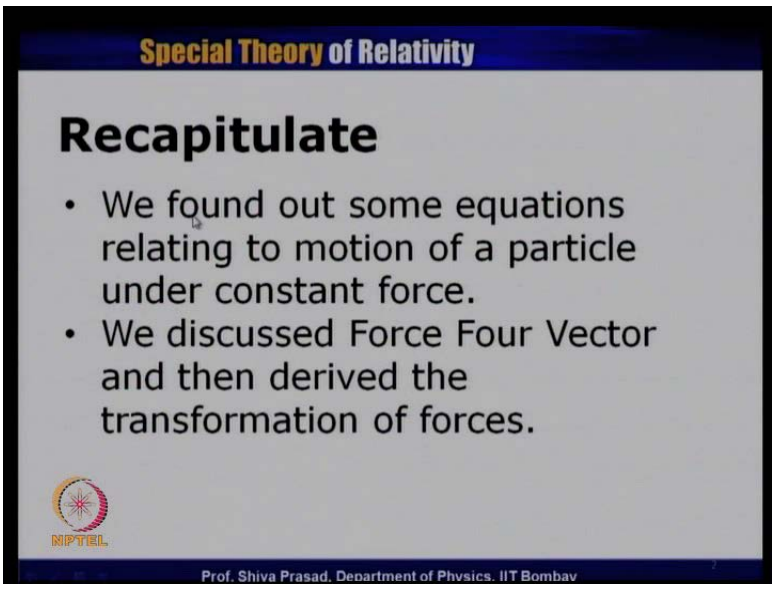
Special Theory of Relativity
Prof. Shiva Prasad
Department of physics
Indian Institute of Technology, Bombay

Lecture - 22
Electric and Magnetic Field Transformation

In our last lecture we had started with giving some equations which were equivalent of kinematic equations in the normal traditional classical mechanics. In which we assumed that force is constant. We assumed that the motion is along a straight line. So, that force and acceleration are in the same direction even in relativity and then these equations could determine the velocity after a given time and the distance travelled by the particle in a given time after that we evolved the concept of force four vector. We had already defined what is a force? We defined in last lecture what is force four vector? And then eventually came to the transformation of forces. We realize that once we change the frames. We the force on the body also changes in relativity, the force is also relative depending upon the frame from which you are looking their force on the particle will also appear to be different.

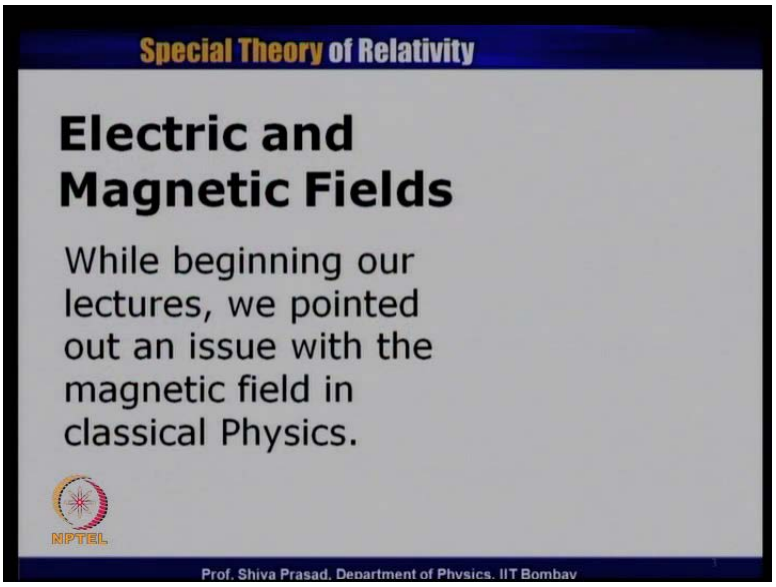
(Refer Slide Time: 01:37)



So, this is what we have mentioned here to recapitulate we found out some equations relating to the motion of a particle under constant force, and we discussed the force four vector and then derived the transformation of forces. Now from today onwards will come

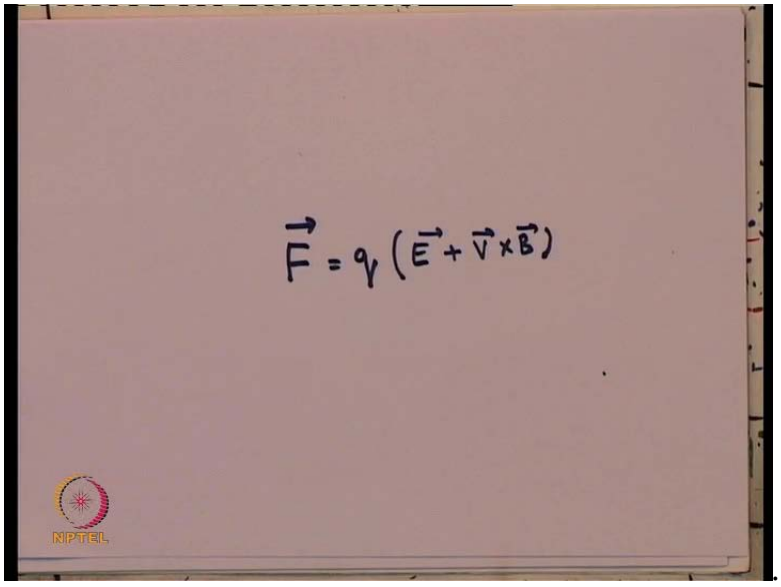
to essentially the last topic of this particular series of lectures on special theory of relativity, which is about the electric field and magnetic field transformation. If you recall right in our first lecture when we are trying to discuss the issues or the problems of the classical mechanics at that time we had mentioned about the Lorentz force.

(Refer Slide Time: 02:28)



So, this what I have written while beginning our lectures, we pointed out an issue with the magnetic field in classical physics. We had mentioned that in classical physics or this is to even relativistically.

(Refer Slide Time: 02:41)



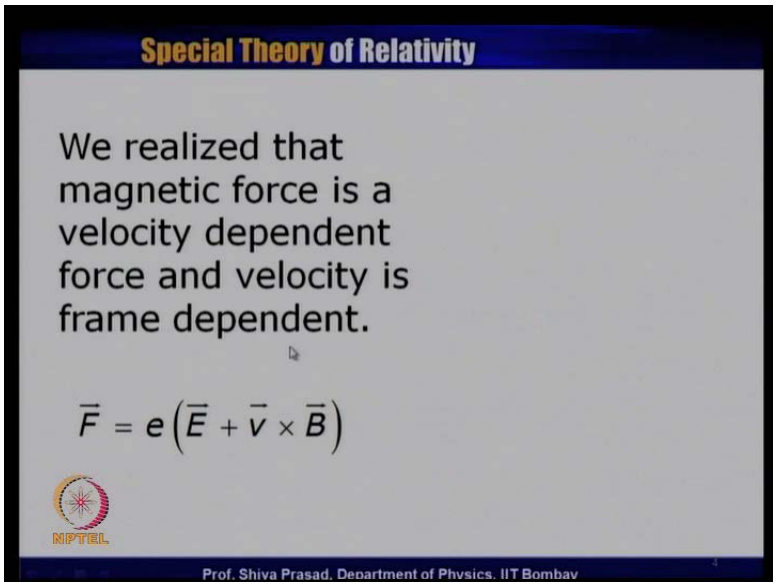
The force on a particle is given by if it is a charge q, when the particle is charged with a charge q, the force is given by q E plus V cross B where V is the speed of this particle in a given frame of reference. Now, we had earlier discussed in the classical mechanics in the traditional classical non-relativistic mechanics that acceleration is frame independent quantity. It means if you go to different frames, the acceleration in the traditional classical mechanics, non-relativistic classical mechanics will turn out to be same. But, the velocity of the particle may differ and in general they will be different.

So, we had said that if this force is a velocity dependent force velocity being dependent on different frames. The force would also appear to be different in different frames take for example, a frame of reference in which V is 0. It means the particle is at rest. In that particular frame of reference, the force on this particular particle because of the magnetic field will be 0. Let us assume that E is 0.

So, it means there is no force but, on the other hand in any other frame of reference when v is non-zero in principle the particle will experience a force which is v cross v or the charge particle or experience a force which is v cross v. So, it looks funny of course, we know that force is frame independent quantity, but we do not expect you know that relationship to be obeyed when a force is zero E one frame of reference another frame of reference it is a finite thing. At that time we also mentioned that some people can always rise a question having our traditional classical mechanics knowledge that after all V is also due to a motion of charge carriers and if I change the frame of reference. I also expect that problem and if you do change and that time we had mentioned that see actually the correct. The presentation or correct transformation of magnetic field and for that matter electric field comes when we discuss special theory of relativity.

So, that is what we are going to discuss today. In fact, what we will be essentially telling is that once we change the frame of reference both magnetic field and electric field would change. So, in principle as far as relativity is concerned, there is no difference between electric field and magnetic field depending upon the frame of reference. The field may appear to be an electric field or may appear to be a magnetic field or a combination of the two. So, let us start our discussion on this particular aspect today and then will give eventually problem in some of the later lectures some examples we are we can discuss this particular aspect little bit in more detail.

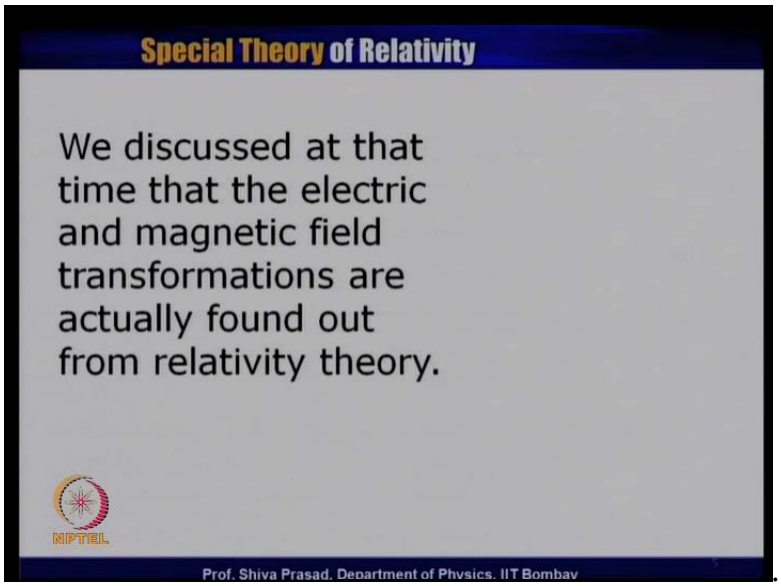
(Refer Slide Time: 05:54)



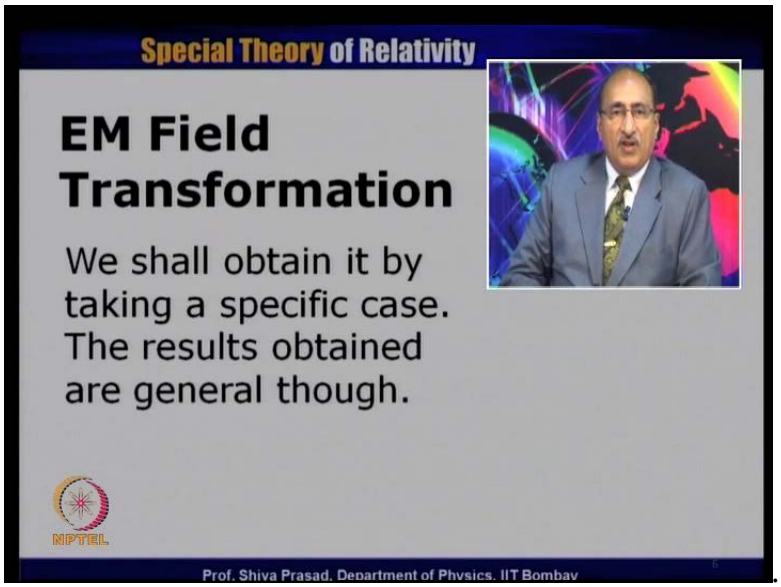
So, this what we have had written. We realized that the magnetic force is a velocity dependent force and velocity is frame dependent quantities. I have just now discussed that. This is a Lorentz force which is F is equal to E multiplied by E which is E is electric field plus V cross B where V is the velocity of the particle and B is the magnetic field.

So, this is the force which is being which will be experienced by a particle which has a charge q or in this particular case I have written e, so a charge e. So, now let us look for the transformation and for this particular transformation we would essentially need the force transformation that we had earlier discussed in our last lecture and we will see that how the electric field and magnetic field feels to transform. So, that this particular force transformation is obeyed. So, we will take some specific examples and using those examples, we will derive the electric field and magnetic field transformations which appear to be somewhat more general. So, this is what I have written we discussed at that time the electric and magnetic field transformations are actually found out from relativity theory.

(Refer Slide Time: 06:54)

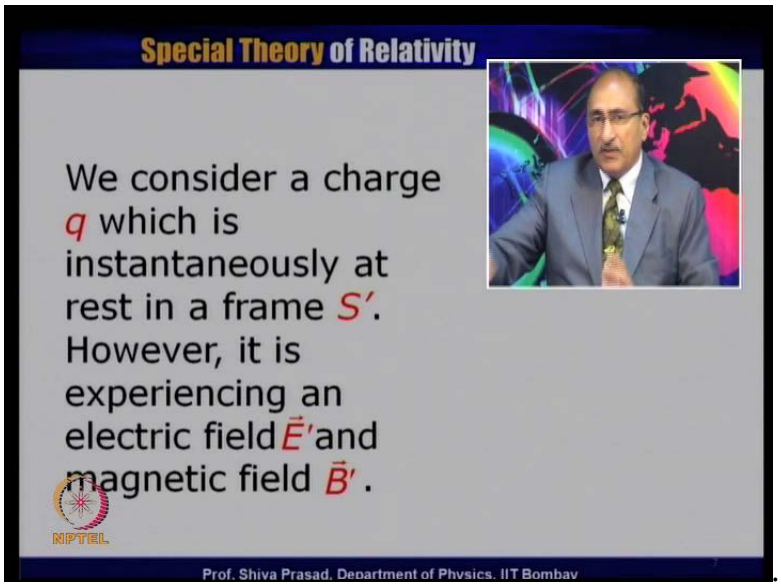


(Refer Slide Time: 07:05)



So, that is what we are going to do today. So, this is what I say electric electro-magnetic field transformation. So, EM essentially represents electric field as well as a magnetic field. So, as we said that in relativity we do not differentiate strictly between electric field and magnetic field. Because, what be appear to one frame of reference observer in a frame electric field be appear to be mixture of electric field and magnetic field to some other observer. So, we shall obtain it by taking a specific case the results obtained are general.

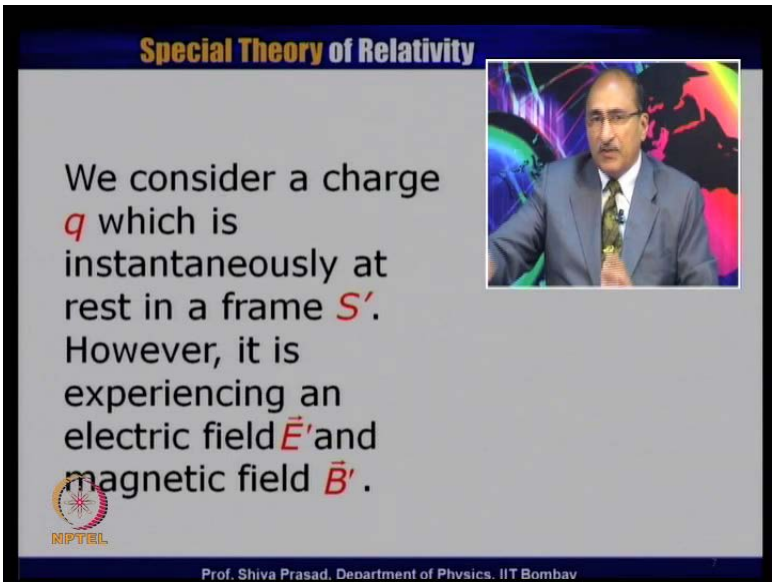
(Refer Slide Time: 07:36)



Now, let us consider the case, let us consider that there is charge q which is instantaneously at rest in frame S prime. We can always assume that that though this particular charge could be moving or could be accelerating or whatever it is but, there is a frame of reference in which this particular particle or this particular charge instantaneously at rest if you remember in our last lecture when we discussed the force transformation also we considered this particular case.

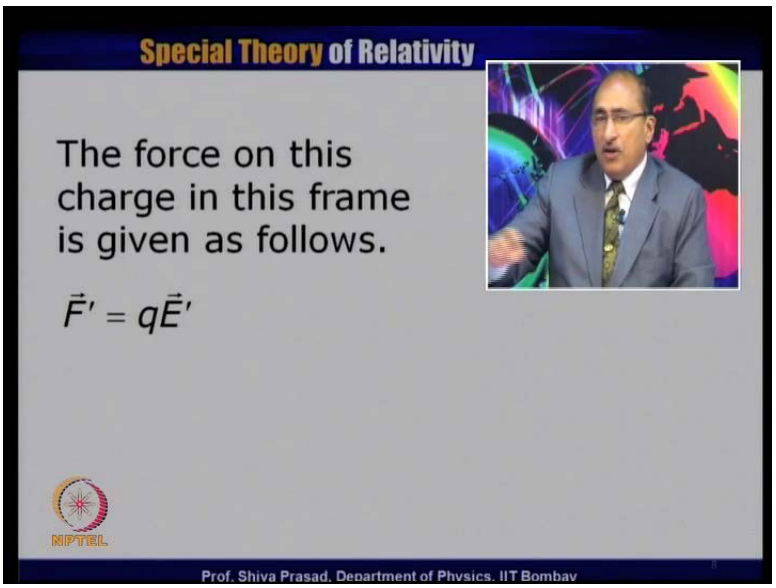
So, of course these frames or initial frames, so if actually the particle is accelerating then if it is instantaneously at rest in this particular frame of reference at related time it may not be. Because the frames are actually initial frames and all are requiring that at instant of time when I have discussed described in the situation this particular particle is at rest in this particular frame at a later time it may not be. Now in this particular frame I assume that there is a the particle is experiencing in electric field which I am writing as E prime and it is also experiencing a magnetic field which I am writing as B prime. So, the particle is instantaneously at rest in this particular frame and its experiencing electric field and magnetic field which are given by E prime and B prime.

(Refer Slide Time: 07:36)



So, let us see read this again we consider a charge q which is instantaneously at rest in a frame S prime; however, it is experiencing an electric field E prime and magnetic field b prime once I know this particular aspect I can always write what will be the force on the particle which will be given by the Lorentz force and now, let me use the relativistic notation because this is a particle which is experiencing this particular direction.

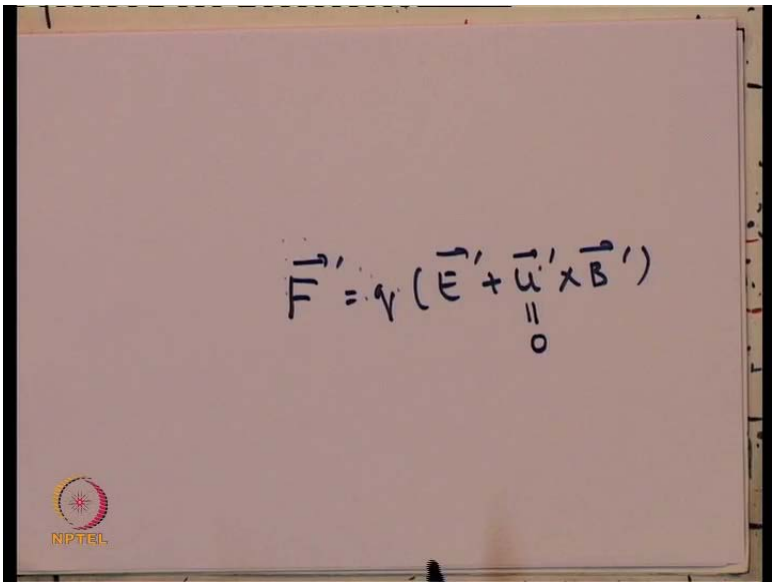
(Refer Slide Time: 09:50)



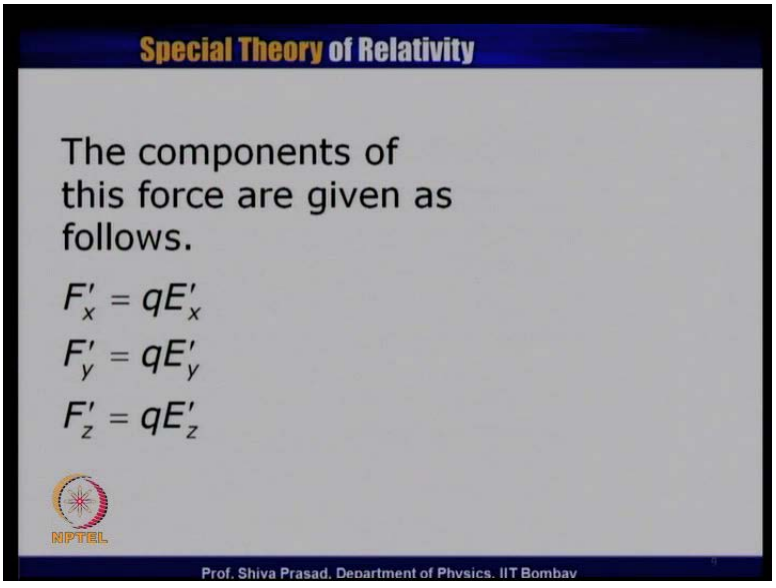
So in fact, this speed which will be given will be u prime because it is S prime frame of reference remember the symbol v we have reserved for the relative velocity between the

frames and u and u primes are the speeds of the velocities of the particles in respective frames. So, let us write the force on this charge S the speed of the particle the velocity of the particle is 0 at that instant in this particular frame. So, at that instant v cross b is 0, this was my force v cross b of course here we have written u cross b.

(Refer Slide Time: 10:12)



(Refer Slide Time: 11:12)

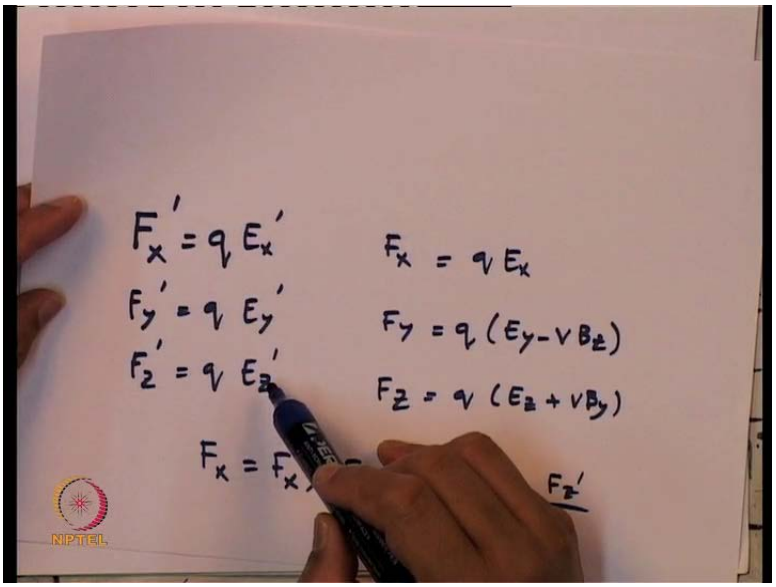


We write again F is equal to q E u cross b prime. I am in frame S prime. So, everything is been primed. This is the force which is being experienced by the particle in S prime frame of reference. This was the electric field E prime and this is the magnetic field B

prime all I am assisting that is u prime is 0 at this particular instant of time. So, this was 0. Therefore, the particle is experiencing only the force is due to only electric field. The magnetic field would not cause a force on this particular particle, because the instantaneous velocity happens to 0. So, the only force is due to the electric field.

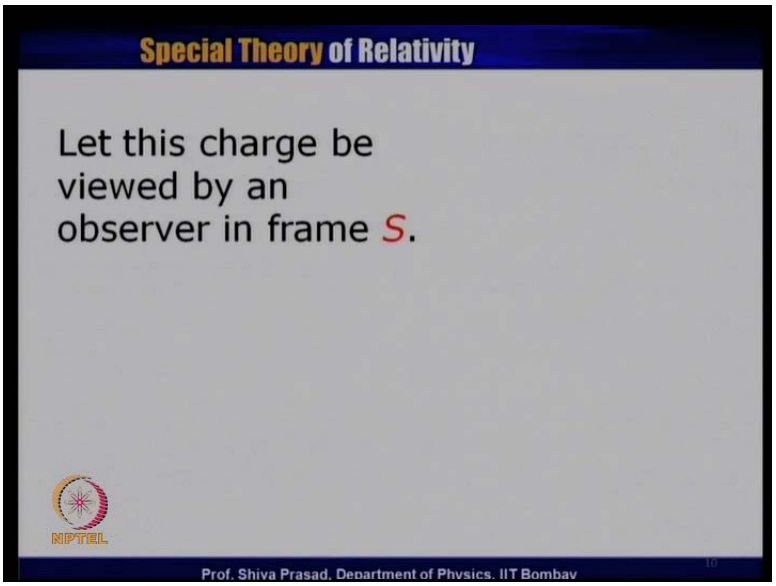
So, this is what I have written S prime is equal to q E prime. I can write this component wise because it will help me. So, I will have to do this in components because, electric field I can always write as E x prime i plus E y prime j plus E prime z prime k. I just take the components. So, I am writing F x prime which is the x component of the force which should be given by q times x components of the electric field y component of the force is q times the y component of the electric filed and the z component of force is q times the z components of the electric field and I am going to use this particular equation. So, what I have done. I have written this particular thing in this particular piece of paper, so that I can keep on using it whenever I want. So, now this is the equation which I have written here on this particular piece of paper.

(Refer Slide Time: 11:56)



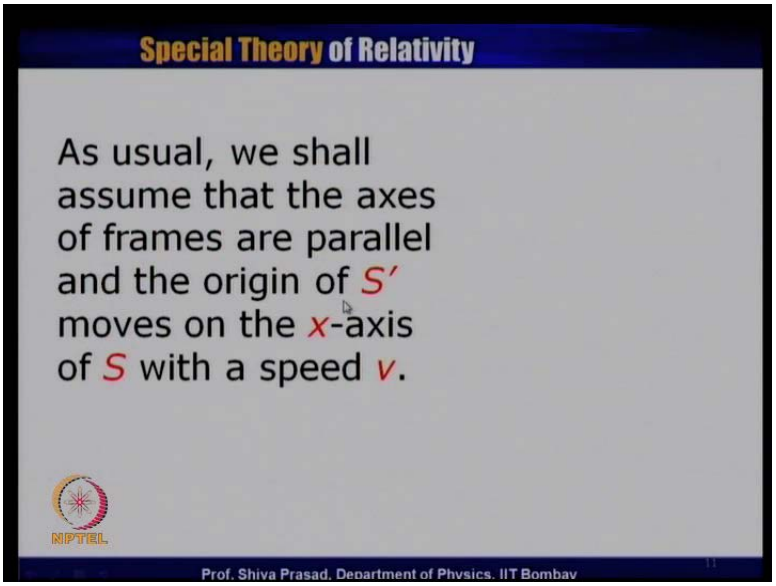
F X prime is equal to q E x prime F y prime is equal to q E y prime F z prime is equal to q E z prime then look at other equations later. Now let us assume that there is another observer S in another initial frame S and this is viewing this particular charge. So, there is another observer S which is viewing this charge this is I am writing here.

(Refer Slide Time: 12:30)



Let this charge be viewed by another observer in frame S and let me assume that this S and S prime would be exactly the same condition which we had been using right from beginning from Lorentz transformation. It means x axis's and the y axis's parallel to y prime axis parallel z prime axis. The relative motion is along the x direction and its along the x axis and x and x prime axis are always coinciding this is the standard thing which I have written here.

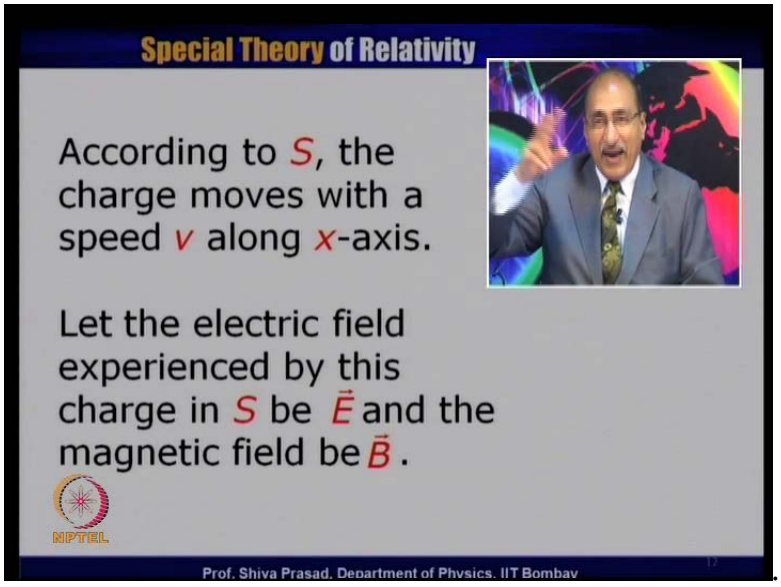
(Refer Slide Time: 13:03)



As usual we shall assume that the axis of the frames are parallel and the origin of S prime moves on the x axis of S prime with the speed V the relative speed between the frames we have always reserved the single v. So, this is still I am using this same now when this particular observer looks this particular particle at this particular particle at the same instant of time. It will assume that if we find out that the velocity of this particular particle is not 0 but, because this particle is at rest at that instant of time in S frame.

So, therefore, this particular particle velocity will turn out to be V which is the relative velocity with the frames. This also one can use the velocity transformation we have taken done from similar examples earlier this is very straight forward. We will always get if a particle is at rest in S prime. Its velocity will turn out to be equal to v which is the relative velocity of the frames in S prime frame of reference. So, the velocity of the particle is v in S frame of reference that is what I had said according to S the charge moves with the speed v along the x axis and of course this is velocity along x axis.

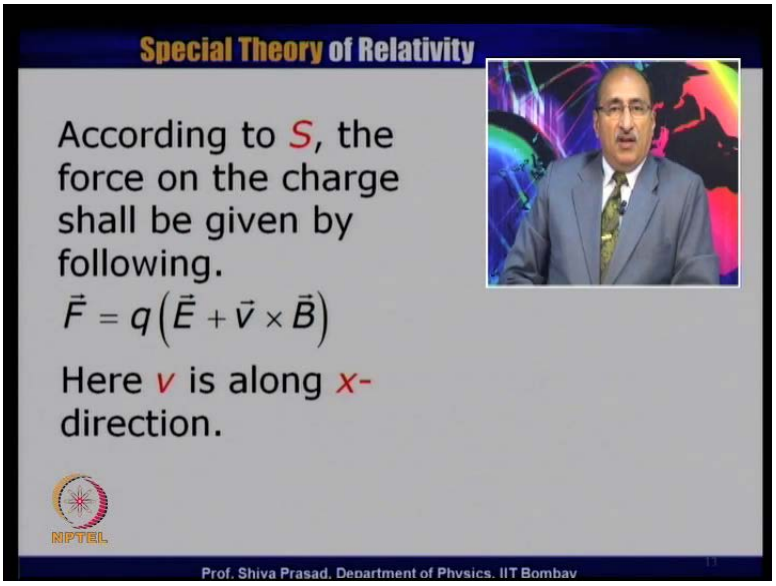
(Refer Slide Time: 14:05)



Now we have said that in general I expect that the force on the particle the electric field sorry, electric field on the particle will turn out to be different and let us assume that the field that is seen by this particular charge in this particular frame S will be E and B; electric field will be E and the magnetic field is B. So, these feels in general will be different from the feels which have seen by S prime observer used in the arguments which we have given in the beginning of the lecture. So, what I can do what is our

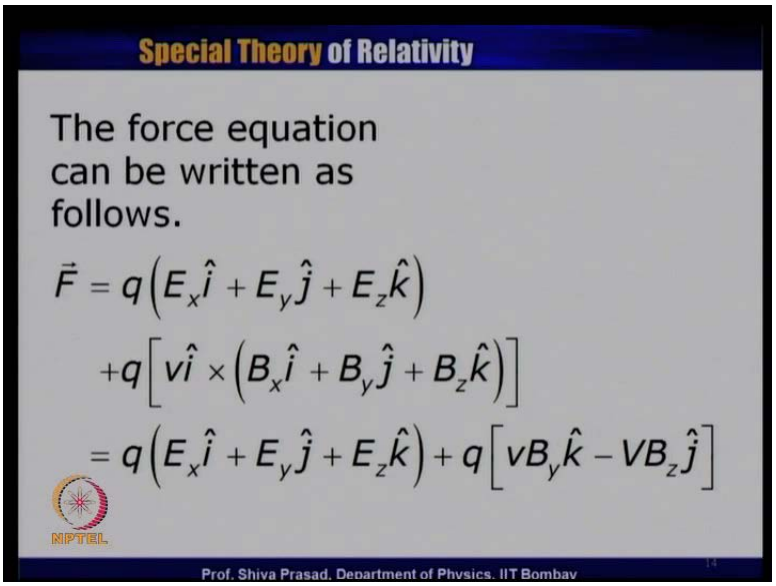
methodology is that I can always calculate what will be the force on this particular particle in S frame. If I assume what will be the value of if I take the electric field to be E and magnetic field to be B then I know how the forces will transform from S prime to s. Because we have already derived the transformation post transformation equation use these but the same equation and using those equations. I will be able to find out the relationship between the electric field and magnetic field as seen in S and as seen in S prime.

(Refer Slide Time: 15:47)



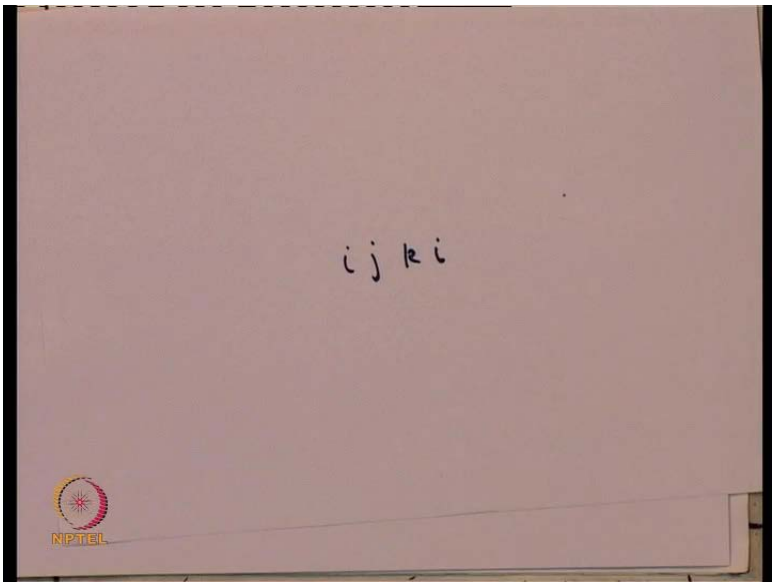
So, this is my logic; this is the way I am going to proceed. So, what I am going to do now is to calculate what will be the force assuming the electric field to be E and magnetic field to be B in this particular frame and then I know how the forces should transform using that I will calculate how the magnetic fields and the electric fields use to transform. So, let us go to the next transparency as I said according to S the charge particle is not at rest therefore, it would experience a magnetic force also not just the electric field not just force because of electric field, but will also experience the force because of the magnetic field and that particular force will be given by F is equal to q E plus v cross B because and this v of course, is along the S direction and this magnitude is going to the relative velocity between the frames. So, I am written according to the S the force on the charge shall be given by F is equal to E plus v cross B. here v is along the x direction.

(Refer Slide Time: 16:44)

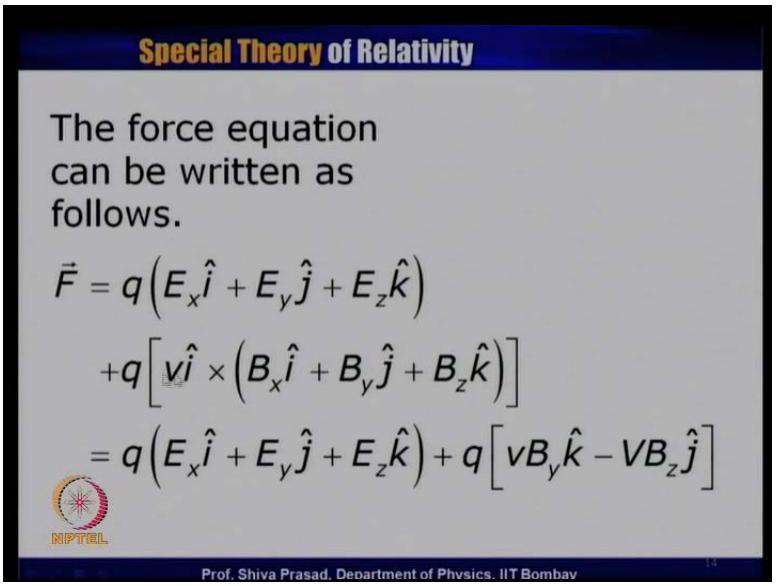


So, let us this particular equation in a proper vector form and try to extract what will be f x what will be f y what will be f z. That is why I am doing in the next transparency. Here I want to written out careful therefore; because, there is a magnetic field terms also. Therefore, I am writing in explicit form F is equal to q times E x i plus E y j plus E z k then it turn to magnetic field plus q v is heavy component only in x direction therefore, I am writing as v i cross this is the magnetic field which in general can be in any direction I write B x i plus B y j plus B z k. But, as far as this first term is concerned it remains unchanged. I have just copied it here. This is exactly identical to this but, now I can write this cross product and simplify the second term. I know that i cross i is 0. So, when I take v i cross B x i this will give me 0. So, that term will not can written then I take cross product with the second term which is i cross j i cross j would give me k therefore, v i cross B y j would give me v multiplied by B y in k. So, this what I have written v multiplied by B y in k direction. Now i cross k is a minus a if you remember you have to go to i j k i j k i. So, if you go i to i j, then I will get k if I go to j to k will get i, but here I am getting and k to i then I must get j, but here I am going from i to k. So, it will be minus j.

(Refer Slide Time: 18:22)

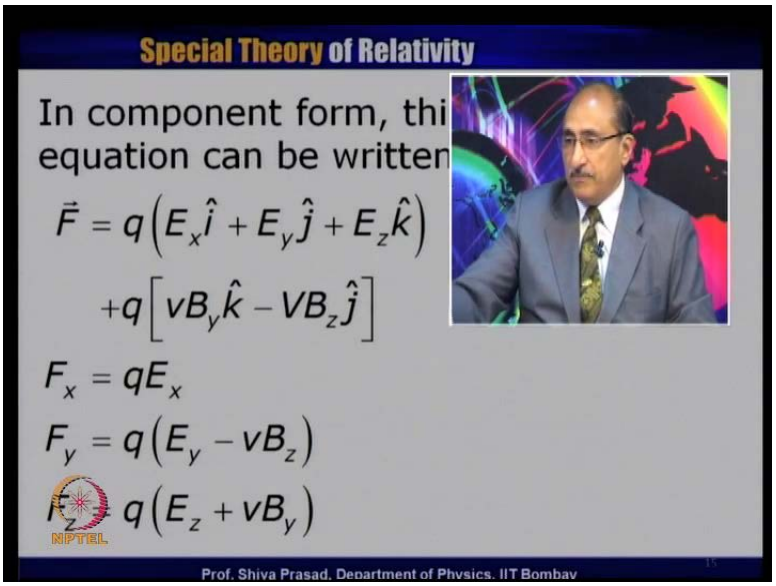


(Refer Slide Time: 18:40)



So, this is what I have written here v multiplied by B z i cross k is minus j which brings this minus sign and this is j. So, this is what I have written here. Now like before I will pick up the x component of the force y component of the force and z component of the force. So, these will be the force which we experienced by the particle in S frame of reference. If the electric field was E and the magnetic field was B both at this moment of time, I do not know what is E and B.

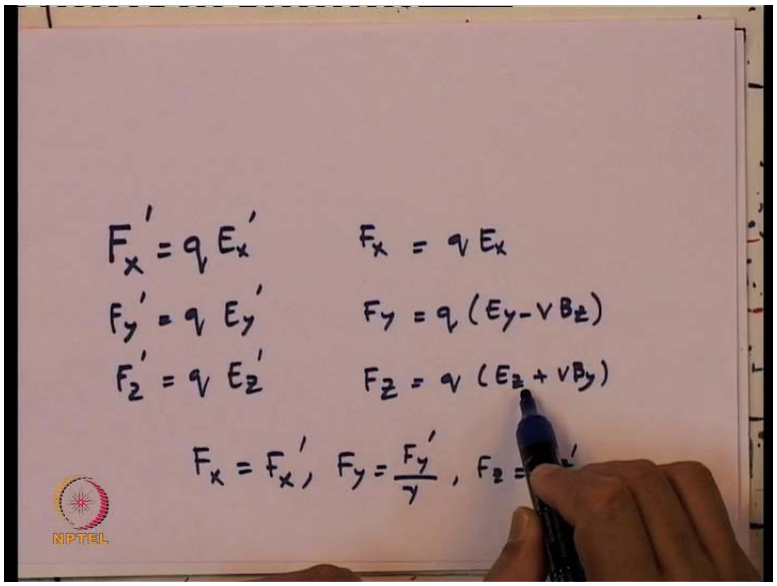
(Refer Slide Time: 19:20)



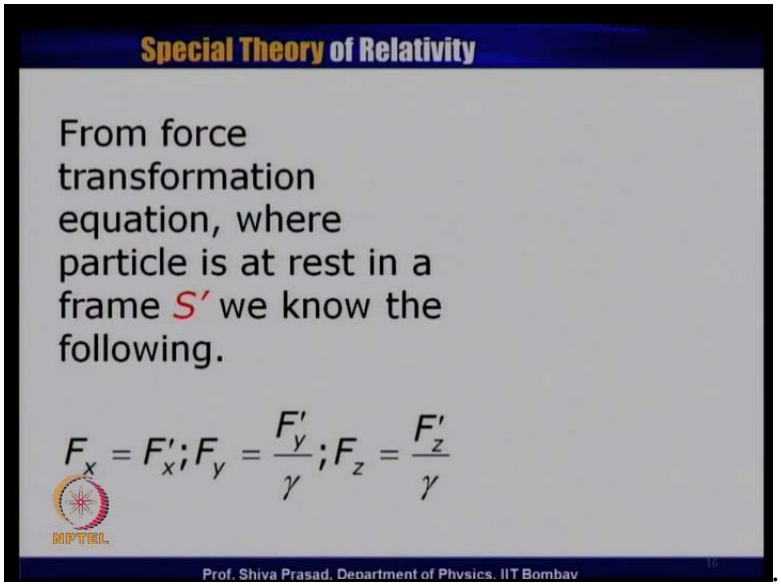
I all am only writing the force component and only if we which I know is the transformation of the forces. Same equation, which I have written in the last transparency have copied it here and I picking up the different components. So, I am writing x component of the force which will be the only i part. See i is appearing only in this term nowhere else i is appearing. So, I just have picked up one term is just q E x i. I am not writing because am just writing the x component of the force; x component of the force is given by q E x. Let us write the y components of the force force for that I have to pick up the terms containing j. This term contains j this terms contains j. So, they are both contributing in a force in the y direction.

So, this is E y minus B v B z will be the y component of force multiplied by q. So, F y will be q E y minus v B z. Now let us pick up the z component of the force. So, I have to look only the k terms. There is a k here. So, this is E z k there is a k here there is a plus sign here. So, the F z will be E z plus v B y of course, multiplied by q. So, F z will be q E z plus v B y this is what I have written again here, because I am going to use immediately after that in this particular piece of paper. F x is equal to q E x F y is equal to q E y minus v B z and F z is equal to q E z plus v B y.

(Refer Slide Time: 21:05)



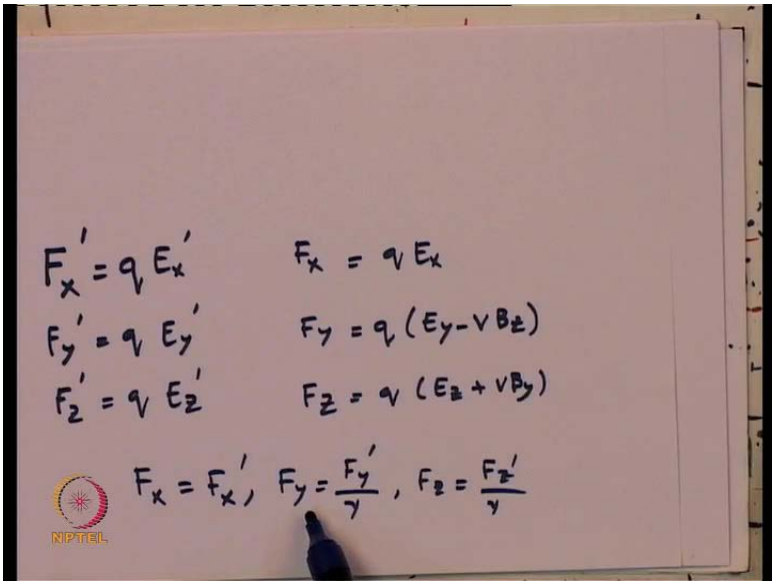
(Refer Slide Time: 21:20)



Now, I know that what are the transformation equations? Remember the particle is at rest in S prime frame of reference and in that particular case our force transformation equations were somewhat simpler. So, those equations I can write now. Because, the particle was at rest in a S prime frame of reference. So, force F x of course, does not change will be explain. F x prime force in y direction F y will become F prime y divided by gamma and force along the z direction F z will be F z prime by gamma. This is a special case of the force equations which we have that force transformation which I had used last time. In which I have said that if the particle is happens to be at rest

instantaneously in a given frame then the transformation equations may not somewhat similar, and this precisely the case in this particular situation where the particle is at rest in S prime frame of reference.

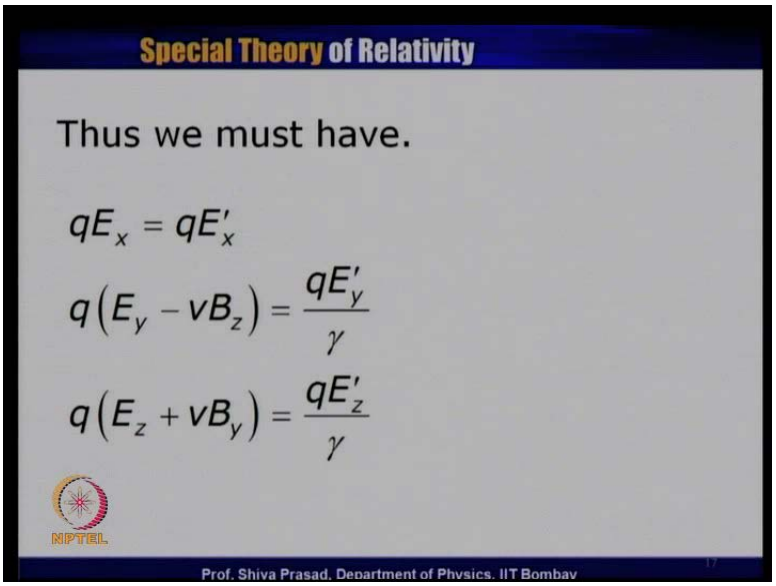
(Refer Slide Time: 22:32)



So, I can write these equations in rather simpler form and this is what I have written here and this is also I have written this particular piece of paper. Here where I have written F x is equal to F x prime F y is equal to F y prime by gamma and F z is equal to F z prime by gamma or let us look at this particular paper carefully. I know that my force transformation equation must obey these because, this is the way we have derived the force. If we have to be consistent these equations have to be obeyed.

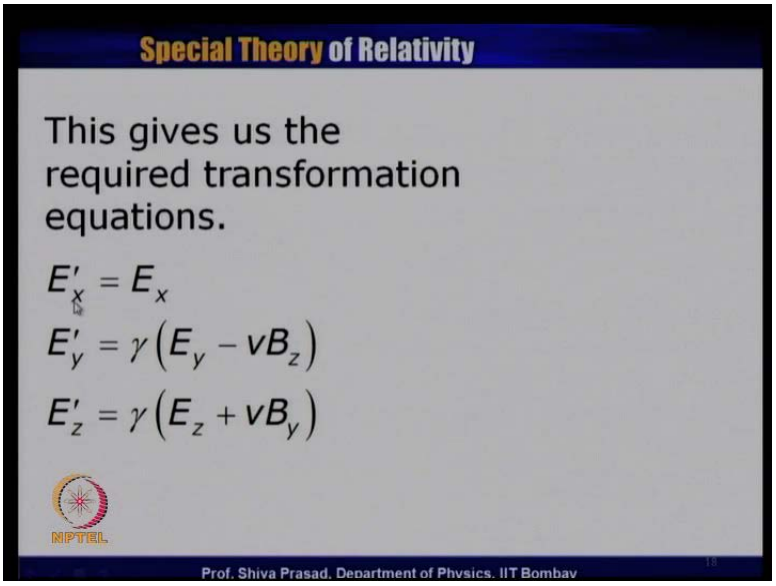
I do not know what is the relationship as yet between E x and E x prime and E y and E y prime and other fields other components of the fields. But I can always use in this particular thing that effects must be equal to F x prime F y must be equal to F y prime by gamma must be equal to F z prime by gamma because they have to obey these force transformation equations. So, if I use this particular thing I can quickly find out the relationships between the electric fields and the magnetic fields as we see in the next transparency.

(Refer Slide Time: 23:37)



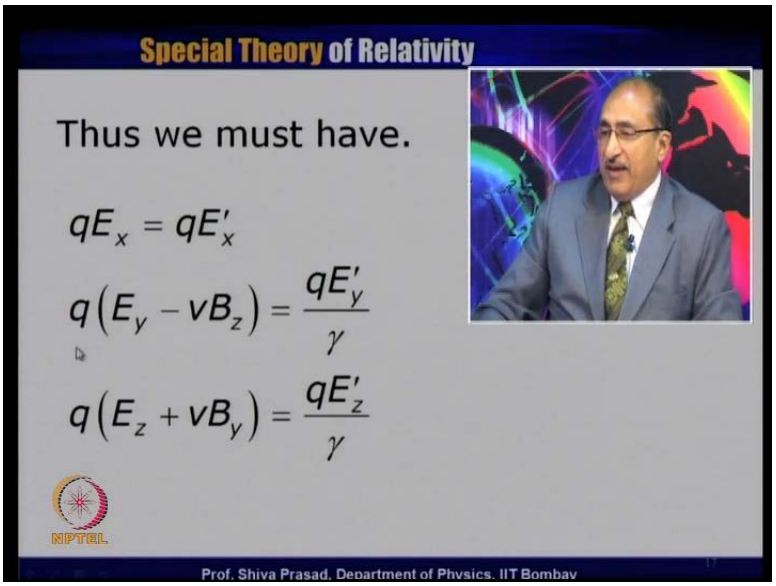
So, this is what I have written. This is the first equation which is q E x must be equal to q E x prime which because F x was equal to F x prime. The second equation was F y is equal to F y prime by gamma; this was i F y F y prime was just q E y prime divided by gamma. This is F z must be equal to F z prime which is q E z prime divided by gamma.

(Refer Slide Time: 24:15)



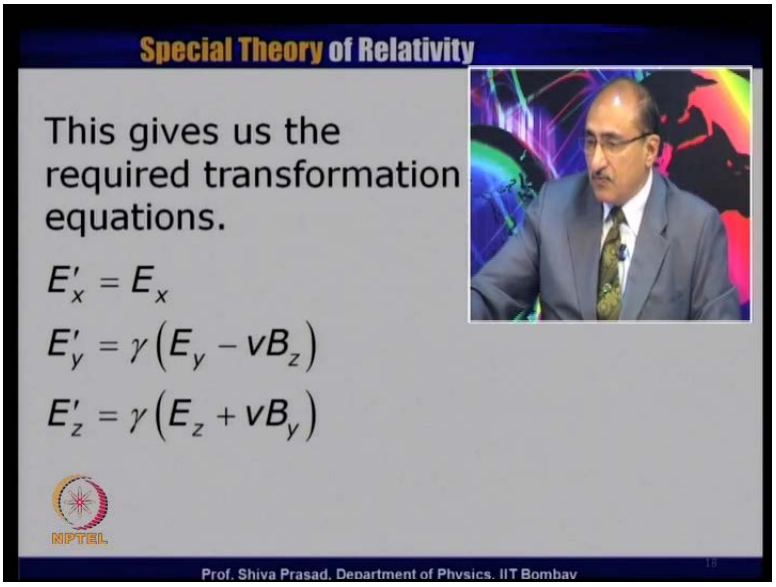
So, these are the three equations. First equations is straight forward q will cancel gives you E x is equal to E x prime. Let us look at two equations and see what what do they give us. This is just writing is equations simple form as E x prime is equal to E x.

(Refer Slide Time: 24:27)



If you look at back equation the q will cancel out here, E y prime this gamma will come here on this particular equation and this will give me E y prime is equal to gamma E y minus v B z. Similarly, if you look at this particular equation q will cancel this gamma I can take on this side. So, E z prime will be equal to gamma times E z plus v B y. So, these are precise the equations that I have written in the next transparency.

(Refer Slide Time: 24:55)

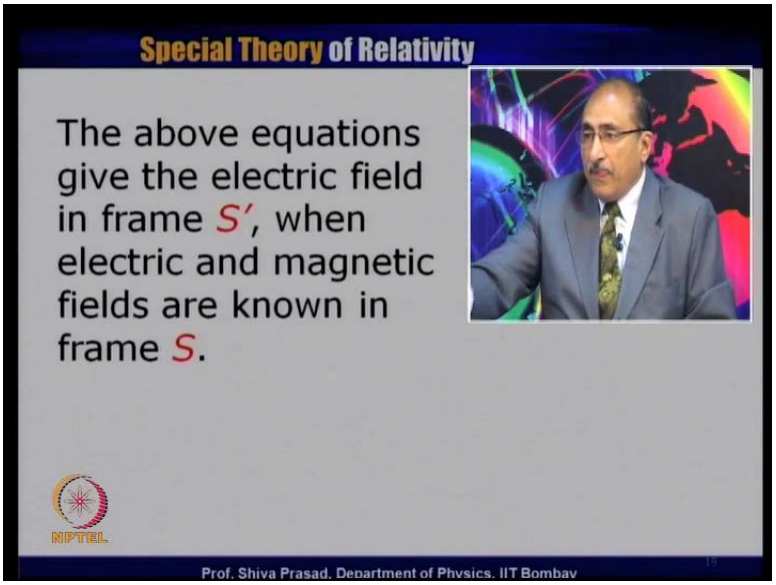


E x prime is equal to E x E y prime is equal to gamma E y minus v B z and E z prime is equal to gamma E z plus v B z v B y I am sorry. So, what do this equations give us this

equations tells us that if my force transformation equation has to be obeyed, then the x component of the electric field must remain identical between S and S prime frame of reference as far as the y component of the force is considered if electric field is concerned y component of the electric field will be change. It will become I mean in the y component in the x prime component it will be given by E y prime if E y and B z happened to be the y component of the electric field and z component of the magnetic field in S frame of reference then the electric field in the y component of the electric field in S prime frame of reference will be given by this particular equation.

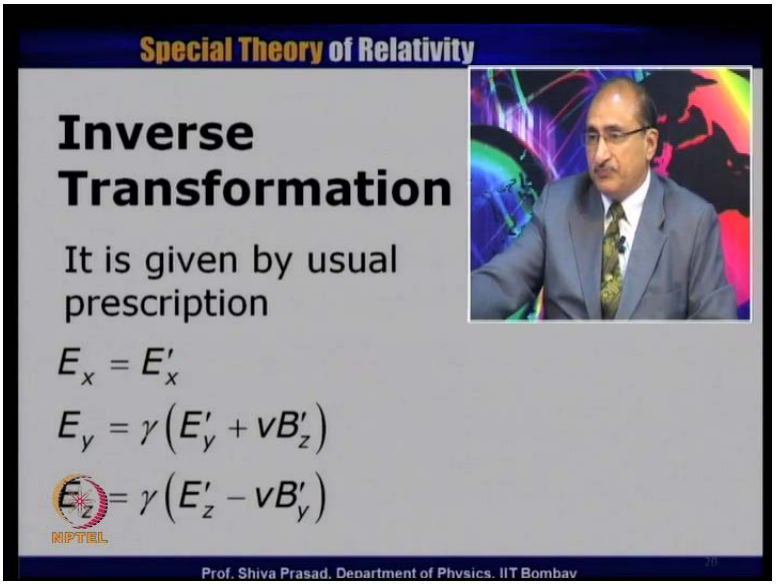
Similarly, the z component of the electric field will be given by this equation where E z is the z component of the electric field in S frame and B y is the y component of the magnetic field in S frame. So, it means if I know what is E x E y E z and if I know what is B x B y B z. I will find out I will find that it is will be the electric field. So, I get this equations will tell me electric field in a given frame if I know the electric field and magnetic field in a different frame what is interested to realize then this electric field thus depend on the magnetic field with the other frame. Similarly, this electric field also depends not only the electric field but, also the magnetic field in the other frame that is what we admen by saying that relativity electric field and magnetic fields have they do not have really that sense separate context because what happened this particular magnetic field is contributing. So, to see to electric field in different frame of reference. So, electric field does depend on magnetic fields in a frame.

(Refer Slide Time: 27:00)



So, this is what I have written above equations give the electric field in frame S prime when electric and magnetic fields are known in frame S always do a inverse transformation.

(Refer Slide Time: 27:35)

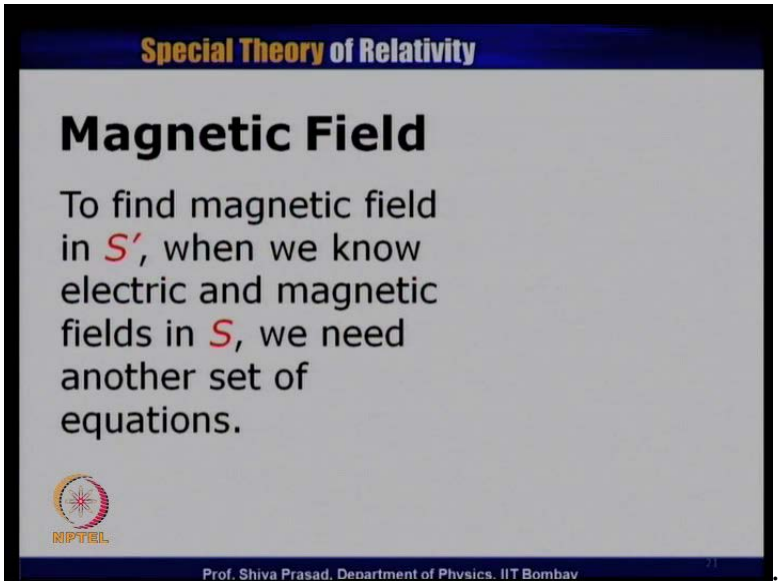


If I know what is electric field what are the electric field and magnetic field in S prime frame of reference. I can always find out the electric I means at the moment electric field in S frame at that particular matter all we have to do is to change prime to unprimed quantities change the sign of v the standard prescription of going from a direct transformation to inverse transformation. So, this I have written inverse transformation it is given by usual prescription. So, I have written E x is equal to E x prime because that does not change. Here I have changed the sign here E y is equal to gamma E y prime plus v B z prime. Here also I have changed the sign E z is equal to gamma E z prime minus v B v B y prime. So, if I know electric and magnetic field in S prime frame of reference. I can find calculate what will be the electric field in S frame could there is a missing component you might have realized in these equations done. This equation is to evaluate the electric field.

But I have not given the transformation; I have not given the equations what will happen to the magnetic field. We also realized the magnetic field will also change that is what we are saying. So, these equations only give me the values or the components of the electric field in a frame. if I know electric field and magnetic field in a different frame. I

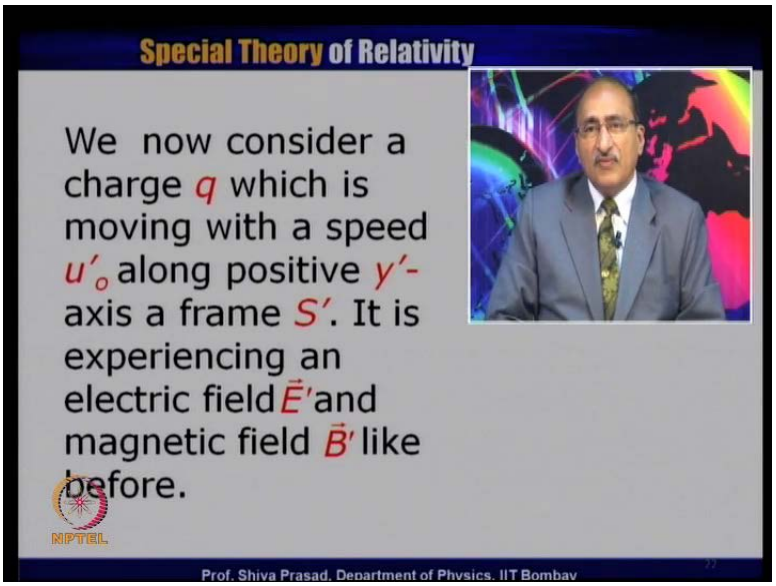
must also I have to calculate the components of the magnetic fields also in this particular frame even happened to know electric field and magnetic field in a different frame. So, for that I have to do little more exercise which is not that simple as the way we have done it but, that is do it because that is also equally important. Because, in principle I must be able to calculate both electric field and magnetic field in a given frame. If I know electric field and magnetic field in a different frame that is what actually the electric magnetic electro-magnetic field transformation would mean.

(Refer Slide Time: 29:18)



So, this what I have written magnetic field; I have to find out the magnetic field components. To find magnetic field in S prime when we know the electric and magnetic field in s. We need another set of equations that we have not yet found out. This is what I will do now to find out what are the components of the magnetic field if I know electrical magnetic field in a different frame.

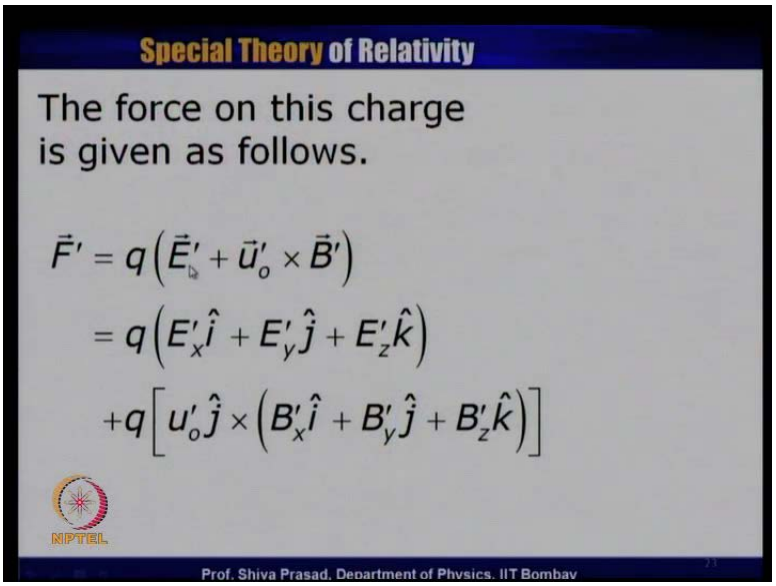
(Refer Slide Time: 29:63)



Now, I look again to any special case and then results happened to be general enough. Now the I alter this equation little bit which makes things little more difficult mathematically little more difficult in earlier case I had assume that the particle was at rest in S prime frame of reference. Now I will assume that this particular particle is actually moving along the plus y prime direction of S prime frame of reference. It is not at rest in a S prime frame of reference. But, its moving with a velocity but, that velocity is directed along the y prime direction of S prime frame of reference.

So, we now consider a charge q which is moving with a speed u naught prime along y prime axis in a frame S prime of course, like before I have assumed that this experiencing if electric field E prime and magnetic field B prime in this particular frame. Exactly this situation the only thing is which is different now that I am assuming that the particle is not at rest and therefore, the Lorentz force also have a term because of the magnetic field. So, let us write that particular term again we do exercise the flow is exactly identical. I take this particular force write the components of the force then I go to S frame of reference then I will wait what will be the velocity of the particle which will not be little more complex assume what will be the electric field and magnetic field in S frame and E and B then again like the forces then I know what are the transformation of forces in the two frames and of course, in this case neither of the frames you will find the particle to be at rest.

(Refer Slide Time: 31:40)

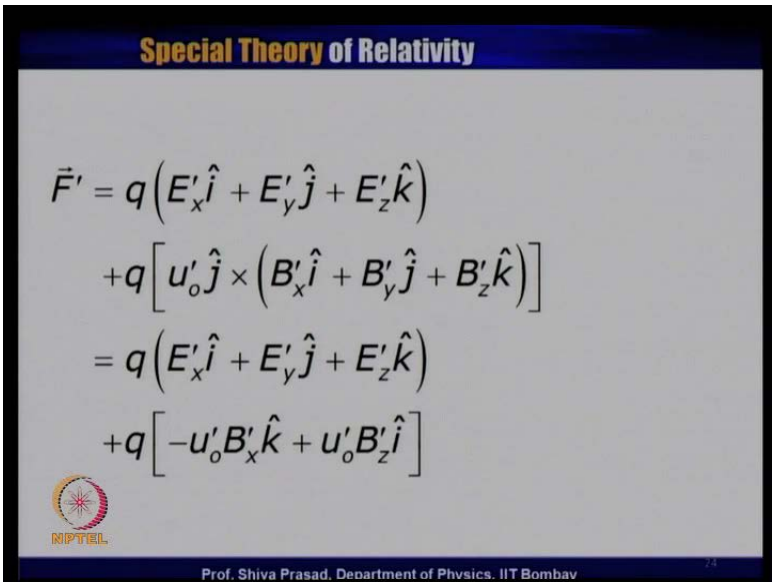


So, I have to use generic most most general form of transformation and using that I will be able to find out magnetic field equations. So, let us do that. So, this is the force on this charge which has been given as F prime is equal to q E prime plus u naught cross B prime this is S prime frame of reference. In my earlier situations, this term was not there but, now it is there because I am assuming because that the particle is moving along the z direction. This first electric field term I have written just as a in the component form E x prime i plus E y prime j plus E z prime k. u naught prime I realizes purely in the y direction. So, I have written u naught prime j cross this B prime again I have written in the component form B x i plus B y B x prime i plus B y prime j plus B z prime k then I will expand this particular thing I take this particular cross product I will write all this terms just like before then take out x component of F prime y component of F prime and z component of F prime just like before.

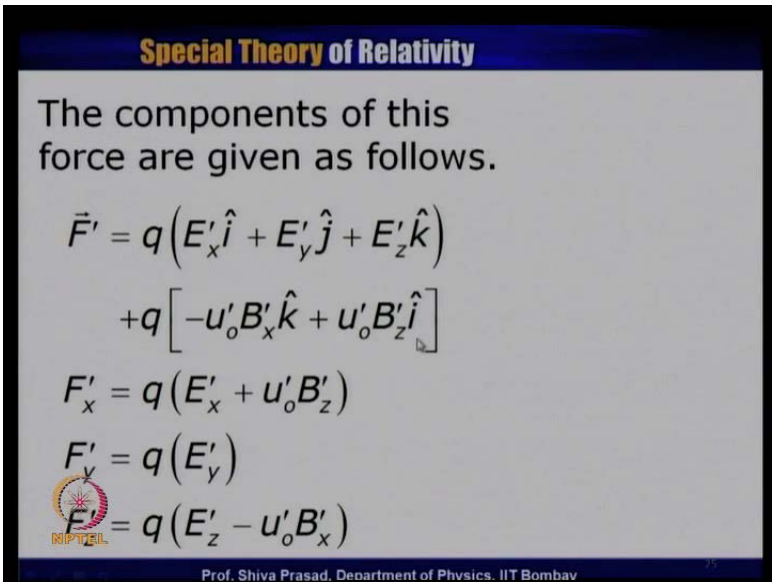
So, this first term is exactly identical of what I have written this first equation in exactly identical of what I have written in my last transparency. Let us try to evaluate this if I take j cross i. This will be minus k, Because, i cross j k j cross i will be minus k. So, you will have u naught prime B x prime minus k. So, this what I have written here there is a negative sign u naught prime B x prime; j cross j will give me 0 then j cross k will be i. So, u naught prime multiplied by B z prime i and there is a plus sign here. Because, j cross k is plus i. So, plus u naught prime B z prime i or using this equation I can resolve I can take out the x components y component x component y component and z component

of the force in S prime frame of reference. Same equation which I has been written here without any change. Let us pick up the x component it means F x prime.

(Refer Slide Time: 32:40)



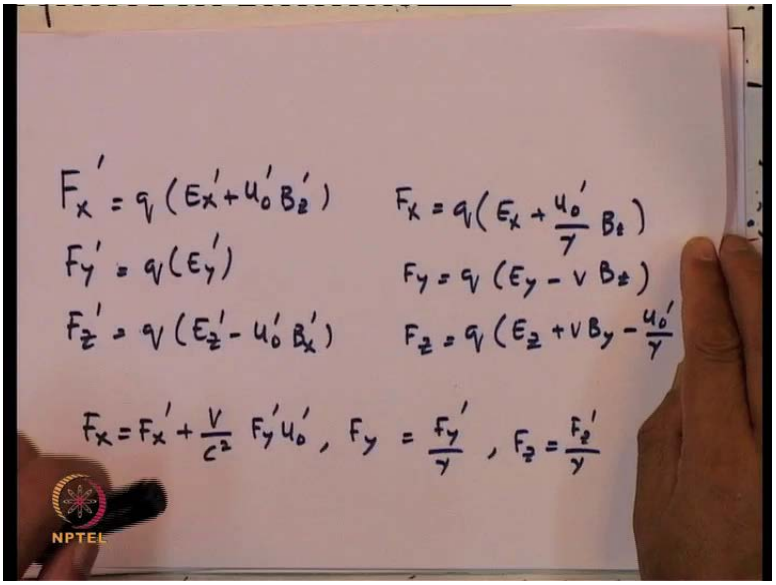
(Refer Slide Time: 33:45)



So, let us look at those terms which contain i. There is E x prime which contains i. There is a u naught prime plus B z prime which contains i. So, this becomes q E x prime plus u naught prime B z prime which is forms x components of the force as for the y component of the force is concerned there is only one term which contains j. There is no other term which contains j. So, F y prime is just equal to q E y prime. Become the z component z

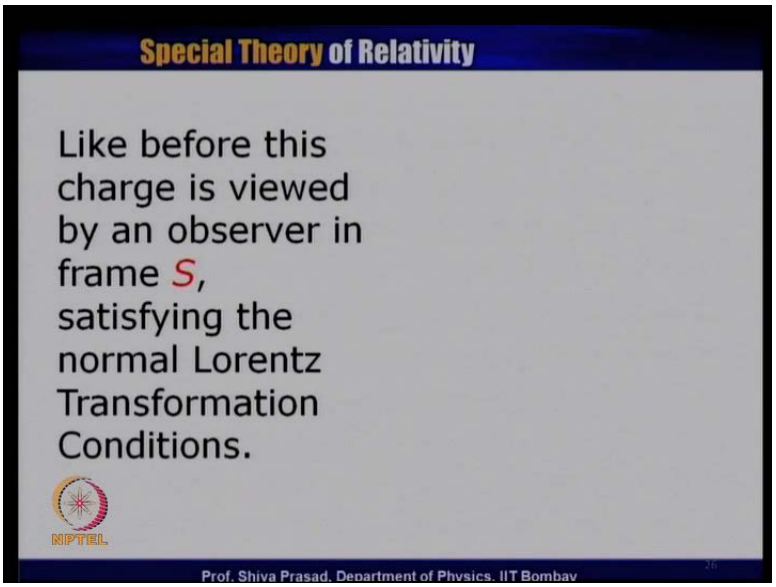
component this term contains k this term contains k. So, z component is q E z prime minus u naught prime B x prime. So, these are the x y and z component of the forces as seen by an observer in S prime frame of reference. I have just copied like before these

(Refer Slide Time: 34:54)



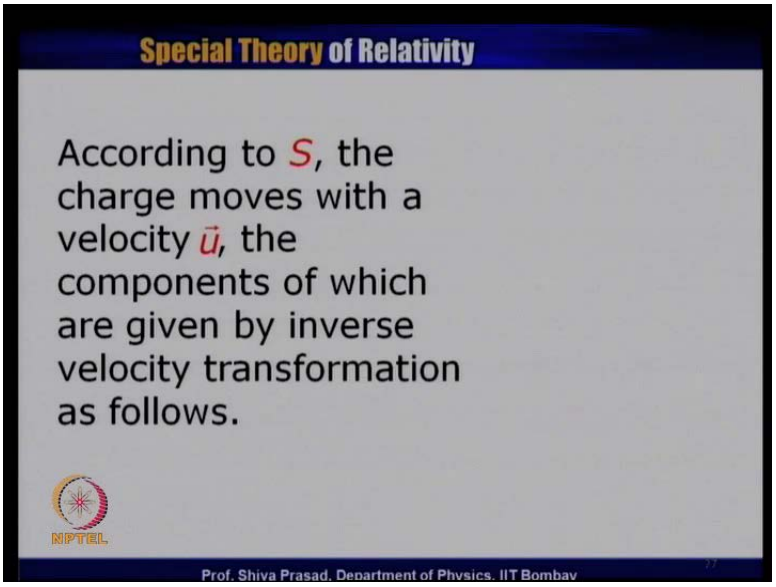
equations in this particular paper because I am going to use this things F x prime is equal to q E x prime plus u naught prime B z prime F y prime is equal to q E y prime f z prime is equal to q E z prime minus u naught prime B x prime these equations will see later.

(Refer Slide Time: 35:15)



So, like before this charge is being viewed by an observer in frame S satisfying the normal Lorentz transformation condition and of course, I will assume that in this particular frame S the field seen by the particle or seen by the charge is E electric field is E and the magnetic field is B and again write the force and do the force transformation equations.

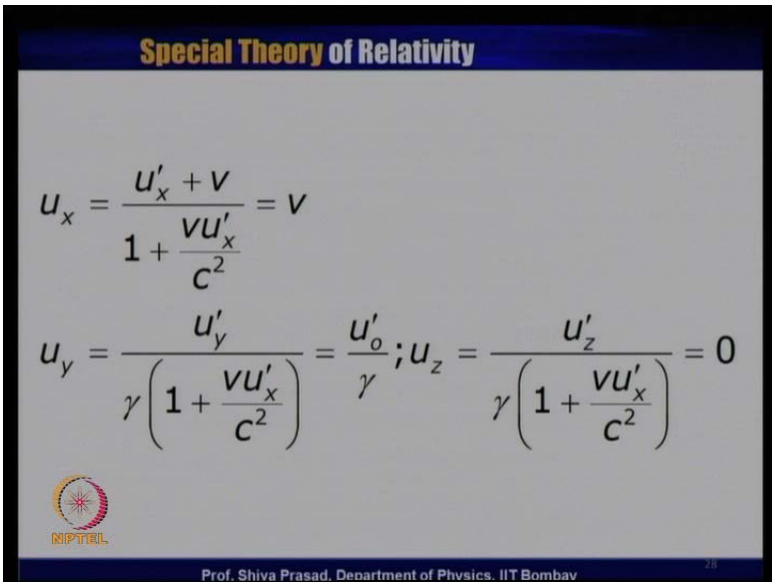
(Refer Slide Time: 35:37)



According to S the charge moves with a velocity u which now have to obtain by inverse velocity transformation. Because I know the velocity in S prime frame of reference. I want to find the velocity in S frame of reference. So, I must use inverse transformation and if I use inverse transformation equation. I can find out what will be the value of u in the vector form or rather x y x y and z components of u.

Once I know what are the components in S prime frame of reference. So, let us do that first. These are the inverse transformation equations x component u x prime plus v divided by one plus v u x prime by c square. Because, the charge is moving only along the y direction in S prime frame of reference.

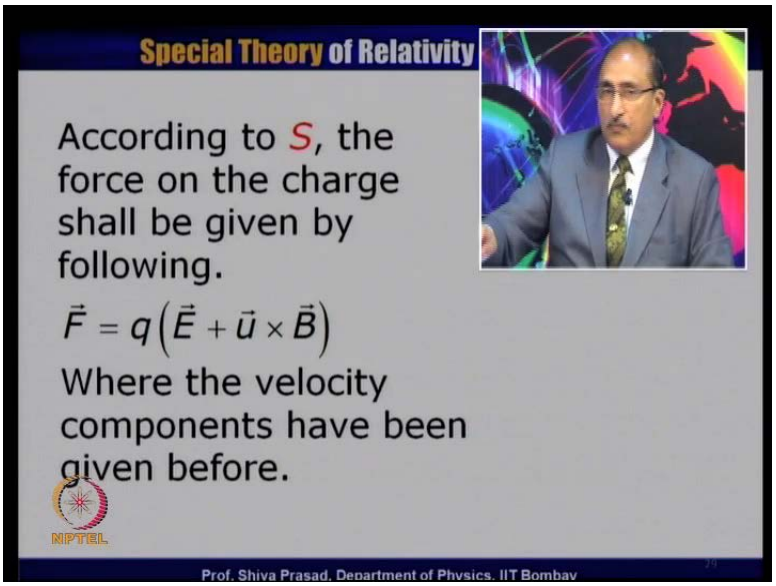
(Refer Slide Time: 36:08)



Therefore u x prime is 0. Because it happens only a non-zero component along the y direction. Therefore, u x prime is 0. So, denominator gives you only one this component is 0. So, u x is equal to v it just picks some of the x component is the same as a relative velocity in the frames as far as the y component is concerned of course, like before because this term is 0. So, there is only one here u y prime is non-zero and there is a gamma. So, this will be given by u naught prime divided by gamma.

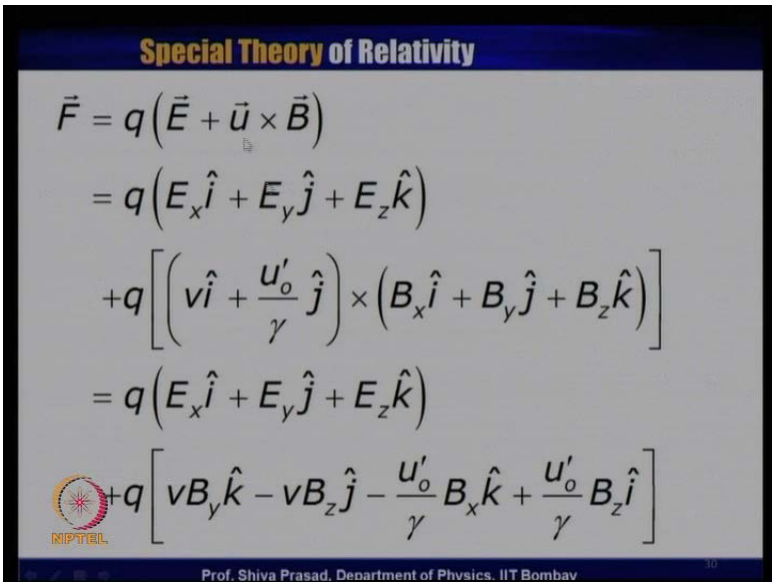
Because we have written the speed along the y direction to be u naught prime. So, u naught prime divided by gamma is what is the velocity along the y direction. u z is equal to u z prime divided by all this things and I know that u z prime is 0. Because the particle is moving along only the y direction. So, this gives me 0. So, I have velocity u components the x component is equal to the v v is equal to v and the y component u naught prime by gamma. So, my u has two terms as I have said or we using these two terms.

(Refer Slide Time: 37:37)



I can again like the force. So, as I said according to S the force on the charge shall be given by q E vector E is field seen in this frame plus u which we have just now evaluated by inverse transformation cross B where B is the magnetic field as will be seen by an observer in this particular frame of reference set where the velocity components have just been given. I mean just given in obtained. So, I will now write this particular equation vector form and then eventually get x y and z components of the forces then compare with what I have obtained as F x prime F y prime and F z prime and I must know that they must obey the force transformation equation.

(Refer Slide Time: 38:20)

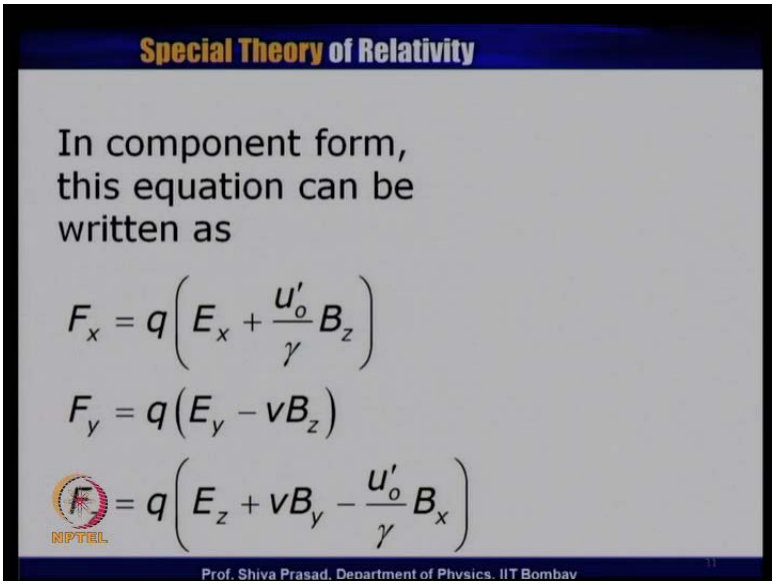


This rather Lorentz transparency. q is equal to E plus u cross B. E like before I have written is E x i plus E y j plus E z k plus q u I have written x component and y component x component was v I have written that as v i y component of u naught component divided by gamma.

Also this I have written plus u naught prime divided by gamma j cross again B like before B x i plus B y j plus B z k. First term we just write like that exactly the same way here. Second term let us take the cross product and write it open it up. So, let us first look at it v i cross B x i. This will give me 0. Because, i cross i is 0 then try take the second term i cross j will give me k. So, first term will be E v B by k.

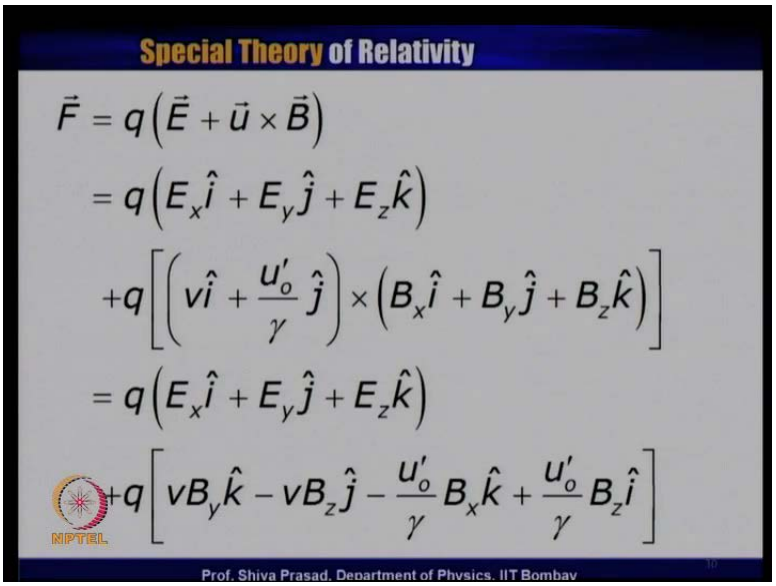
So, this what I have written v B y k then I take i cross k i cross k which will be equal to minus u therefore, I write this as v B z with a negative sign here j. So, its minus v B z j. Now I try to take cross product of this second term j cross i will be giving minus k. So, this is minus there is a minus sign here u naught prime gamma B x k; j cross j will give me 0 j cross k will give me i plus i therefore, plus u naught prime divided by gamma multiplied by B z. I pick up the x y and z component of this particular equation like before, let us look at the x component.

(Refer Slide Time: 40:12)



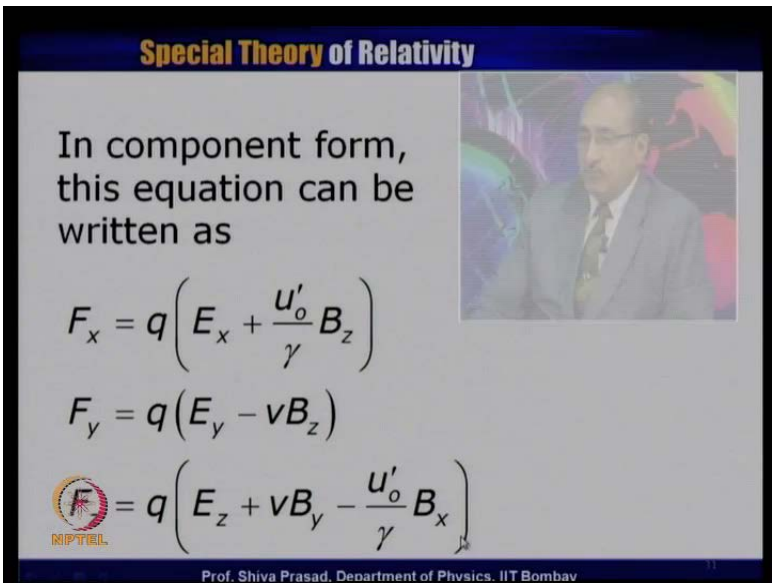
x component will have E x term and will have a term u naught prime divided by gamma B z which I have written here; y component will have a term E y here it will have a term v B z here.

(Refer Slide Time: 40:20)



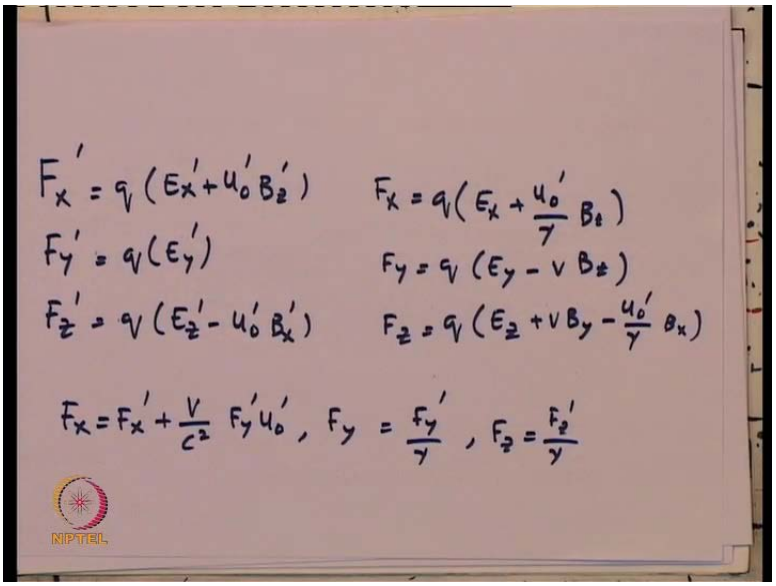
There is no term other term which contains j term. So, you will have E y minus v B z. This what I have written here E y minus v B z. Let us look at the z component z component have three term which contain k this term this term as well as this term.

(Refer Slide Time: 40:44)



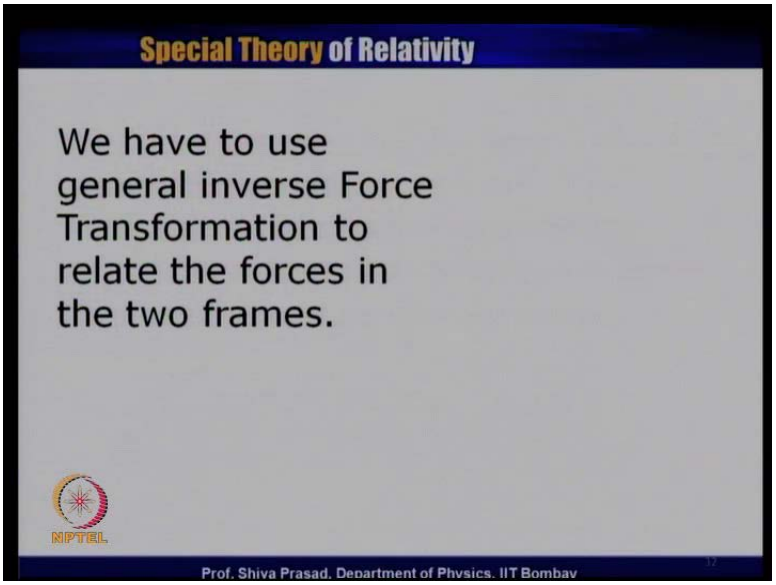
So, this will be E z plus v b y minus u naught prime gamma B x. This what I have written here. So, the three components have been written here which I am writing again here for the convenience here.

(Refer Slide Time: 41:15)

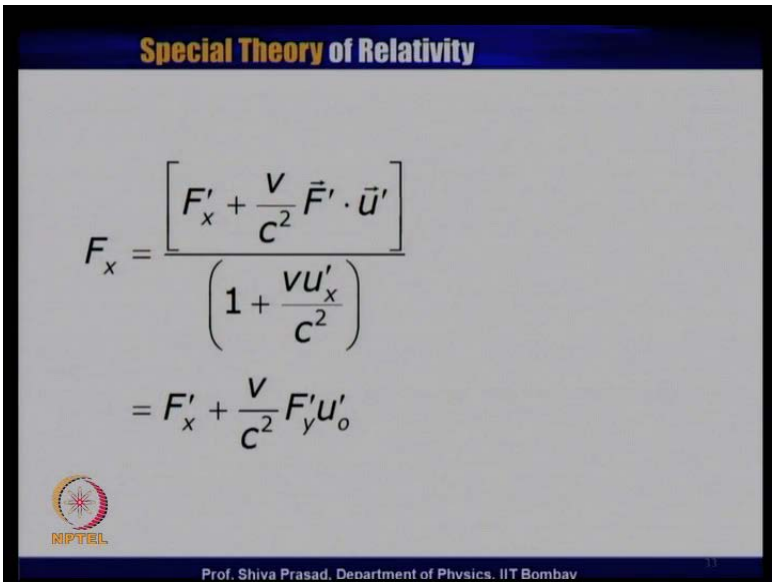


F x is equal to q E x plus u naught prime by gamma B z. F y is equal to q E y minus v B z. F z is equal to q E z plus v B y minus u naught prime by gamma B x. Let us look at just this particular equation here and here now when I am writing the relationship between F x and F x prime. I cannot use those simple relationship which I could use in earlier case because the particle happened to be at rest of mass frame of reference I will use the generating relationship. So, let us write that we have to use that general inverse force.

(Refer Slide Time: 42:03)

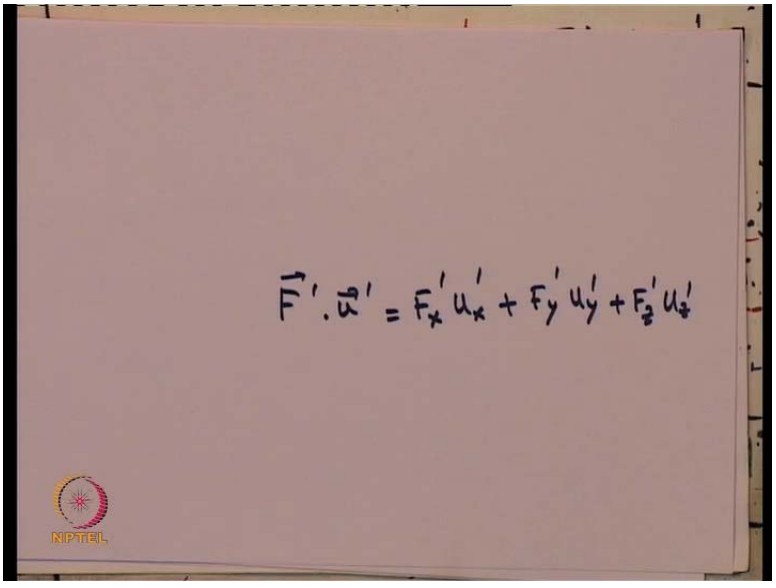


(Refer Slide Time: 42:15)



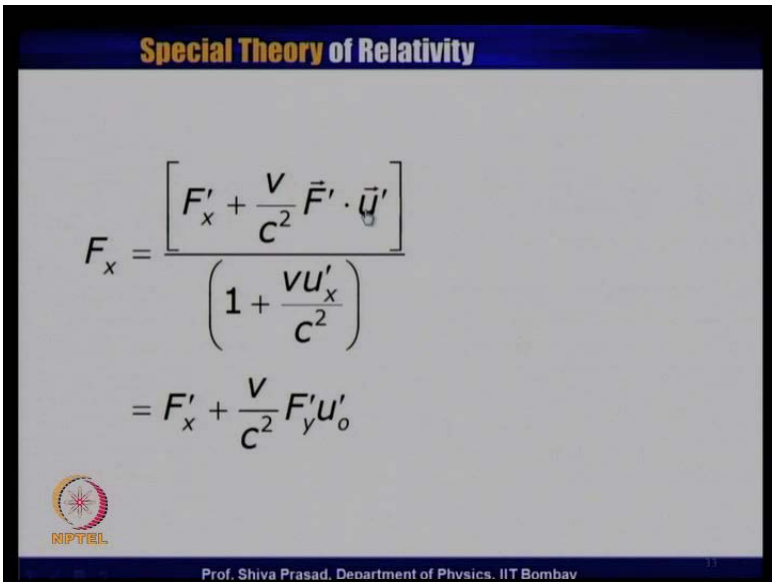
Transformation to relate the forces in the two frames see if you remember this was the x component transformation of the force the force term of the four vector was work related to F prime dot u F x was equal to F x prime plus v divided by c square F prime dot u prime if I take the dot product.

(Refer Slide Time: 42:37)



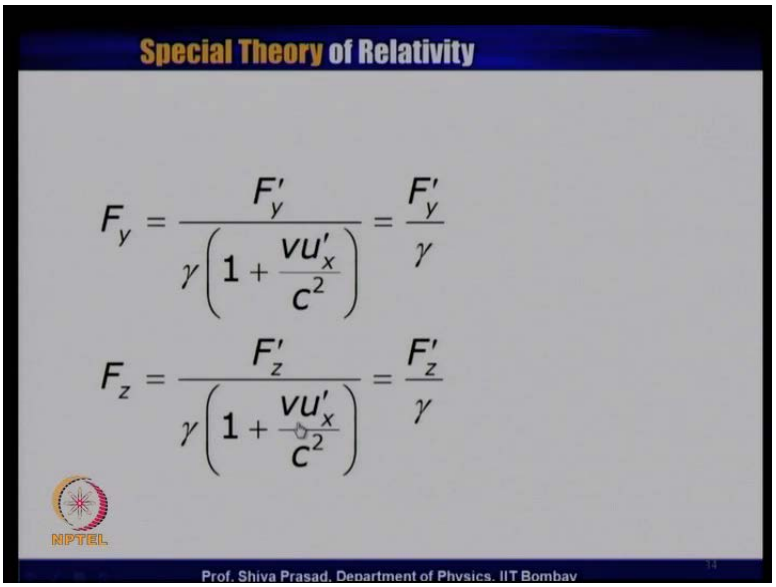
I am sorry, this will be given by F x. Now because, this is 0 this is 0 only one term will remain which is F y prime therefore, I can write this particular term as just F prime multiplied by u y prime of course, denominator because u x prime is 0.

(Refer Slide Time: 43:20)



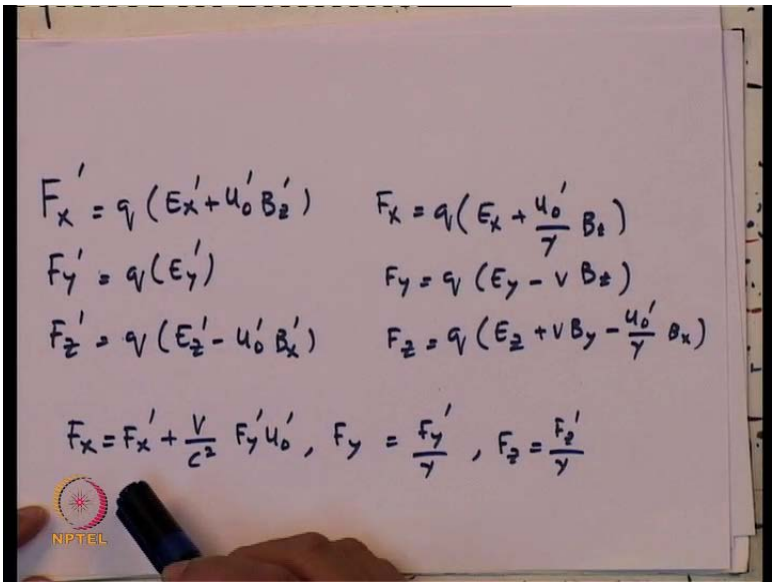
So, this denominator denominator there is only 1.

(Refer Slide Time: 43:40)



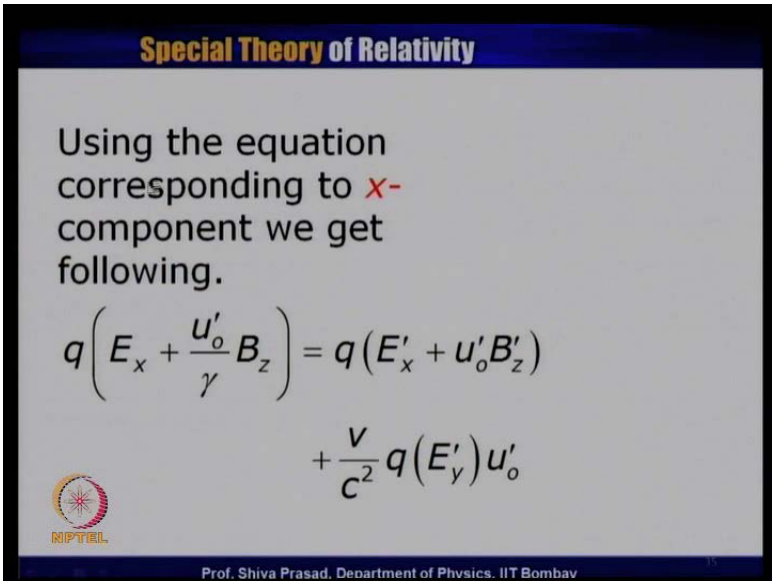
So, I can straight way write only the numerator which gives me F x prime plus v divided by c square F y prime u naught prime. So, this is relationship between x and x prime relationship between F y and F y prime cannot be. So, formed found from the relationship; this term happens to be 1. Because, where x prime is 0. So, this gives F y prime by gamma F z F z prime by gamma which is similar to what we have done earlier because u x prime happens to be 0. So, these are the equations which I have written here.

(Refer Slide Time: 44:07)



So, what we have just now seen that F x is equal to F x prime plus v by c square F y prime u naught prime F y is equal to F y prime by gamma F z is equal to F z prime by gamma we already evaluated F x prime F x F y prime F y F z prime F z all we have to do substitute in these equations.

(Refer Slide Time: 45:00)

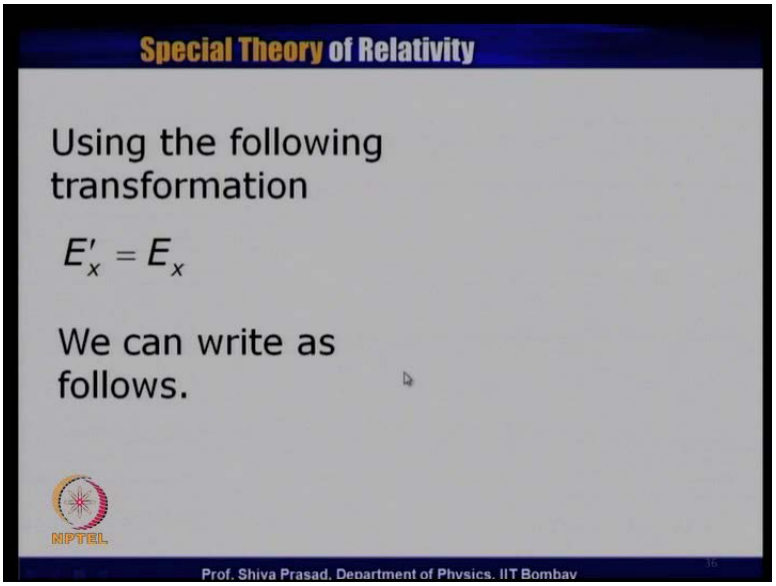


So, let us first take only the x component of this equation substitute this value of F x here this q E x plus u naught prime by gamma B z here F y prime is equal to q E x prime plus

u naught prime B z here. There is F y prime F y prime we put q E y E y prime. So, I substitute things in this particular equation and let us see what we get.

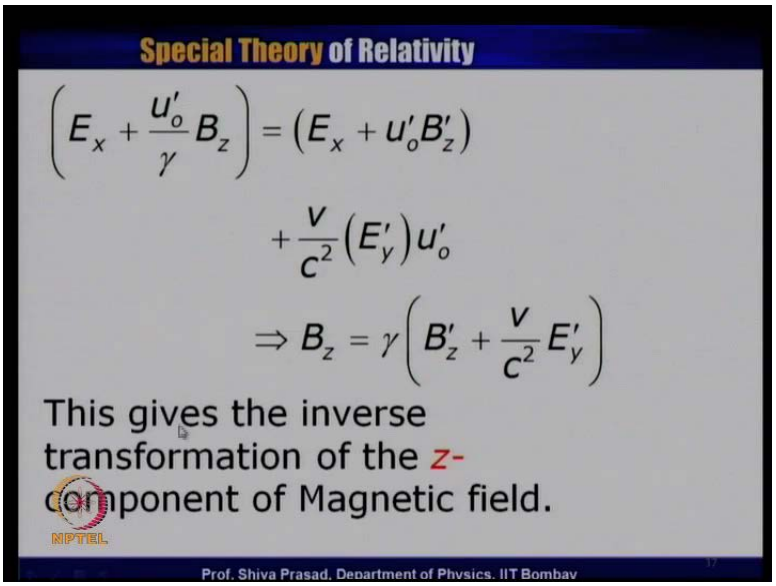
So, this what I have written using the equation corresponding to the x component we get q E x plus u naught prime by gamma B z which happens to be F x. This what is F x prime plus v by c square this is what was F y prime using this particular equation you can see that q gets cancel out and I can write this particular equation with E x prime is equal to E x because E x prime is equal to E x has already been evaluated has already been found out when we did transformation obtaining E x E y and E z components. So, E x prime we knew is equal to we know is equal to E x. So, what I will do cancel this q substitute E x prime is equal to E x.

(Refer Slide Time: 45:47)



So, using the following transformation that is E x prime is equal to E x we can write the equation as follows which I am writing here q has been cancelled out instead of E x prime I have written E x as you can see that this E x would cancel out.

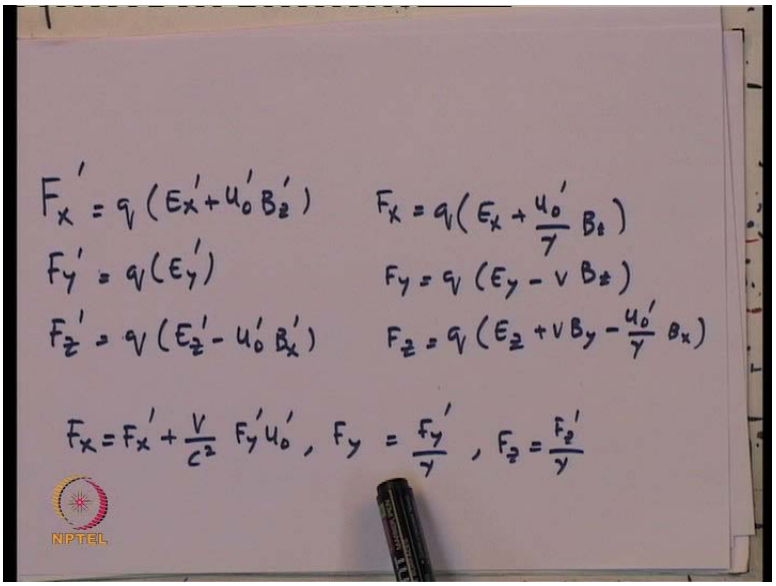
(Refer Slide Time: 45:54)



Here, we have u naught prime divided by gamma B z; once this E x has been cancelled out there is a u naught prime here there is u naught prime here there is u naught prime here this u naught prime will also get cancelled out. So, here I will only get B z by gamma just gamma I can take on the right hand side and multiply it. So, B z will become equal to gamma which is this gamma which gets multiplied here this term has already cancelled out in this particular term u naught prime has already cancelled out. B z prime which is B z prime plus v by c square which I have written here u naught prime has already cancelled out E y prime which I have written here. So, as you can see I get a equation B z is equal to gamma B z prime plus v by c square E y prime which tells me the z component of magnetic field in S frame of reference.

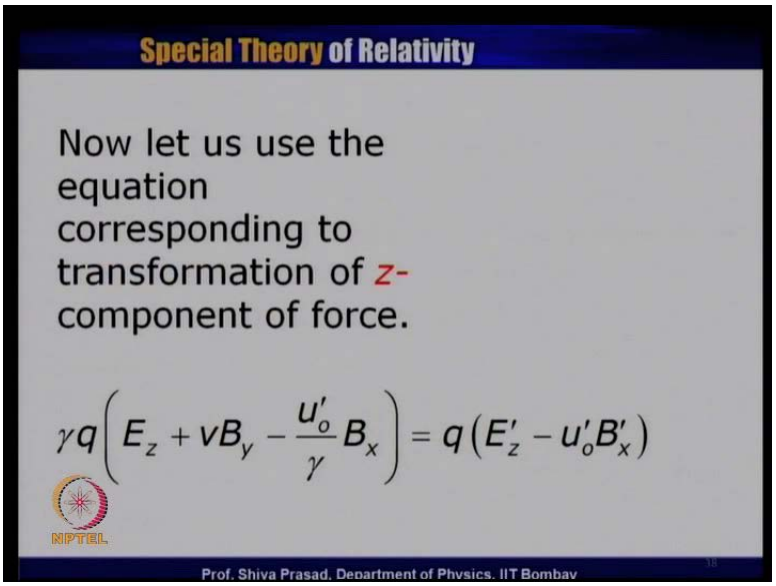
If I know the z component of the magnetic field in S prime frame of reference and electric field y component of electric field in S prime frame of reference. So, this is one of the equations you have obtained using this force equations which is part of the velocity magnetic field transformation you mean magnetic field.

(Refer Slide Time: 47:33)



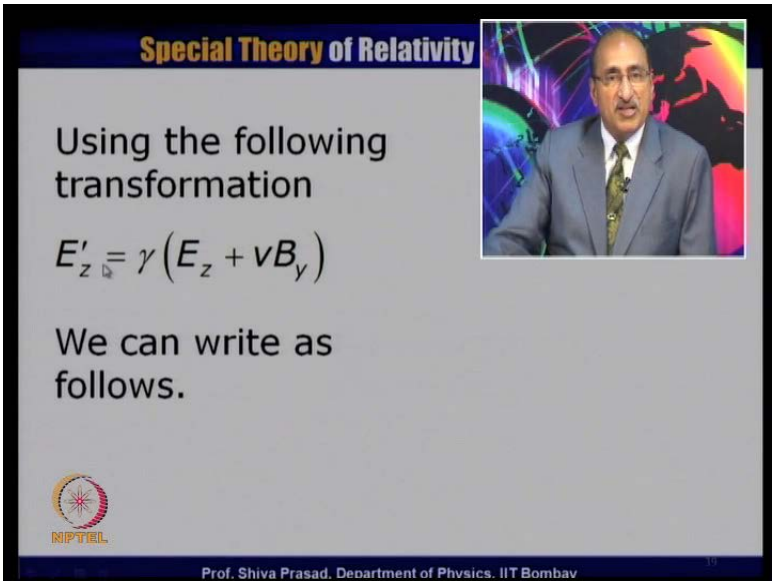
So, as I have written this gives the inverse transformation of the z component of the magnetic field. Now, when I look back at this particular paper. This particular F equation F y is equal to F y prime by gamma. You can try it does not lead to immediately to way new equation I will use this third equation. F z is equal to F z prime by gamma. So, for this F z you know that there are three terms q E z plus v B y minus u naught prime by gamma B x and for F z prime we have two terms q E z prime minus u naught prime B x prime. So, in this equation I will substitute this value of F z here I will substitute this value of F z prime this is what I am writing in the next transparency.

(Refer Slide Time: 48:10)

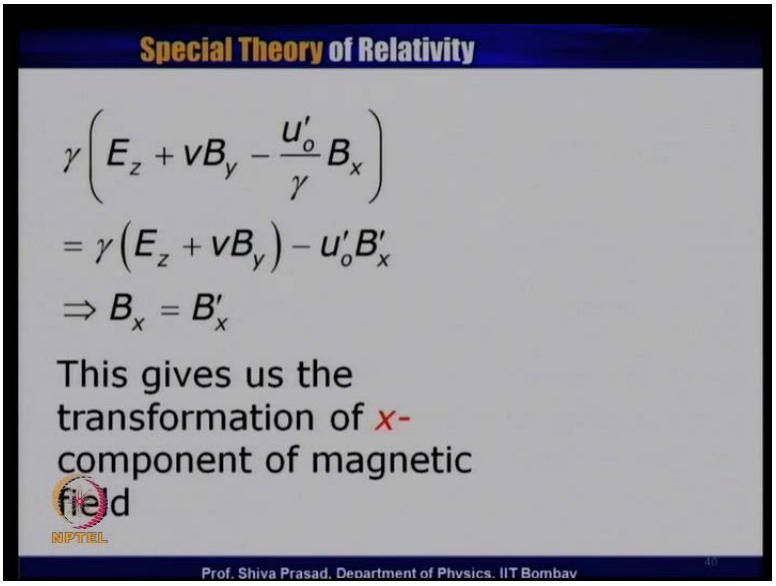


So, let us use the equation corresponding to the z component of the force. So, this is this gamma I have multiplied because on the both sides. So, I get gamma times q E z plus v B y minus u naught prime by gamma B x. This quantity was equal to was equal to F z and this was F z prime which was actually divided by gamma; gamma I have taken on this side. So, this becomes q E z prime minus u naught prime B x prime this E z prime I know how to convert in terms of E z by using the transformation of electric field which we have just now obtained.

(Refer Slide Time: 48:52)



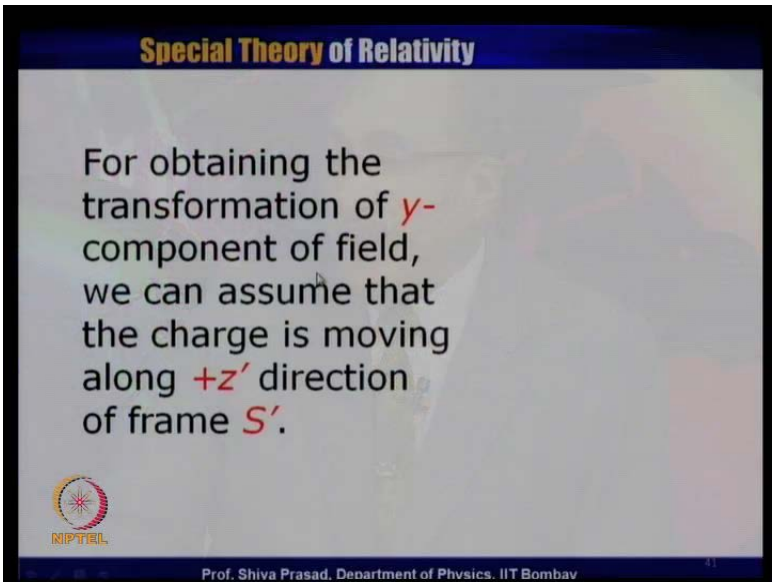
(Refer Slide Time: 49:03)



So, this what I will be substituting in the next thing using the following transformation E z prime is equal to gamma E z plus v B y which we have just now derived I will substitute in this equation and I will get this equation left hand side remains exactly identical u has been cancelled out for E z prime I have written gamma E z plus v B y and this term remains as it is or if you can see very clearly this gamma E z will cancel of this E z there is a gamma v B y which cancels of this gamma v.

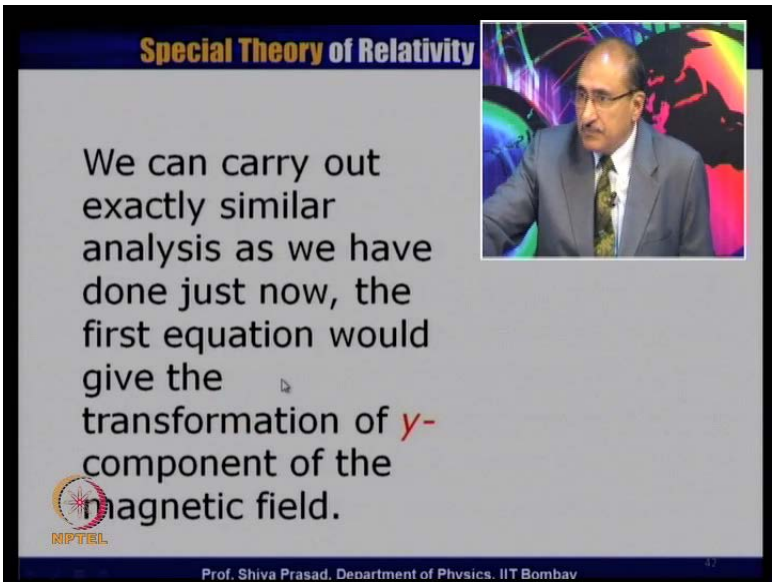
If you look at this term this gamma will cancel with this gamma you have minus u naught prime B x on the right hand side you will have minus u naught prime B x prime u naught prime and u naught prime will cancel but, this minus sign is you cannot be cancel this gives me B x is equal to B x prime. So, like we had E x is equal to E x prime I get B x is equal to B x prime. So, this gives us the transformation of the x component of magnetic field now I know how to find out B z and how I know how to find out B x.

(Refer Slide Time: 50:18)



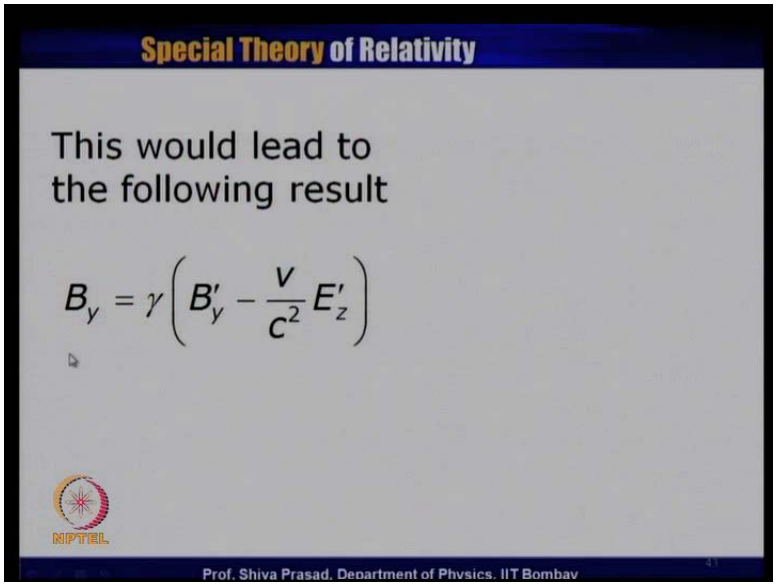
I have not still would find in equation corresponding to y which is quite little but, more afford for obtaining that transformation of y component of the field we have to assume that the charged is moving in the plus z direction plus z prime direction like we have assumed in plus y prime direction in S prime frame of reference. So, you do exactly the same calculation again but, now we assume that the charge is moving in plus z prime direction in S prime frame of reference.

(Refer Slide Time: 50:45)



Otherwise we do all the calculations exactly identically. We can carry out exactly similar analysis as we have done just now the first equation would give the transformation of y component of the magnetic field just like in this particular case it had given me the transformation of the z component of the field if I would have assumed that the charge is moving in a plus z prime direction in S prime frame of reference.

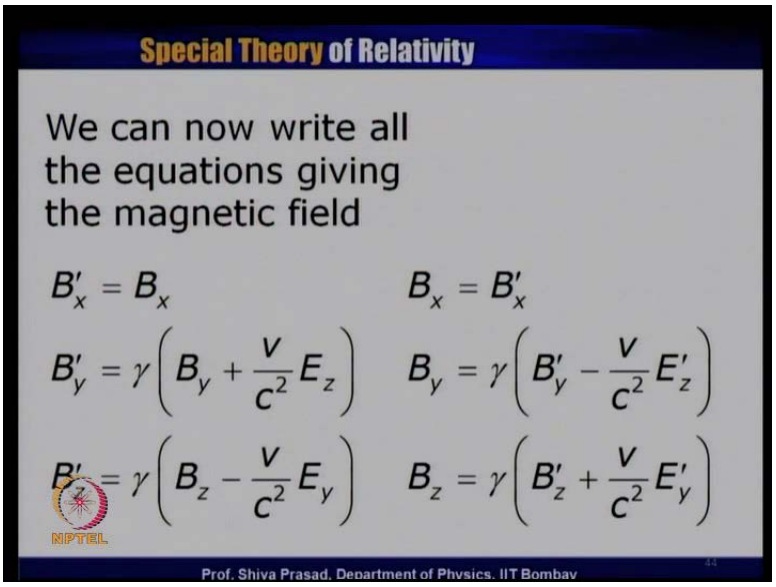
(Refer Slide Time: 51:16)



The first equation will give me the transformation correspondent to the y component which we will not work out here. One can try to do it yourself and the result that we will

obtain will be equal to B y is equal to gamma B y prime minus v by c square E z prime. Now, I have found out all the three inverse transformation corresponding to the magnetic field; I can always find out the direct transformation by doing the same prescription replace v by minus v prime by unprimed unprimed by prime.

(Refer Slide Time: 51:42)

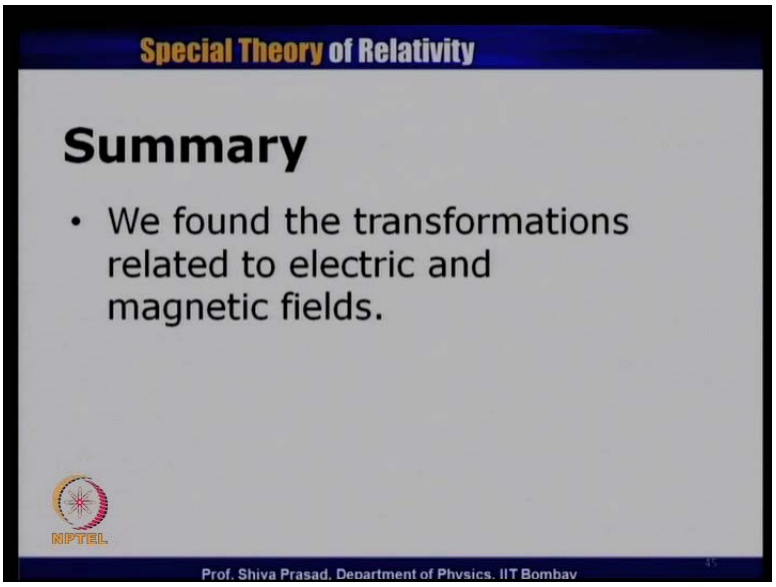


So, these are all by equations which are relating to the magnetic field this is what we call as the direct transformation. I know electric field and magnetic field in S frame of reference. I will find out the magnetic field in y S prime frame of reference. Here, I know the electric and magnetic field in S prime frame of reference. I can find out the magnetic field components in S frame of reference. We have already found out the relationship which gives me E x E y E z E x prime E y prime E z prime essentially means that for example, if I know what are the magnetic field and electric field in S prime frame of reference; I can always find out what will be the electric field and magnetic field in S frame of reference or vice versa using these equations.

So, essentially we have got three sets of equations corresponding to the electric field components and three equations set of equations corresponding to three equations corresponding to the magnetic field components. So, if I know electric field and magnetic field; we I can find out electric field and magnetic field components in any other frame of reference and as you can see very easily from these equations they are all

interdependent for evaluation of electric field I also need of the magnetic field for evaluation of magnetic field in general I also need to the electric field.

(Refer Slide Time: 53:06)



So, here I come to end of this particular lecture. I have just summarize this particular lecture we have done lot of special calculations to find out the transformation related to electric and magnetic field.

Thank you.