prime, rho prime would depend on J x, J y, J z and rho by this particular matrix equation transformation equation. Here of course, this gamma, beta, all these depends on the v the relative velocity between the frame between S and S prime; the standard way we have been dealing in relativity so far. So, now, I can expand this particular thing; we have done it a number of times. So, we have to spend too much of time in expanding this particular thing. I will just write the explanation of this particular equation which I shall deal in the next class.

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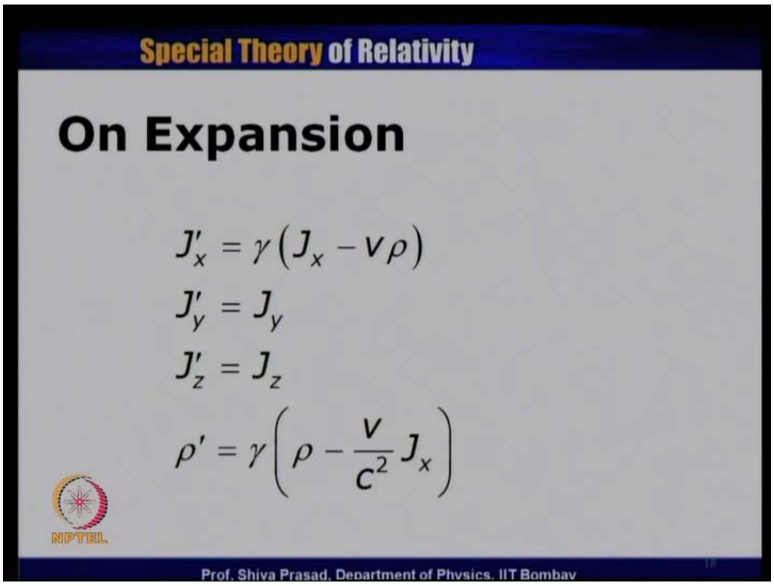

This is what it becomes. J x prime becomes equal to gamma times J x minus v rho, J y prime remains equal to J y, J z prime remains equal to J z, and the charge density now will be different in a different frame S prime as given by this particular equation rho prime will be gamma times rho minus V J x upon C square. So, this is somewhat like your x, y, z and if you want to remember; somewhat like x, y, z, t-transformation where x prime turned out to be gamma x minus V t. So, it is a similar type of equations if you go back in that particular case J x prime is gamma times J x minus V rho and rho prime is rho gamma minus V upon C square J x.

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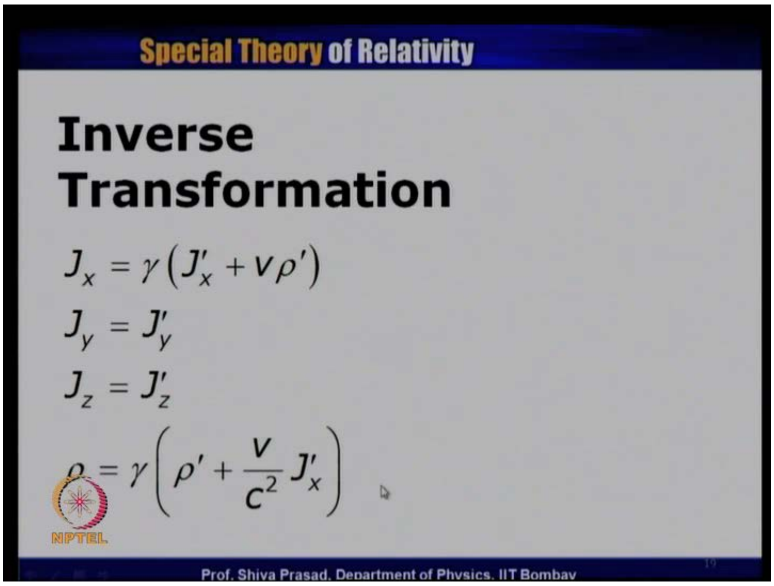

I can always write the inverse transformation equation. For that particular thing as we have said, change prime to unprimed quantities and vice versa; make v as minus V. So, this is what happens. As the inverse transformation, it means that if I know the current densities and the charge densities in S prime frame of reference, I can find out the current density and the charge density in S frame of reference by using these inverse transformation equations. So, this is what has happened to my transformation of current density. Now let us come back to the situation of current carrying conductor.

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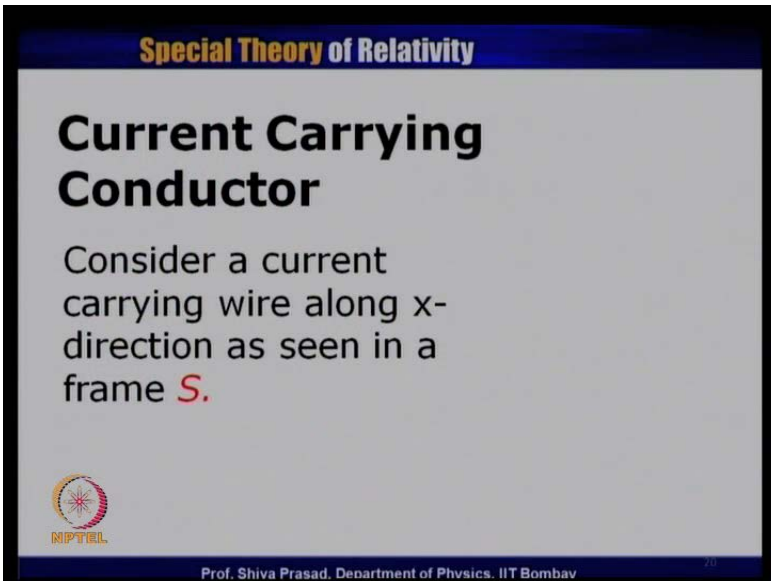

My idea of using this particular example of giving current carrying conductor is only to look little more deeply into the origin of these electric fields and magnetic fields as we change from one frame to another frame of reference. And this particular example, generally reasonably illustrative because many times we have a question; that suppose there is a particular frame of reference in which we see only magnetic field and you say that you go to a different frame of reference. Now in addition to this magnetic field, you also have an electric field. So, from where this electric field has come? Because once we say that the equation, the Maxwell's equation which are the basic equations do not change the relativity.

So, what is the origin of this electricity field? So, let us what we are trying to do is by giving this particular example to go little deeper into this particular aspect, try to somewhat understand these things. So now, let us come to a realistic conductor and let us assume that it is carrying a current in plus x-direction as seen in frame S. So, there is a frame S in which you see a current flowing in a particular wire or a conductor along the x-direction. Let us assume that charge carriers are actually the electrons. It means actually the electrons must be drifting in minus x-direction and that is the reason that the current is actually in the plus x-direction. As we have said, the direction of the motion direction of the drift of the electron is opposite to the direction of the conventional current.

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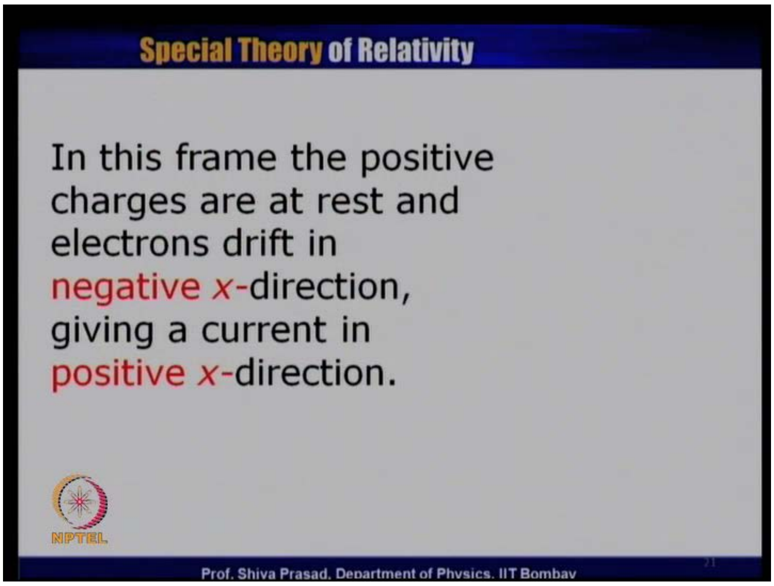

Now when we talk about the current we do not talk about this so clearly but let us be very very specific. I am looking this particular thing from a frame S and in this frame S, I assume that the positive charge carriers; remember the conductor overall is electrically neutral. It is a slightly different situation from what we have discussed now. I am talking now a realistic conductor. See, if there was no electric field, this particular conductor was electrically neutral and the negative charges are there, the positive charges both are there. Now these electrons tend to be somewhat free or approximately free and when we apply these electric fields, these electrons are the one which actually give rise to the current.

But those positive charges which are sort of immobile which are ions which have been left behind; they remain at rest, that is what is the normal picture of current which is always told when I was in high school. So, I am looking at the same frame in which this picture is correct. It means I am assuming that the positive charges are at rest and it is the negative charges which are drifting in negative x-direction to give rise to a current in the plus x-direction. That is what I have said here; in this frame, the positive charges are at rest and electrons drift in negative x-direction giving a current in positive x-direction, alright. Now let me assume we all of us know that, if there is a current carrying conductor; let us assume that is a long conductor. This conductor will generate electric magnetic field around it.

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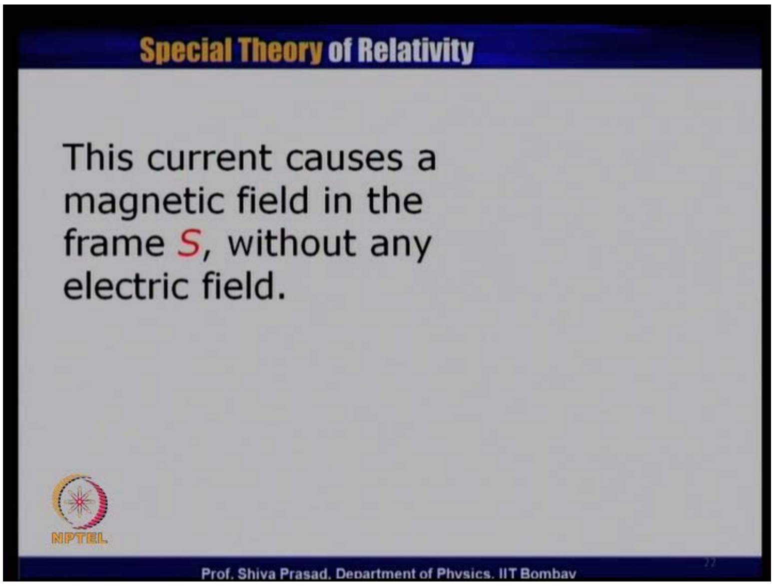

So because of this particular current, there is a magnetic field which is created around it. We know how to find out the direction of the magnetic field if the current going like this and you can find out. The magnetic field lines of forces may be circular with the wire passing through the center of the circles. These are well known; actually it is not discussed in detail. What I am also insisting is that all is generating only the magnetic field without any electric field. So, there is no electric field and the reason there is no electric field because they are positive charge carriers as well as negative charge carriers and in a given volume whatever small elements we have taken, we assume that there are same number positive charge carriers and same number of negative charge carriers.

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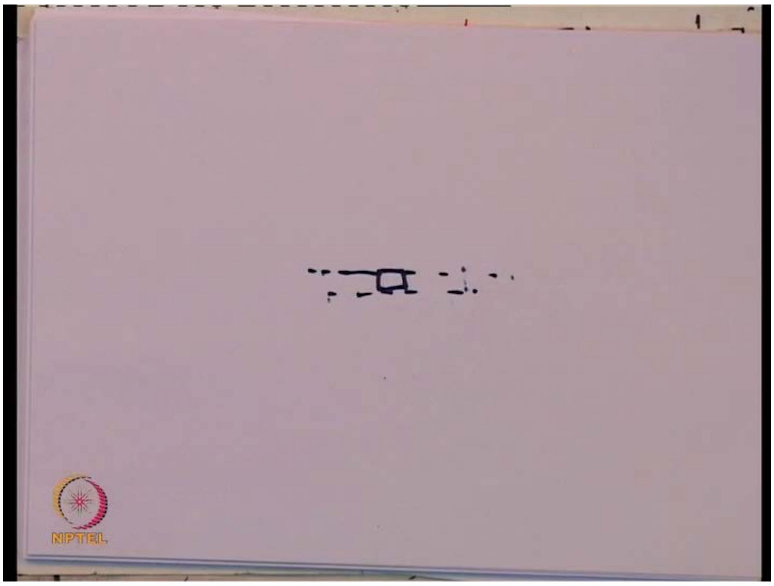

So, the net, let me just write it here. If we take a small element in a wire, in this element negative charge carriers or positive charge carriers, both are same. Therefore, if you take a small section of the wire, you do not find net charge density here and therefore, there is no electric field which is generated outside. There is only a flow of electron and at all the times, this charge neutrality is maintained in a small section of this particular wire. Therefore, there is no electric field, but there is a magnetic field. So, this is picture with which we are quite familiar in our high school, when we have discussing various laws related to magnetism; at that time we have been talking about this type of behavior.

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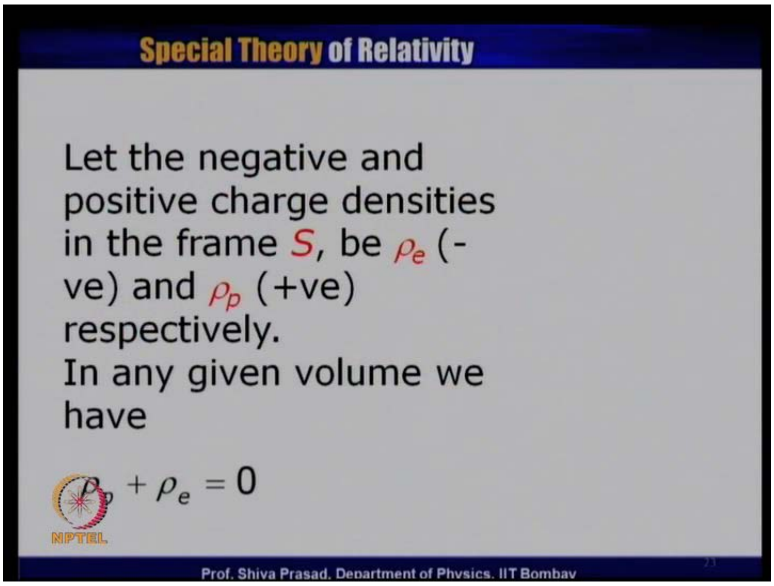

So, this is what I said. Let the negative and positive charge densities in frame S be rho e and rho p; this e symbol I have said for negative charge carriers, and this p symbol I have said for positive charge carriers. So, let us assume that the charge density for negative charge carriers is rho e and for positive charge carriers is rho p; of course, because this is charge density. So, this is negative because the charge is negative, the number density is not negative. Number density has to be always positive. While positive charge density is positive because this charge is positive, and because we are saying that there is no electric field in this particular frame of reference, I expect that if I take any small unit volume, necessarily a small; in fact it is a very small volume. You always find that this rho p plus rho e is equal to zero.

So, there is no charge density in a smallest volume; a small volume which you take along the wire. I am not writing in the differential format sector because that is not the idea. We want to make thing simple; so just want to say that I expect that this rho p plus rho e must be 0 in this particular frame because I do not see any electric field, I see only the magnetic field. There is a current, there is a current density, but there is no electric field. This current and current density causes the magnetic field. Now we realize that because in this particular frame, it is only the negative charges which are moving. So the current density, the net current density is being caused only by the motion of negative charge carriers; positive charge carrier would not contribute to any current density which is obvious, because they are not moving in this particular frame of reference.

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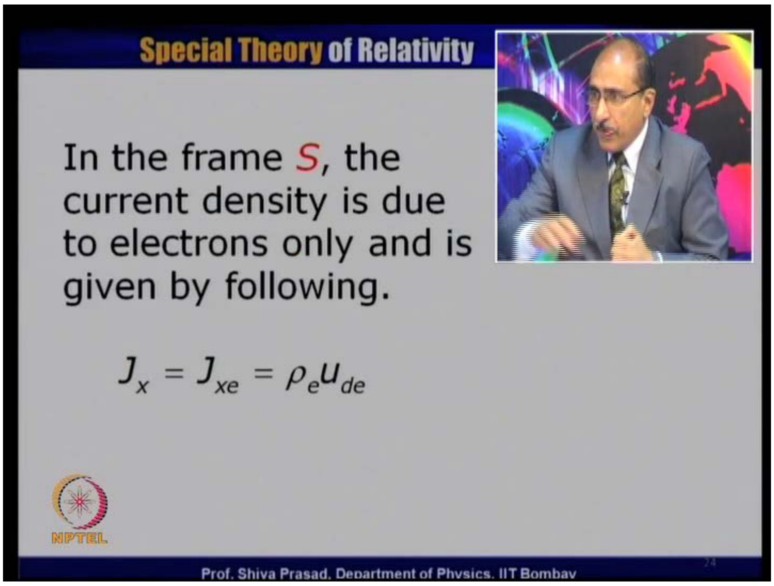

So, this is what I said. In this frame S, the current density is solely due to the electrons. Therefore, net current density I can write as equal to J xe; I am writing only x component because current density current I have been told that is only in the x-direction, current density also in the x-direction. J xe I am using to mention the x component of the current density for negative charge carriers and as we have said and J x is the overall net; the total current density. As we have said because positive charges are immobile, so this J x is solely because of J xe and that will be given by rho e multiplied by the drift velocity of the electrons or negative charge carriers; negative charge carriers we mean electron. So, this is what it will be given as rho e times U de where rho is the charge density, U de is the drift velocity we have already defined; this particular thing is exactly the same thing.

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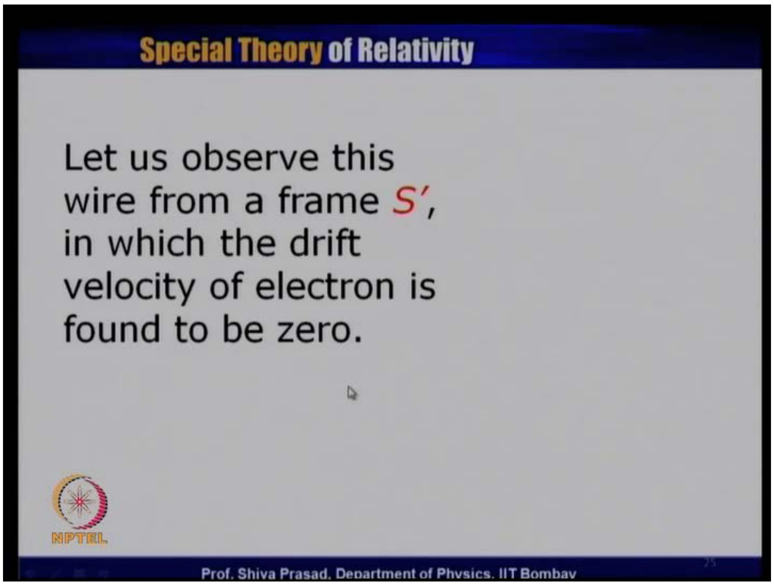

Now let us go to a different frame of reference S prime in which the drift velocity of electron is found to be zero. So, I have already found out what is the drift velocity. If I know the current density, I know what is the drift velocity; if I know the drift velocity, I can always go to a frame of reference, an inertial frame of reference which has their same relative velocity with respect to S as the drift velocity of the electrons.

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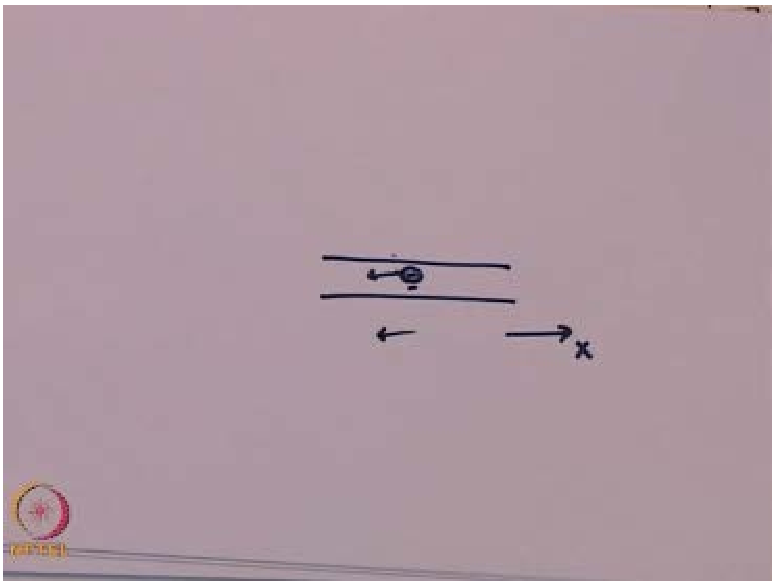

What it means is that, if we have wire in which these electrons are moving in this particular direction; this is plus x-direction. Let us consider one electron which has exactly the same speed; let us not bother about that. Just consider a drift velocity and assume that overall electrons are drifting in this particular way. I go to a particular frame of reference which has exactly the same velocity; of course this frame of reference also has move in this particular direction so that they find that overall, there is no drift of electrons in this particular frame of reference.

So in this particular S prime frame of reference, I am defining this particular frame of reference S prime as one frame of reference which of course is inertial frame of reference, because drift velocity is supposed to be constant for a given electric field. So long, current density is constant; the drift velocity is also constant. So, I am looking at this particular aspect from a frame of reference S prime in which electrons are not found to be drifting. So that is what I said, let us observe this wire from a frame S prime in which the drift velocity of the electron is found to be zero. Let us see what an observer in S prime frame would notice; would it notice that the current has become zero and therefore, there should not be any magnetic field. No, that is not a correct picture.

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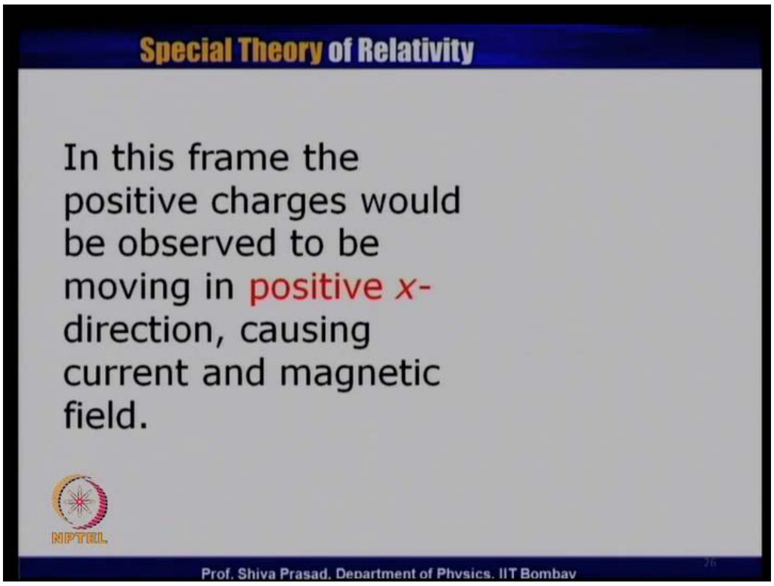

Actually in this frame, the positive charges will be observed to be moving in plus x-direction. See remember, it was in S frame that your electrons were drifting at the positive charges were stationary. But once I have changed my frame of reference to make the electrons stationary in that particular frame of reference; in that particular frame of reference, the positive charges will move and because as we have said here, this velocity is in the negative direction.

So, the positive charges which were mobile earlier which would appear to a person sitting here to be moving in positive x-direction. Therefore, an observer in S prime would feel that all these positive charge carriers are moving in plus x-direction. So, there is a current in that particular frame of reference but that current is being caused by the positive charge carriers and this positive charge carrier will actually produce a magnetic field. Therefore, a magnetic field is expected to be present also in S prime frame of reference. What about electric field?

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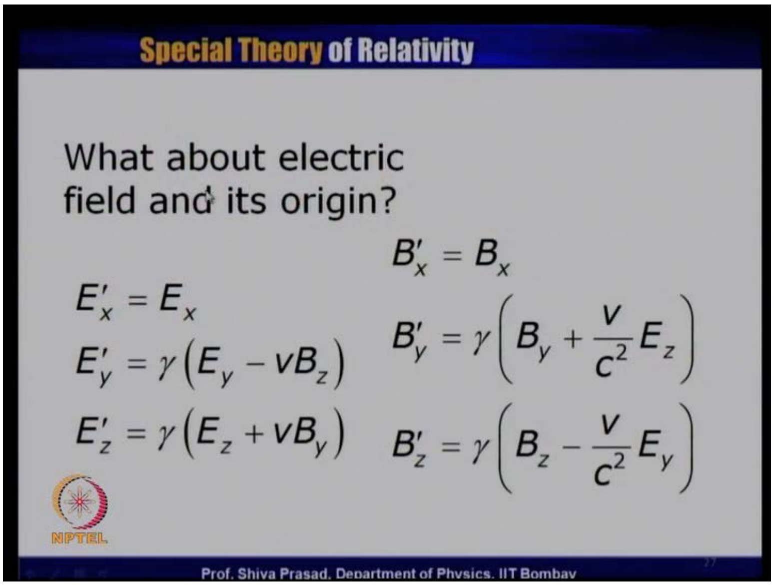

If you look at these equations, these are the transformation equation which I have written from S to S prime frame of reference. Of course in this case, we have x to be negative; lets forgot about it and I am just writing in the number form. If you look at this particular equation, of course E x prime will turn out to be equal to Ex, but E y prime will depend on V B z. E z prime will depend on V B y. Depending upon which point you are looking or you are looking at the field which point; of course the current in the x-direction. You will always find; you may always find at least a component of B y or B z or both.

Even though E y and E z are zero because we have said in S frame there is no electric field. But there is a magnetic field and this magnetic field has to be in some direction; of course it cannot be in x-direction because the current flows in the x-direction. And the magnetic field is being caused only by that current flow and I am assuming this to be infinite wires or sought of infinite wire. So therefore, at least one of these will be non-zero. It means in this particular frame in addition to the magnetic field, the observer would also find electric field. This is going to happen if my transformation equations are correct.

So, these electric fields are present in the particular frame of reference. From where are they arising? Earlier we have said that their charge densities were just neutralizing at each small volume of the wire and therefore, there was no net charge in any small section of the wire and therefore, there was no electric field earlier. From where the electric field is coming; so now we will realize, that is what we will do just now, to show that it was too earlier in S frame. In S prime frame of reference, if you take a small section of the wire; now you will find it to be charged and if you find it to be charged, it is this charge which will generate this electric field. So, electric field will be present in this frame of reference because that particular section; I mean sections of wire being charged. How do I do that? For particular I must know a charge density transformation; that is what I am going to do next.

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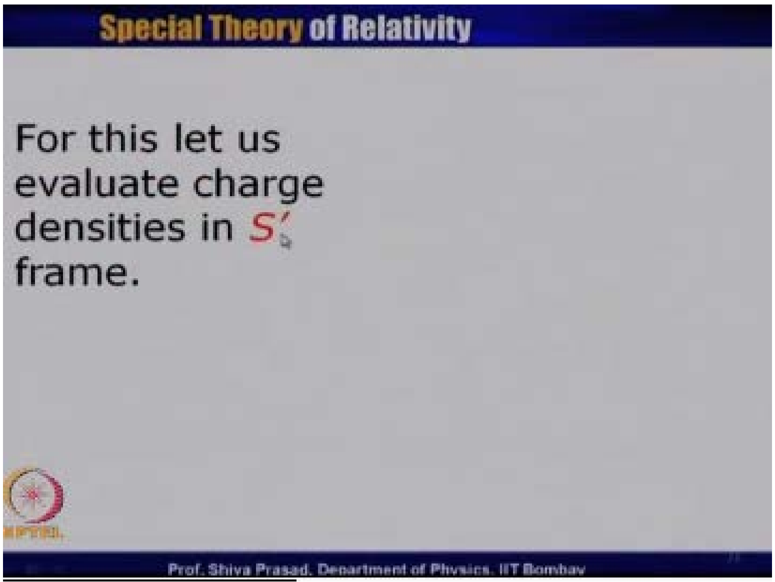

For that let us evaluate the charge densities in S prime frame of reference. I have just now evaluated; I just now found out the transformation equation relating to the charge densities. Let us do the same thing here, find out what will be rho prime if I know rho and S, what will be rho prime.

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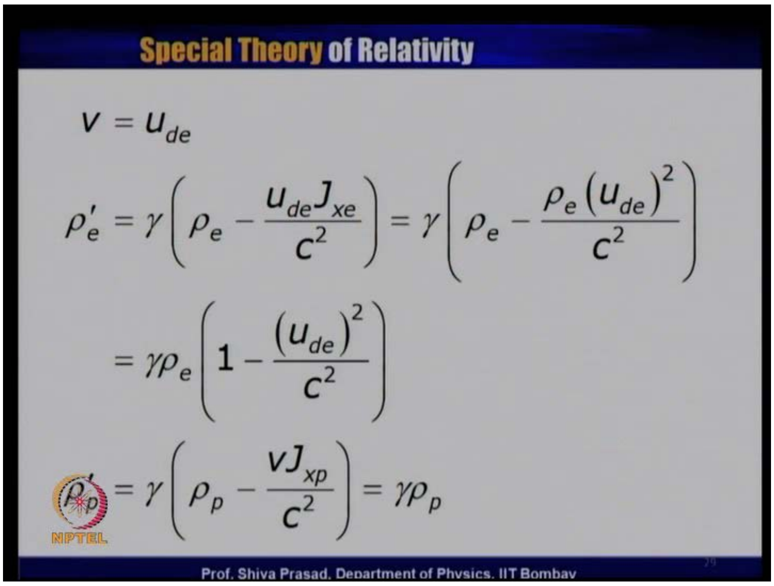

Of course, relative velocity of between the frames I have taken specifically equal to U de; of course I have reserved symbol U instead of V. Though when we originally defined charge densities we had used symbol V, but I am using U because V we have as I will say always reserved for the relative velocity between the frames. Of course in this specific example, I am taking V is equal to U de because only in that particular case, the electrons or the negative charge carriers will be at rest. So, this is my transformation equation for the negative charge carriers rho e prime is equal to gamma and rho e minus U de because V is equal to U de J xe whatever was the current density divided by C square.

So, this is gamma rho J xe I can write as rho e into U de. So, this rho e I have written here, this becomes U de square divided by C square. This rho e I can take it out, this will become gamma rho e multiplied by one minus U de divided by C square; put this is also squared; U de is the drift velocity of electrons as was seen in S frame. So this will be the charge density of electron which will be seen in S prime frame of reference. What will happen to the charge density of positive charge carriers? I will write exactly the same equation, gamma rho p minus V J x p divided by C square where J x p is the current density of positive charge carriers in S frame.

But I know that this J x p is zero because the charges are not moving in S frame of reference; therefore, current density of that particular frame has to be zero. Therefore, this quantity will be zero and this I can write as gamma rho p. As we can seen that the charge densities as seen in S prime frame of reference are not same because of this transformation; therefore, this is not equal to this. They were earlier same; the magnitudes were earlier same, of course signs were different. Their magnitudes were same in S frame, but in S prime frame of reference even their magnitudes have become different. So, let us evaluate the net charge density in S prime frame of reference which is just the sum of these two which earlier was zero.

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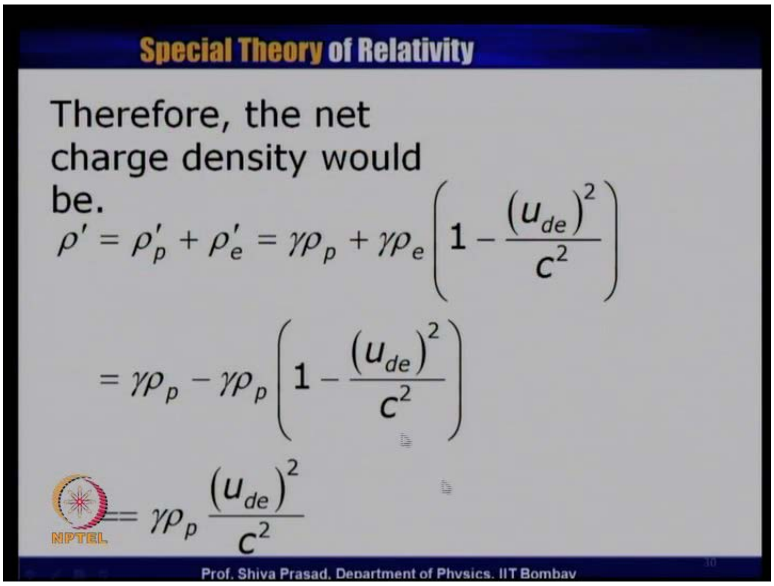

So, I have just used this particular equation which is from this particular transparency this plus this; this is what I have written here. This gamma rho E would cancel out here. Remember this rho E was equal to minus rho p in S prime frame of reference. So this rho e has been changed as minus rho p here. So, once I do that this gamma rho p and this will cancel out and this will become gamma rho p U de square upon C square. So, we find that there is a net charge density in this particular wire. So in S prime frame of reference, it will find that the wire is charged' if you take one particular section of the wire, it will be charged and if it charged of course, we generate electric field.

Now what happens to charge conservation? Some of you may ask that, does it mean to say that this wire overall has become electrically non-neutral. Originally we have said that the conductor was electrically neutral. Now we are saying that there is a net charge density. Does it mean to say that, it is no longer electric neutral? What happens; from where we got the additional charges? Is the charge conservation obeyed or not? The question is that see normally you will not have, strictly speaking, infinite wire. You will always have a loop. So once you are supplying the current, now there will be current flowing and then eventually the current has to flow back into a particular direction.

So, whatever is the situation, you will always have situation somewhere where you have wire and this wire eventually has to close. Here there has to be some current source. Now as you can see, the current direction in this section of the wire is going to become different from whatever is here. If I am looking at positive x-direction, the charges are going; let us say the current is going on this particular direction on this way, while here going this way. So inverse section of the wire if I am finding it positively charged; the other section of the wire I will find negatively charged.

So, overall the wire will still remain to be electrically neutral. So nothing happens to that particular thing once we realize that thing. But if you take a specific section the way we have happen defining this particular thing, we feel only because of this particular wire that comes, because of this positive charges which are being in that particular frame; we use the particular charge carrier, this particular wire as a charged charge now.

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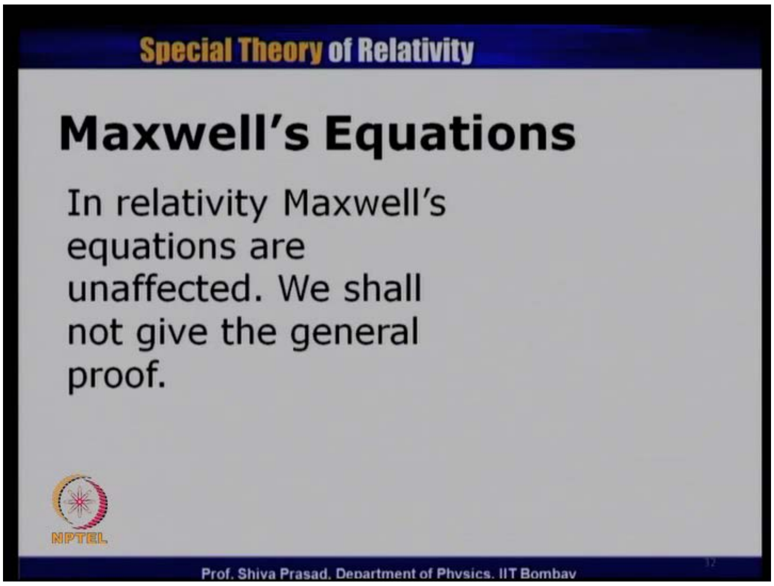

Now with this particular thing as I said, this is probably the last section that we have wanted to cover formally; I want to just introduce how the Maxwell's equation we expect them to remain invariant. In fact if you remember the electric field and magnetic field transformation, we had obtained from the force equation; the Lorentz force equation and saying that this force must obey a force transformation law. In fact, they can also be derived by maintaining that the Maxwell's equations are unaltered or unaffected when I change my frame of reference. We do not change any magnetic equations unlike we have changed many of the classical mechanics equation; the equations pertaining to electromagnetic, electromagnetic theory.

They do not change, must be changed by frame of reference. The equations which are the basic Maxwell's equation, four Maxwell's equation, they do not change. So, I will not proving general in general which is beyond the outcome of this course which is required; if you want a general proof much more, many more details. We will just take one simple example and convince you and I am always trying to say that I am trying to convince that Maxwell's equations are expected; nothing like that, nothing more than that. So, this is what I have said in relative to the Maxwell's equations are unaffected; we shall not give the general proof, we are just trying to convince you.

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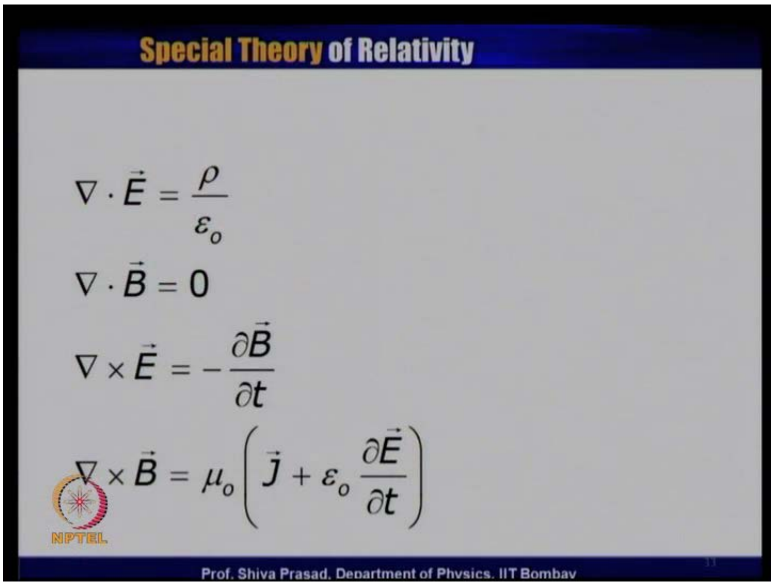

So, these are the four Maxwell's equations which are very very standard equations. The first equation in essentially what we call as a Gauss's law which talks of divergence of electric field in terms of the charge density. This is a similar Gauss's law for the magnetostatics which is divergence of B equal to 0 and this quantity 0 because it always says that there is no monopole. This is what is the curl of electric field which is called

Faraday’s laws of electromagnetic induction and this is Ampere’s law with the connection of Maxwell pertaining to the displacement current which gives you current B, so the two equations are having divergence.

There are two equations which are having curl, and these are the four equations which we call as Maxwell’s equation. So, I will take one of these equations; this particular equation, the third equation and out of that I will take one component and try to convince you that this does not alter once I go for a frame S to S prime frame of reference. So, this is what I am trying to do in this particular way as a last part of this particular course

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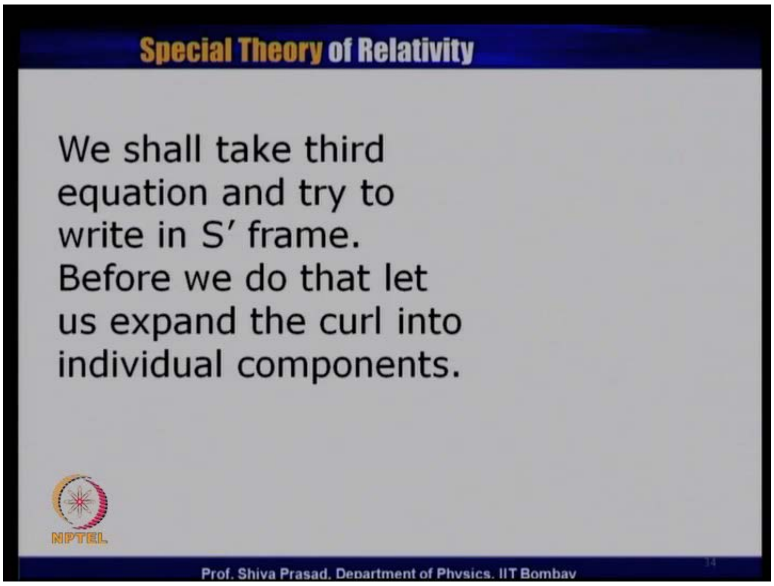

So we said, we shall take third equation and try to write this in S prime frame. But before we do that, let us expand this curl into individual components; remember it has a curl here. There is a curl of electric field and here you have derivative of time, derivative of magnetic field. So, let us first write this into the component form. Then this components will of course be in the form of x, y, and z and on the right-hand side, you will have time derivative. Now I will change these x, y, z to x prime, y prime, and z prime; this t to t prime.

Assuming that x prime, y prime, z prime, t prime, are related to x, y, z, t by Lorentz transformation because that is what is the relativistic transformation. And try to see that I can write exactly similar equation in S prime frame of reference where E will be replaced by E prime and B will be replaced by B prime and this curl will be all with respect to x prime, y prime, z prime, and this t will be with respect to t prime. First I may have to write that, I have show that is Maxwell's equation remain invariant.

Of course, we can do similar type of things for all other equations. We know how the charge density transforms; we have just now discussed. We have also known how current density transforms, that also we have discussed. So, we know the transformation of all these equations. So in principle, you could have taken any equation and try to proof it. But as I said I am not doing that in general, just taking this particular equation trying to convince you. So, let us first expand the curl del cross E.

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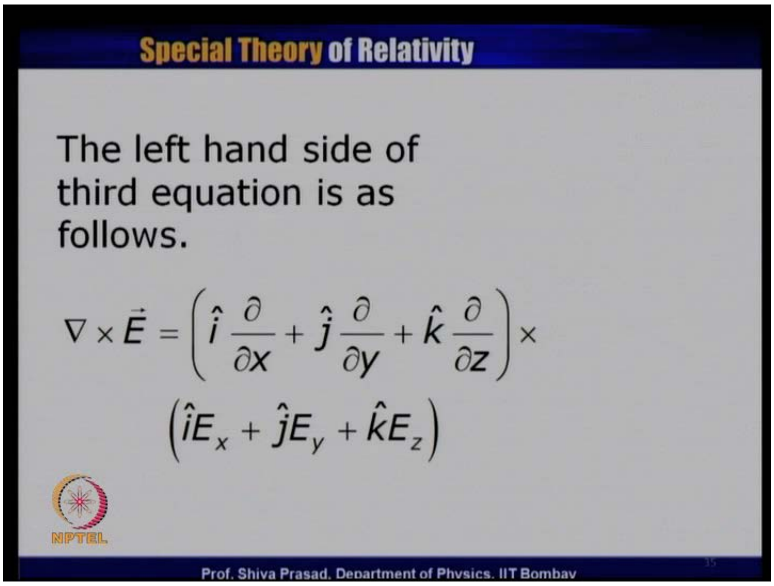

So, left hand side of that equation had just curl of E; that is del cross E. Of course, you can use any method of expanding it; some people will try to write in form of determinant; then try to expand it. I have written in a particular vector form which I find generally much more easy to remember but in any way one prefers for this you can write; this particular way also we can expand this curl. So, this del operator as we call can be written as i del del x plus j del del y plus k del del z, where I, j, k are the unit vectors. Then cross E i can write in the vector form as i E x plus j E y plus k E z; again i, j, k are unit vectors in the x-, y-, and z-direction, this standard way.

So, I have to expand this curl; it means I have to take, this cross this, this cross this, this cross this, then plus this cross this, plus this cross this, plus this cross this, then this cross this, this cross this, this cross this. If I take first term i del del x, when you take i cross i, I will get zero; i cross j will give me k; and i cross k will give me minus j. Similarly j cross i will give minus k; j cross j will give me zero; and j cross k will give me i. Here also k cross k will give me zero.

So all I wanted to say that normally if you would have expanded this, you will have got nine terms; because one has to operate on three, second also has to operate on three, third also has to operate on three; we will get nine terms. But out of those, three terms will be terms zero. One because of i cross i, another because of j cross j, and third will be because of k cross k. So, eventually you will be landing only with these six terms. I am not giving the details of these things, I can think it and mark out simply and try to convince yourself that whatever I have written is probably correct.

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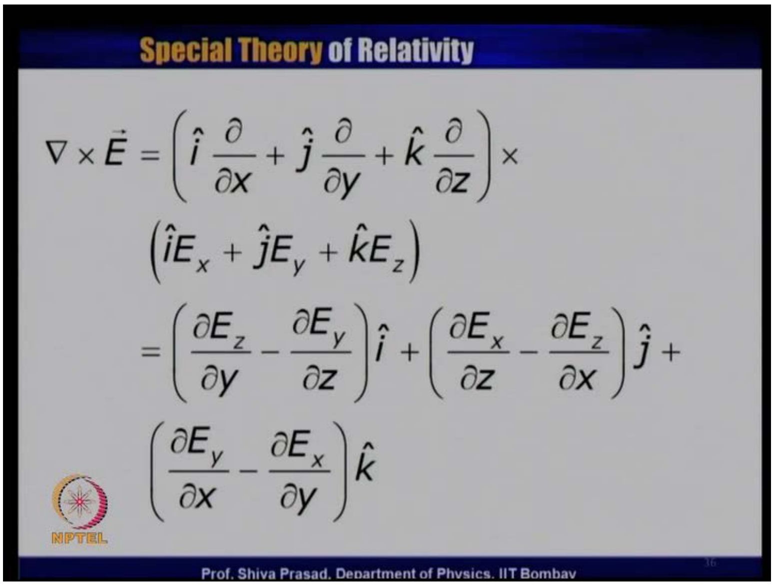

So, these are my cross products and all I have done is expanded into this particular form. As I have said that this explanation you could have your doubt in any other way which is comfortable to you. So all I have written is six terms and content kept those terms along the x-direction here, y-direction here, and the z-direction here. So, this del cross E i can writen like this; that this has to be equated to the right-hand side which had del B del t.

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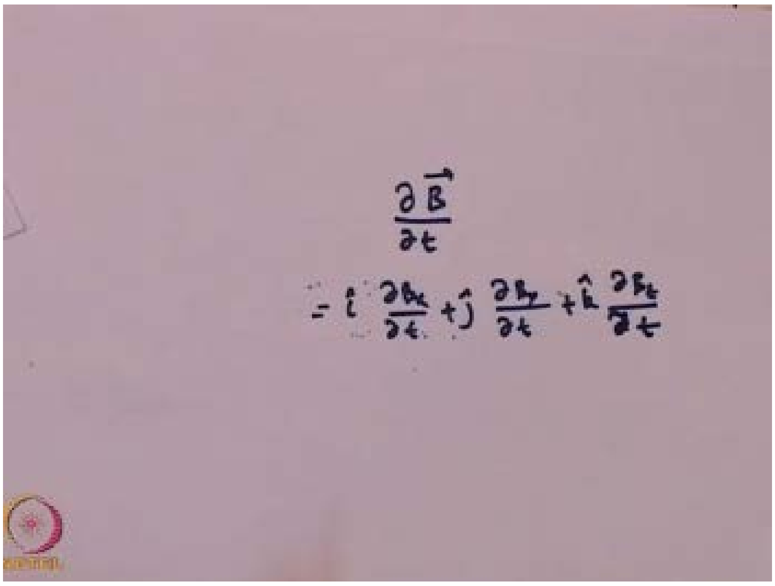

The x component and y component and z component of this equation, in fact can be written as i del B x del t plus j del B y del t plus k del B z del t. Then this x component can be equated to x component of the curl that we have obtained; y component can be equated to the y component of the curl that I have found out; z component can be equated to the z component. So, I will get three equations which I am writing in the next transparency.

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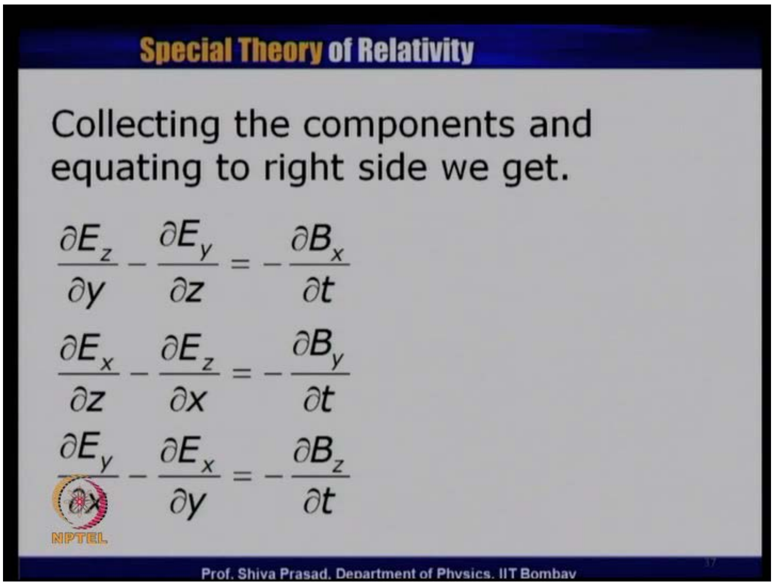

So, these are collecting them; we have collected all the components and equated to the right-hand side. So, I get these three equations. Now I will use only one of this equation; this particular equation, the second equation. Try to transform into S prime frame of reference by using this standard equation corresponding to the partial derivatives. I will not go into the details because this requires lot of time but as I say my idea is only to convince you and not to give you too much of details of the mathematics, which I say that is beyond the scope of this particular course.

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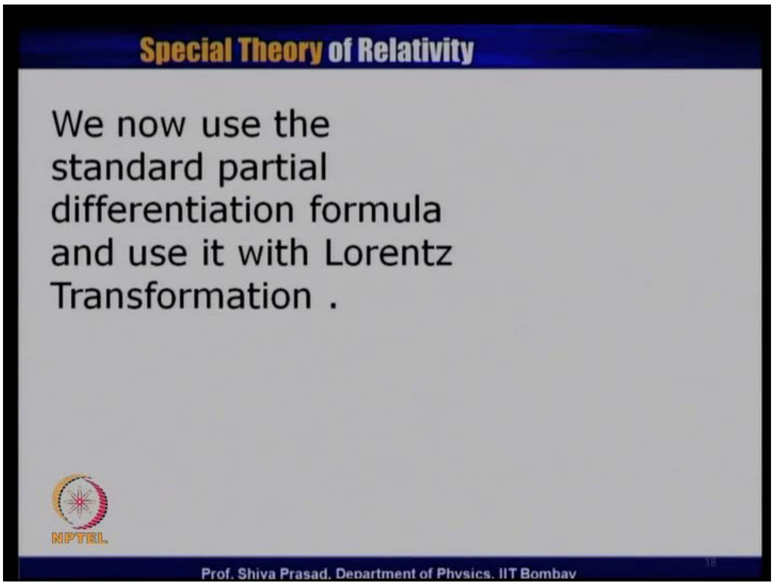

So, we will use the standard partial differentiation formula and use it along with the Lorentz transformation. What is this particular formula? This del del x can be written as del x prime del x del del x prime plus del y prime del x del del y prime plus del z prime del x del del z prime plus del t prime del x del del t prime. This is the standard partial derivative formula. Then you have therefore variables on which it is dependent. I know from Lorentz transformation x prime is equal to gamma x minus V t, y prime is equal to y, z prime is equal to z, t prime is equal to gamma t minus Vx upon C square.

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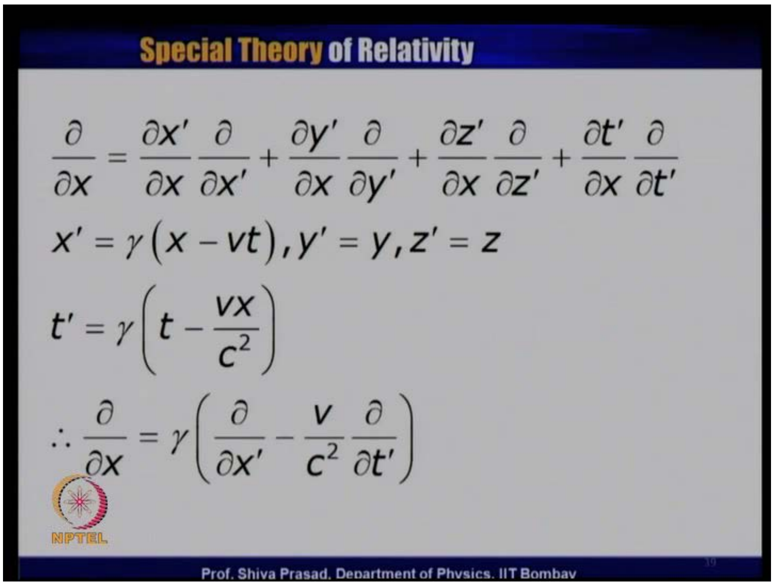

So, let us examine this particular term. If I take del x prime del x, this is x prime, this is partial derivative; it means all other variables I have take them as constant. So, t will turn on constant. Once I take the partial derivative of x with respect to x prime with respect to x, so I will just get gamma. This term will give be zero. So, this becomes gamma into del del x prime. Similarly del y prime del x if I take partial derivative with respect to x, it means y has to be treated as constant.

So, this will give me zero, del z prime del x will give me zero. Here del t prime del x, t prime does depend on x. Once I take partial derivative, t has to be taken as constant. I take derivative, so I will get minus gamma t V upon C square. So this is gamma not t, minus gamma V upon C square. So, minus gamma V upon C square into del del t prime. So, this operator del del x can be replaced by this operator once I change my frame from S to S prime frame of reference.

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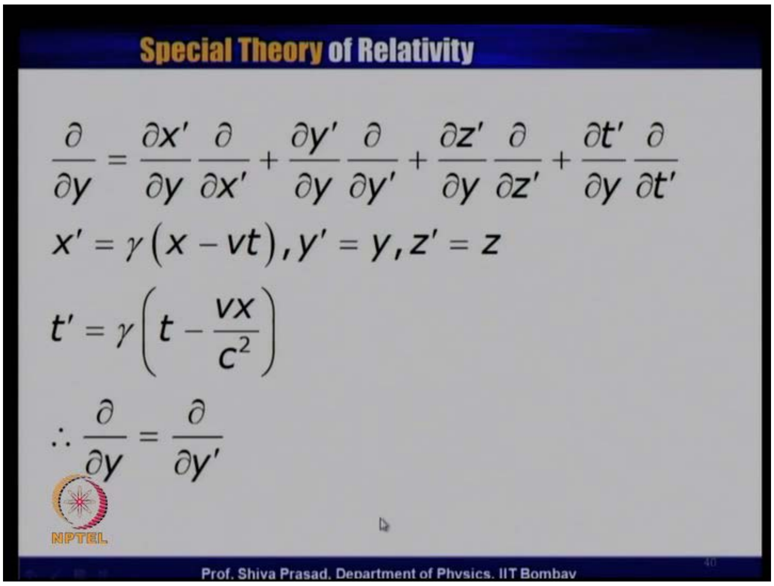

I can write similar equations for y which will give me very clearly del y is equal to del y prime. This you can write to work it out yourself and exactly, similarly I will get del del z is equal to del del z prime.

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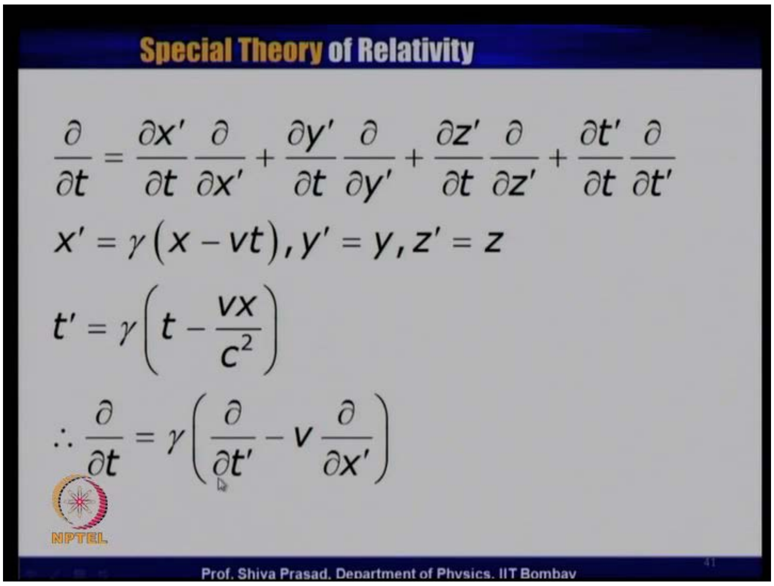

Again del del t will be different from del t prime. I use exactly the same thing, again you can work it out yourself; del del t will turn out to be equal to gamma del del t prime minus V del del x prime. So now, I know how to replace these partial derivatives from S frame to S prime frame of reference.

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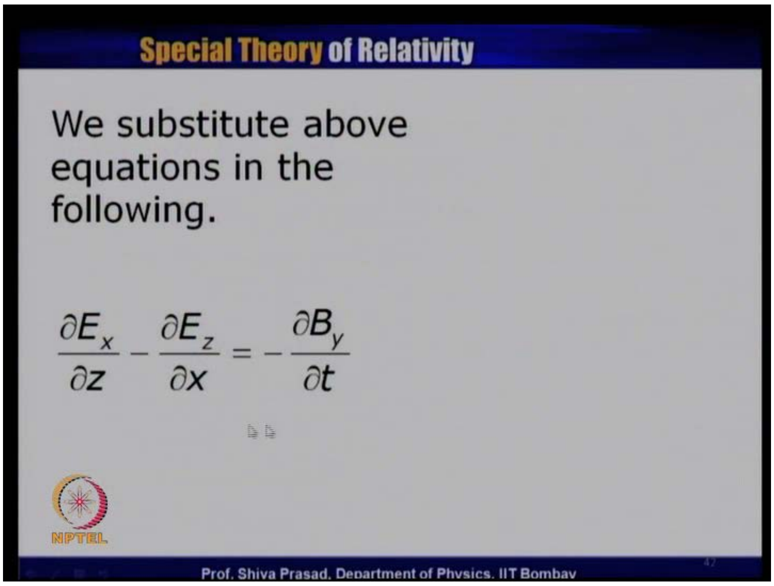

So, these things I will substitute in this particular equation. So this del del z prime del del z, I will replace by whatever we have valued it earlier. Of course as delta z is concerned, it is just that z becomes z prime. Here there is a partial derivative with respect to x. So this once I change to del x prime, I will get another term. Similarly when this partial derivative with respect to time, I will get another term which I am doing in the next class we will see.

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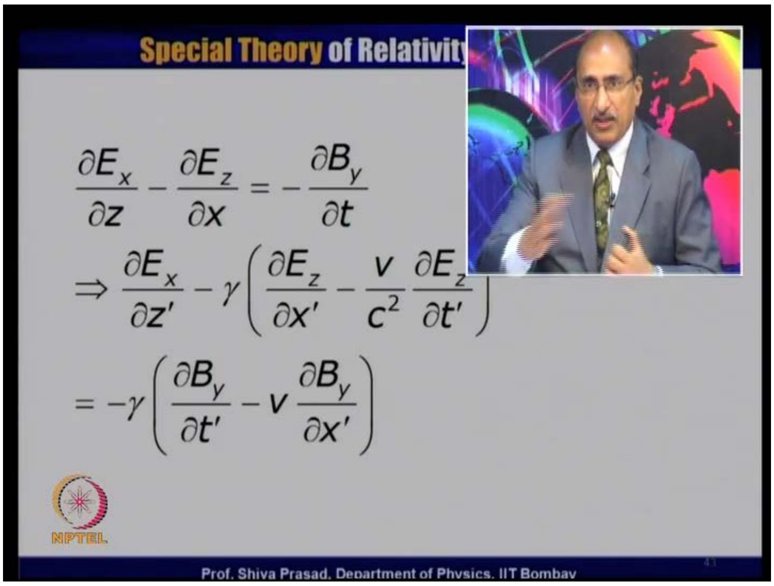

I am rather going fast, but idea as I have to say is to convince you. So, this equation as I said remain identical; del del x I am replacing by this equation. So, once I go to prime frame of reference, this will become del E z del x prime minus V upon C square del E z del t prime. This del t also gets changed and this becomes del B y del t prime minus V del B y del x prime. So, this equation now becomes in this particular form. What I will do, I will collect terms correspond to partial derivative with respect to x and with respect to partial derivative with respect to t.

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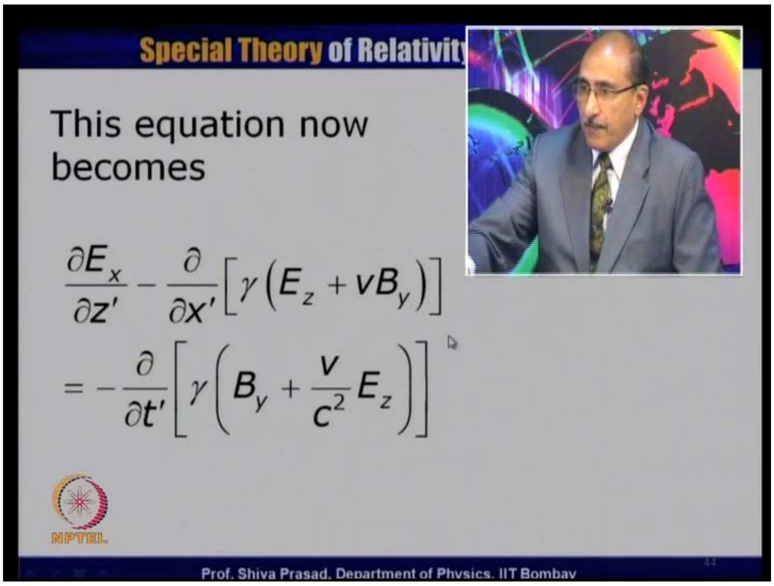

So if I do that, this equation becomes in this form. This is interesting because normally if a similar equation has to be obeyed in S prime frame of reference.

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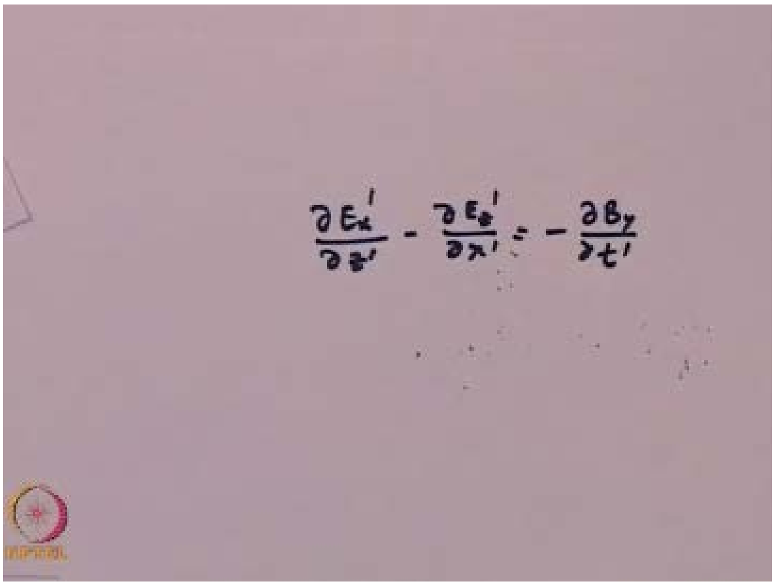

I expect del E x prime is equal to del z prime minus del E t prime by del x prime is equal to del B y by del t prime. This is an equation which I expect to be true because this is exactly the same equation. I am sorry there should be a prime here. Because this what I expect if the same equation I am able to write in S prime frame of reference, I will be able to say that things are consistent in S prime frame of reference. This is the equation which I have written here. So if I write this equal to this, this happens to be equal to E z prime and this happens to be equal to B y prime, I know that these equations will be identical in S prime frame of reference and this is precisely what I know is true from electric field and magnetic field transformation.

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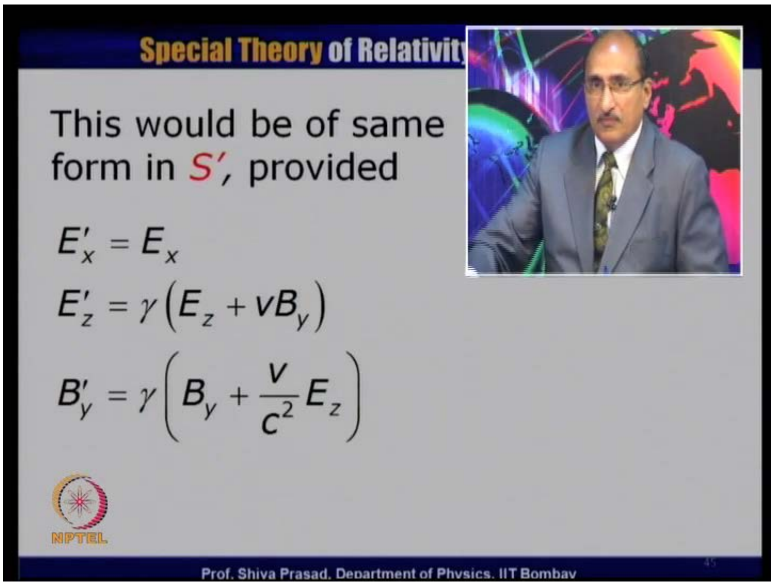

And of course, E x must be equal to S prime. So, from this I can write these equations. So, these equation will be consist ant in S prime provided we have these equations valid. I know from electric field and magnetic field of transformation that these equations are correct. So, I expect this equation to be valid also in S prime frame of reference. So, what I have only done is shown only for one equation. You can try to show for other equation; you have to know somewhat more mathematics and try to convince yourself that Maxwell's equations are actually consistent in all both the frames. They remain unaltered; we do not change Maxwell's equation once I go from S frame to S prime frame of reference.

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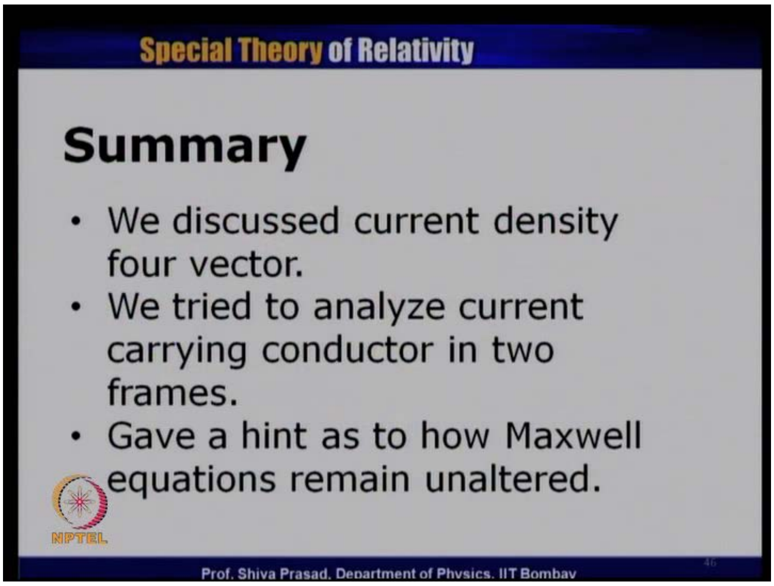

So, this is my summary. We discussed current density four-vector. We tried to analyze the current carrying conductor in two frames. Especially discussed how the electric field is generated in that particular frame of reference and given a hint how Maxwell's equation remain unaltered. So, this happens to be the end of the course. Best of luck all the best.