Special Theory of Relativity
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Module - 1
Lecture - 21
Force Four-Vector

Hello, in our last lecture we had described the concept of force, in classical mechanics force is well known. We discussed how we redefine it when we come to the special theory of relativity then we realized that the way we define force in the special theory of relativity the acceleration And force need not be in the same direction, while in classical mechanics, if I apply force in a particular direction the acceleration is also in the same direction, but in relativity it is not like that it need not be like that.

Then we consider two special cases; one when we are talking of only one dimensional motion. It means that the force is being applied in the same direction as the velocity of the particle, and a situation in which the force is applied perpendicular to the direction of velocity. In these two cases we found out that the force and the acceleration will turn out to be the same, will turn out to be in the same direction, sorry will turn out to be in the same direction; these are the things that we discussed last time.

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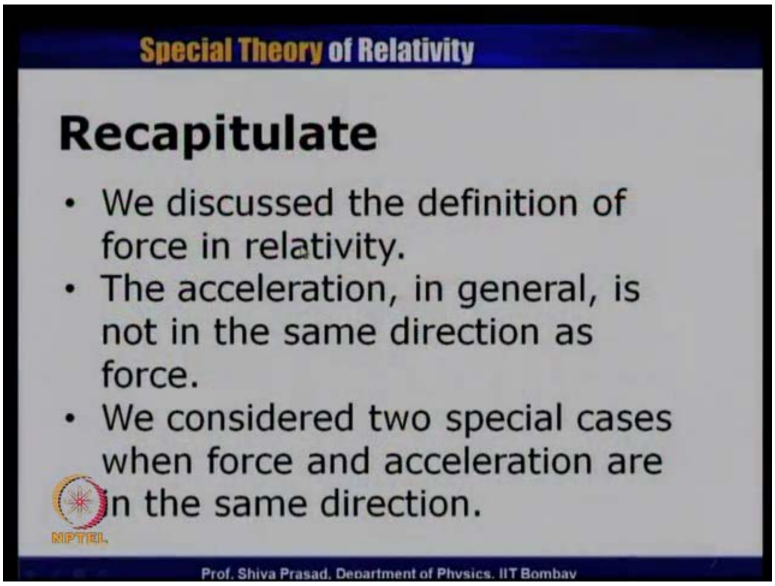

So, we recapitulate, we discussed the definition of force and relativity, the acceleration in general is not in the same direction as force. We considered two special cases when force and acceleration are in the same direction and this is what I have just now told these two are the two special cases; one where the motion is along the line, and the force is applied along the direction of velocity.

And the second condition, when the force is applied perpendicular to the direction of velocity of the particle. In the last lecture, we had sort of said that we should we will find out the equivalent of what we call is parametric equations using these definition of force in relativity. So, we will spend some time today to discuss that particular aspect then of course we will talk about the force transformation.

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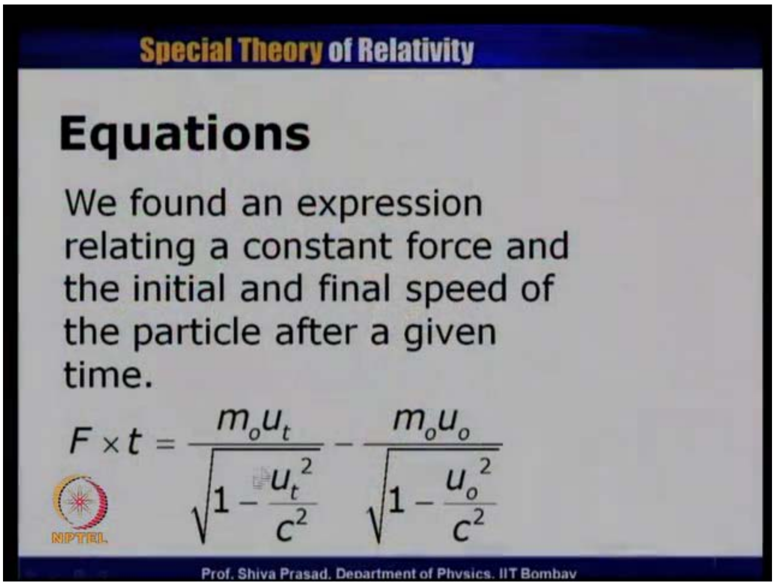

So, this is the equation that we have derived earlier force multiplied by time. Let us just recapitulate the nomenclatures u naught is the initial velocity of the particle. If force is applied in the same direction of velocity of the particle was the force is applied in the same direction as the velocity of the particle and the motion is only in one direction. We are not talking about the vector signs, so we have remove the vector signs. So, u naught is the initial velocity of the particle or we now call initial speed of the particle and a constant force is applied for a time t and after time t, u t is the final velocity of the particle. This u here, there is a u t here there is a u naught here, there is a u naught here and m naught of course, the rest mass of the particle.

So, this is the equation which we had derived last time which relates force to the final velocity of the particle and the initial velocity of the particle of course, it assumes that force is constant as a function of time. So, my next step would be to find out this u t as a function of time, so that I can directly get one equation from which I can find out the speed of the particle this is the way the traditional kinematic equations in classical mechanics are derived. So, let us attempt to do that.

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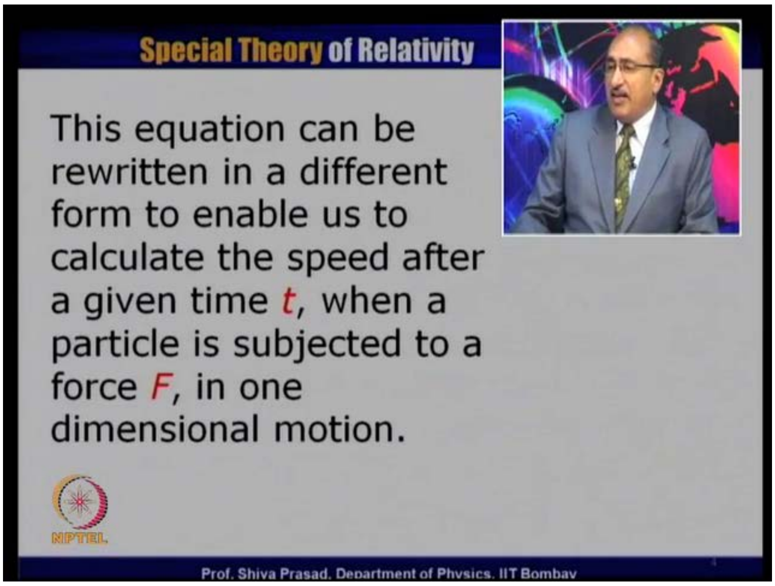

This is what I have written this equation can be rewritten in a different form to enable us to calculate the speed after a given time t, when a particle is subjected to a force F in one dimension motion which essentially means, reorganization of these terms because the equation has already been obtained. It is only sort of reorganizing the term, so that by direct substitution I can find out the velocity at a particular time when a constant force is applied to the particle. So, let us try to reorganize these terms.

So, this was my equation which we have just now written and explained. I define a quantity A is not a very standard symbol, we just force divided by m naught rest mass of the particle this I am calling as A which is some sort of acceleration. You know in some sense because it has the dimension of acceleration, but I am writing it as capital A. If I take this define f by m naught as capital A then this m naught which is appearing in both these equations can be brought here. We brought it here this becomes F of one m naught and this quantity will become A this 1 upon under root 1 minus u naught upon u naught square upon c square.

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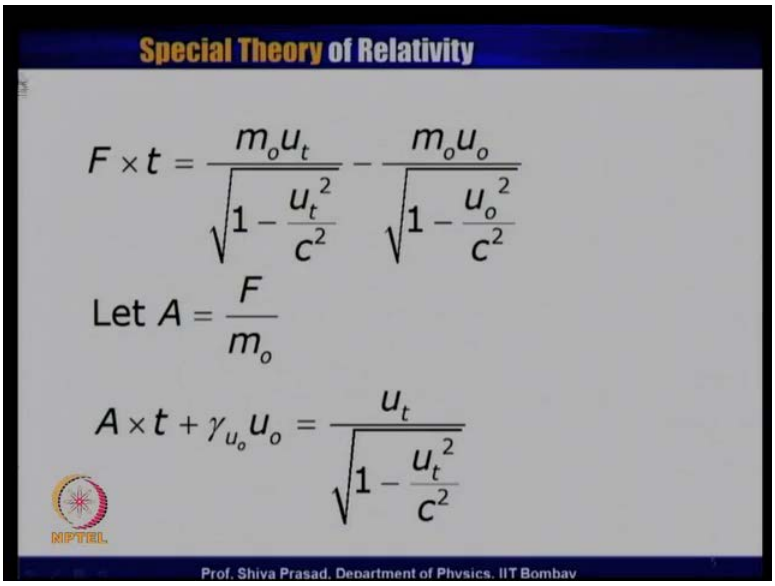

I can write as gamma u naught which is the standard nomenclature which we have been always using it if this quantity can be written A gamma u naught. This will become gamma u naught multiplied by u naught this quantity I take on their right hand side because this I had divided by m naught. This becomes this quantity becomes A t m naught disappears from the right hand side of these equations.

This equation I take it this side and write in the form of gamma u naught, it is the left hand side become A multiplied by t plus gamma u naught u naught. So, this term remains exactly as becomes except for m naught which we have already taken by replacing f by A from the right hand side, we just have u t upon under root 1 minus u t square by c square.

My idea is to write u t in terms of everything, so what I will do I will square this quantity and try to solve for u t. It is a simple very simple algebra, but just arrive at the equation which is comparative simplified equation top determine final velocity in most of the kinematic problems that we are used to the classical mechanics. Essentially, you are given force and given acceleration and you have to find out the velocity of the particle at a particular time.

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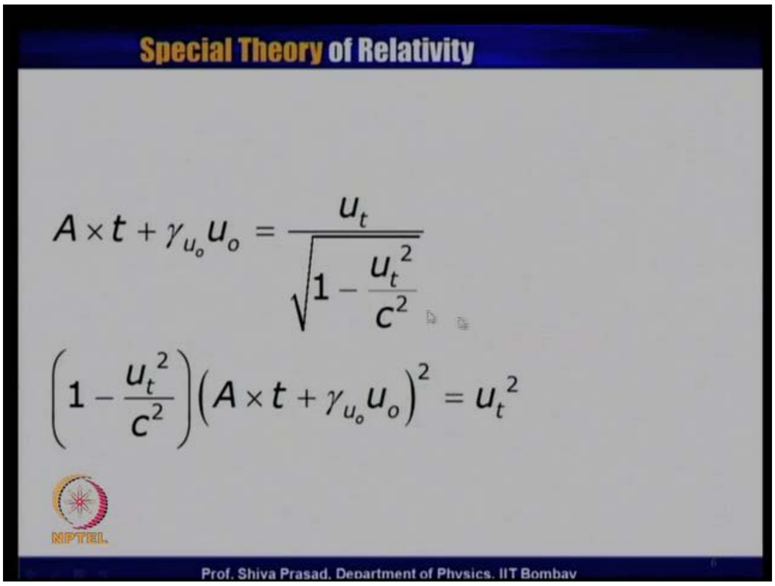

This is what I am doing in the next transparent sheet; this is the same equation what I had written in the last transparent sheet exactly the same I have squared it. I have squared it this becomes this square A t plus gamma u naught u naught A t plus gamma u naught u naught squared, when I square the right hand side this will become u t square and in the denominator the under root will go away. So, it will become just 1 minus u t square by c square this I take on the left hand side this will get multiplied to this particular quantity c squared this is what is happening here.

This quantity after squaring and bringing to the left hand side becomes 1 minus u t square upon c square this quantity squared is put here becomes equal to u t square. Now, it is simple we have tried to organize the terms with u t square collect all the terms that have u t square and those terms, which do not have u t square and solve for u t square this is what I am doing in the next transparent sheet. So, this u t square term has just been taken on the other hand side.

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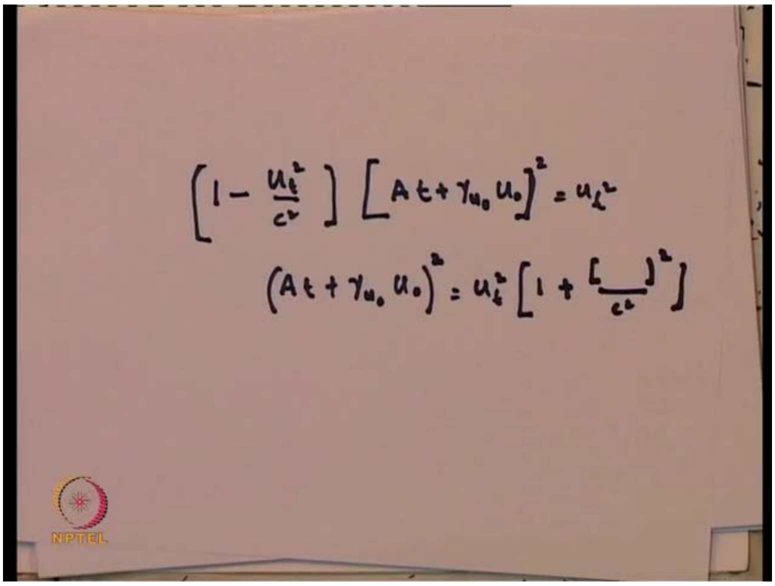

Let me just work it out we had 1 minus u t square by c square and here we have this quantity A t plus gamma u naught u naught square, whole square is equal to u t square. If I take one and multiply this, this becomes a constant quantity and this in retain on the left hand side this becomes A t plus gamma u naught u naught square, this contains u t square. So, I take it on the other hand side, so this becomes u t square this term will give you one and because this I am changing to right hand side. So, this negative will become plus this whole quantity divided by c square this is what this equation would look like this is what I am writing in the next transparent sheet precisely.

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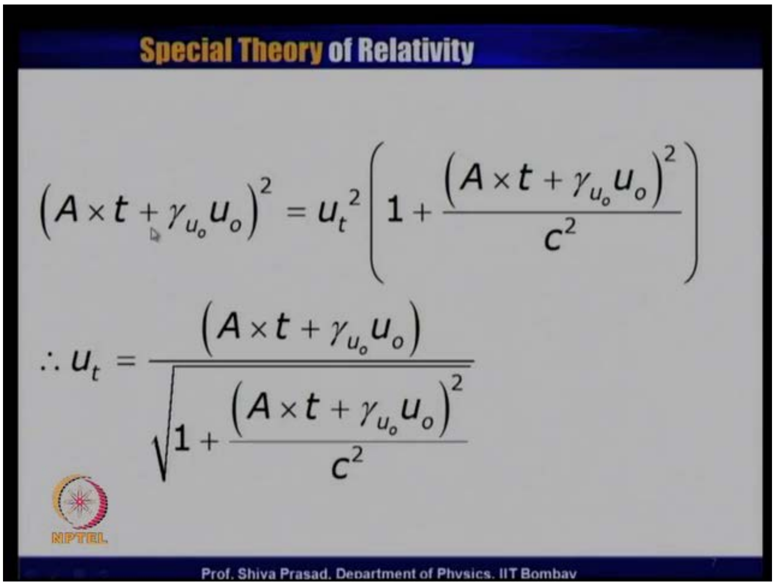

If you look here at the transparent sheet, A t plus gamma u gamma u naught u naught square and this u t square 1 plus A t plus gamma u naught u naught square divided by c square, after that simple I divide this quantity on the both both the side. So, left hand side becomes this divided by this square, and then I take another root once I do that this is the value of u t which I get u t once I take the under root this just becomes A t plus gamma u naught u naught this is on the numerator this gets divided.

So, this becomes 1 plus A t plus gamma u naught u naught square by c square and of course, because I am taking another root, so this becomes another root. So, this equation would tell me if I know what is the value of A that is f upon m naught, if I know what is the force and if I know what is the initial velocity u naught. Then if you tell me what is going to be the speed of the particle after time t of course it assumes very clearly that motion has to be only along a single line, because if the motion is not along the single line these equations will not work unlike kinematic equations where you could take the components and write those equations here.

It is very clear in this case that force and acceleration would not be in the same direction. So, in this case one has to be careful it is only for the one dimensional motion, that I can write this type of equation. And as you have seen this equation is much more complicated than the usual kinematic equation that we have been used to writing v is equal to u plus a t. Suppose, we are opting this equation of velocity as a function of time and also like to find out the distance travelled by the particle in the same time t and at the same conditions.

This is an equivalent of the kinematic equation that we write s is equal to u t plus half a t square, which tells you that if you know the acceleration then you know what is the distance travelled by the particle in a given time t. Of course, knowing the initial velocity u equivalent of that equation can be d found out by integrating this particular equation and that is what I will do, next to find out the distance travelled by the particle in a given time t.

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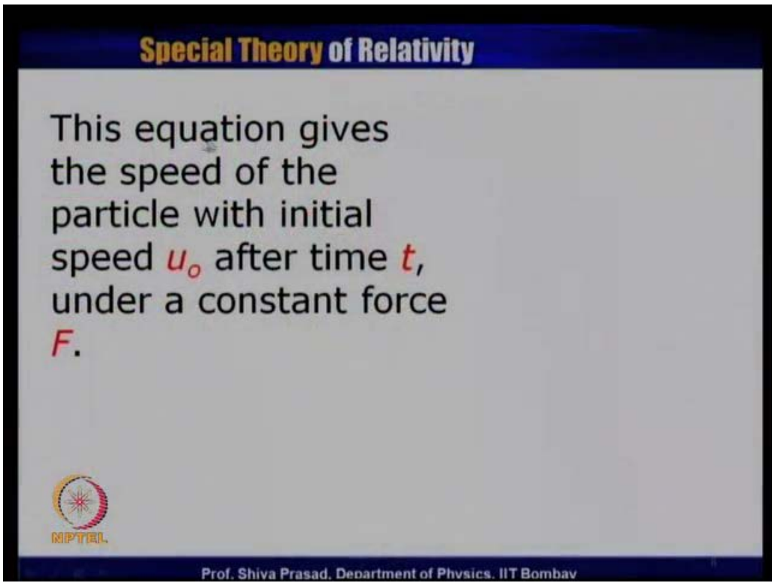

This is what I have written here the equation gives the speed of the particle with initial speed u naught after time t under a constant force F. Force is very clear that may repeat I assume that the motion is in one dimension, force is applied in the same direction in which the velocity of the particle exist.

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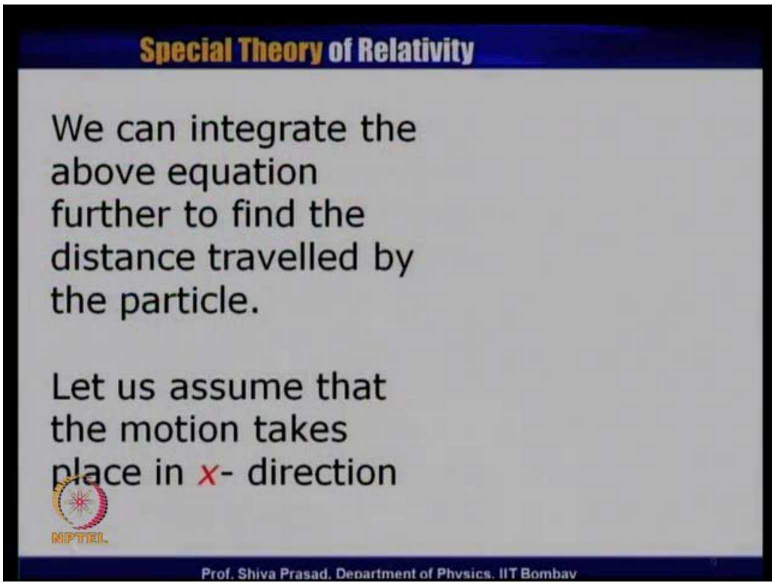

We can integrate the above equation further to find the distance travelled by the particle that is what I said and going to do next. For that matter, let us just assume for simplicity that this is along the x direction, because we are talking only of one particle frame, there is no question of transformation as of now. So, you have just a frame and you would like to know you have a particular particle which is moving in a particular direction. Let us assume that this particular dimension direction is x direction just for simplicity.

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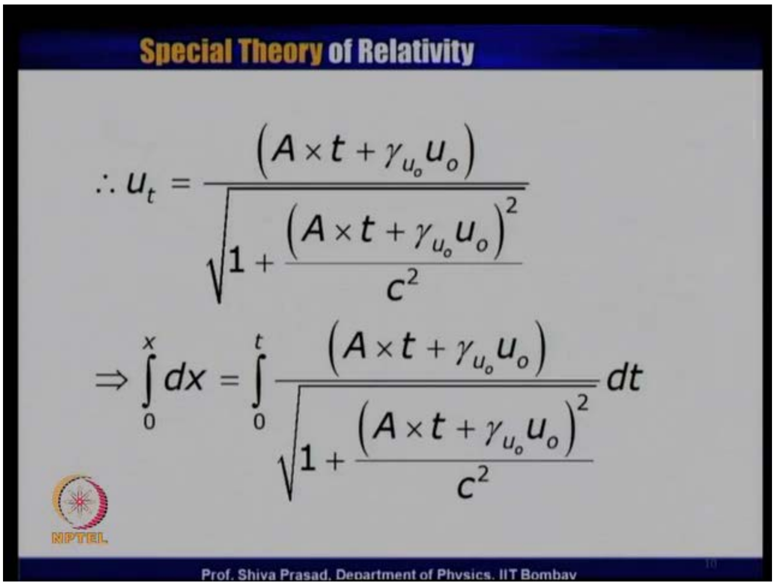

Therefore, once I try to integrate this equation this u t can be written as d x d t, because the motion is along the x direction.

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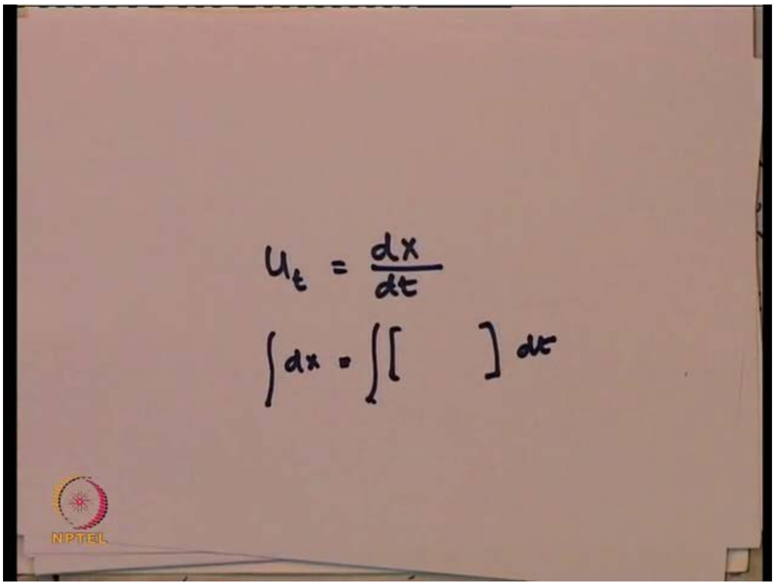

Therefore, if I write this u t as d x d t this equation becomes d x and I take d t on the right hand side, so this whole thing to d t after that I can integrate it, this is what I have written. This particular transparent sheet 0 to x d x assuming that the particle starts at time t is equal to 0 from x is equal to 0. Of course, we assume the time starts from the time when we are trying to measure the distance and goes after time t here, x is total distance travelled by the particle.

I have to integrate this particular equation once I integrate this equation then I can find out what will value of distance that the particle travels at this particular time t, like before I would like to take the indefinite form of this integral and solve. Of course, the left hand side is very simple because this just gives you the value of x because once I integrate, I will get x and if you put the limit this will just be x. So, the left hand side is obvious trivial only the right hand side has to be integrated though the integral looks complicated, but you can see that this integral as you will see this integral can be evaluated. So, let us go to your next transparent sheet, where I write indefinite form.

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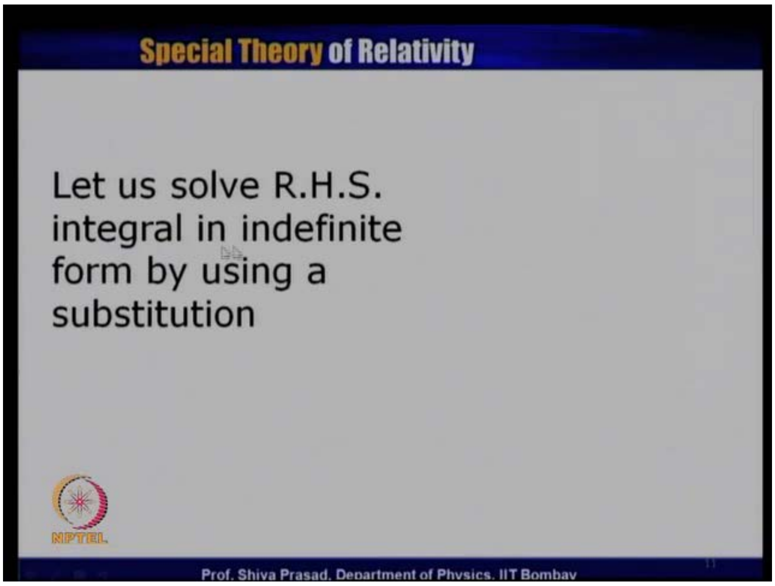

Let us solve right hand side integral in indefinite form by using a substitution, I take a substitution and using that particular substitution I will able to solve this integral.

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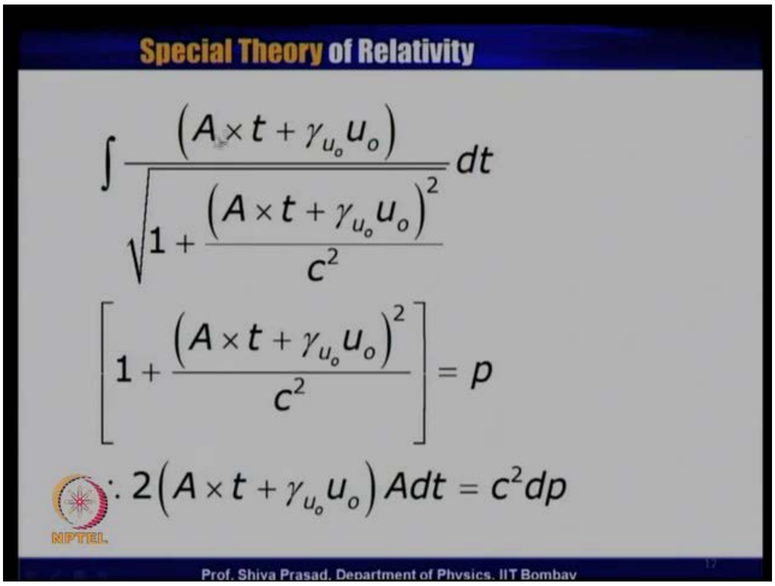

So, this right hand side integral which I have written, I have removed the limits because I want to solve this in indefinite form just to make things easy. You could have done the other way also if you like it and I out the substitution this particular quantity which is under the root 1 plus A t plus gamma u naught u naught divided by c square. Of course, this quantity is also squared this whole quantity within the bracket. I write as sub variable that I am just taking this variable to be p to sub variable which is put here, which is put inside the bracket.

Suppose, I take this for substitution I have to change this by variables integration also from d t to d p, for that I have to differentiate this quantity. Once I differentiate this quantity on the right hand side I will get d p, but let me try to first differentiate this particular quantity. If I differentiate 1 I will get one times m, so I will get 0, so that does not contribute to the differential.

Once, I start differentiating this, this whole square is multiplied by constant 1 upon c square or rather multiplied by a constant c square, this constant c square will remain as it is in fact that is a reason this c square have taken on the right hand side. Then I have to differentiate this quantity which is on the numerator, if I differentiate that particular quantity this is f x square or rather f t square. So, first I have to differentiate assuming f t to be one single variable this becomes two times f t, so this becomes 2 multiplied by this quantity, which is in the bracket which is a multiplied by t plus gamma u naught u naught.

Then I have to differentiate this quantity in the bracket itself, once I differentiate this quantity in the bracket you know I have to find out d f d t and for that this quantity is constant, initial velocity is constant then gamma u naught is also constant. Then this quantity when I differentiate I just get A, so this A this 2 is because of this differential this A is because of this differential and divided by c square.

I have taken on the right hand side, so what I get 2 A t plus gamma u naught u naught A d t is c square d p what I will do here. I have A t plus gamma u naught u naught this particular quantity I will replace by c square d p. Of course, this multiplied by d p, I will replace by c square d p, d by 2 A this what I am doing in the next transparent sheet.

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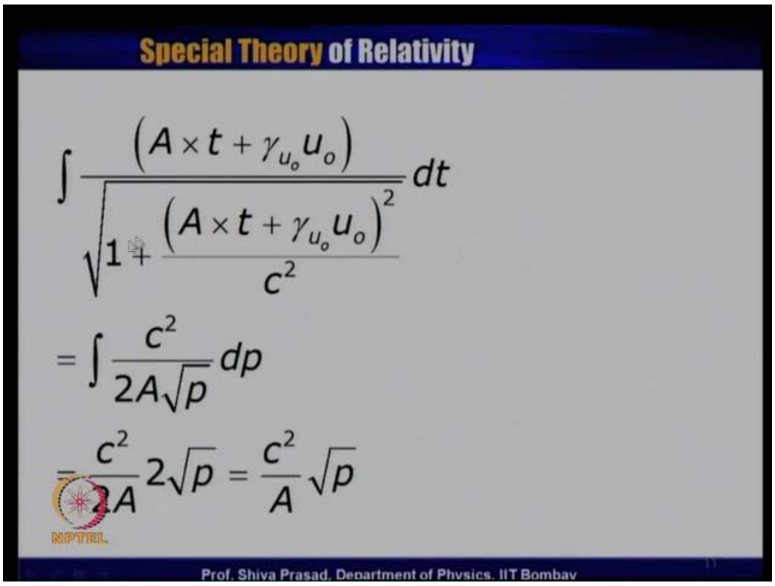

This was my original integral this quantity I have written as p, so this becomes under p. And as it is discussed this multiplied by d p, can be written as c square divided by 2 A multiplied by d p this is what I have written c square upon 2 A is constant.

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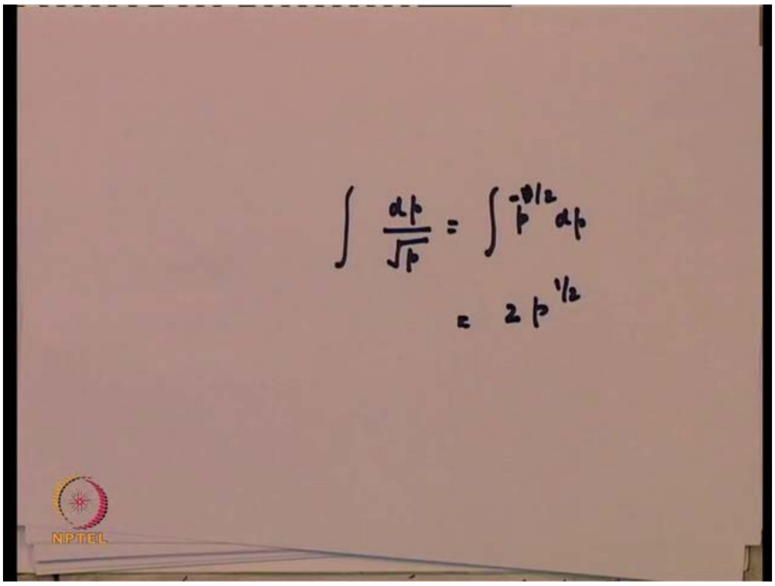

So, I can take it out and what remains here is the integral of 1 upon under root p, which becomes integral of p to the power minus half d p. This will become n plus 1 which becomes 3 by sorry minus half this becomes p to the power half must be divided by half. So, this becomes 2 p to the power half, so this is p to the power half is at the root p, this what I have written here, in this particular transparent sheet c square divided by 2 A multiplied by 2 p.

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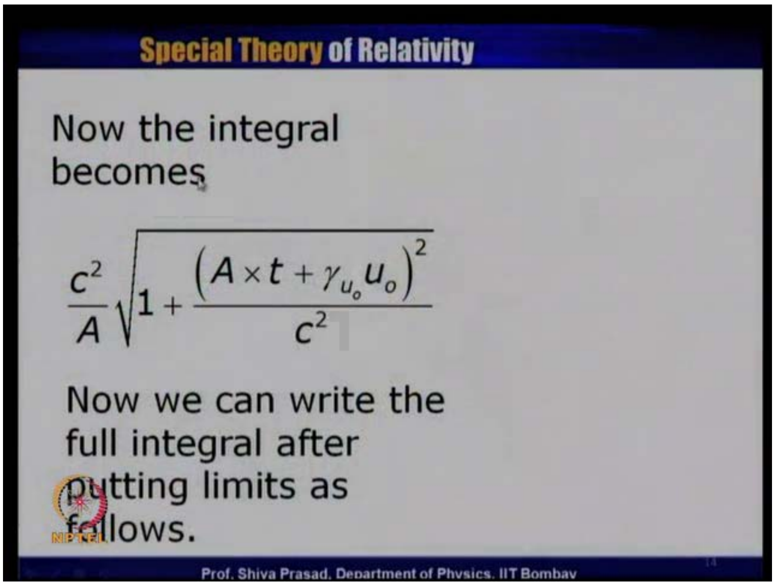

This two of course, cancels out and then what I get is c square by 2 A root p. So, this rather easy integral to solve not all that difficult I substitute the value of p and then put that into the integral, then I will get x as a function of p in fact p contains time. So, now the integral as I said becomes c square by a under root p, so this quantity was p which we had defined it of course, depend on time and depends on A which is f upon m naught. We can substitute that into the integral and we can write the full integral and the solution of the full integral by putting limits, if I put the limit I have to I mean the left hand side was very clear because it was integral of d x.

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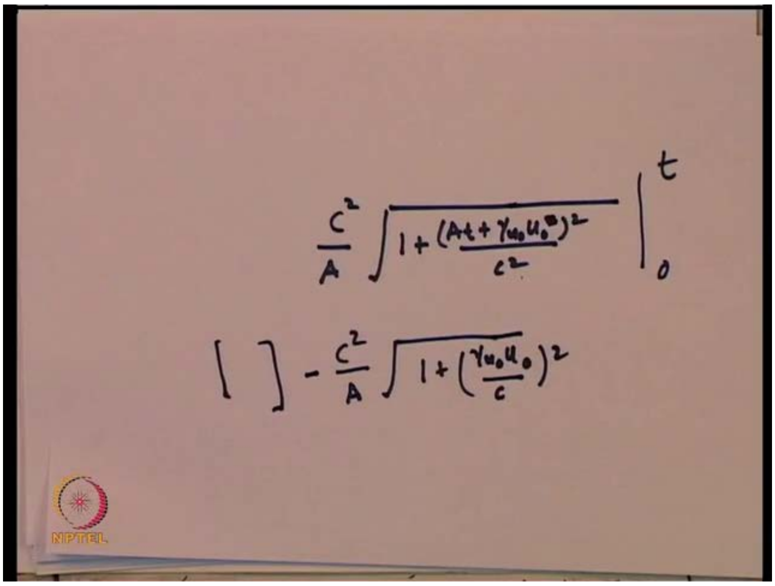

So, first you put the limits you just get x, but here when you put the limit you have to put the limit t and subtract minus put t is equal to 0 and then get the answer. So, this is what I will be doing in the next transparent sheet taking this write it. So, that it makes little clear quantity written here was just c square by A 1 plus A t plus gamma u naught u naught square divided by c square, first quantity when I put the limit t this quantity remains identical. So, when I put 0 then have to subtract whatever I get substitute u 0, if I substitute 0 you will get c square by A and the root 1 plus gamma u naught u naught, from there is no square here gamma naught u naught divided by c square. This is what I am writing in the next transparent sheet here, that x is equal to c square by A.

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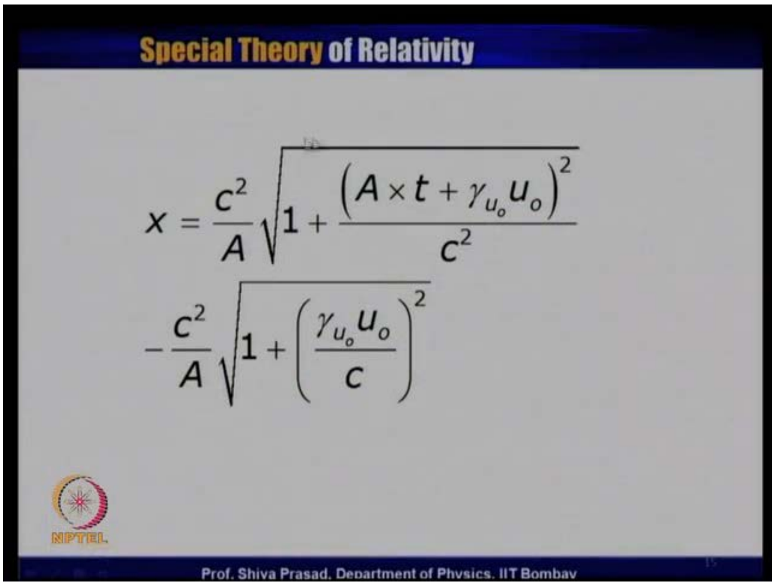

This quantity remains as it is, minus c square upon A under root 1 plus gamma u naught u naught upon c whole square. As you can see that this equation is also much more complicated than simple equation s is equal to u t plus half a t square, which we have been using for kinematics in the traditional classical mechanics. Now, what this equation can give me x the distance travelled by the particle in time t, if the particle starts with a speed u naught and is under the influence of a force F and it has a rest mass m naught.

So, a is equal to F upon m naught of force assuming that the motion is purely in one dimension this is where the equivalent of the kinematic equations, we are used to in kinematics in classical mechanics. Now, let us go to the next step which is crucial for us to arrive at the transformation of electric field and magnetic field that we are going to discuss, as the last topic for this course. So, question is that if I know the force of the particle in a given plane what will happen to the force as we viewed by another frame.

This what we call then transformation we have been talking about momentum transformation, we have been talking about momentum transformation, but it means that I know the velocity components in a given frame and I want to find out the velocity components of the same particles in a different frame. Similarly, I know the momentum components in the given frame and I want to find out the momentum components in a different frame.

Now, my question is if I know what are the force components F x, F y, F z, I would like to find out what are the F x, F y, F z in different frame. In classical mechanics they are expected to be same, but as we will be seeing in relative mechanics they do not turn out to be same the force components all four are framed. Remember, F is equal to d v d t, so once you are talking of transformation of momentum how we are going to transform the force.

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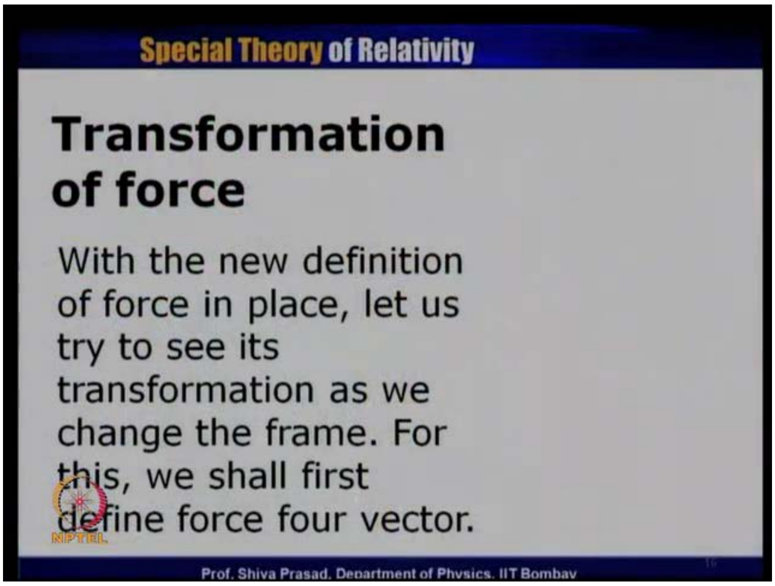

This in have written as transformation force with the new definition of force in place. Let us try to see if it is transformation to seek its transformation as we change the frame, I know the components of force in a given direction what are the components of a force in a different frame on that, what we will do? We will go back to the concept of fierce vector, which we have been using earlier we had defined the Lorentz transformation in terms of four vector.

We have defined velocity transformation in terms of force vector and we defined energy momentum in terms of force vector. So, let us try to construct a force four vector and if I am able to construct a force, four vector obviously that will give me the transformation of force, so that is the way I will look into it, I will first try to define the force four vector.

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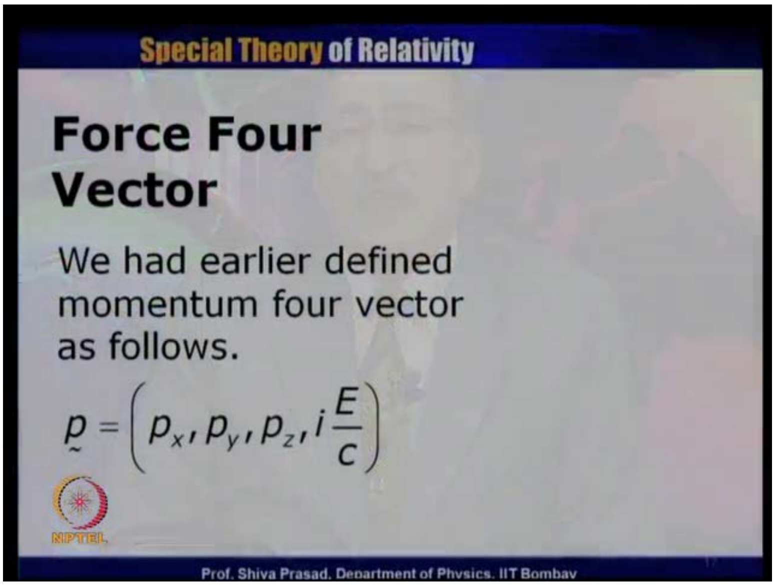

As you have said earlier define the momentum four vector, as this the notation that we have used if you remember is that these are the four components of the four vector, first three components are x, y and z. Components of momentum of the particle, first component is p x second component is p y third component is p z and the fourth component of momentum called momentum or normally call momentum energy. Four vectors are dependent on the total relativistic energy of the particle and in the notation that we have used.

We write it as i E by C, where E is the total energy, total relativistic energy of the particle. Obviously, if I have to construct a momentum for force, I have to differentiate with respect to time or I have to differentiate differentiate something; I have to differentiate this with respect to something that has a dimension of time. If I differentiate this with respect to d t again we land into the same problem, where we are trying to construct from a displacement four vector to a velocity four vector because d t is a frame dependent quantity, time interval.

If we take a particular time a moment of the particle and at a little later time the moment of the particle is the difference this delta t time is a frame dependent quantity. If I differentiate this particular thing with d t, then I will not be able to get the four vectors because in that case d t will be frame dependent quantity. If I have to construct another four vectors out of this I have to multiply or divide with something which is a four scalar which does not depend on the frame, this we have discussed earlier.

If you realize if you remember it is a proper time interval which is frame independent quantity. Therefore, if I have to construct a force four vector from a moment four vector I cannot differentiate with respect to d t, which is different in different frame I must differentiate with respect to d tau. The proper time interval because d tau is a frame independent quantity, if you change your frame d tau will not change. Therefore, if I have to take a force four vector I prefer to differentiate this with respect to d tau.

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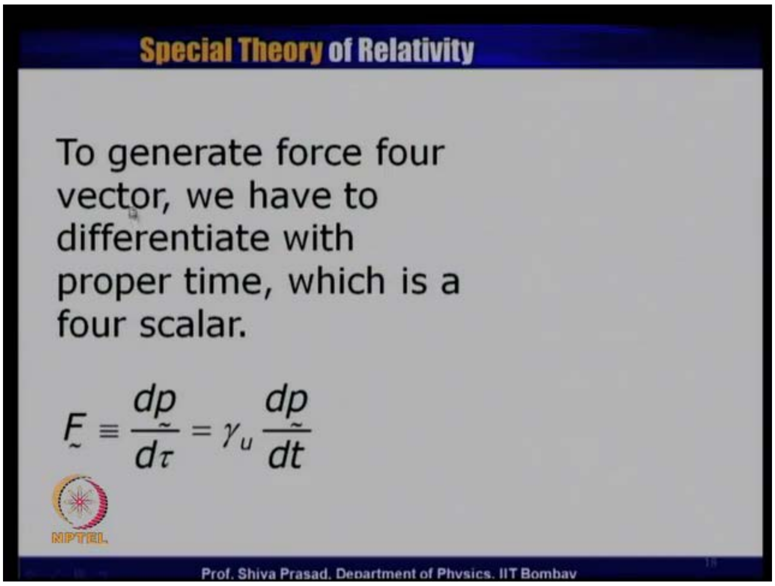

That is what I have written to generate a force four vector, we have to differentiate with proper time which is four scalar therefore, I define force vector, four vector force four vector as d p d tau where tau is the frame, tau is a frame independent quantity. If you remember whatever we have done at the time at velocity transformation eventually, I will write to write in terms of the force component in my own frame. If I have a particular frame and in my frame I would prefer to write this in terms of d t and as that is my own frame, but my definition gives the differential in terms of d tau. I can always convert this d tau into d t, so if I know the velocity of the particle at that instant of time then I can calculate gamma u.

Then I know d tau is equal to d p divided by gamma u this also wee have earlier used in the case of force four, I am sorry velocity four vector. So, this d tau can be written as d p divided by gamma u, this quantity force can be written as gamma u d p d t the advantage is that now I write this in a particular frame. If you are in a particular frame you would like to know things in your frame to construct a force four vector, so it is better if I know things in terms of my own frame.

So, my prescription is that I have to calculate the moment four vector in my frame then differentiate with respect to time with my own frame, because this is going to a frame dependent quantity. So, what I have to do I multiply this by gamma u, where gamma is 1 upon under root 1 minus c square by c square, where u is this speed of the particle measured in my own frame. If I know the speed of the particle the moment of four vector, I know gamma u and using that particular d t I can find out what is force four vector.

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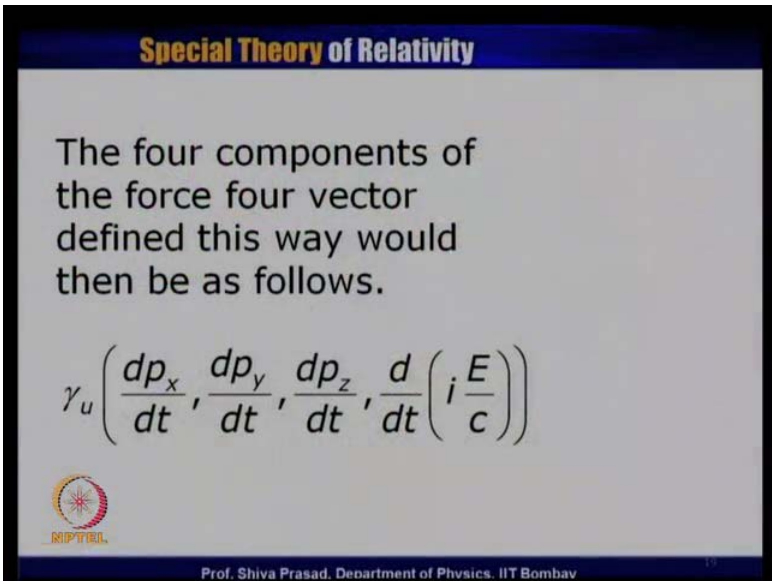

This what I have said the four components of the force four vector, vector defined in this way would then given me like this I calculate with my own frame, what is the x component of momentum differentiate with respect to time as measured with my frame. This is this quantity take the y component of momentum differentiate with respect to time as measured in my own frame, this will give me the second component.

I take the z component of the momentum differentiate with respect to time in my own frame I will calculate and get the third component. I know the energy of the part in my frame, I differentiate with respect to time in my own frame all these quantities are given in my own frame then I do what is the speed of the particle, I calculate gamma u which is the under root 1 minus u square by c square. I know gamma u, I have multiplied this gamma u by these quantities then these four vectors are now written in with respect to everything that is known in my own frame.

So, I know how to write it, so the force four vectors will be given by this particular quantity all these quantities multiplied by gamma u will generate the force four vectors. If you remember exactly the way we had calculated or we have we have constructed a velocity four vector, now once we know that this is a velocity four vector, I am sorry force four vector then I know how it is transformed. Once I go from one frame to another frame because this is the way force transformation four vector class formation takes place through a standard equation.

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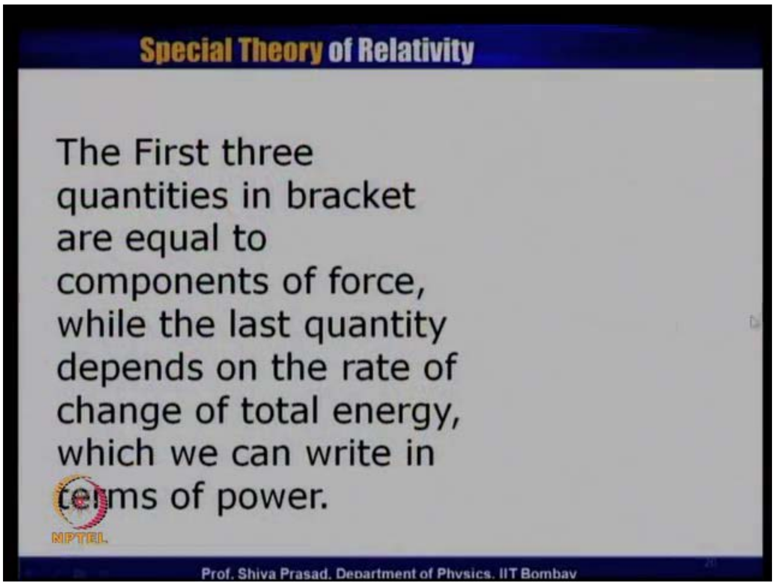

Let me just first read this thing the first three quantities in the bracket are equal to the components of the force once I said d p x d t, we have already defined as the force d p y by d t. I have defined as f y d p z by d t, I have defined as f z and the last quantity depends on the rate of change of the total energy which we can write in terms of power, which we have done earlier in the last lecture. So, before going to the transformation let me just rewrite this thing this components in terms of the force and d e d t.

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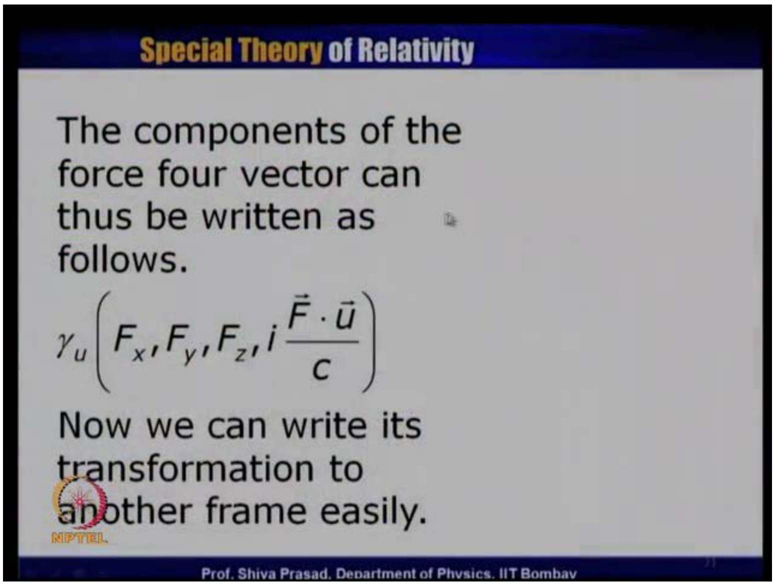

Therefore, the components of force four vector can be written as gamma u, first component was d p x d t which by new definition is f x d p by d t is the new definition is f y d p z d t. If I call this by new definition is f z. If you remember we had calculated d p of E in the last lecture and that we have found out be power F dot u. So, d p d t I have written F dot u divided by c, so energy happens to be the forth component of energy moment four vector time happens to be the fourth component of the displacement four vector for the force four vector, the force component turns out to be power.

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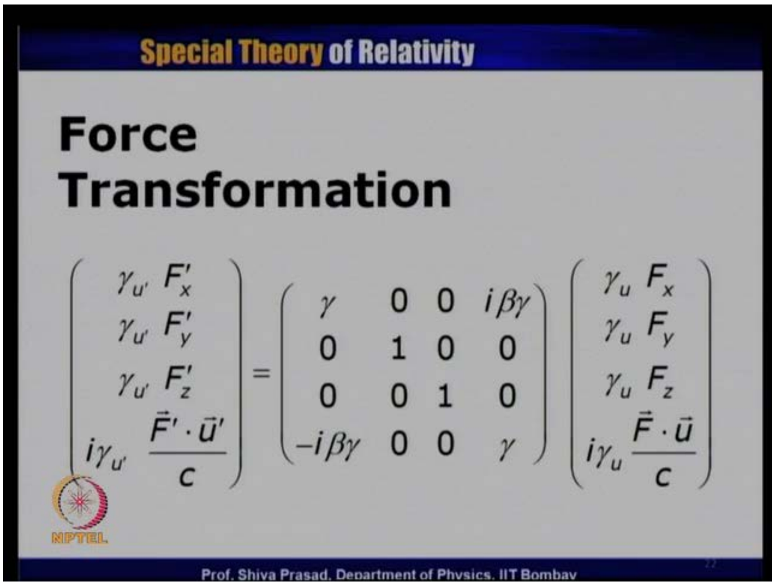

So, once we have generated everything in terms of forces, if I go to different frame in general f x is likely to change f y will change f z will change. Of course, u will change and of course, gamma u will change because the velocity of the particle would change. Once I change my frame of preference, I can write the transformation equation as given in this particular transparent sheet.

These are my four components of the force four vectors in the particular frame s prime this is the transformation matrix which depends on gamma, gamma depends on v the relative velocity between the frames. Remember, we have three different speeds, so we should not get confused these are the components that I want to find out in a frame s this a particle which is moving in s its velocity is being measured by an observer sitting in frame s. That is what I am calling as u the same particle is being observed by an observer in s prime and that observer finds the speed of the same particle to be u prime, so that is second speed.

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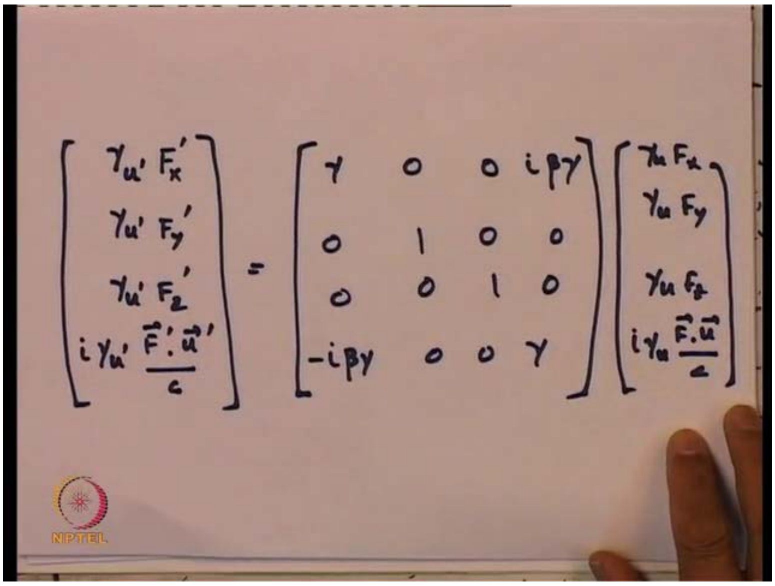

The third speed is that the relative velocity between the force, the two frame which always assumed to be along the x direction and all those things which are the conditions of Lorentz transformation that is v. We have three speeds and correspondingly we have three gamma’s, we have gamma u we have gamma which depend on v, we have gamma u prime which depend on u prime. This is my transformation matrix which I have to open up if I have to find out what will be f x prime relating to other components in s frame that is what we see next.

Let us expand it within this particular transformation matrix here just to take some time to make it clear, if I have write gamma prime f x prime this will be gamma multiplied by this component 0 multiplied by this component plus 0 multiplied by this component plus i beta gamma multiplied by this component. So, that will be the first term, let us just see first term once I multiplied here gamma u, gamma u will come out and f x there will be gamma and gamma u which will also be similar i square will make it minus 1. So, second term will be F dot u by c into beta.

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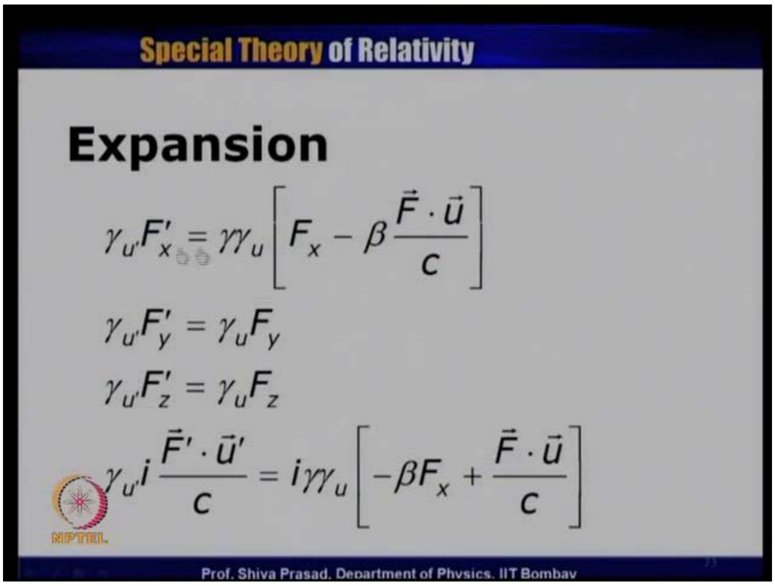

This is what I have written here that is the first term gamma u prime f x prime, which is the first terms on the matrix. The column matrix on the left hand side is equal to gamma gamma u which we have agreed will come out multiplied by f x minus beta F dot u divided by c. Let us look at the second component, second equation here in the paper it is gamma u prime f y prime is equal to 0 multiplied by this one multiplied by this plus 0 multiplied by this plus 0 multiplied by this which gives gamma u prime f y prime is equal to gamma u f y gamma u prime f z prime.

Similarly, will be gamma u f z and the fourth equation will be in gamma u prime this quantity minus i beta gamma multiplied by gamma u f x plus 0 multiplied by this plus 0 multiplied by this plus gamma multiplied by this. Here also you will see that there is a gamma and gamma u there is a gamma and gamma u which will come out of the matrix. So, these are my equations gamma u dot f y prime is equal to gamma u f y gamma u prime f z prime is equal to gamma u f z and this is the fourth equation gamma u prime i F prime dot u prime by c because the fourth term in the column matrix of the left.

This i beta beta with this i gamma gamma u and i comes out and you will get minus beta f x plus f dot u by c, these are expanded this particular matrix equation has been expanded into the four equations. Now, we will try to simplify these equations by using a notation using a equation, which we had opted at the time of velocity transformation to write in a compact form which is a usual form and which probably is much more useful as far as our futures endeavors are concerned.

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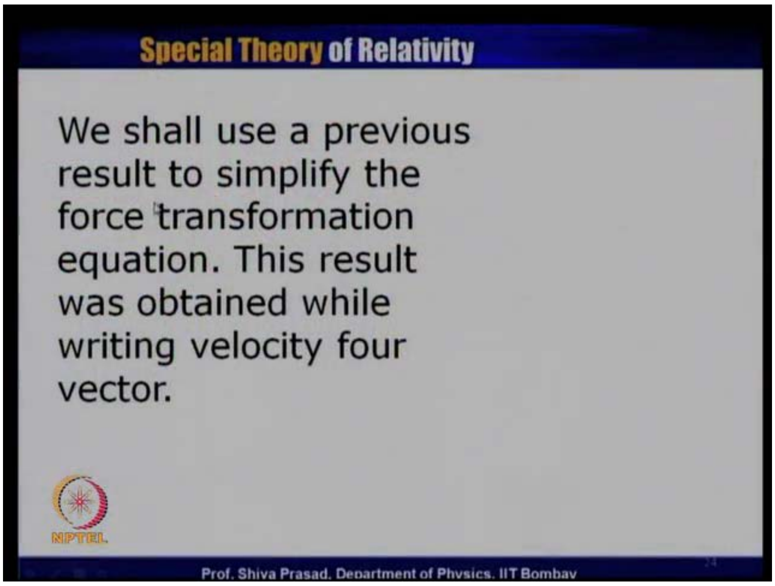

So, this is what I have written, we shall use a previous result to simplify the force transformation equation this result was obtained while writing the velocity four vectors.

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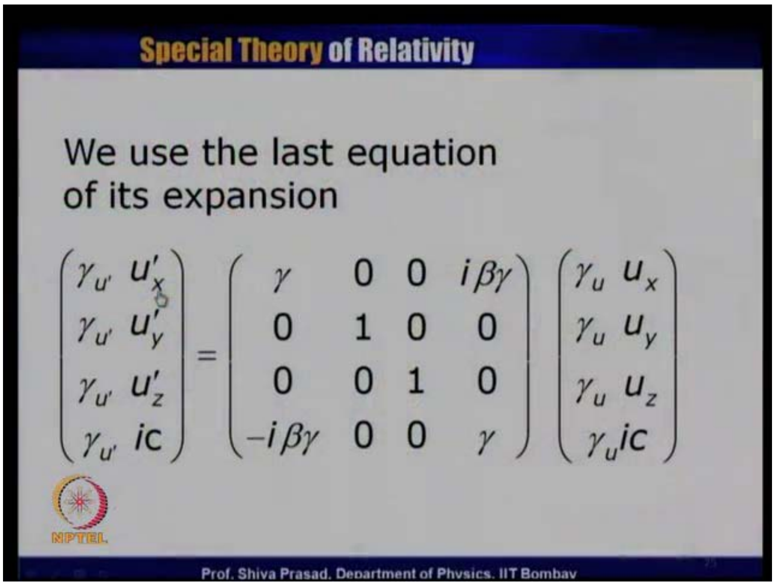

This was my velocity four vector transformation, see remember this transformation matrix is exactly same in that case. If you remember what was forming the four vector was gamma u u x gamma u u y gamma u u z and gamma u i c and when we change the frame it became gamma u prime u x prime gamma u prime u y prime gamma u prime u z prime and gamma u prime i c prime.

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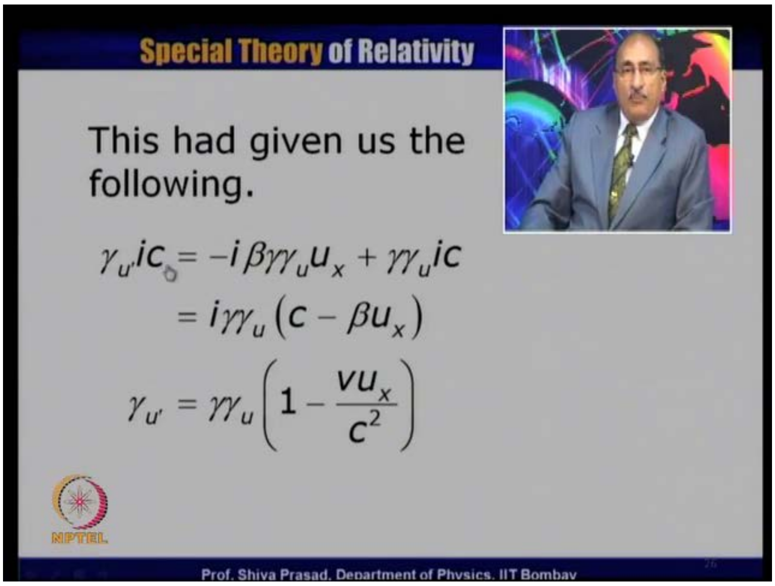

Let us look at only the fourth equation which is this equation, which is gamma u prime i c which is equal to gamma multiplied by gamma u I am sorry minus i beta gamma multiplied by gamma u x plus gamma u multiplied by gamma u i c. I am looking only the fourth equation first three equations will give me standard velocity transformation.

Now, you just look at this particular fourth equation which is here gamma u prime i c is equal to this particular quantity this we have done earlier, discussed earlier in great detail when we are discussing the velocity transformation. And by reorganizing the term on which I would not like to occupy time here is what we this what we get gamma u prime is equal to gamma gamma u 1 minus v u x upon c square.

So, on the left hand side the force transformation equation the four equations that we have got where ever gamma u prime comes I will replace it by gamma gamma u 1 minus v u x upon c square. Therefore, I will be able to get a clean equation writing f x prime in terms of the components this is what I am doing in the next transparent sheet.

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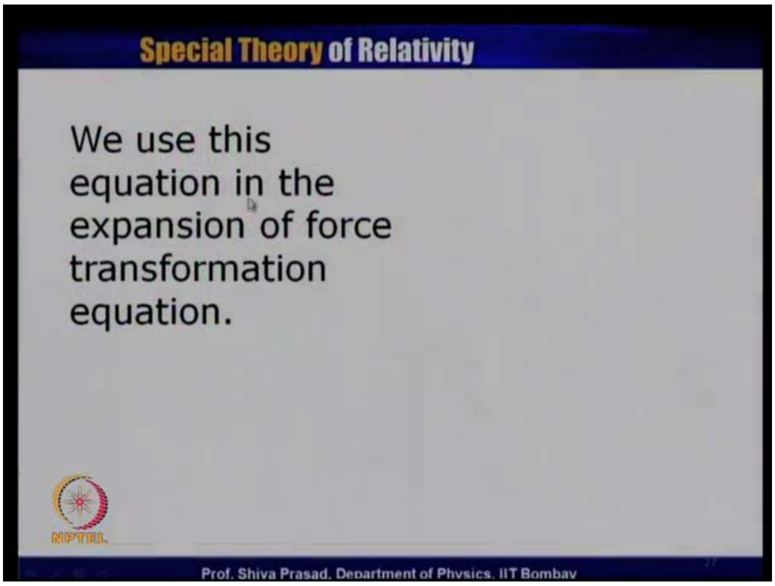

So, I have said we use this equation in the expansion of the force transformation equation. This was my first term which we have opted by expanding that particular matrix equation gamma u prime f x prime is equal to gamma gamma u f x minus beta F dot u by c just now we have obtained by expanding that particular matrix. This is the equation which I have written in earlier transparent sheet, which we had obtained from the velocity transformation which related gamma u prime in terms of gamma gamma u so gamma u prime is equal to gamma gamma u multiplied by 1 minus v u x upon c square.

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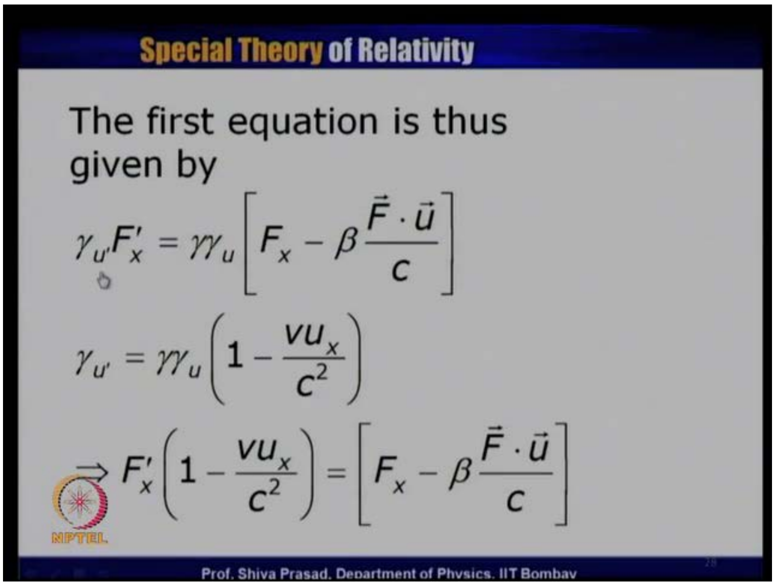

So, this is gamma u prime here I will substitute by this particular quantity gamma gamma u multiplied by 1 minus v u x by c square once I do here. Let us put it here gamma gamma u cancel from here, what I will be getting 1 minus v u x upon c square which will get multiplied to f x prime and right hand side will just become f x minus beta f dot u upon c. Now, it is rather simple I can just divide this quantity both sides and I will get f x prime is equal to a quantity that I am doing in the next transparent sheet.

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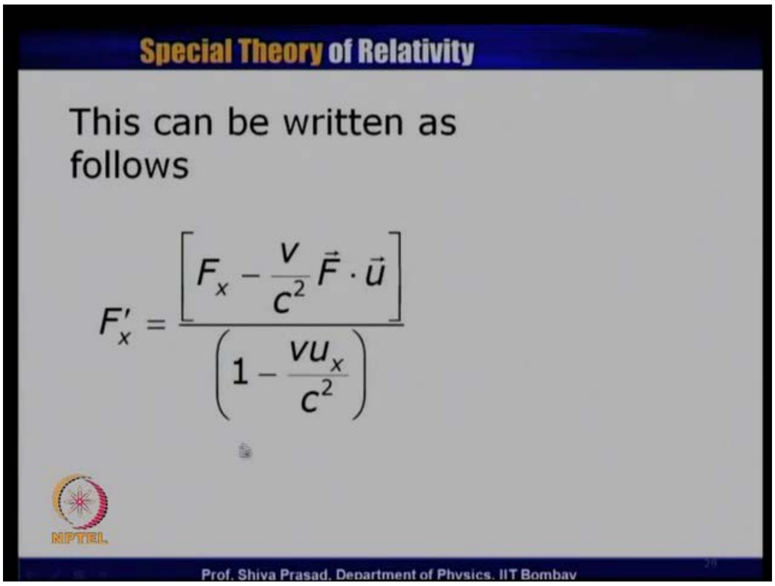

I have divided 1 minus u x upon c square and remember there was a beta and there was a c here, there was beta was v upon c there was also a c here. So, this become v upon c square this becomes v upon c square F dot u divided by 1 minus v u x upon c square may be you will see the resemblance that you will get from the equation. That was the time when we were doing Lorentz transformation, we were doing the time transformation we were getting something like this, and this is something similar to that except the fourth term to be F dot u.

This will be the transformation of the x component of the force, let us look at the transformation of the y component of the force of course, before first let me first discuss it because this equation sometime is written in different form. Let us first work out that particular form then we will do their vital component transformation and transformation just wanted to make one more point.

That member f x prime will also depend on force f x f y and f z see like in the case of moment of transformation x prime also dependent on the energy which depend on E square should depend also on the x y and z component of moment of not just the x component of the momentum. Similarly, here it is also dependent on f y and f z sometimes the equation is written in a different form avoiding to write this F naught. So, let us just try to write that equation in that form before you go to the transformation of the y component and the z component.

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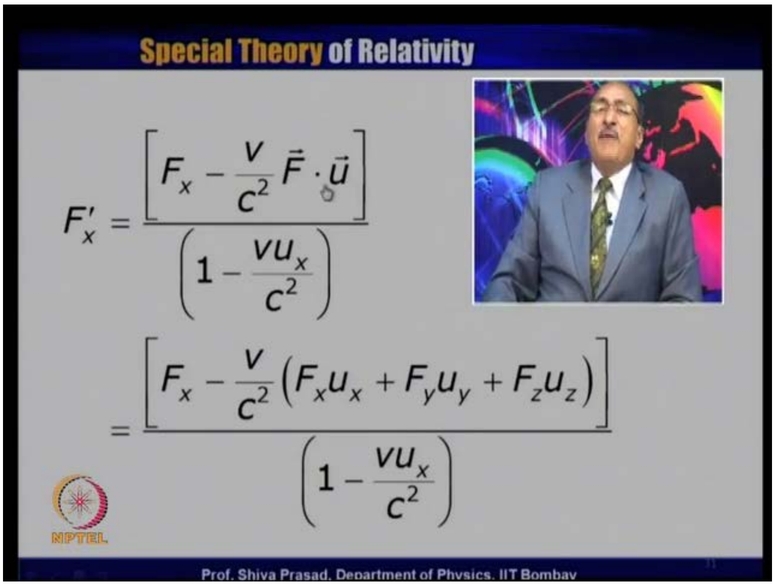

So, what we do we just expand this F dot u f x is v upon c square remains like this and like simple dot product of two vectors. If we have dot product of E and v we have e x dx plus e y d y plus e z d z and if you have F dot u you will get f x u x plus f y u y plus f z u z. This is what I have written minus v upon c square F dot u is f x u x plus f y u y plus f z u z just simply expanding and writing what I will do, I will take this f x common here.

If I take f x common I will get 1 minus v b u c square I am sorry we will get f x 1 minus v u x by c square. So, there is a u x here there is a u here, so I will get a v u x by c square and that quantity will cancel with this particular quantity only as far as the x components are concerned as far as these components are concerned there is no cancellation, so I can rewrite slightly this equation.

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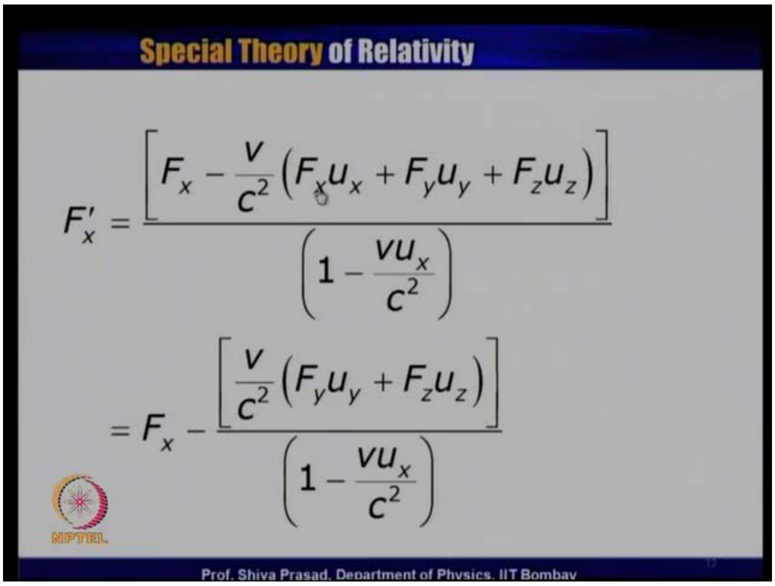

This was my equation, so as I said f x 1 minus v upon c square f x 1 minus v u x upon c square this cancels and you get this simple f x these two quantities, there is no cancellation. If you get f x minus v upon c square which is here f y u y which is here plus f z u z which is here the of course, divided by 1 minus v u x upon c square. So, this tells you that f x prime is equal to f x minus all these quantities.

Remember in classical mechanics we expect v f x prime to be equal to f x. So, we can see very easily that when v is going to be close to c then all these terms will cancel and just f x prime and u will be negligible rather and this will give you f x prime is equal to f x. Now, let us go to the transformation of the y component of the force.

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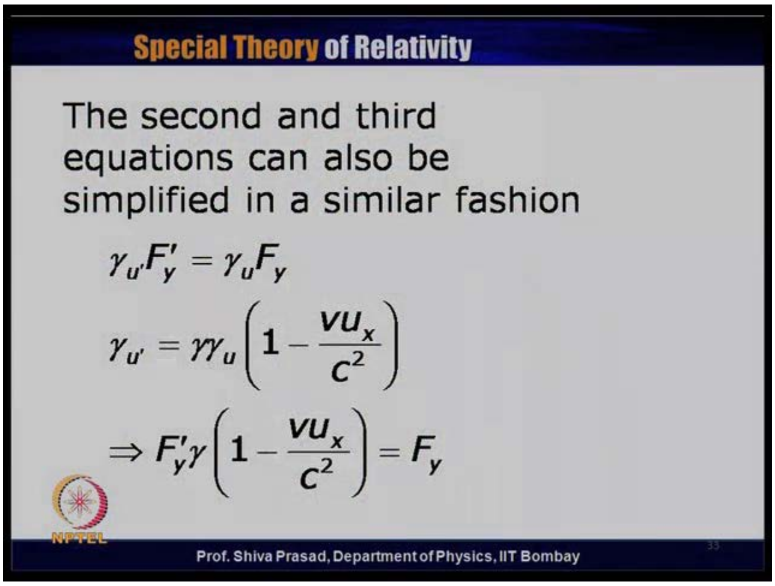

For second and third both are similar, so let us look for simplifying the second term this was the equation gamma u f y prime is equal to gamma u f y. This was my second equation which we had obtained from opening the matrix I use again gamma u prime is equal to gamma gamma u 1 minus v u x upon c square exactly similar way here. There will be gamma this gamma u will cancel from this gamma u, on the right hand side multiply by 1 minus v u x upon c square. So, this gamma will not cancel here, we will just have a f y prime into gamma into the whole of this quantity 1 minus v x upon c square divided by equal to y f y.

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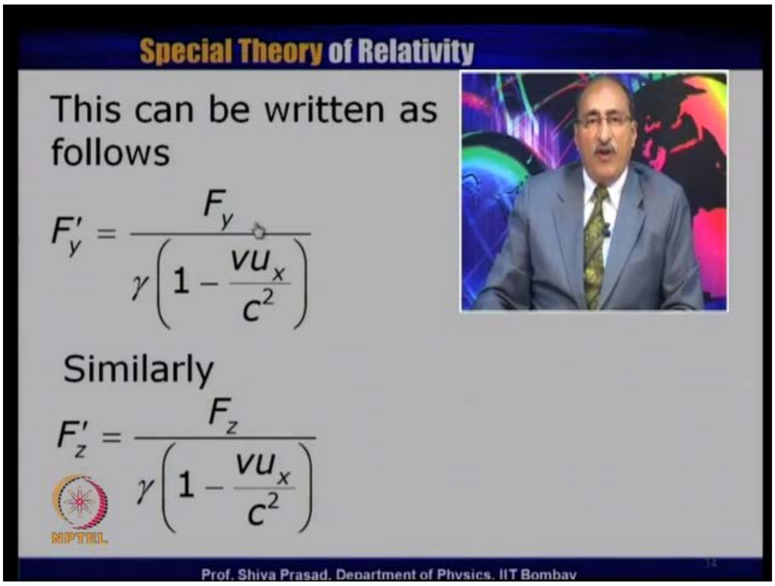

Therefore, f y prime is equal to f y divided by gamma 1 minus v u x upon c square. If you remember this was similar to the velocity transformation of the y component and because the z component equations are exactly identical their f z prime is equal to f z gamma 1 minus v u x upon c square. Remember in the x component there is no gamma there because the gamma has cancelled out here gamma has not cancelled out gamma u has cancelled out. So, gamma is written in the denominator just like the velocity transformation that we had got obtained there.

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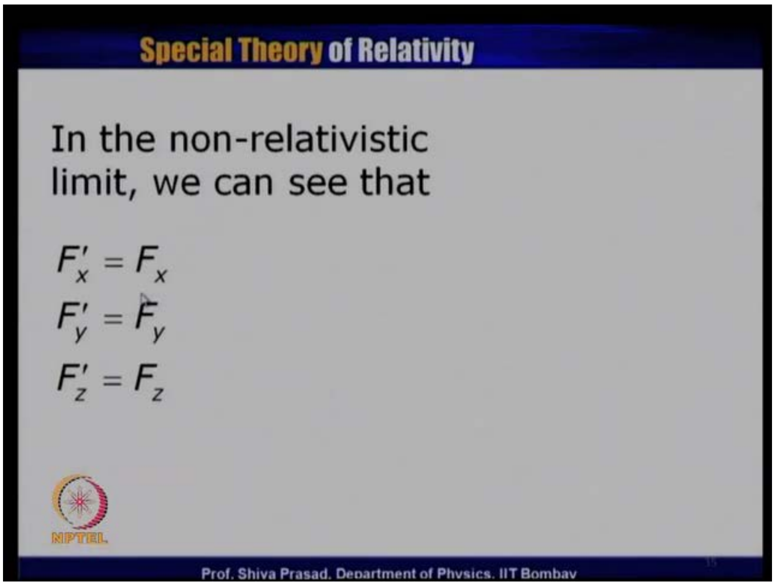

You can see that in non relativistic limit that f x prime is equal to f x, that we have discussed. You can also see in non relativistic limit this will become one this when v and u x are much smaller than c this quantity is negligible in comparison to one f y prime will be equal to f y f z prime will be equal to f z. So, normally in non relativistic limit we do get f x prime is equal to f x f y prime is equal to f y and f z prime is equal to f z, but in relativity forces also transformed in a different frame they appear to be different.

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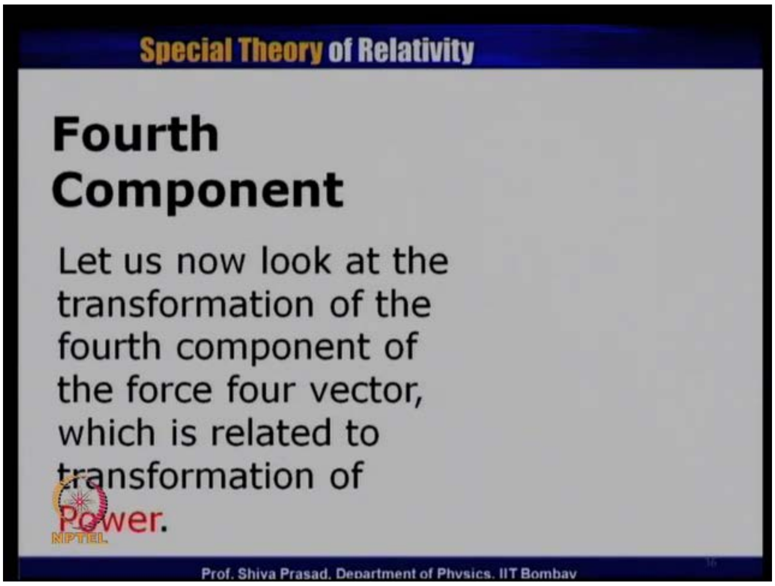

Now, let us look at the fourth component which component which is related to transformation of power, so power also transforms. So, once I go from one particular frame to another frame the power will also transform. Now, let us look at the transformation of the fourth component of force for vector which is related to the transformation of power.

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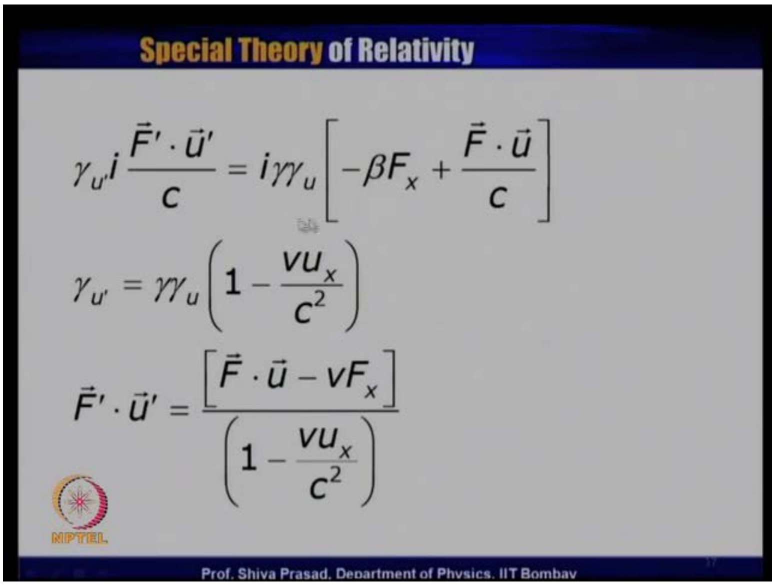

This was my equation which we have obtained by expanding the matrix, which was the fourth component equation again I write for gamma u prime gamma gamma u 1 minus v u x divided by c square here. There is a gamma gamma u which will cancel, it will also cancel which is being to somewhat simpler equation it is F prime dot u prime is equal to f dot u minus v f x divided by 1 minus v u x upon c square. So, this gives me the transformation of power once I move from frame s to s prime how my power changes.

This is the transformation equation that governs that particular transformation equation. Now, let us look at the inverse transformation this tradition whenever we write a transformation, we also write an inverse transformation. It means if my quantities are given in s prime frame of reference, we have to find put the equivalent quantities in s frame of reference how do I transform like for instance in velocity transformation.

My velocity components are given in s prime frame of reference and I want to find those velocity components in s frame of reference that is what we call as inverse transformation descriptions are simple. As we have discussed number of times that all prime quantities have to be changed to unprime quantities and wherever there is a v replace it by minus v this is as simple as that, so that is what we are doing. We are now writing the inverse transformation equation just by interchanging the prime and unprime and replacing v by minus v.

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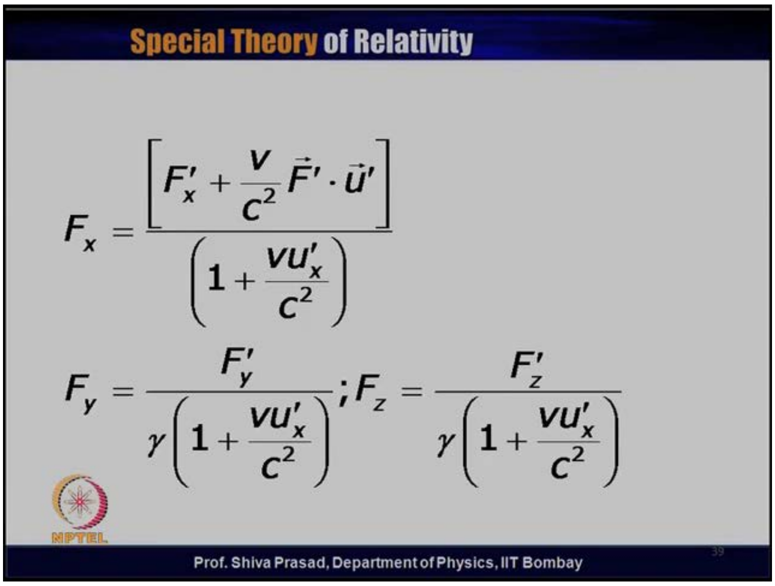

So, this is my force component, so on the left hand side we have f x is equal to f x here there was f x. So, this becomes f x prime there was a minus v because we have replaced a v by minus v, so this becomes plus this becomes v upon c square F dot u. Now, F prime dot u prime should be prime divided by 1 minus v u x prime by c square. Similarly, f y will now become f y prime gamma 1 plus v u x prime upon c square because v has been replaced by minus v. Similarly, here f z will be equal to f z prime divided by gamma 1 plus v u x prime by c square, so this becomes inverse transformation by force.

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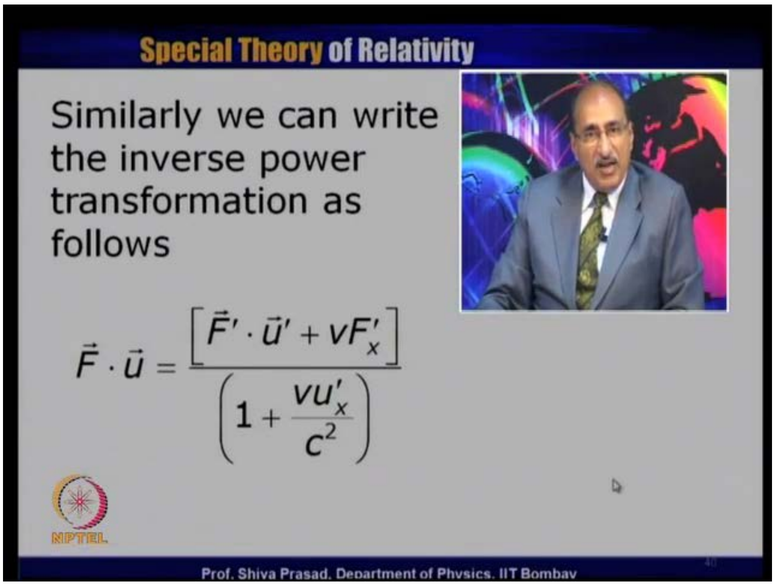

Similarly, we can write the inverse power transformation also exactly identical things here become F dot, I am sorry become F prime dot u prime plus v x prime divided by 1 plus v u x prime by c square. For the left hand side we have F dot u prescription is simple we do not spend too much of time in describing this, we just interchange all quantities as I said make all prime quantities all unprime quantities make them prime replace v by minus v. This case we get inverse transformation before we close the lecture, let us look at a special case which is quite useful and illustrative and we will also be using in our next lecture, we consider a special case of a force transformation.

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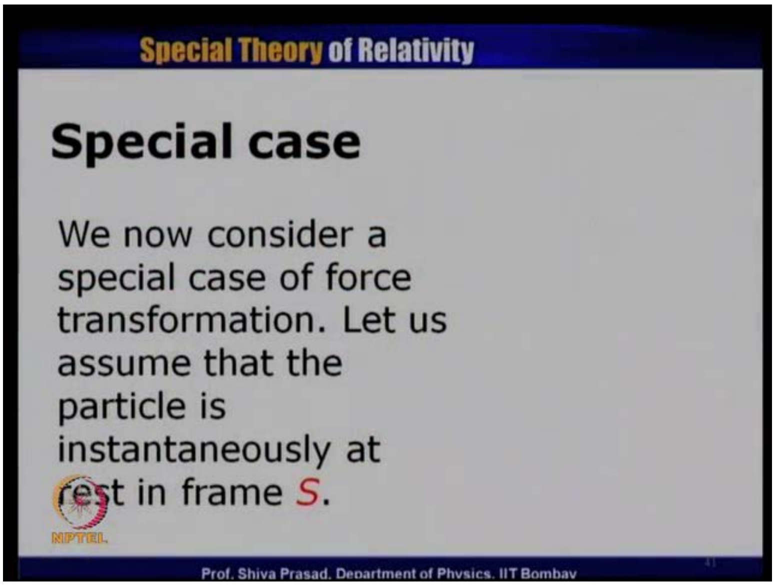

Let us assume that there is a frame s in which this particular particle is instantaneously at rest of course, it is at rest at that given point of time, but a later time because it is under the influence of force it is speed will change. So, no longer it will at rest at that particular frame assume that there is a frame in which at a given instant in which I am interested. In this particular particle is at rest is whatever was the velocity of that particular particle at that particular frame that particular the frame seems to go with the same velocity. Therefore, this particular particle is at rest in that particular frame in that case u would be 0 u will be 0 and the force equations become somewhat simple equations.

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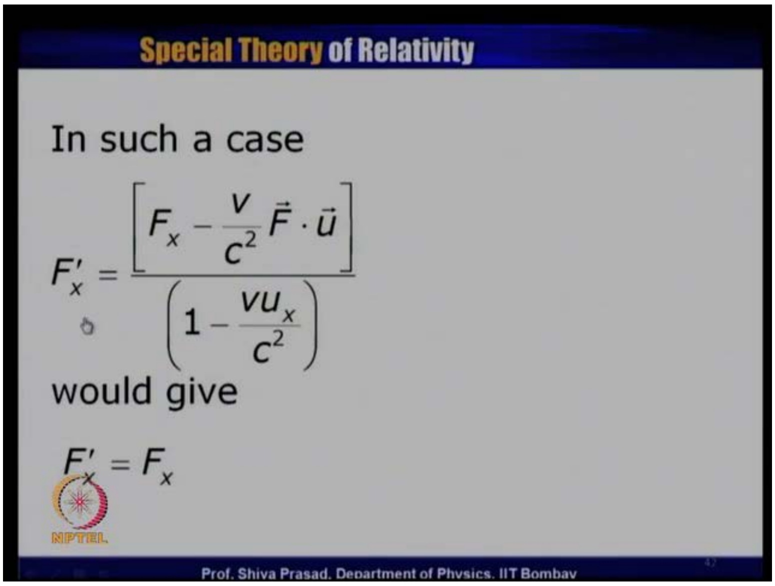

So, this is the f x prime this is the transformation of the x component because this particle is instantaneously at rest in s frame, so u becomes 0 once u becomes 0 this whole quantity becomes 0. Of course, if u is 0 x is also 0, so this quantity becomes 0 and denominator you have only one which gives me f x prime is equal to f x. So, if we measure the force on the particle in the s frame at that point of time when the particle was instantaneously at rest in s prime frame also it will appear to be at rest the x component of the force will turn out to be the same.

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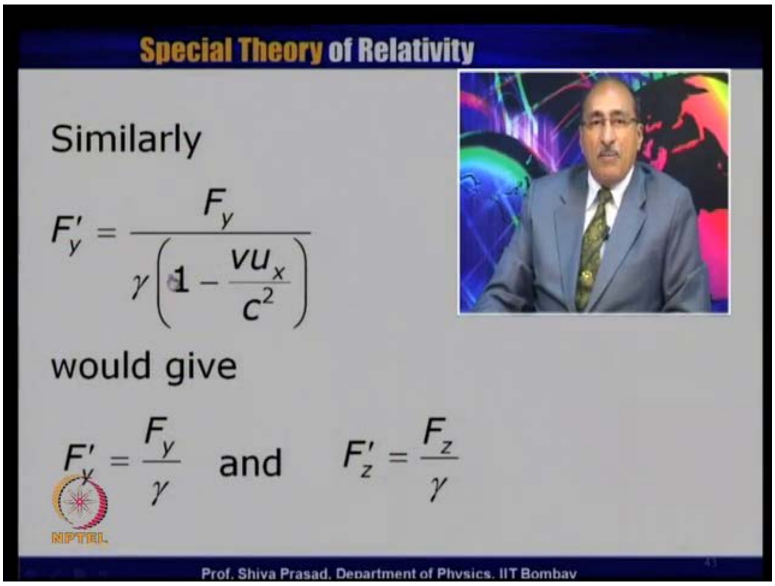

Let us look at y f y prime is equal to f y divided by gamma one minus v u x upon c square u and x being 0 in s frame, you will have this quantity equal to 0. So, you will just have f y prime is equal to f y divided by gamma gamma be always greater than 1. Your f y prime will be smaller than f y and of course, because these transformation equations are similar for component I will also get a z prime is equal to z divided by gamma. So, what get in this particular case an f x prime is equal to f x by gamma f y prime is equal to f y by gamma and f z prime is equal to f z by gamma.

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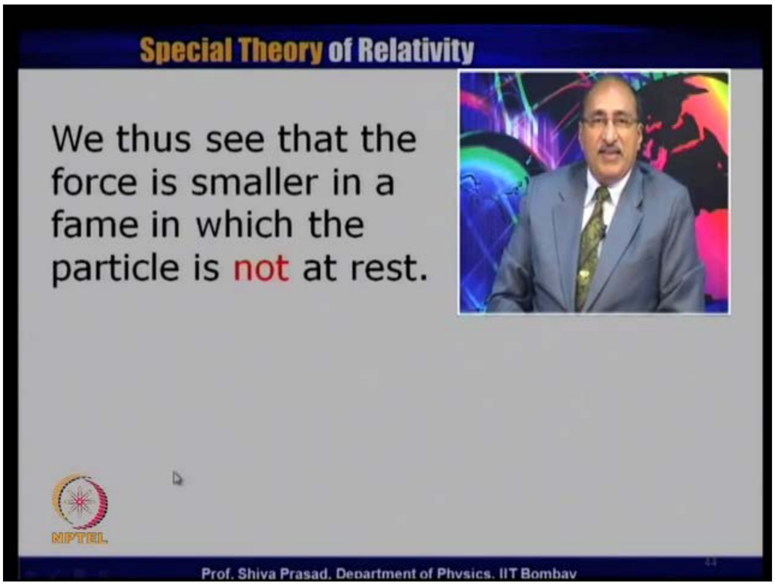

We thus see the force is smaller because the f y components have become smaller in a frame in which the particle is not at rest. So, the force is large in a frame in which the particle is at rest which you can call as a proper frame.

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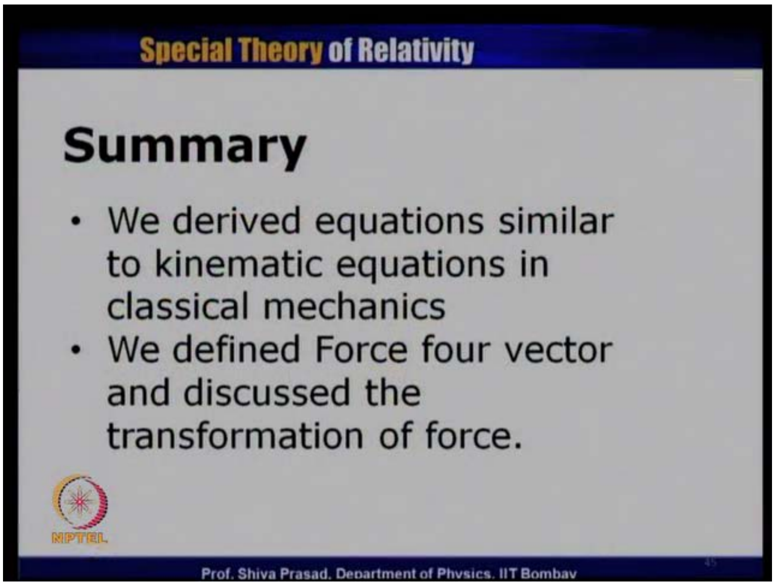

So, to end the lecture we just summarize whatever we have done we derived the equations similar to the equations in classical mechanics. We also defined the force four vector and discussed the transformation of force.

Thank you.