Special Theory of Relativity
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Lecture - 23
Example of EM Field Transformation

Hello, in our last lecture we had described the concept of electric field and magnetic field transformation. We had said that relativity a field can be mixture of the two, what it means that if in a particular frame, the field experienced by a charge is purely electrical it is likely or in general, it will always be that in any other frame it will experience both a combination of electric and magnetic field. So, what we generally call we call as an electro-magnetic field, because is nothing purely electrical purely magnetic it depends on the frame from which you are doing.

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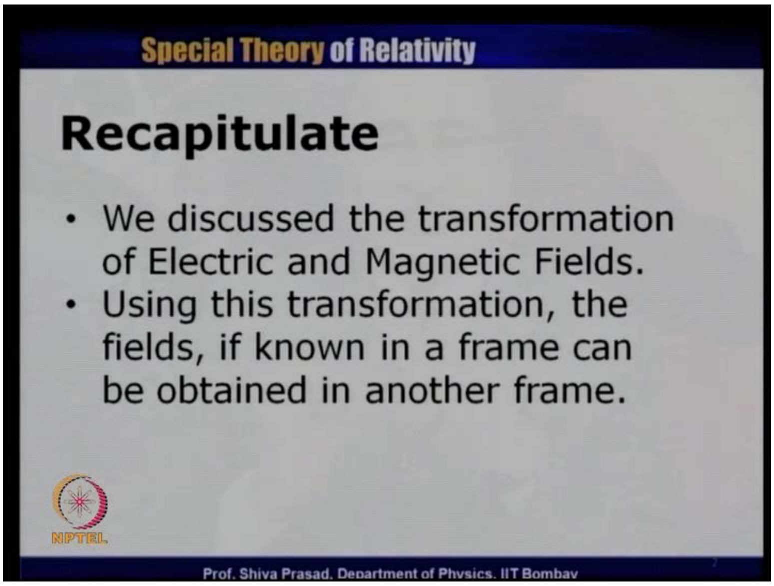

So, this is what I am recapitulating, we discussed the transformation of electric and magnetic fields using the transformation we found out that if I know the field - electric and magnetic field in a given frame how do I find out the electric and magnetic field in a different frame. This is what we have discussed in our last lecture.

Today, what I will try to do is first to tell that some of these equations can be derived in a different fashion. Because they happened to be somewhat interdependent, so that is the first aspect I will discuss then I will give one example, one specific example of how electric field and magnetic fields will transform once I go from one frame to another

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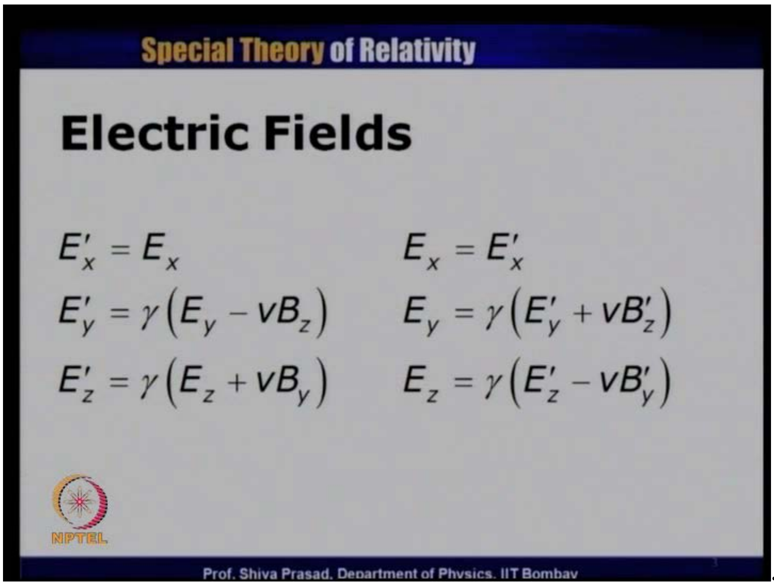

So, this is the electric field transformation that we had written, if I know in a frame s the electric field components it means I know E x, E y, and E z and I know the magnetic field components that is B x B y and B z, then in a different frame s prime of course, s and s prime frames have to have the same relationship as we have discussed in the earlier Lorentz transformation. This E x prime will turn out to be equal to E x, E y prime will turn out to be equal to gamma E y minus v B z and, E z prime will turn out to be equal to gamma E z plus v B y.

Similarly, we can write the inverse transformation, which means that if I know the fields in the primed frame of reference I can find out what is what are going to be the electric fields in unprimed frame of reference or in s frame of reference. Now, these equations tell only half the story, because they only will tell you how to evaluate the x, y, and z component of electric fields, we also require equations which can tell us what will be the x, y and z component of the magnetic field. So, there is a different set of equations which gives the transformation or which enables us to find out the magnetic field in a frame if I know the electric field and magnetic field in a different frame.

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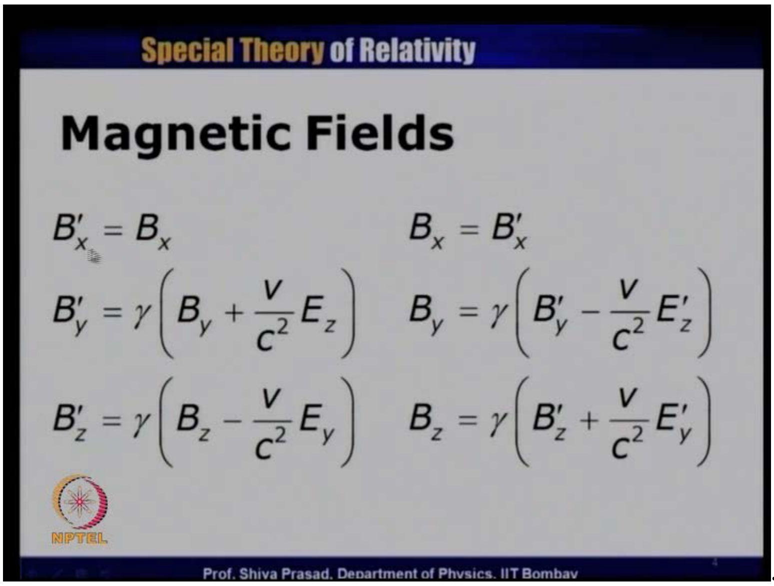

So these are the set of equations so b x prime turns out to be equal to b x B y prime turns out to be equal to gamma B y plus v by c square e z B z prime turns out to be equal to gamma B z minus v by c square E y.

Similarly, this inverse transformation the fields are known in s prime frame of reference and using these equations I can find out what will be the magnetic field in the unprimed frame of reference or in the s frame s frame of reference. So, these are the equations which these are the equations which enable us to find out the magnetic field so this in combination with the earlier three equations which were electric fields I can find out all the components the electric field components as well as the magnetic field components in a given frame.

Now, what I wanted to tell is that these last three equations which we have obtained here for example, can also be derived using the last two of electric field equations, which are these equations so let me first discuss these particular aspect because the some sort of interdependence of these equations on each other so let me first discuss that particular aspect before we come to an actual example.

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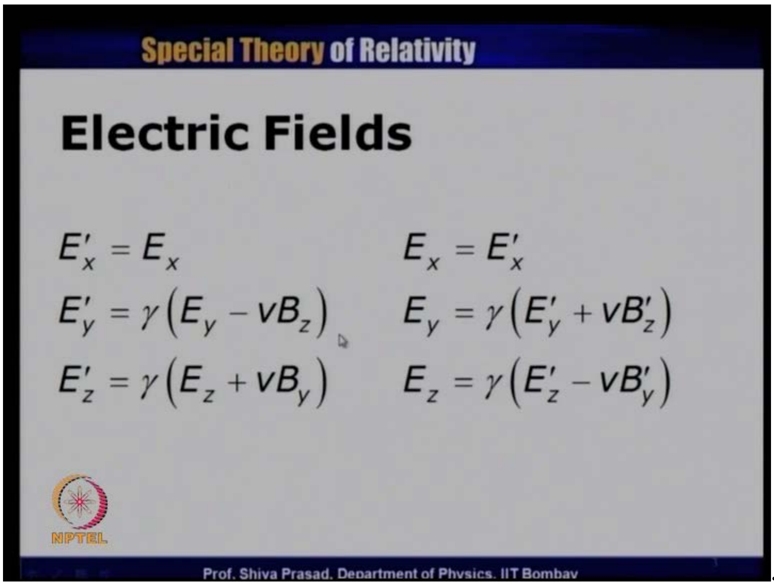

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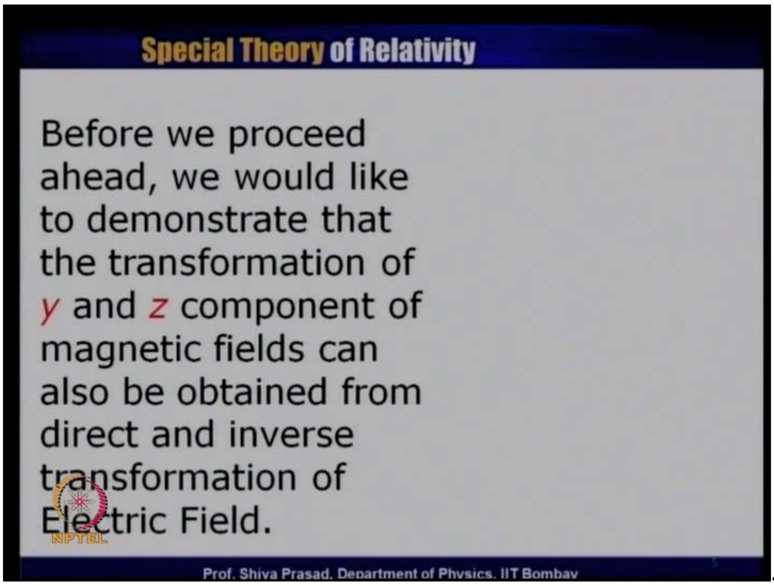

So, this is what I have written before we proceed ahead we would like to demonstrate that the transformation of y and z component of magnetic fields can also be obtained from direct and inverse transformation of electric fields, so this is what we will try initially.

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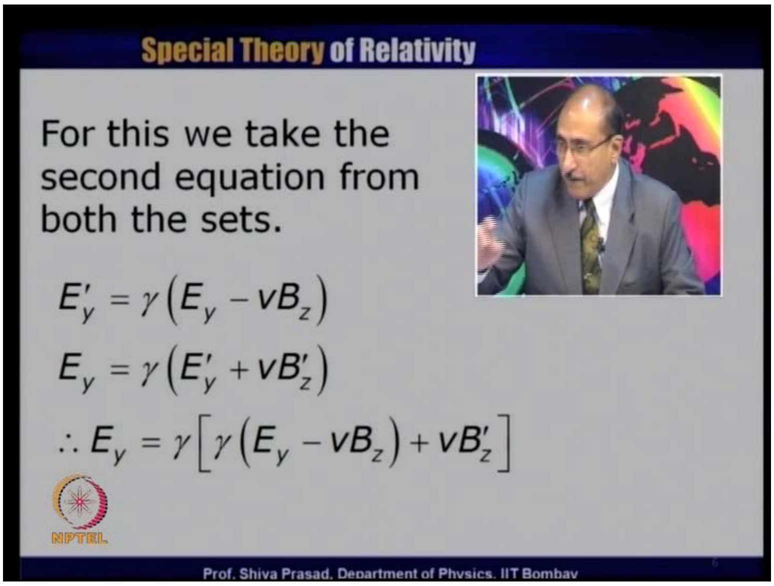

Try to show so what I will do I will first write the second equation as I said, from both the see here on this particular piece of paper I have already written these equations.

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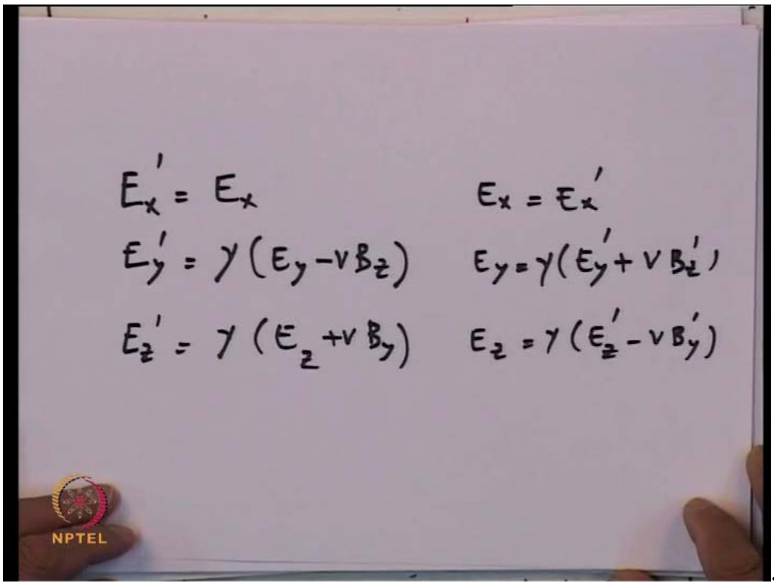

These are the equations which we are which enable us to determine the electric field in a particular frame of reference. Let me look at the second equation this is this equation as well as this equation this equation tells me that if I know E y and B z I can find out E y prime this equation tells me that if I know E y prime and B z prime I can find out E y so what I will do I will try to eliminate E y prime from this particular equation using this equation and as you can (( )) as you would see that from this we can obtain what will be the value of B z prime or the magnetic field transformation equation.

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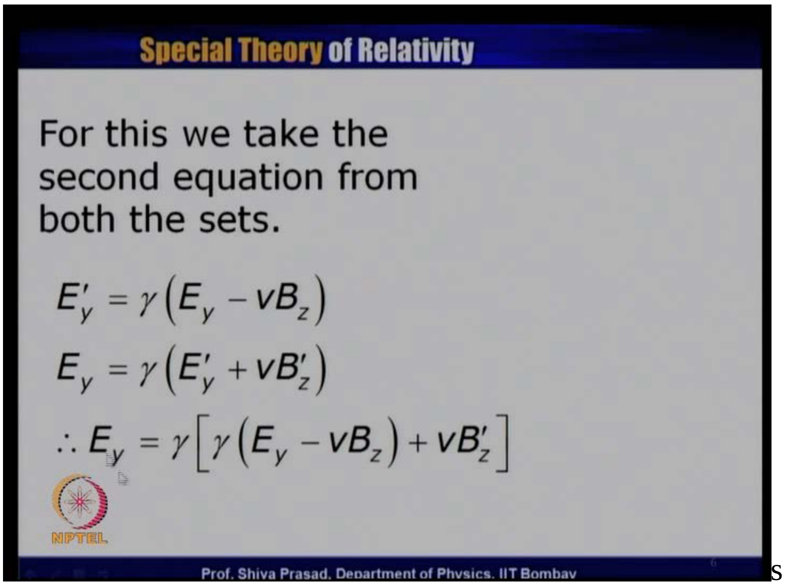

s.

Similarly, if I use these two equations and eliminate e z prime I can find out at transformation which gives me the value of B y prime if I know the electric and magnetic fields. So, let me first attempt that so these two equations as I have said are the second equations which I have just now described so E y prime is equal to gamma E y minus v B z and E y is equal to gamma E y prime E y prime plus v B z prime this E y in this particular equation I will substitute this value of E y prime so here in place of this E y prime I put E y prime is equal to gamma E y minus v B z so this is what i have put it here this is this particular quantity which is here is replacing this E y prime the second term is as it is v B z prime now what I will do I will expand it I will multiply here so once I multiply E y and v B z by this gamma it is to be further multiplied by gamma it will become gamma square so let us look at this particular equation. This is the equation which I have just now written on my last transparency.

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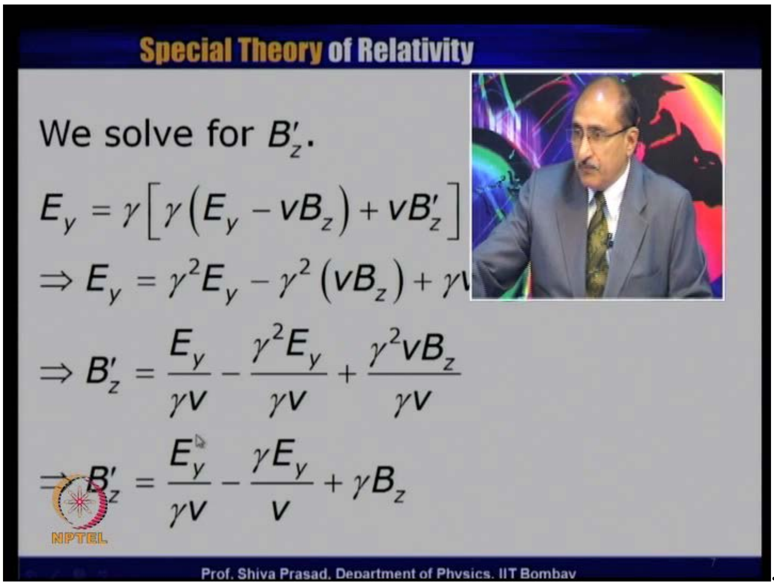

So, as I said when I open it when I expand this particular bracket there is a gamma here there is another gamma here so this will become gamma square this will become gamma square e by similarly, there is a gamma here there is a gamma here there is a negative sign here so it become minus gamma square v B z plus this term remains as it is except that this gets multiplied by gamma, which is here so it becomes gamma v B z prime.

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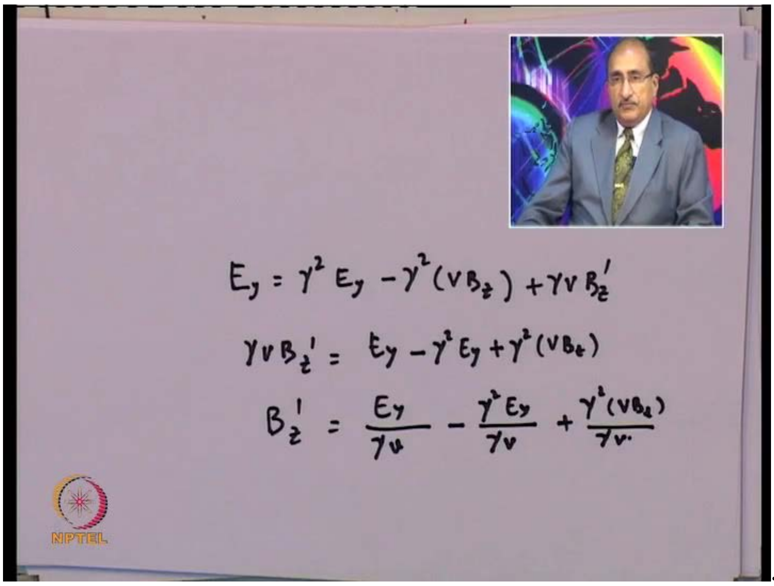

Now what I am trying to do is to find out the value B z prime so I want to take this on the right hand side if I take want to take this particular thing on the right hand side I will get the equation, which is this will become negative so this becomes first look at this if I take this on the left hand side.I am Sorry, and E y on the right hand side, then this B z prime I have to divide by gamma let me write this particular thing it will be more clear. The equation which I had was E y is equal to gamma square E y minus gamma square v B z plus gamma v B z prime.

So, I take this on the left hand side or you can put I take these things on these two things on left hand side because this I want to written as positive quantity so I can write this as gamma v B z prime is equal to E y minus gamma square E y, when this goes to this side this will become plus gamma square v B z I divide the equation by gamma v if I divide the equation by gamma v I will get B z prime is equal to E y by

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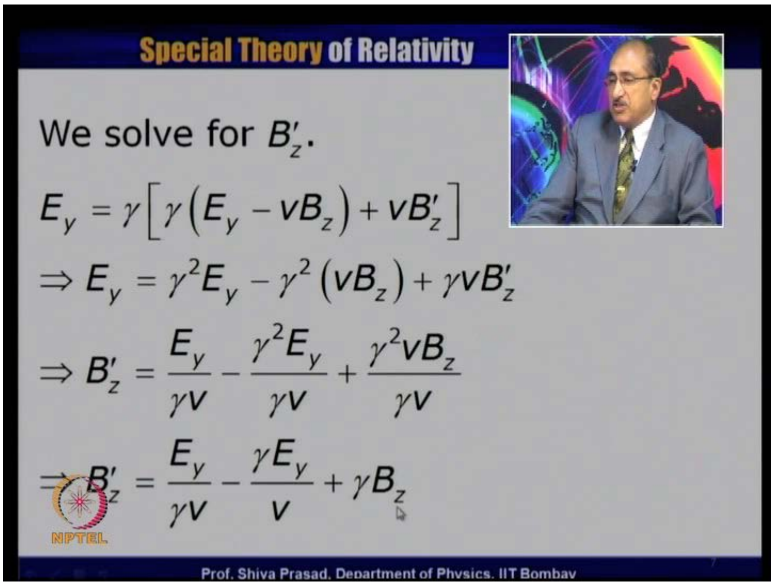

Gamma v minus gamma square E y divided by gamma v plus gamma square v B z divided by gamma v. So, this is the equation which I have written here I am sorry this involved a few steps so B z prime is equal to E y divided by gamma v minus gamma square E y divided by gamma v plus gamma square v B z divided by gamma v same equation which we just now wrote on this particular piece of paper now this gamma square there is a gamma square there is a gamma here so, this will cancel with one of the square and this becomes just gamma similarly, there is gamma here there is a gamma square so this will become gamma here this v and this v will cancel out so this particular term the third term just becomes gamma B z her E you have just gamma E y by v and this term remains as it is E y by gamma v is simple Algebra.

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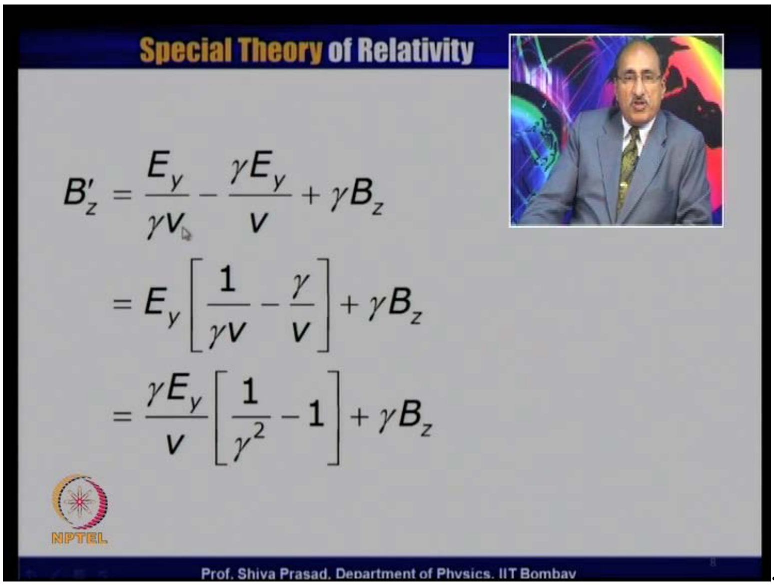

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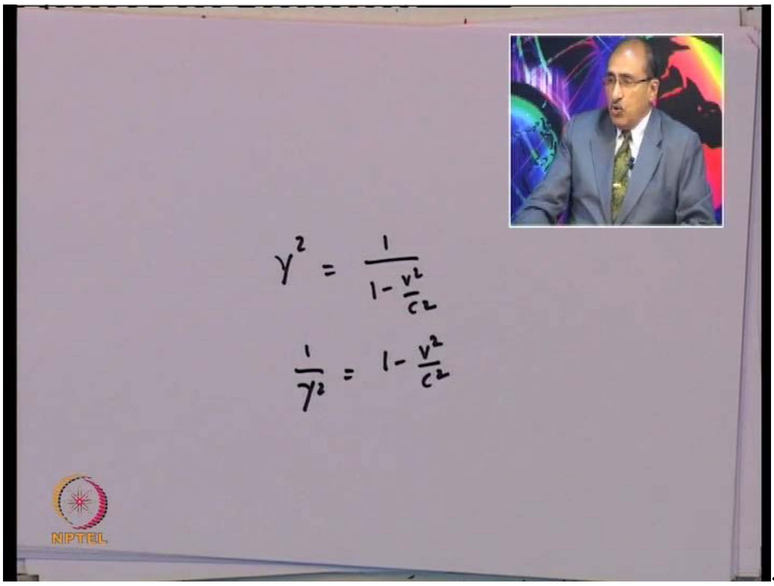

We put this equation here from these two I take E y common so I write E y what is remaining in the bracket is 1 divided by gamma v minus gamma by v this term is gamma B z as it is, now what I do how to this particular equation I multiplied further by gamma so this equation becomes gamma E y by v if I multiply by gamma these things have to be divided by gamma so I divide by gamma when I divide this term by gamma I get one upon gamma square when I divide this term by gamma I just get are this v has been taken out here.

So this equation becomes gamma E y by v v comes from these two equations and 1 minus 1 divided by gamma square minus 1 plus gamma B z now this equation can be simplified because I know what is the value of gamma square and therefore, I can always find out what will be the value of one upon gamma square, which I am doing in the next transparency, as we know gamma square will be equal to 1 upon 1 minus v square by c square so if I take inverse of gamma square this we have used number of times earlier also it becomes 1 minus v square by c square.

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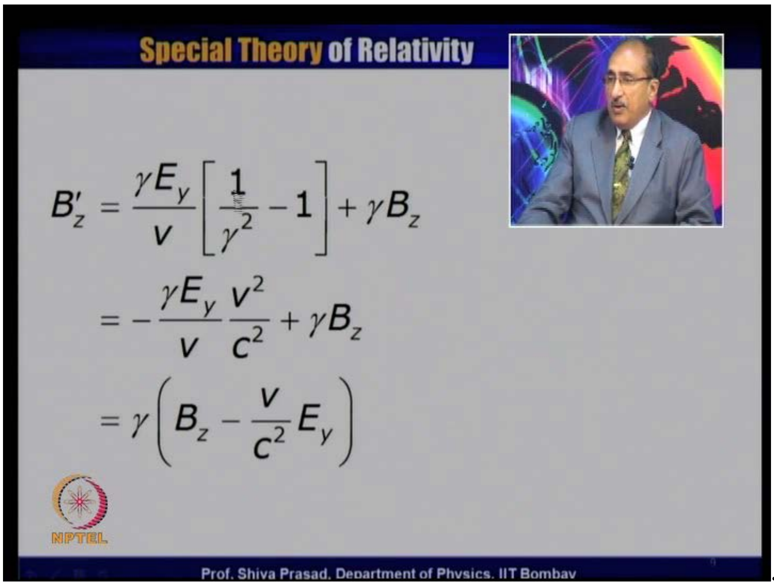

So, here it becomes 1 minus v square by c square this 1 will cancel with this 1 and what will be remaining is just minus v square by c square so this term remains here and this whole bracket gives me minus v square by c square so this minus v square by c square plus gamma B z this gamma can be taken out I get B z minus v by c square E y which as you would have realized that this is the transformation equation, which gives me the value of B z prime in the z component of the magnetic field.

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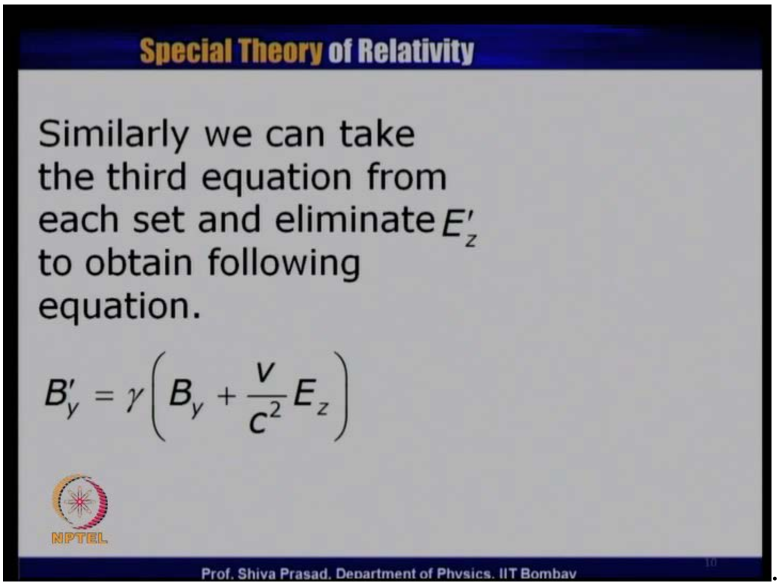

If I know the z component of magnetic field in s frame and the E y component of the electric field in s frame of reference similarly, you could have taken the third set of equations as I described just now and eliminate e z prime from that particular equation and then I could have obtain transformation equation corresponding to the E y component of the magnetic field in that case B y prime will turn out to be equal to gamma B y plus v by c square e z. So, as we have seen that the last two equations of magnetic fields transformation can also be obtained just by using only the electric field transformation and just manipulating the last set of equations of course, you have to use direct transformation as well as as well as inverse transformation and you can manipulate these equations to get this result.

I would just like to mention one more point because many times I mean of course, it is always we always been telling when we started special theory of relativity that the initial conditions that we have put on s and s prime the relative velocity was always around the x direction but, sometimes we will prefer to a little more generic equation which does not have this relative velocity between the frames only along the x direction, and if you want to do that these equations of electric field and magnetic transformations can also be written along the longitudinal and the transverse direction that is a direction parallel to the relative velocity motion direction and perpendicular to the relative velocity motion direction.

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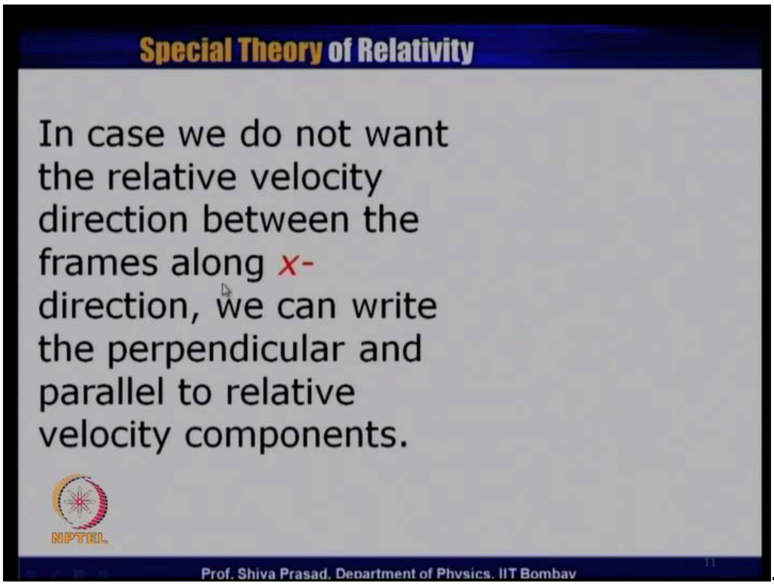

So, this is what I have written in case we do not want the relative velocity direction between the frames along x direction we can write the perpendicular and parallel to the relative velocity components, so when I say perpendicular it will mean perpendicular to the relative velocity between the frames and parallel which means parallel to the relative velocity between the frame the results somewhat obvious but, they were less give because this sometimes we give these set of equations.

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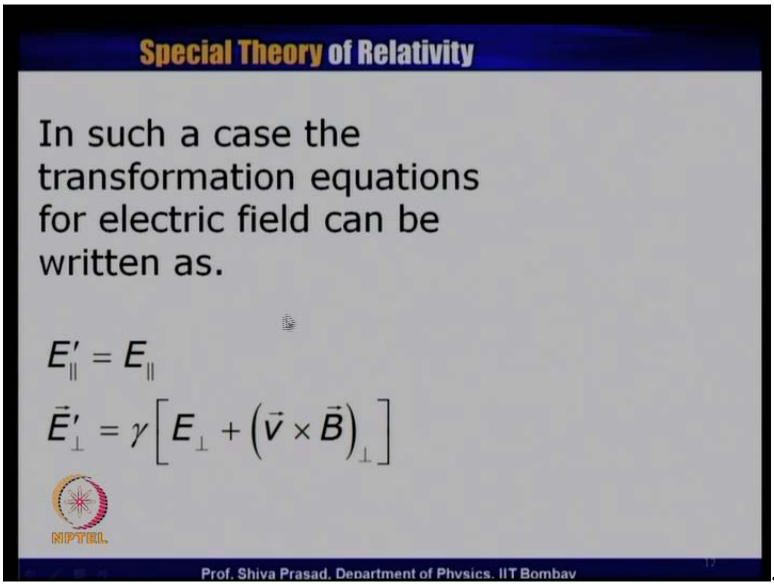

In such a case the transformation equations for the electric field can be written as so many times you will see in the text book they write this type of equations that e parallel prime is equal to e parallel e perpendicular prime is equal to gamma e perpendicular s plus v cross b the perpendicular component of this cross product, so essentially the other two equations which we have written y and z component can be merged into this particular way of writing these equations so this you will have to remember that case only the two equations for the first equation is obvious because along the direction the relative motion these do not change true for electric field as well as magnetic field because even for magnetic fields we got b x is equal to b x prime its only along the perpendicular to the relative velocity motion direction that the fields transformed. So, this is what we call as a perpendicular or transfers transformation of the fields. Similarly, we can write the equation corresponding to the magnetic field also.

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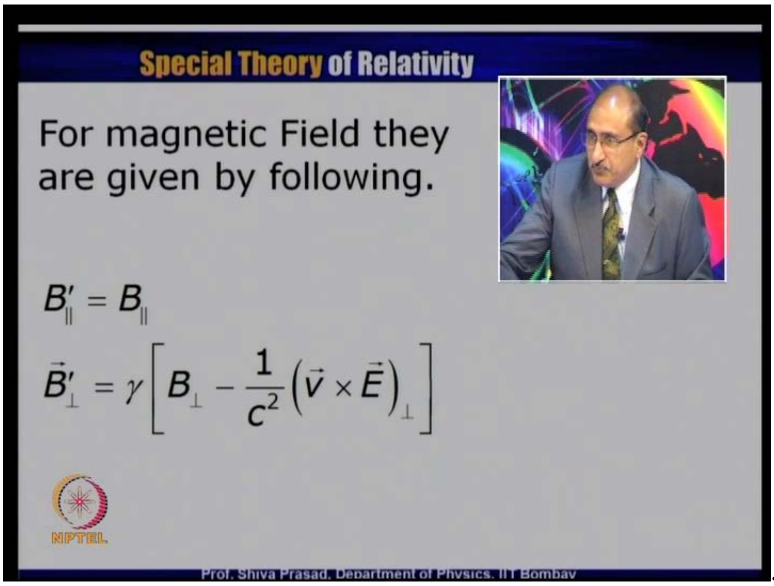

So, for magnetic field I will write b parallel prime is equal to b parallel and b perpendicular prime is equal to gamma b perpendicular minus 1 upon c square just here there is term of 1 upon c square v cross e perpendicular component of this particular thing and if you have any query you can always take the v along the x direction and expand the cross product you will get the same three equations which you have derived earlier, so what I have written this can be verified by taking the velocity along the x direction and expanding the cross product.

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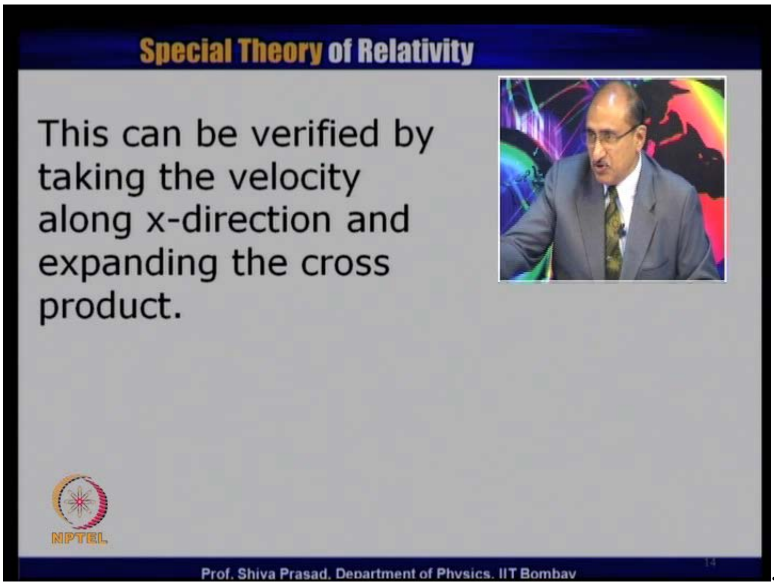

So this was about the electric field and magnetic field transformation let me now take one example one simple example extremely simple example just to give you some idea of how the field transformed. So, we consider three frames of reference s s prime and s double prime.

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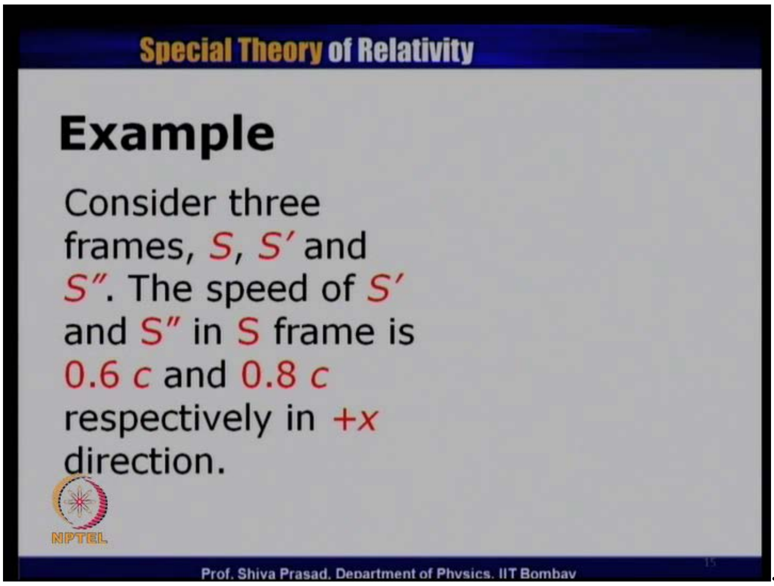

We of course, assume that there y and a y prime and y double prime a these are parallel you know all those things which we have always assume for deriving the original Lorentz transformation now this speed of s prime and s double prime in s frame is 0.6 c and 0.8 c respectively in plus x direction. So what has been given are the velocities of s prime and s double prime in s frame the speed of s prime and s double prime in s frame is 0.6 c and 0.8 c so you have three names.

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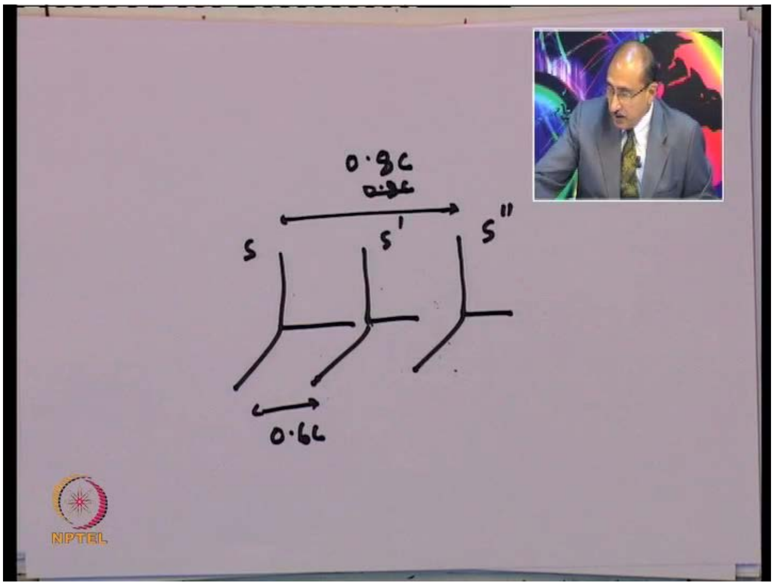

Let us say this is s frame we have s prime frame and we have s double prime frame now the speed of s prime in s frame has been given is 0.6 c so s prime in s frame appears to be moving with a speed of 0.6 c of course, along the plus x direction. Similarly, s double prime is speed has often given in s frame which happens to be 0.8 c.

So, the speed between these two frame is 0.8 c, so both the speeds have been given in s frame. Let us go ahead with a question in the frame s prime magnetic field is 0 so what has been told that in s. Prime frame the magnetic field is zero while there is an electric field along the y prime direction because the direction of y prime is same as the direction of y because now that is what way transformation equations have always been driven and its magnitude is e 0 prime or E naught prime, so this is the magnitude of the electric field which is happening which is only in the plus y direction.

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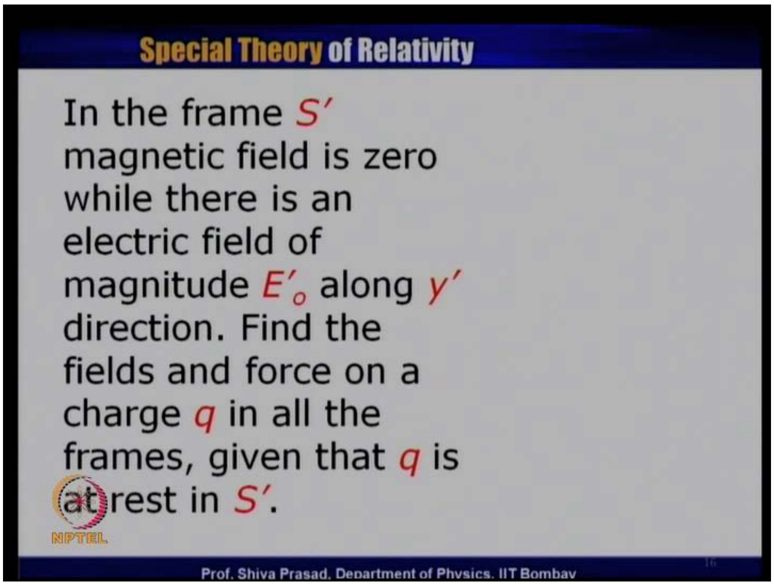

So a person in the frame s prime says that there is no magnetic field there is only electric field which is pointing out in plus y prime direction and it has a magnitude of E naught prime now what I have to do is to find the field in s frame find the field in s double prime frame and we have to also find out the forces between all these three frames s frame s prime frame and s double prime frame and see verify ourselves that the forces actually obey the force transformation equation the way we have written it earlier of course, this is obvious because these equations we have been derived by using the force transformation so this has to be valid but, the idea is to take a simple example, and convince our self because nothing like working it our own self and try to convince our self that whatever I am getting is consistent as I would expected.

So, this is what I am trying to do these particular example, so as for the field equation is concerned it has been given in a s prime frame of reference we have been told that the magnetic field is 0 essentially means that b x prime is equal to 0 B y prime is equal to 0 and B z prime is equal to 0 we have also been told that electric field is only in the plus y direction, it means there is no component of electric field in x direction and z direction therefore, E x prime is equal to 0 and E y prime is equal to E naught prime which has been given is a constant field which has been given as E naught prime of course, we are not mentioned about master but, does not matter and e z prime is equal to 0.

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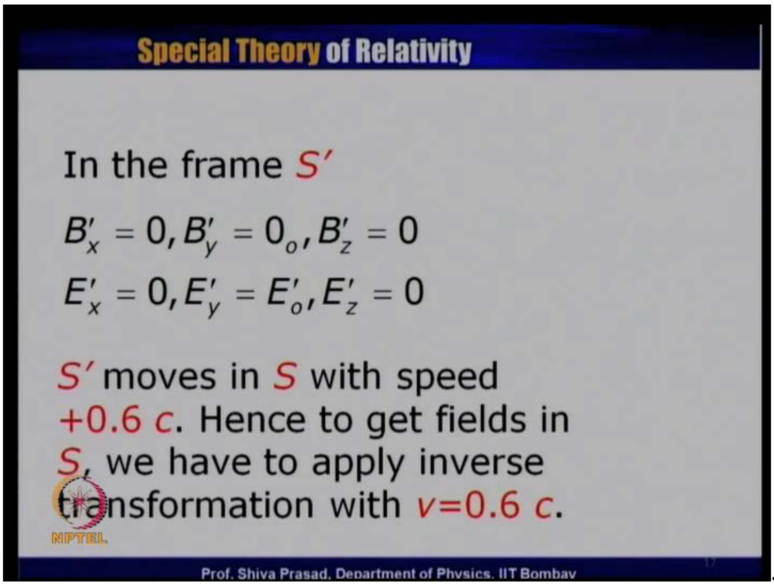

The only field component which is existing in s prime frame is e prime is equal to E naught prime all other components are 0. Now, we have said that s prime moves in s with a speed plus 0.6 c hence to get the fields in s we have to apply inverse transformation with relative speed v is equal to 0.6 c. let me just look back at this particular field which I have written it earlier in this particular piece of paper.

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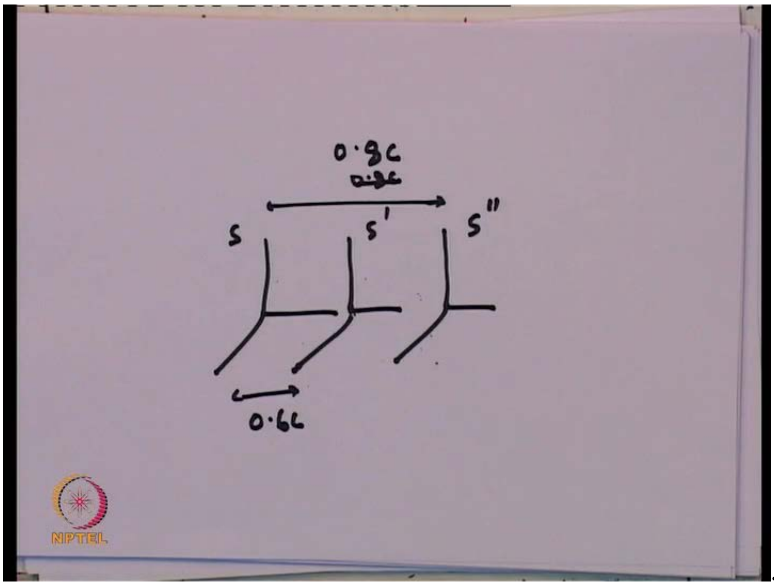

Here, we have written in this particular piece of paper that if we look at this particular frame s and s prime frame of reference s prime moves with a relative velocity of 0.6 c in frame s therefore, if any information can which is given in a s has to be transformed to s prime, we lose the direct transformation if the information has been given in a s prime and I want that information equivalent information in s then I have to use inverse transformation now here the fields have been given in s prime frame of reference therefore, I must use inverse transformation to find out the field in s frame the relative speed is 0.6 c in s prime frame.

So, I still use b is equal to plus 0.6 c but, use an inverse transformation because the fields have been given in s prime frame of reference, so if you remember the original Lorentz transformation the way we have derived this comes from that that if things are given in s prime frame of reference in s frame if I have to find out I have to use inverse transformation.

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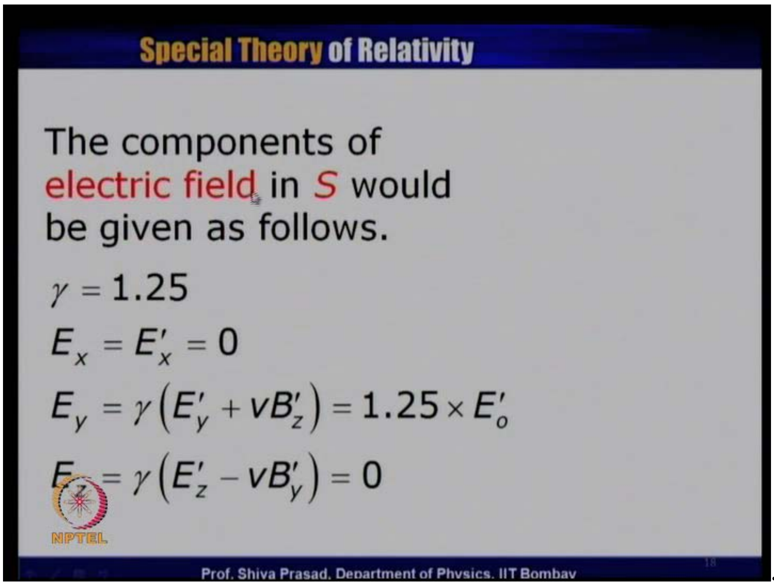

So this is what i will do the component of the electric field in s would of course, we have done number of times a 4.6 c, the gamma will turn out to be equal to 1.25. So I do not want to derive it again gamma is equal to one upon under root 1 minus v square by c square you can. If you do not remember you cannot work it out that if I take v is equal to 0.6 c gamma turns out to be equal to 1.25.

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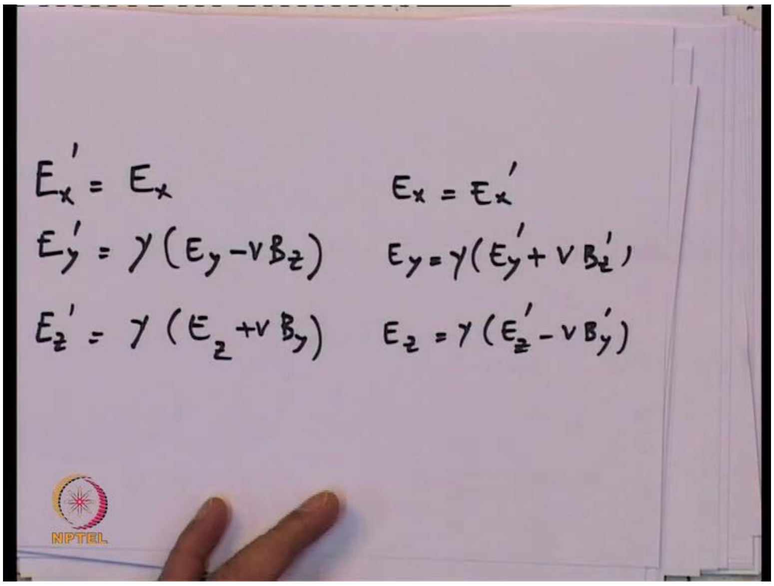

Now, I have to just use the inverse transformation equations which I had written it earlier here on the right hand side from this particular piece of paper are the inverse transformation equation E x is equal to E x prime.

So this is what I have written E x is equal to E x prime we have already seen that E x prime is equal to 0, so E x also has to be equal to 0, E y I have written this equation E y is equal to gamma E y prime plus v B z prime this what I write here E y is equal to gamma E y prime plus v B z prime here only E y prime is non-zero quantity e v B z prime is 0 that is what we said earlier E y prime is equal to E naught so this becomes as gamma times E naught gamma we know is 1.25, so this becomes e point e is equal to 1.25 E naught of course, there is a prime similarly, for e z we write this equation because this is 0 this is 0.

Therefore, e z is equal to 0. So what we have seen is that only the electric field by in they component would be present out of the three electric fields E x will be 0 e z also will be 0 but, E y would appear to be changed in principle E y will turn out to be.

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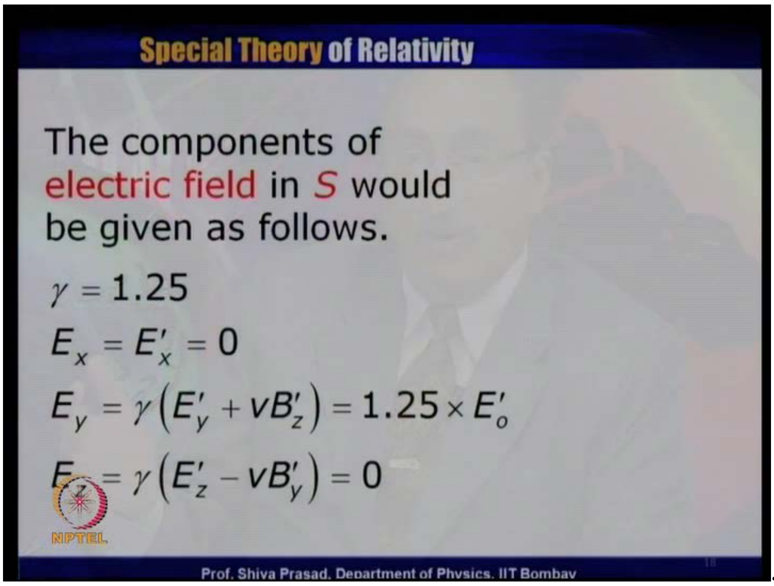

Larger than E naught prime it will become 1.25 times E naught prime but, that is not the complete story because what I have told related only the electric field I must also calculate the magnetic field to find out if there is a magnetic field in s frame in s prime frame there was no magnetic field but, in s frame there could be a magnetic field.

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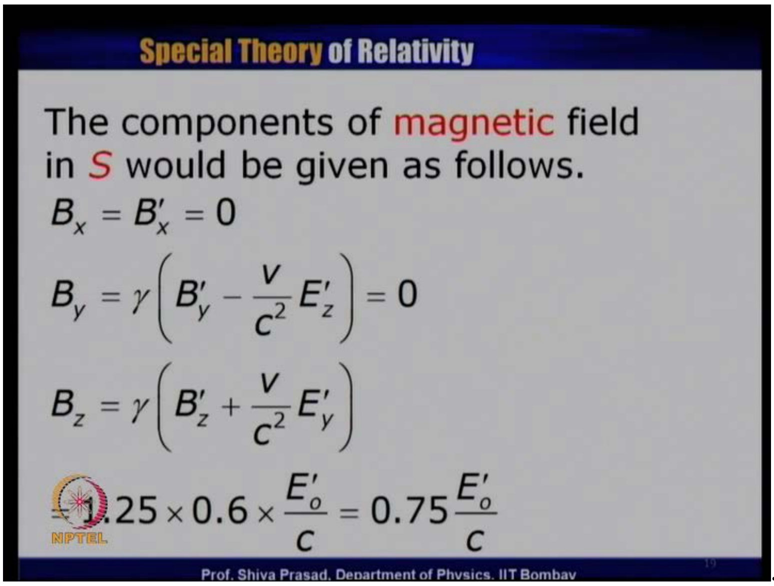

So the components of the magnetic field can be derived or can be found out by using these equations let me just remind these equations which we have written it here in this piece of paper these are my equations I have to use a reverse transformation.

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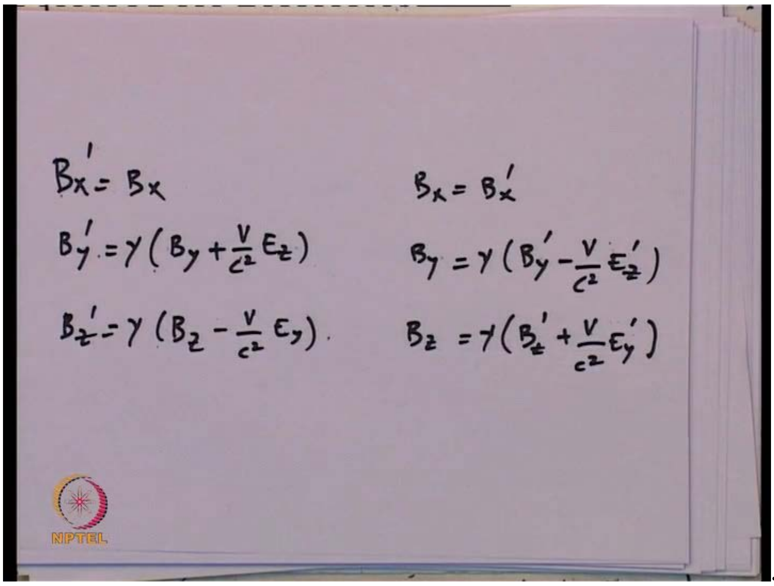

So, I would use this set of equations b x is equal to b x prime as we have said along the x direction which is the relative motion velocity direction between the frames the fields do not change so b x remains equal to b x prime b x prime was anyway equal to 0 so b x is equal to 0.

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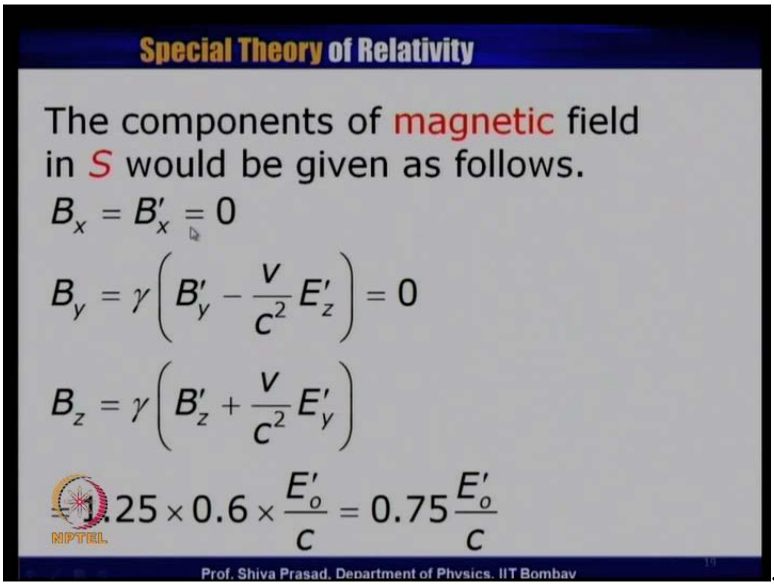

So, this is what I have written here as for second equation is concerned B y prime is equal to gamma B y prime minus v by c square e z prime e z prime is 0 B y prime is equal to 0 so this is also equal to 0 as for x component and the y component of magnetic fields are concerned they are 0 in this frame also but, let us look at B z which is non-zero B z if you look back in this particular paper B z was equal to gamma B z prime plus v by c square E y prime same thing I have written here B z prime 0 but, E y prime is not zero is actually equal to E naught prime gamma is equal to 1.5 v i o 0.6 c so this is 0.6 this c will cancel one of with one those squares.

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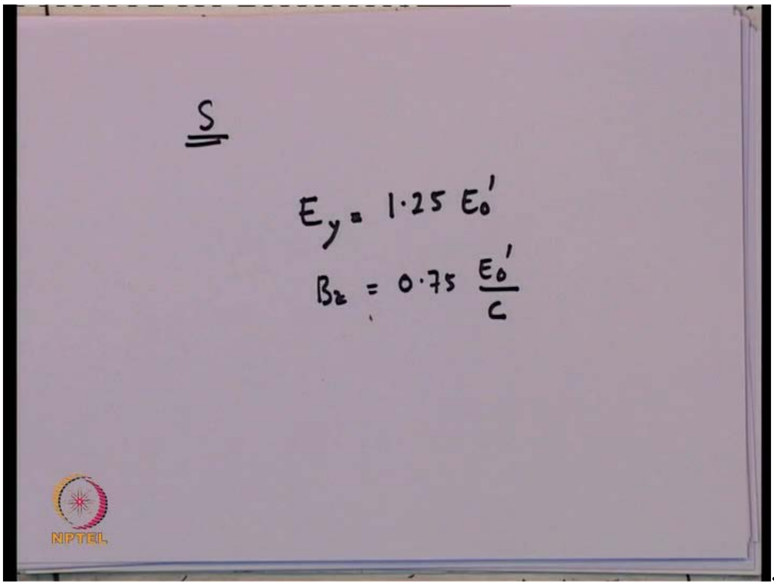

I will get 0.6 multiplied by E naught prime divided by c. If I just calculate this number this terms out to be equal to 0.75 E naught prime by c, So what I find that in this particular frame of reference s. The electric field was equal to 1.25 times E naught prime which you have just now written it earlier, and you have B z which is equal to 0.75 times E naught prime divided by c so, what I see that in s frame of reference they charge or the person in the frame would also experience in magnetic field in the z component direction pointing out in the plus direction it will be equal to 0.75 times E naught prime by c so the field which was purely electrical in s prime frame of reference turns out to be partly electrical partly magnetic in s frame of reference.

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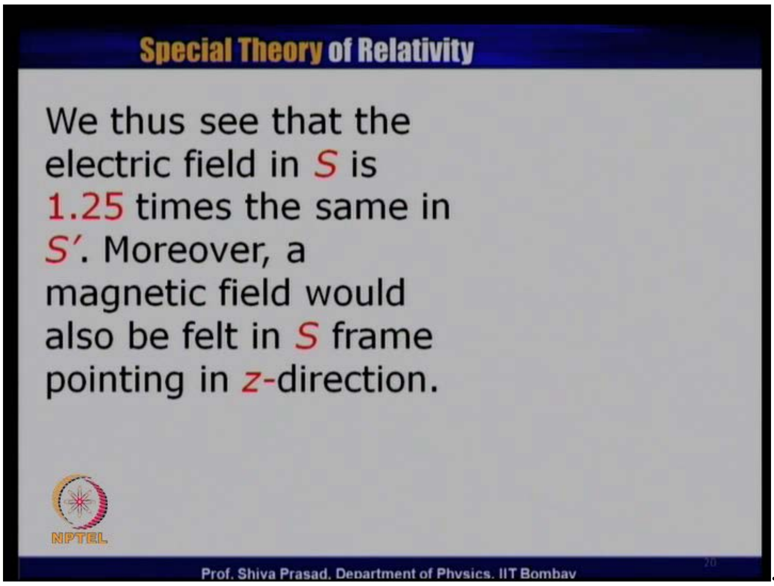

So, this what I have written it here we thus see that the electric field in s is 1.25 times the same in s prime moreover a magnetic field would also be felt in s frame pointing out in plus z direction.

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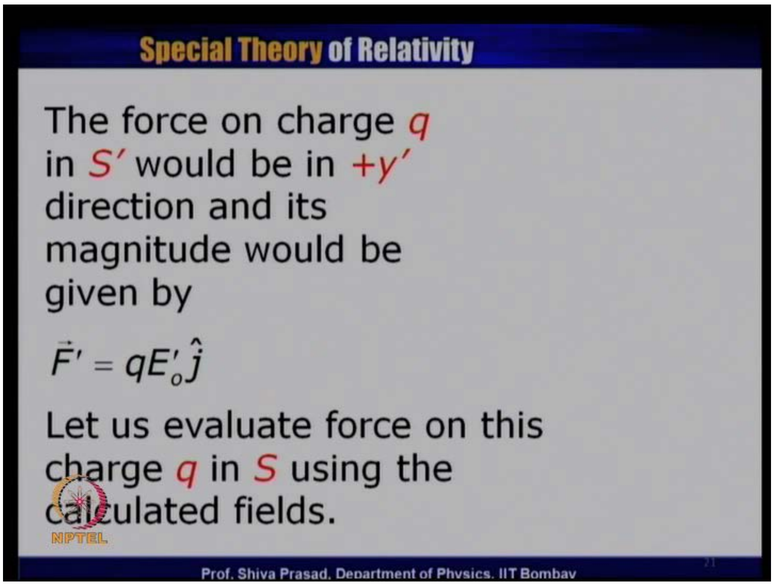

So, this is what is what comes out from the relativity. Now, let us look at the forces if I look at the force in s prime frame of reference because there is only electric field and also it has been given that the charge is at rest but, anyway there is no magnetic field is there which has not been given in so we can write the force which I can write as e.

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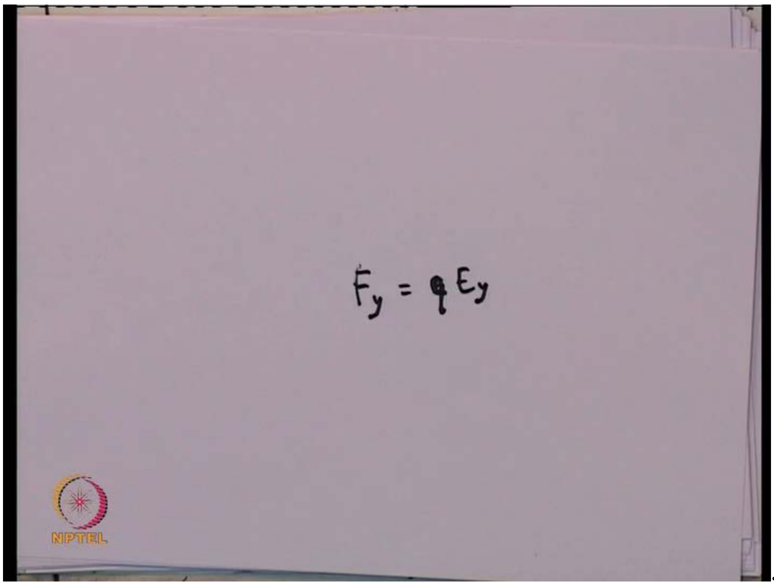

y once I write E y and because I am sorry let me write the force not the field has been given I am writing f y this will be given by just e times E y because that is only force because of there is a electric field in the y direction and there is no negative field charge is anywhere at rest so comes a comes e let me write q because q is the value of charge which has been given to me so the force will be the just given by q E y.

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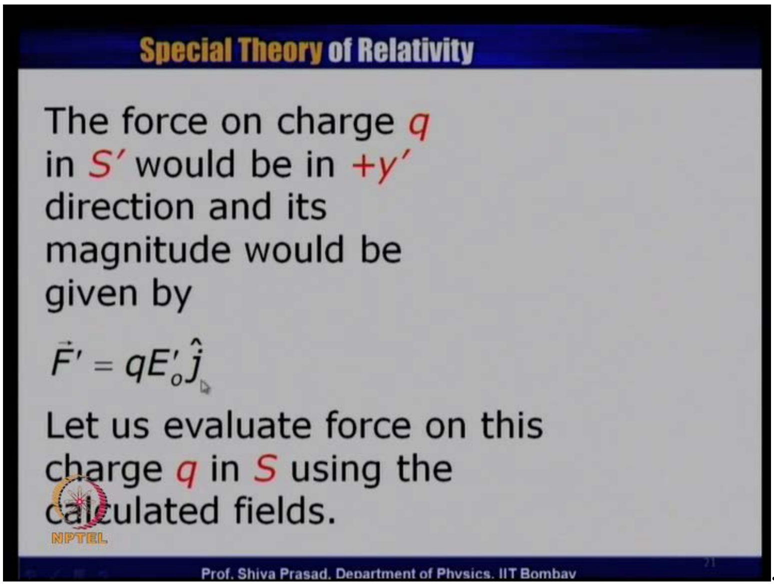

Purely electrical field force which will be pointing out in the y direction so this is what I have written f prime is equal to q E naught prime j this I have written in the vector form of course, i j k point out the same direction they just give you unit vectors irrespective of the frame they give the unit vectors in directions pointing out along the y direction x x direction y direction and z direction and because this is j therefore, they are pointing out in the y direction or let us evaluate the force in this charge in s using the calculate fields now as far as s frame is concerned there is also an magnetic field and though the charge is at rest in s prime frame of reference charge is not at rest in s frame of reference because s prime moves relative to s therefore, the charge is also moves relative to s therefore, if magnetic field in s the magnetic field will also cause a force on this particular charge so let us calculate that particular force. Now, we are looking at the force on the particle so let us first look at this speed of the charge because the charge is at rest in s prime frame of reference so whatever is the speed of s prime in a s frame of reference same would be the speed of the charge.

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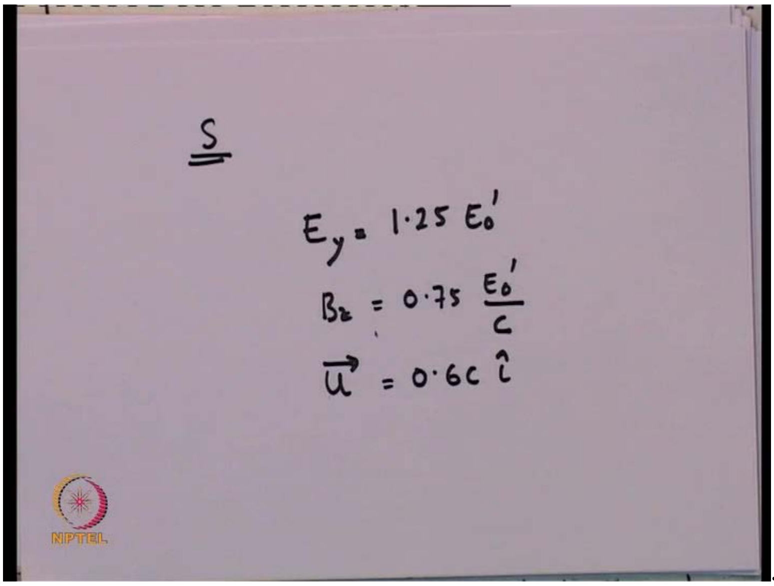

So u which is the speed of the charge as seen in s frame of reference will be just 0.6 c that will pointing at the i direction because s frame of reference is other s prime frame of reference moves in s with a speed of 0.6 c in plus x direction therefore, this will be u I can write the force will be given by.

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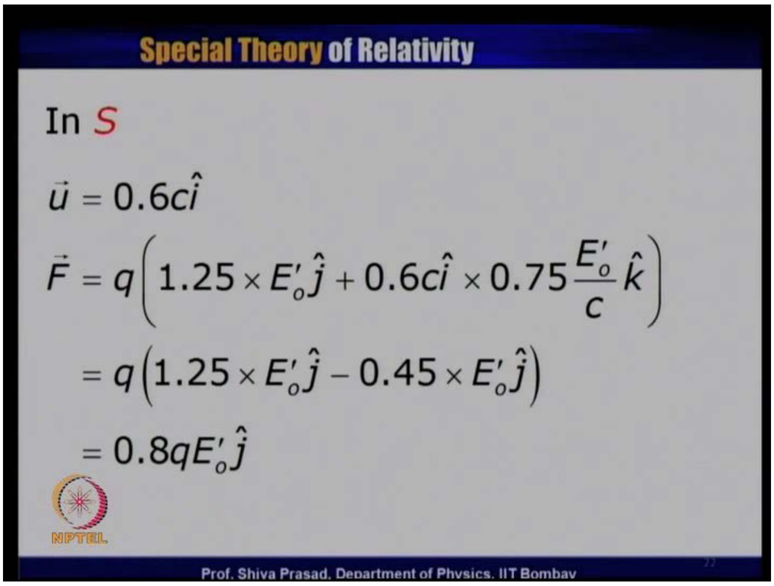

q times e electric field we have already seen is 1.25 times E naught prime plus v cross b v is 0.6 c i you have already seen that there is a magnetic field pointing out plus direction plus z direction and the magnitude of that particular field is 0.75 E naught prime by c and because it is a plus z direction I have written k so this will be the entire Lorentz force on this particular particle this is because of the electric field this is because of the magnetic field first term I keep as it is let me look at this cross product this is i cross k so i cross k will give me minus j, I just multiply 0.6 c with 0.75 this c will cancel with this point this c you will get minus 0.75 this minus sign is because i cross k is minus j so this I write as j so this gets subtracted from this and i find that the force which should be experienced charge in s frame of reference will be 0.8 q E naught prime 0.8 q E naught prime j.

So, this piece of paper gives me what are the field values that I see what are the speed of the particle and what will be the force on this particular charge remember this is q of course, I except that this is force must obey the transformation equations that we have found out in x frame in s frame the force in the y direction, so it is an s prime of reference so let us look only at the equation corresponding force transformation corresponding to the y component and if you remember if charge happens to be at rest or if the particle happens to be at rest in one of the frame the transformation equations comparatively simple.

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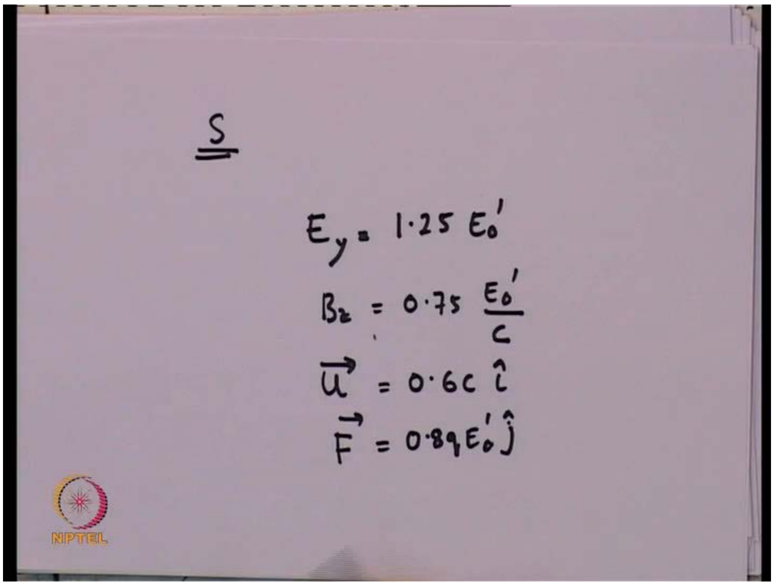

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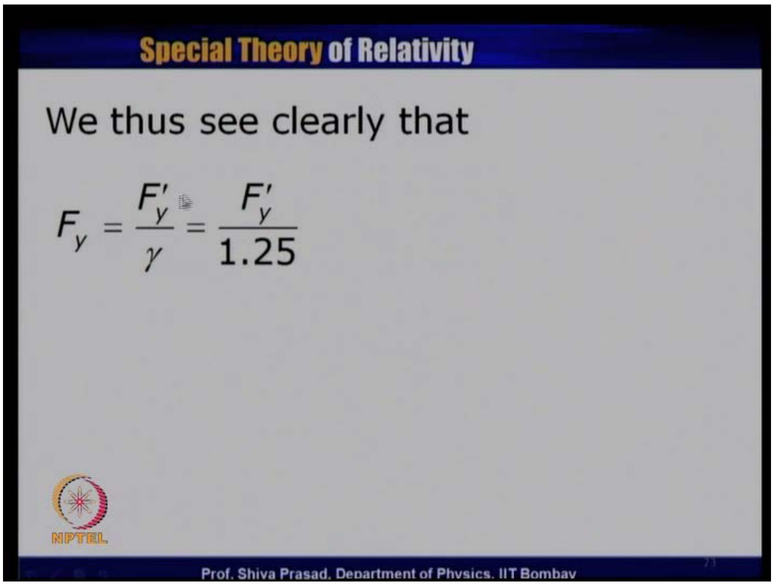

So, I will use those simple equations this is very obvious here because this is f prime frame of reference that the charge is at rest so force the y component of the force will be just given by f y prime by gamma and gamma is 1.25 one upon 1.25 is just 0.8 and this is what I have obtained the force has become 0.8 times the force on the charge in s prime frame of reference, so this obeys that transfer force transformation equation as I would have explained. Now, it is time to calculate the fields in s prime frame of reference double prime frame of reference I could have to gone either from s prime frame of reference or I could have gone from s frame of reference so at the moment I am transforming from s frame because now I do fields in both the s frame and as well as in s prime frame of reference so I could have used any transformation let me first calculate the fields starting from s frame of reference where I have calculated the fields, just now at later part I would also transform from s prime frame of reference to see that we are getting everything right which we must get we must get ourselves that we are whatever we are doing is deal turning out to be correct.

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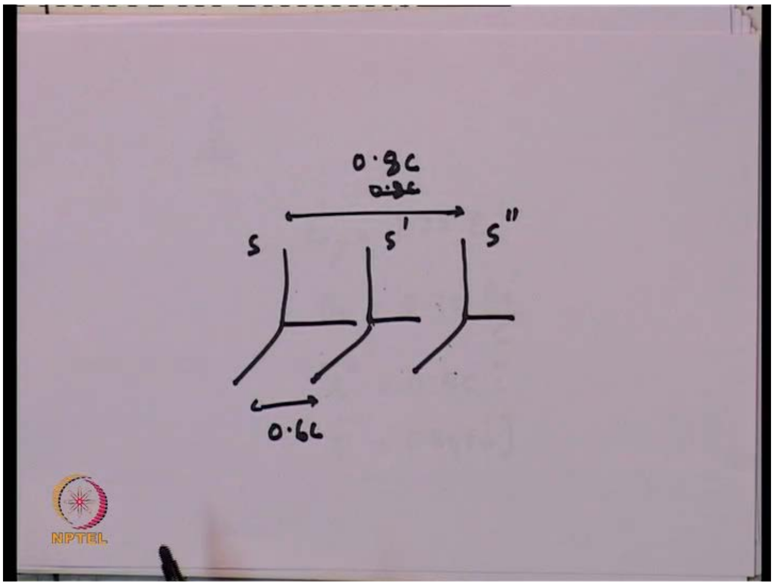

Now, I if I go from s frame to s double frame of reference I go back to this particular piece of paper here, this is s this is s double prime we have said that according to an observer in s frame of reference s double prime frame moved with a speed of 0.8 c, if I have to find out certain quantities in s frame I have to transform to s double prime frame of reference therefore, I will and I will use a direct transformation with a v is equal to 0.8 c because this is the relative velocity between the frame so remember the symbol v was reserved for relative velocity between the frames. So, here I will use v is equal to 0.8 c because that is the relative velocity between s and s double prime and I will take v is equal to plus 0.8 c and I will use a direct transformation.

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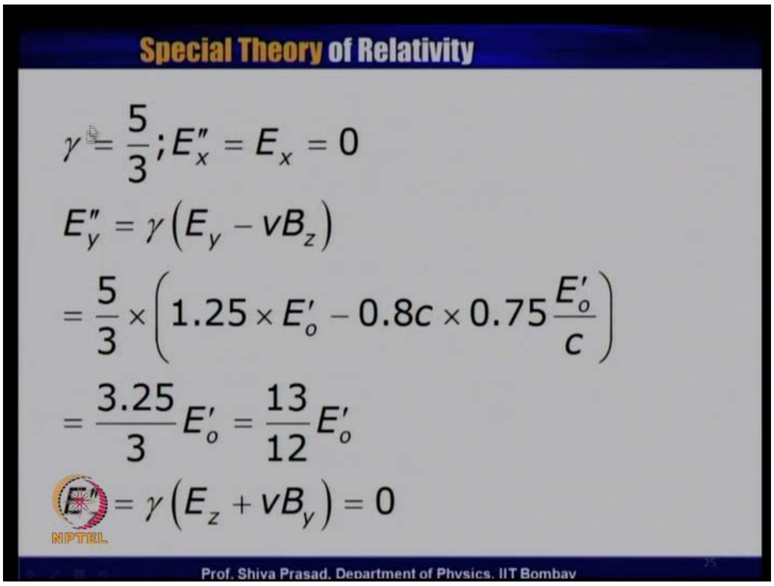

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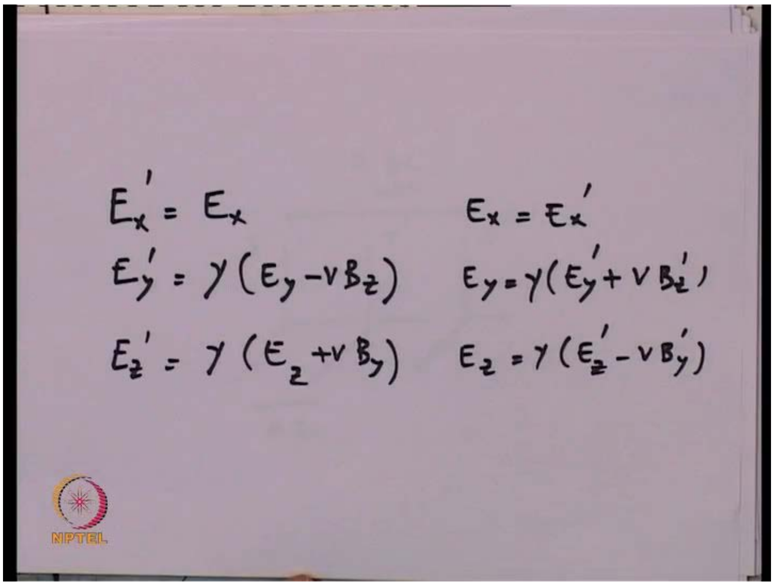

Because, information is known as prime and I want them to be in s double prime frame of reference now again v is equal to 0.8 c gives me clean number for gamma and this gamma happens to be 5 by 3 this also we have done many of our earlier examples we have used corresponding to 0.8 c gamma turns out to be equal to five by three now E x double prime is equal to E x there is no problem because as i have said along the relative velocity direction the fields would not change so relative velocity is along the x direction. So the x component of the fields do not change so E x double prime is equal to

E x is equal to 0 unless we get E y double prime here I am going to use the direct transformation equation as i have mentioned earlier.

See if I look at this paper earlier when I get from s prime to s prime I use this equation when I go from s to s double prime i use this equations because now I am using a direct transformation. So I use E y prime is equal to gamma E y minus v B z and remember in s frame we have non-zero E y and non-zero B z both are non-zero, so I should calculate E y prime this what I have done here E y double prime is equal to gamma E y minus v B z we have just now calculated the electric field in s frame which is 1.25 times E naught prime minus v, v is the relative velocity between the frames which is 0.8 c B z is the magnetic field.

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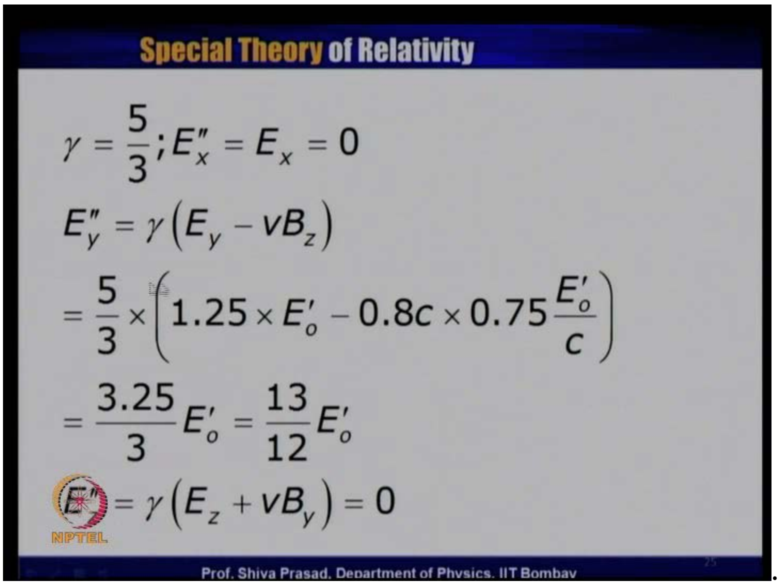

As seen in s frame or the z component of the magnetic field which is 0.75 times E naught prime by c this c and this c will cancel so her E you will get 0.8 multiplied by 0.75 E naught prime just sort of calculate this simple number you will get 13 by 12 E naught prime so what will find out that an observer in s double prime frame of reference will notice an electric field of 13.12 E naught prime.

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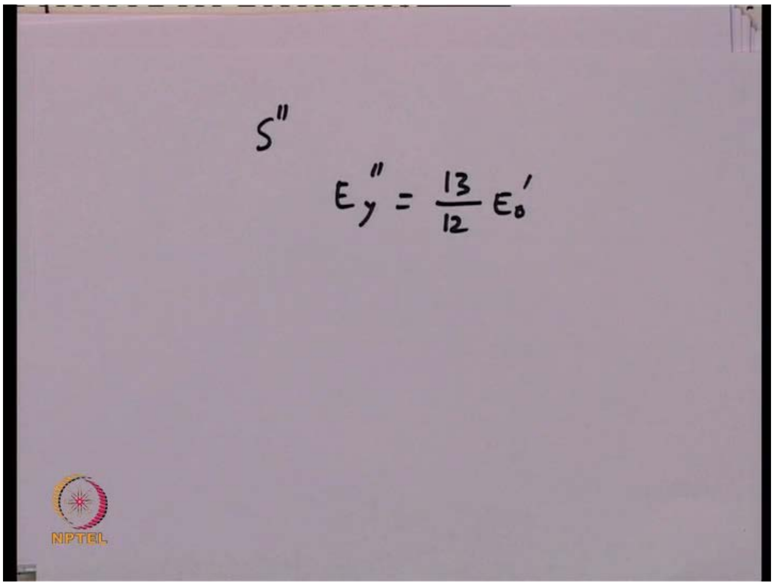

So will write it here in s double prime frame of reference E y double prime turns out to be equal to thirteen by twelve E naught prime.

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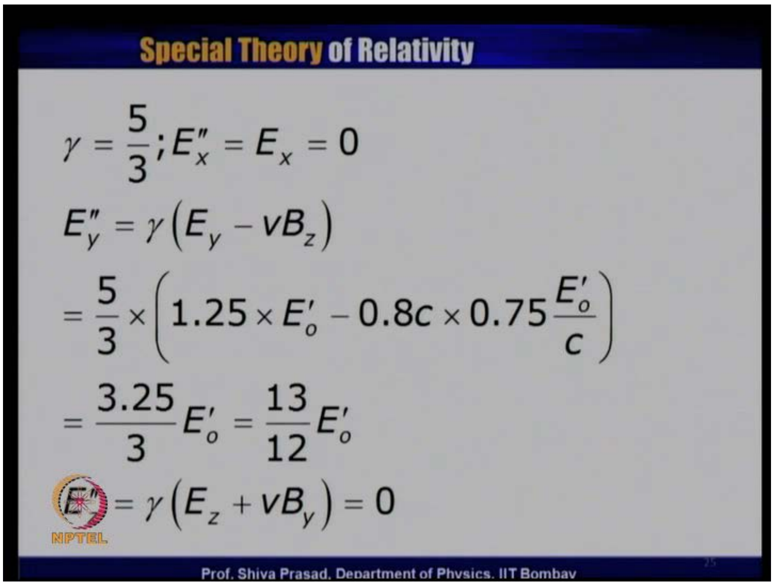

Let us look at e z double prime e z double prime is here gamma e z plus v B y e z is 0 in s frame B y is 0 in s frame so e z double prime is equal to 0 therefore, the only field only component of electric field that you see is only along the y direction which happens to be 13 by 12 E naught prime which is of course, larger than what you have seen in s prime frame of reference.

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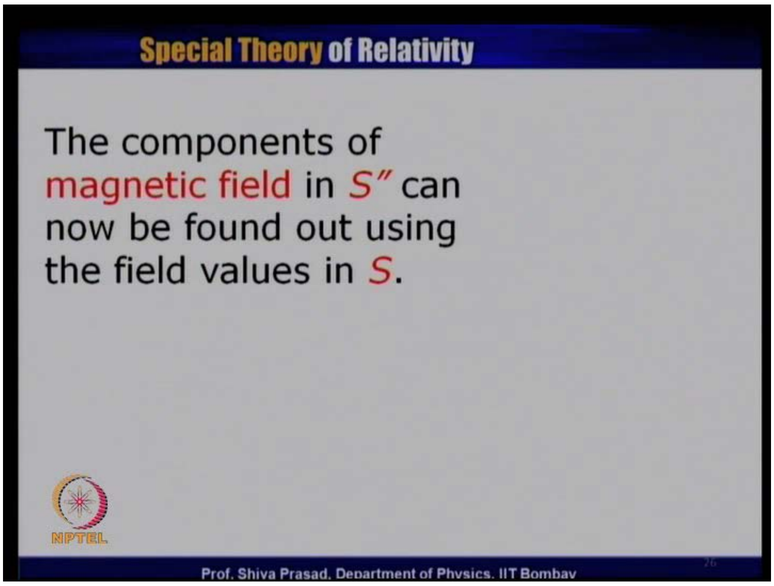

But, as I have said this is not the complete history I should also calculate the magnetic fields now let us calculate the components of the magnetic field in s double prime can now be found out using the field values in s because I am using direct.

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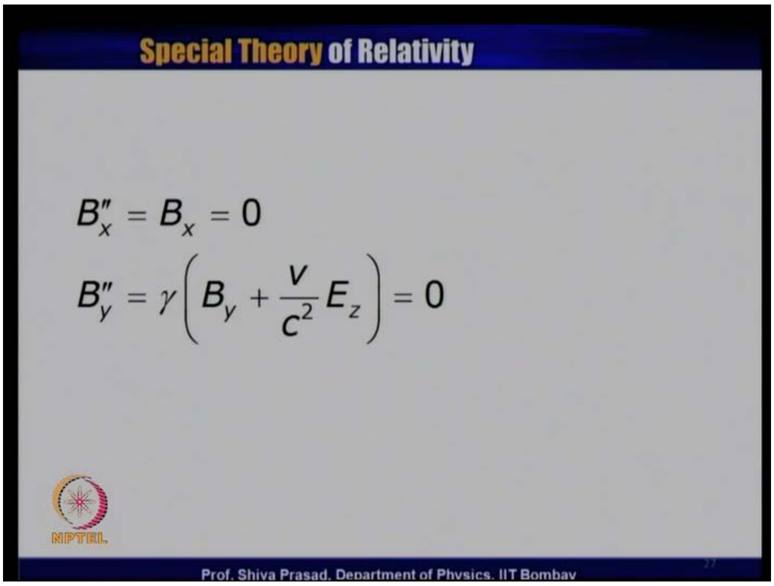

Transformation from x frame along the x direction of problem b x double prime is equal to b x is equal to 0 let us look at the y component I am using a direct transformation remember this is equal to gamma B y plus v by c square e z B y is 0 e z is equal to 0 this is 0 so x and y component of the magnetic fields are 0 as they were also 0 s frame.

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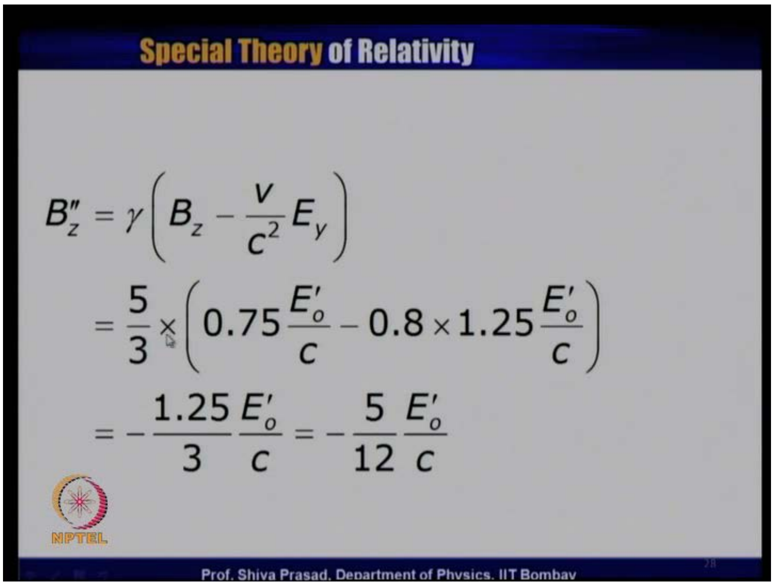

Now, let us calculate B z double prime B z component of the magnetic field in s double prime frame. B z prime is equal to gamma B z minus v by c square E y using the direct transformation gamma is equal to 5 by 3 I have put 5 by 3 the z component of the magnetic field B z is calculated is 0.75 times E naught prime divided by c electric field we have calculated 1.25 multiplied by E naught prime by c this is relative velocity 0.8 c this c cancels with one the square so this just remains E naught prime by c just calculate this number this turns out to be equal to minus 5 by 12 Minus 5 by 12 E naught prime by c.

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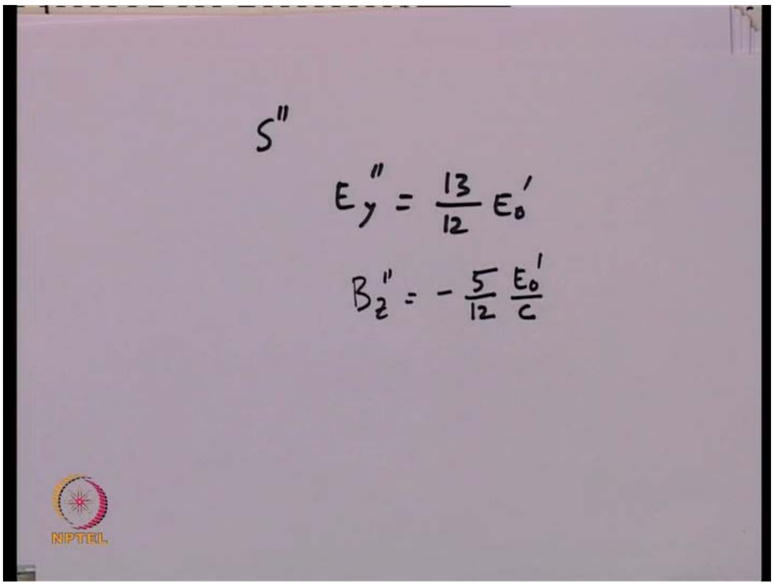

So we see that now in s double prime the magnetic field appears to be in minus z direction because there is negative sign here, so an observer in s double prime field that in s there is a field electric field in the y direction and a magnetic field in minus z direction. let us calculate the forces let us see whether they obey the transformation law.

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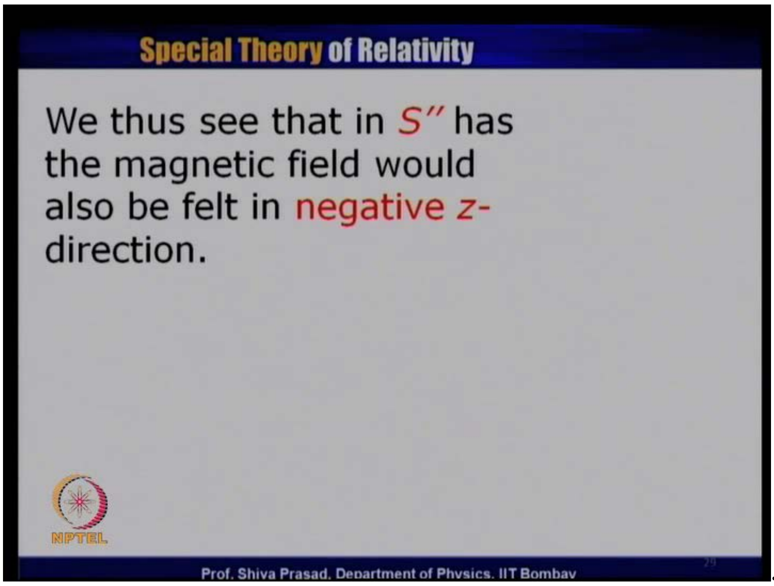

So we thus see the s double prime has a magnetic field we thus see that in s double prime has the magnetic field would also be felt in negative z direction.

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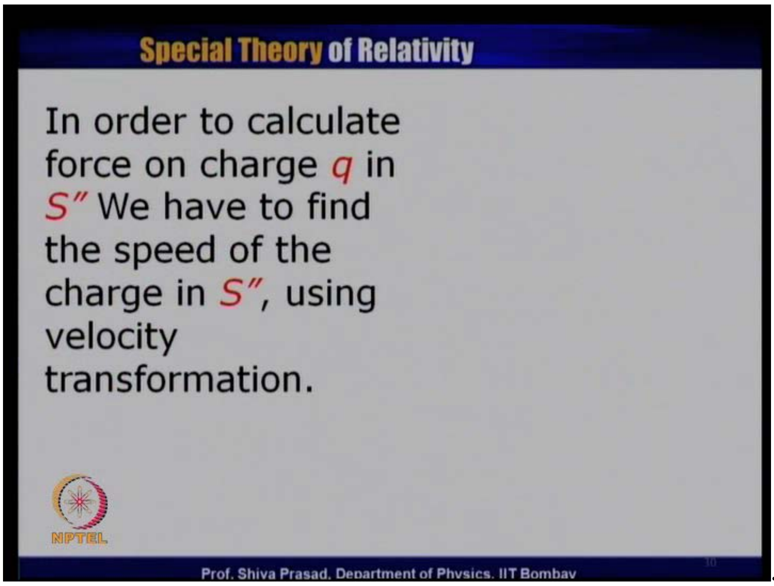

You know to calculate the force on charge q in s double prime we have to find the speed of s double prime because what has been given are the speeds of frame s prime and s double prime I mean s what has also been given is that the charge is at rest in s prime frame of reference. I do not know the speed of the charge see from s to s prime I can find out because in if the speed of the charge is point of the s prime frame is 0.6 c and the charge is at rest in s prime therefore, obviously u is equal to 0.6 c but, here this particular case I have to find out by using a velocity transformation see I know what is the speed of the frame s double prime in s I also know what is the speed of the charge in s I have to find out what is the speed of s in s double prime I have to use relative velocity transformation information about the speed of the charge and the speed of the frame has been given to us in s s frame if I want to find that information in s double prime s double frame because I have to calculate how the force in s double prime of reference and because there is a magnetic field in that frame therefore, force will depend on what is the speed of the charge carrier in that frame I must find out the speed of the charge in s double prime frame of reference and for that I must use relative velocity transformation which I am using in the next transparency.

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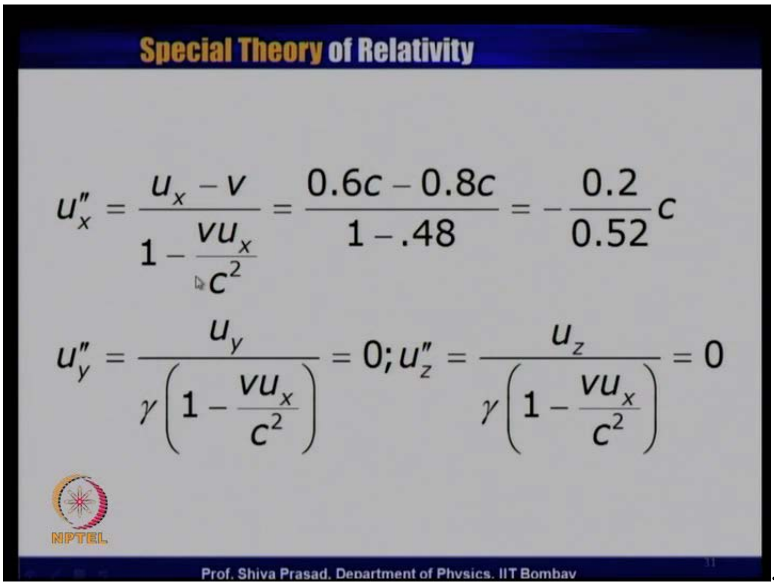

This is the standard relative velocity transformation equation which you have used in many of the examples earlier u x minus v divided by 1 minus v u x by c square see remember u x is the x component of the particle of the speed of the velocity of the particle the particle is the charge which moves with a speed of 0.6 c in s frame of reference so u x is equal to 0.6 c. v is the relative velocity between the frame I want to transform from s to s double prime and that speed is 0.8 c.

So, v plus 0.8 c divided by 1 minus v is 0.6 c I am sorry 0.8 c u x which is 0.6 c square will cancel with c square u get 0.6 multiplied by 0.8 which is 0.48, so this becomes 1 minus 0.48 which will give you minus 0.2 divided by 0.52, so according to an observer in s double prime frame the charge is moving in minus x direction which is sort of sensible because in s frame the frame s double times with the moves larger velocity than s prime therefore, whatever is at rest in s prime would appear to be moving backwards.

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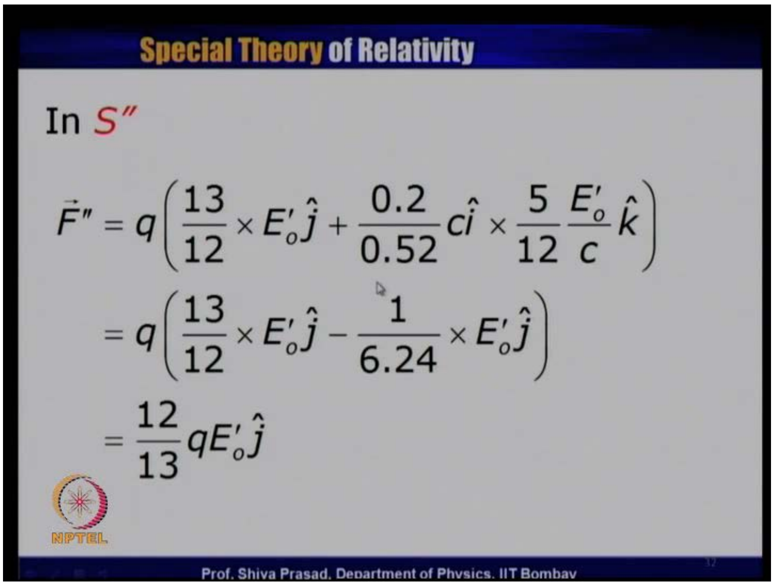

Therefore, it is expected that the external prime would turn out to be negative which it indeed turn out to be turns out to be as par the y axis components are concerned they will give me 0 so i need not bother about it because u i is 0 u z is 0 as far as the charge s concerned u is only along the first x direction in s frame these components are 0 only the x component is not 0.

I do exactly the same equation I use the Lorentz force equation in s double prime frame of reference write force, force equal to q times e which I have just now calculated 13 by 12 E naught prime g which I have just now calculated is minus let us remember we know that is this was minus speed therefore, minus 0.2 divided by 0.52 but, the magnetic field was also minus 5by 12 we E naught prime by c. So these two minus makes it plus so this becomes 0.2 divided by 0.52 c in i direction cross 5 by 12 E naught prime by c in k direction.

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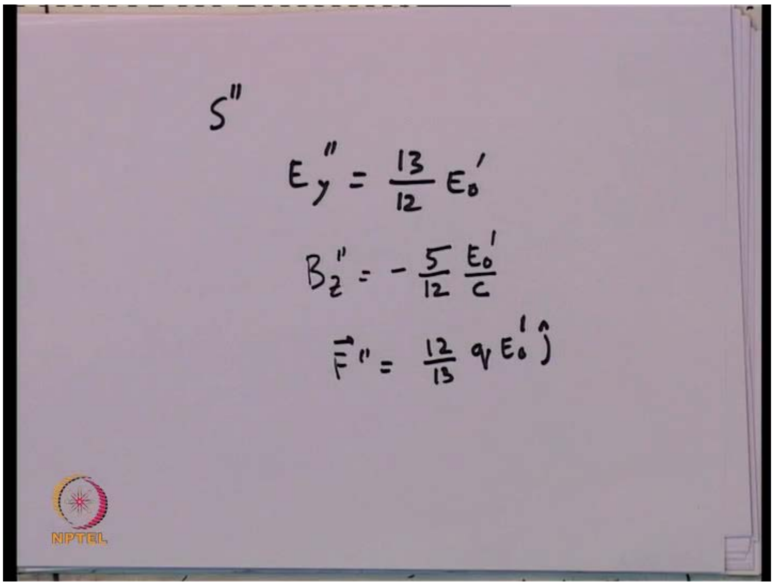

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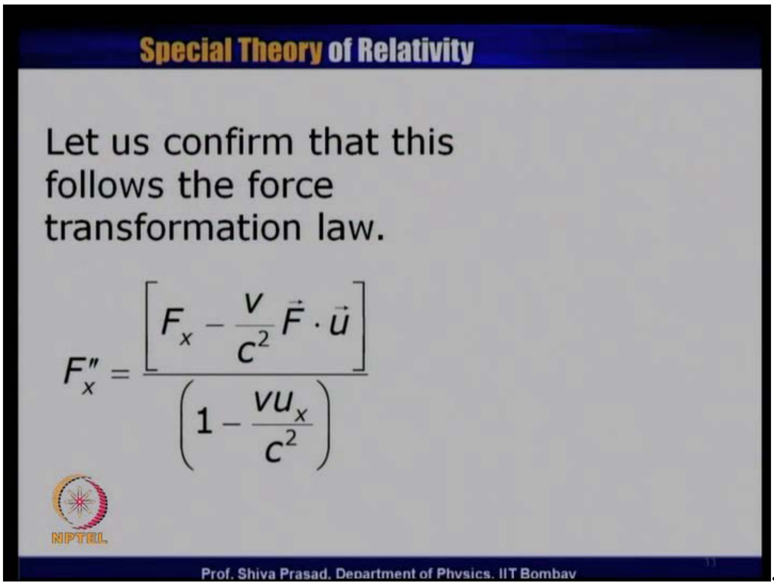

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I take the cross product i cross k this will give me minus k so here there will be minus sign then we just simplify multiply this particular things and you get s double prime is equal to 12 by 13 q E naught prime j even s double prime frame of reference the charge they that the force on the charge will be given by 12 by 13 q E naught prime j just remember smaller than q E naught prime remember we have said that this force seem to be a largest in a frame of reference in which the charge the particle happens to be at rest so as you can see that these two forces are smaller that the force that this particular particle have experienced in s prime frame of reference.

Now, let us confirm that this follows from the force transformation law so let us look at the transformation of the x components of the force here when I am transforming from s frame to s double prime frame of reference particle is neither at rest at s prime frame of reference not at double prime frame of reference so I cannot use those short equations I have to use the complete equations, and complete equations as far as x component is concerned see here, its little boring but, nevertheless we can do because I know all these splits so let us calculate as far as the f x is concerned this is 0 let us look at f dot u f dot u will be f x u x plus f y u y plus f z u z, so i have tried to calculate it in the next transference .

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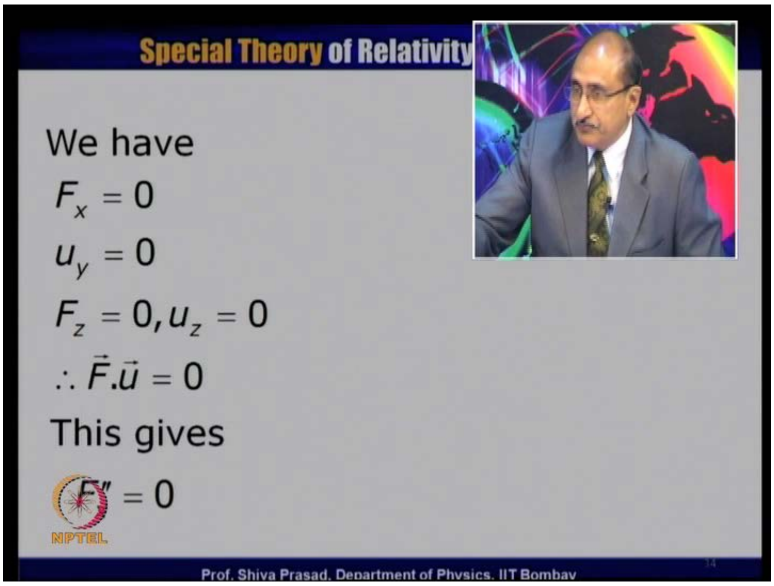

Now, we have seen f x is equal to 0 you know the charge also moves in s frame only along the x direction therefore, u y is equal to 0 the force is only along the y direction so f z is also equal to 0, velocity is only along the x direction so u z is also equal to 0 so remember f x u x will make this particular term 0 f y u y because u y being 0 f y u y also will be equal to 0 f z being be equal to 0 of course, u z is also equal to 0 a dot product f dot u will be 0.

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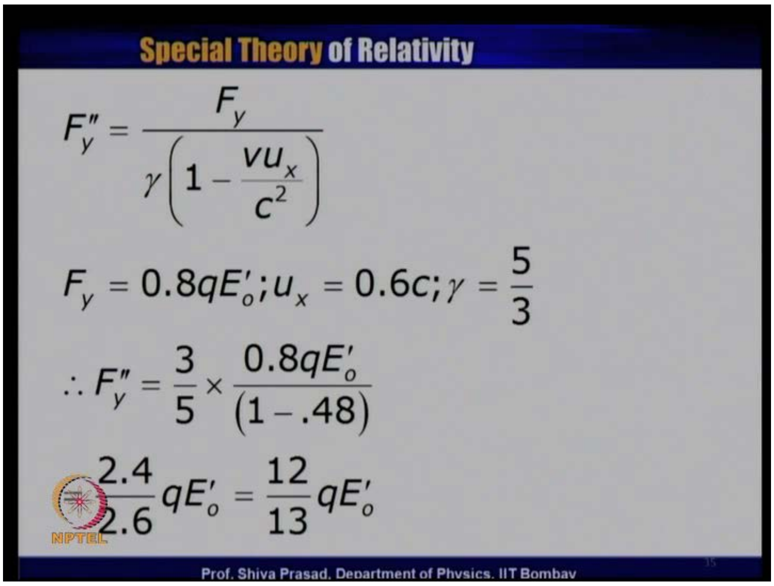

So this will give me the x component of the force to be equal to 0 this anyway I had expected its the y component it is the more interesting which let us write it here complete equation f y divided by gamma 1 minus v u x by c square as far as f y is concerned this is 0.8 q E naught prime this is what I have written as far as the frame s is concerned, u x the speed of the charge along the x direction which is 0.8 c gamma we have already said is 5 by 3, so in this particular equation gamma is 5 by 3.

So I will write this as 3 by 5 but, is one upon gamma so this is 3 by 5 for f y I have written 0.8 q E naught prime so this 0.8 q E naught prime divided by 1 minus v into u x which is 0.6 into 0.8 which will be must 0.48 just substitute rule of mathematics you find that the y component of the force indeed turns out to be 12 by 13 E naught prime as we have seen. In the earlier this piece of paper I have written that f double prime is 12 by 13 which I also get from the force transformation equation which I have expected.

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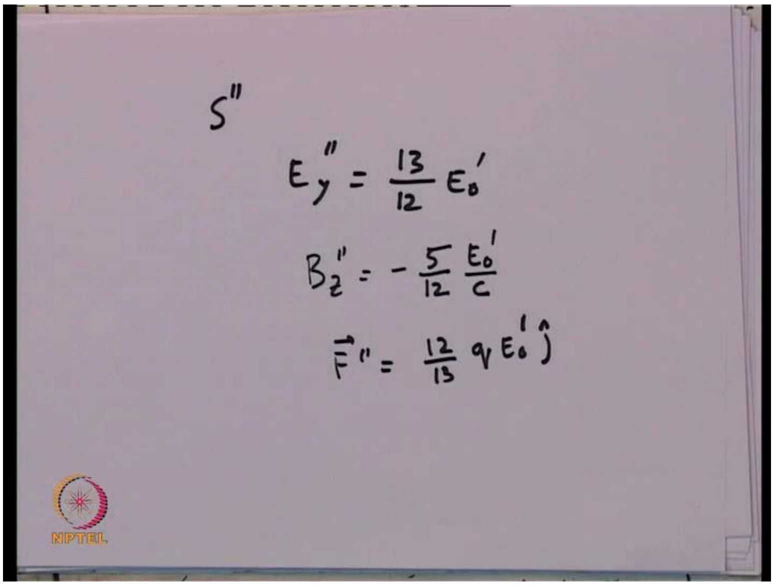

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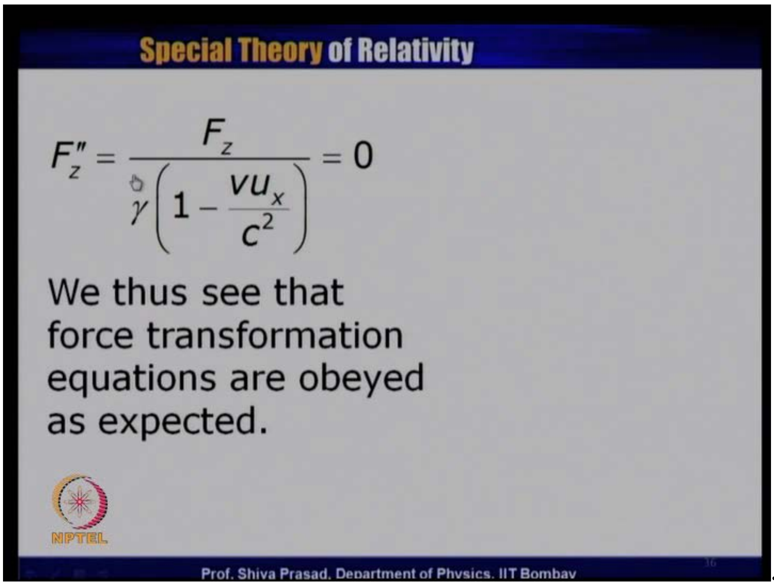

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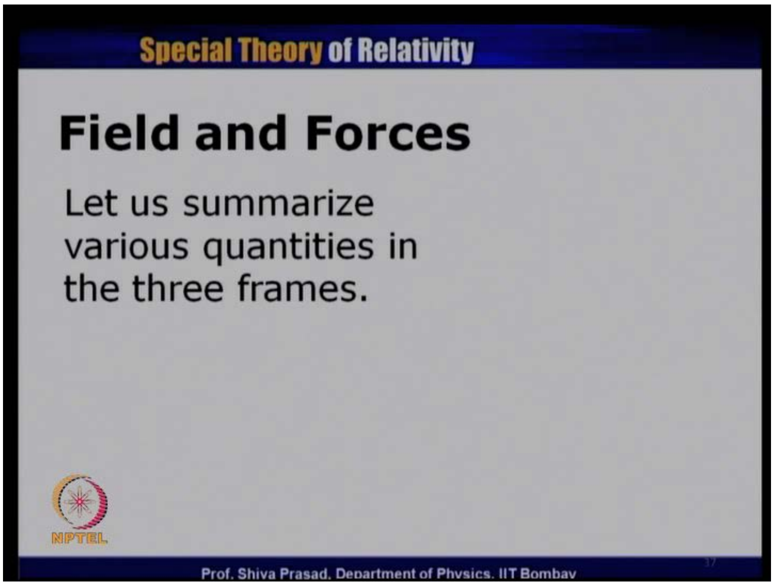

Now, as far as f z prime is concerned this anyway turn out to be equal to 0 we thus see that the force transformations are obeyed as was expected. So I am just summarizing all the fields and the forces that I have got in all the three frames.

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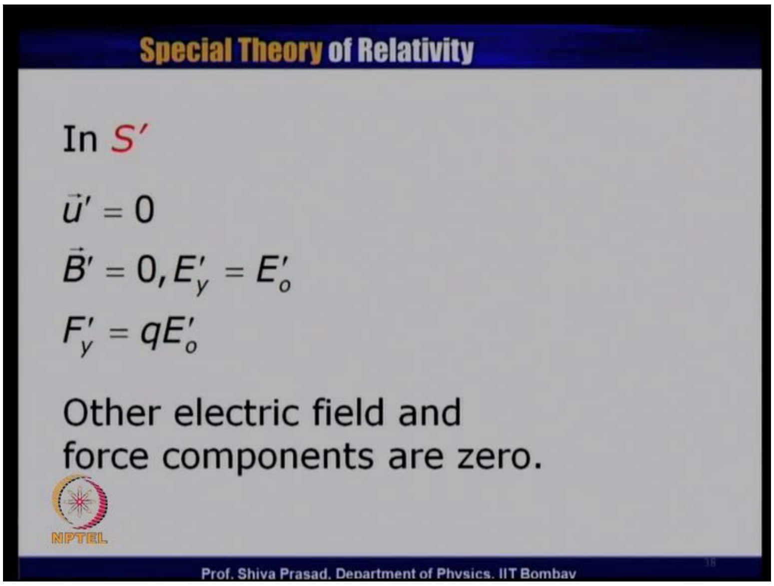

This is an s prime frame which we had started we had written the speed of the particle was 0 b the magnetic field was 0 electric field has only y component, which is E naught prime the force on the particle was q E naught prime other electric field and the force components were 0.

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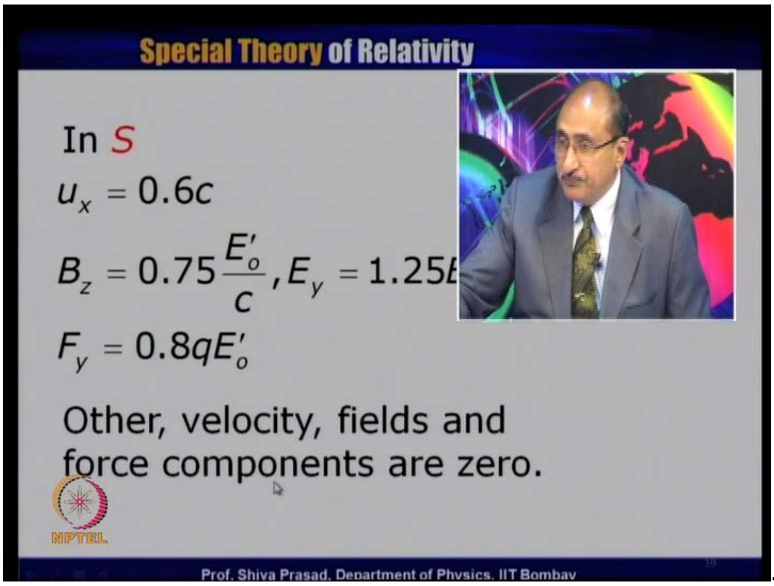

If we go to s frame of reference things change unless the particle is along a restless which is speed is 0.6 c the particle also experiences is z component of the magnetic field which is in the plus z direction the magnitude of which is 0.75 E naught prime by c it also experiences electric field in the y direction which is now somewhat larger than what we have seen by s frame s prime frame this is equal to 1.25 E naught prime, and a force on this particular charge would turn out to be 0.8 q E naught prime which will which is a complete Lorentz force the force which is because of electric field as well as of the magnetic field other velocity fields and the force components will all be 0. Now, let us write in a s double prime frame of reference in s double prime frame of reference I had to calculate the speed of the charge by using the velocity transformation.

And that I found out to be equal to u x double prime was equal to minus 0.2 divided by 0.52 c in this particular frame of reference there was a z component of magnetic field but, it was pointing out in the negative z direction which was minus 5 by 12 E naught prime by c and then the electric field as usual in the y direction and the magnitude of that will be 13 by 12 E naught prime. So as we have seen that the electric field also has changed magnetic field also has changed and the force on the particle is 12 by 13 q E naught prime these forces is also changed but, as we have seen that these forces would be the force transformation law as it was expected.

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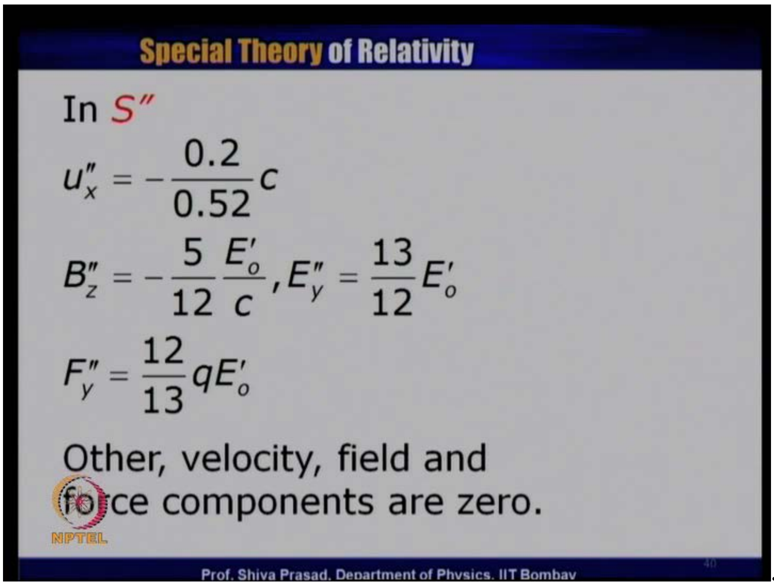

So I said other velocity field and force components are 0 in this particular frame of reference so this is just a summary of what will see what will be the field in which will be seen s s prime s prime frame of reference and also the forces as we can see that a field which happened purely electrical turns out to be electrical and magnetic combination of electric and magnetic field both in s frame and s double prime frame. As you can see the picture is very different, so there is nothing like a pure electric field nothing like a pure magnetic field it depends on the frame from which you want to look into a combination the two before we close let me do one more thing.

Let me try to make it transformation directly from s prime frame to s double prime frame of reference that is what we have said that where I have to calculate the field in s double prime frame of reference either I could have started from s or i could have started from s prime what I did I started from s prime to s double prime frame of reference the advantage here was that i knew the relative velocity between the frames but, I did not know the charged velocity in s prime to s double prime this calculate I will not go to the relative velocity of the frames because that has not been given to me I have to move by velocity transformation and find it.

Now but, the charge I know is at restless prime frame of reference and the advantage is that because the charge is at rest so that I can use the simple simplify the force Equation.

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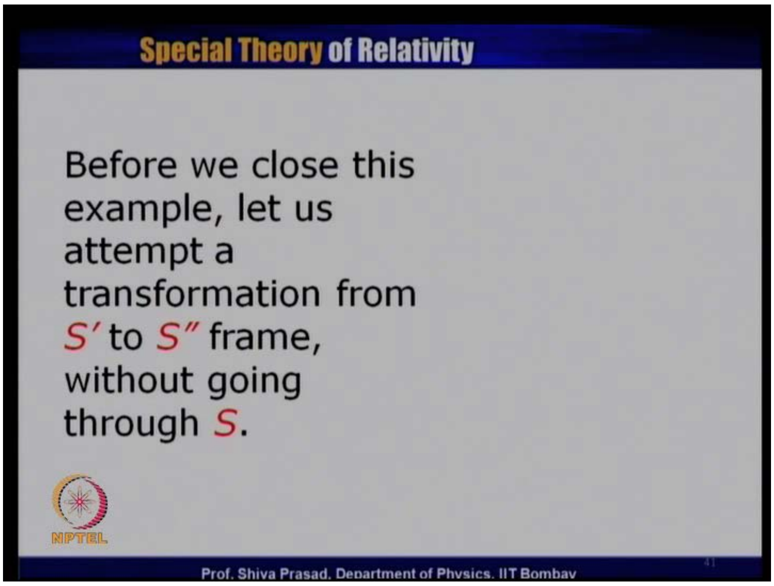

Let me just do this particular thing quickly so before we close this example let us attempt a transformation from s prime to s double prime frame without going through s, so if I go from s prime frame to s double prime frame of reference I must know and if I want to use the direct transformation I must know the speed of s double prime frame of reference in s prime frame of reference alright.

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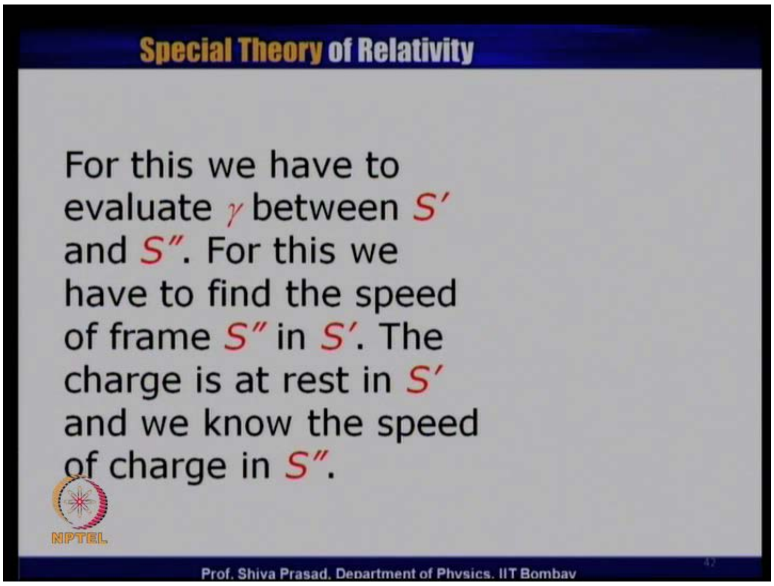

As I said for this I have to evaluate gamma between s prime and s double prime frame for this we have to find put the speed of s double prime in s because I am transforming from s prime to s double prime frame of reference. So one must know what is the speed of s double prime in s prime frame of reference and formation being in an s prime and transforming to s double prime I would use a direct transformation however I need not calculate this because I have already calculated this in a way I was told but, the charge at rest s prime frame of reference we have already calculated the speed of the charge in s double prime frame of reference which was minus whatever, it is so remember what is the speed of the charge in s double prime frame will also be the speed of s prime frame in s double prime frame of reference because the charge is at rest in s prime frame. therefore, whatever I had calculated in 0.2 or 0.52 will be the speed of s prime frame in s double prime frame of reference but, if I want to use direct transformation I must know what is the speed of s double prime in s prime frame of reference which is just be opposite in the sign.

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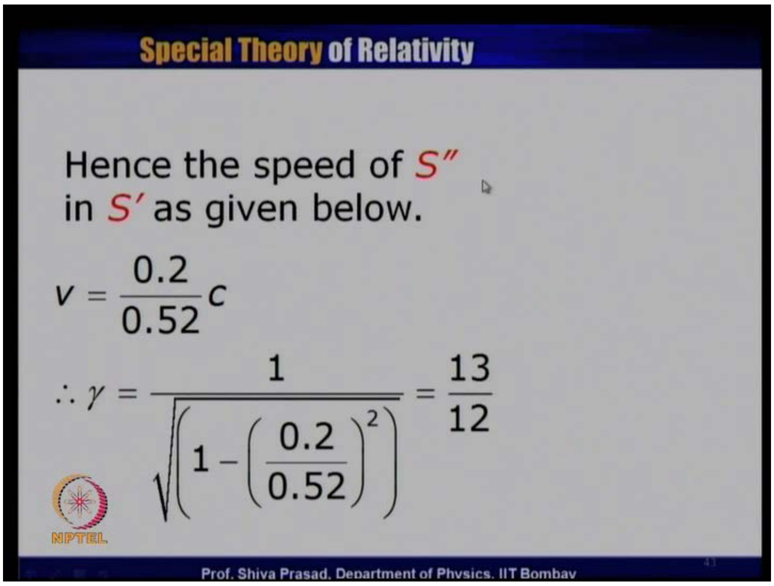

This what I have written hence the speed of the s double prime in s prime frame will be given by v is equal to plus 0.2 divided by 0.52 c because when I had used minus that was the speed of s prime in s double prime I want to find out what is the speed of s double prime in s prime is just opposite in sign if I am going in this way the speed is like this if this at rest this gives the other way it is as simple as that I calculated gamma using this this turns out to be 13 by 12.

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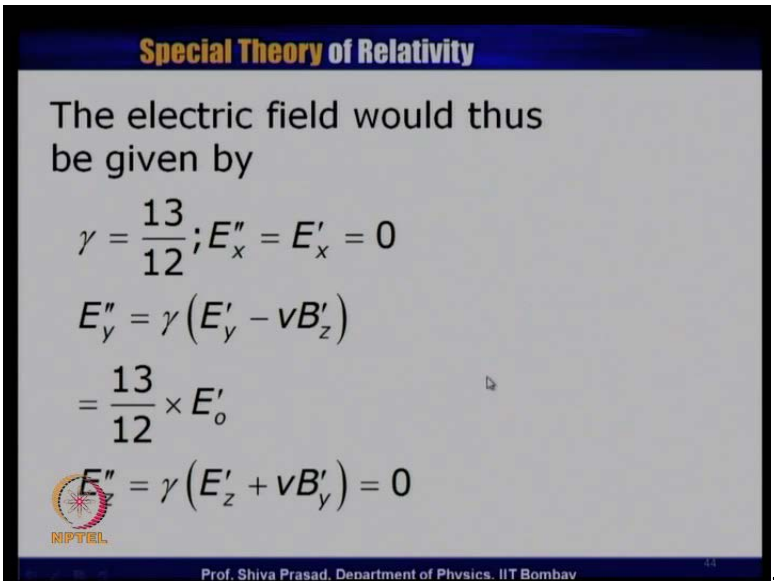

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Now, I can make electric field transformation as far as x components is concerned straight from 0 I am using direct transformation I use gamma is equal to 13 by 12 E y prime minus v B z prim E you see just be given by 13 by 12 E naught prime as I expected e z prime will turn out to be equal to 0 as I expected. So I could have directly gone from s prime to s double prime and calculating the electric fields without any mistake.

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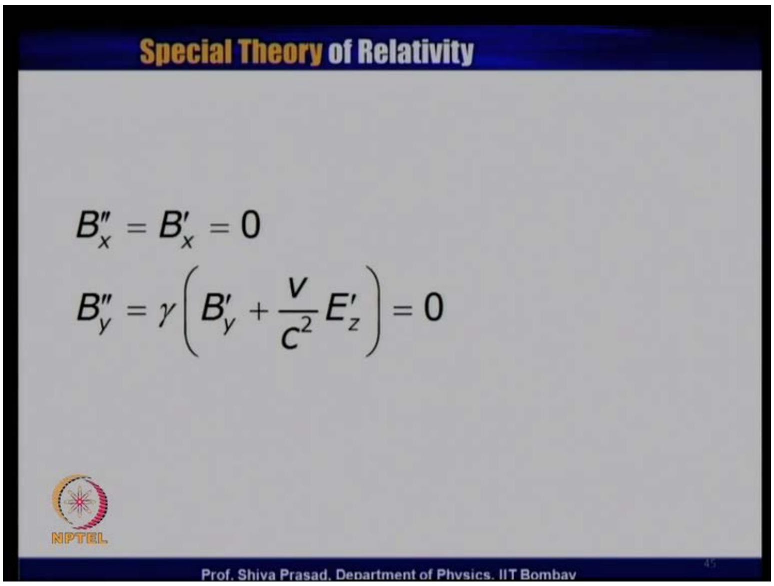

Then we did feel x component is 0 y component is also.

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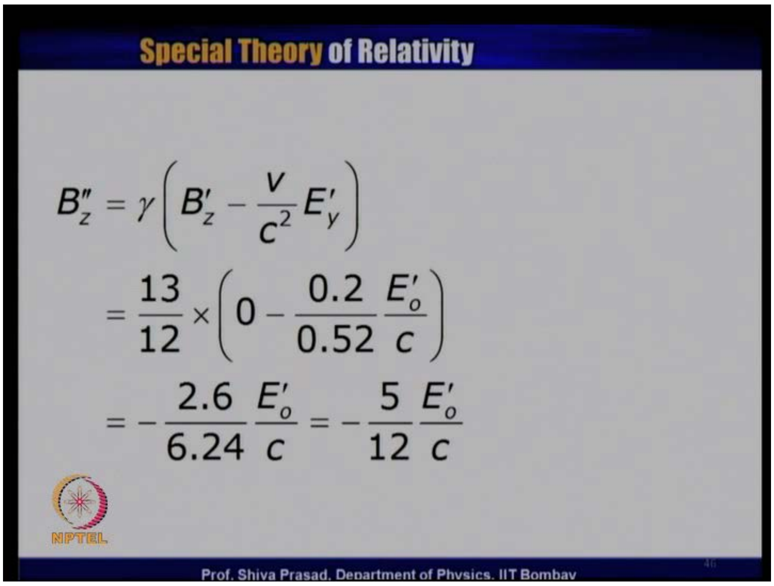

0 let us go to the z component z component is B z prime which happens to be 0 minus v by c square E y prime v by v is given by 0.2 divided by 0.52 c, this c will cancel here the field as far as s prime frame is concerned is just E naught prime I put it the E naught prime just work it out I get exactly minus 5 by 12 E naught by prime by c as I had expected so I could have gone from s prime to s double prime also or s to s.

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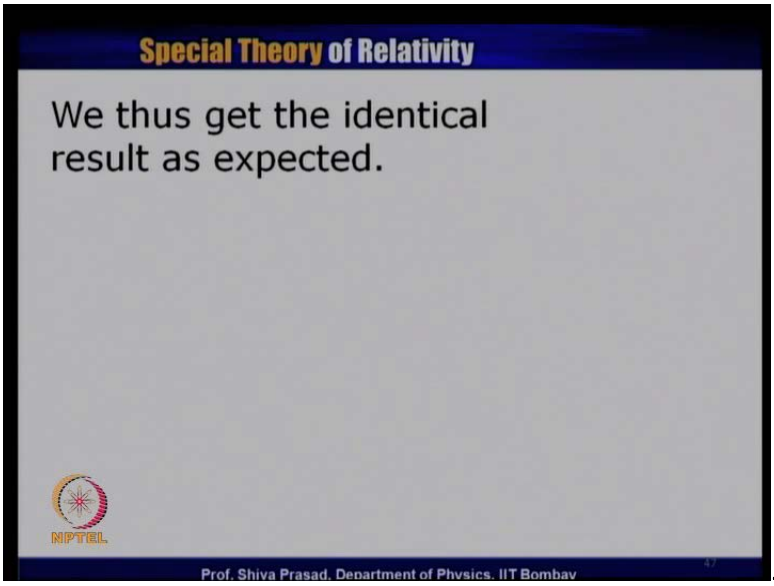

Double prime there is no difference we thus get the identical result as expected. So in conclusion we had used another method to derive perpendicular magnetic field transformation and also worked out an example of electric and magnetic field transformation.

Thank you .