**Special Theory of Relativity**
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**Lecture - 20**
**Force in Relativity**

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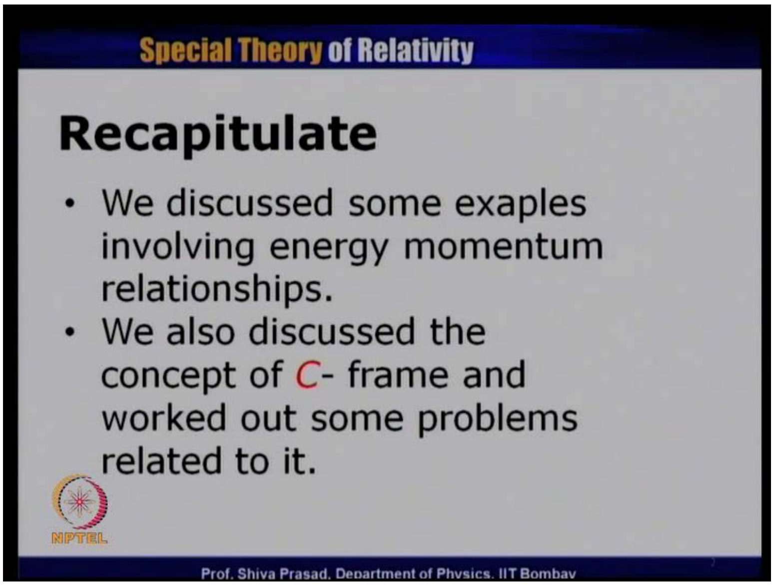


In your last lecture we had discussed some examples which involved momentum and energy, and the relationships how we change from one frame to another frame. Specifically we discussed the center of mass frame, which we call C frame. There is in contrast to C frame we have a l frame, which we call as a laboratory frame, which need not be the frame in which the initial momentum 0, initial momentum of the set of particle is 0, well in C frame it is 0.

We also said that some problems become easier to solve, if we go to center of mass frame. We also discussed that some of the experiments high energy experiments are perform in the center of mass frame in order to save energy. So, this what we have? Said about the last lecture, we discussed some examples involving energy and momentum relationship. We also discussed the concept of c frame and worked out some problems related to it. Today we will like to go little ahead and introduce a new concept which of course in classical mechanism is well-known, which is the concept of force.

In Newtonian mechanics, force is very important thing. We all know what is the force? So, let us discuss what happens to the definitions of force once we comes to the special theory of relativity, if you remember there two ways which we can talk of force in classical mechanics.

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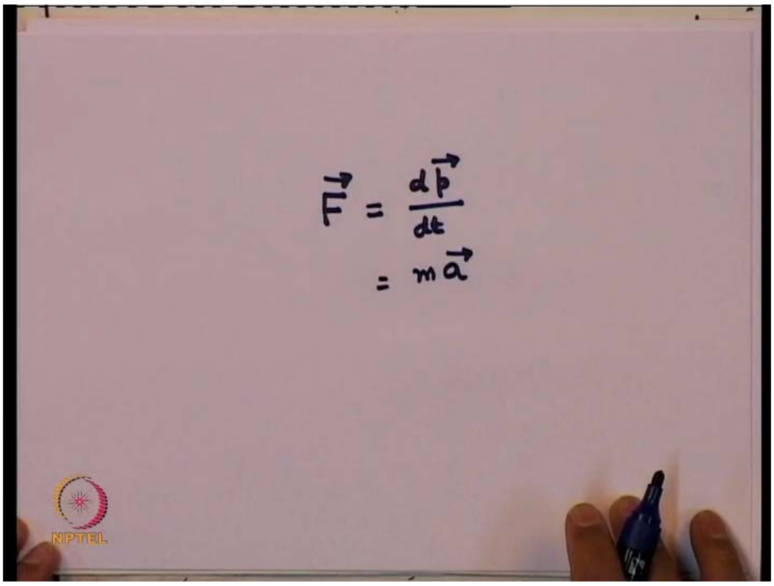


One of course the force is the vector quantity. So, always like with this, the vector sign F. we can write that F is equal to the rate of change of momentum. When we say rate of change, it always means that time rate as for its definition is consistent. So, force can be written as time rate of change of momentum because in classical mechanics p is given by mass into velocity, where mass is suppose to be costing.

Therefore, equivalently we can write this relationship in the form of mass into acceleration because rate of change of velocity with respect to time is given as an acceleration. So, in classical mechanics we can use either of the two ways, to express the force, either as the rate of change of momentum or mass into acceleration.

We will shall we shall just now see that in a relativity, we have to choose one of the two and in peaceful in fact in relativity we choose the first one. In relativity, we define force as the rate of change of the momentum. So, we take the first definition not the second definition because as we shall see that the two definitions are not same. Specifically as you know, that in momentum relates to something, a in fact relates to the mass, in which the gamma u factor, we just now discuss that.

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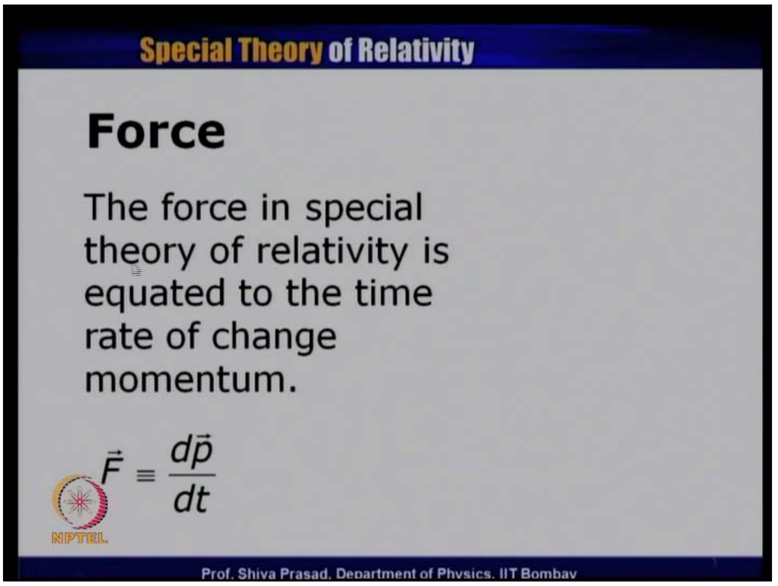


So, this what we said the force in special theory of relativity is equated to time rate of change of momentum. So, the definition that we have taken in relativity for the force is force is dp dt, were p is the momentum of the particle. Let us see what does it mean, let us try to take this example forward, this definition forward and try to get little bit more inside into, how our concept will change with this particular definition of momentum this definition of force and also the wave relativity defines momentum and other quantities.

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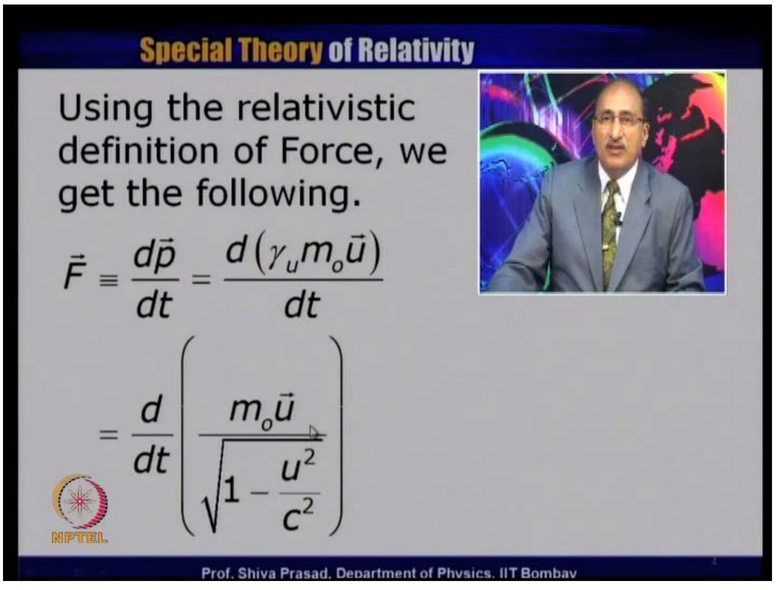

So, as we have said that, force in special theory of relativity, we have said earlier is defined as gamma u m naught. So, u is a particle is moving with a speed or with a velocity u in a given frame preference and m naught is the rest mass of the particle, then gamma u m naught u is defined as the momentum of the particle, this what we have discussed earlier in our lectures.

So, I must take the rate of change of this particular quantity, if I except this definition of force which is force equal to rate of change of momentum, which essentially means that force has to be written as d dt which is a time which is the rate of change of have that time derivative. m naught which is here, u which is here and gamma u have express as one upon under root 1 minus u square by c square.

So, force must be written in this particular fashion which essentially means that have to take a time derivative of this particular quantity, where now you appear at two different places, one here and another here. In classical mechanics in non relativistic mechanics this was only place where you was appearing and must be took a time derivative just take give a m into acceleration m into du dt which I can write an acceleration. But now if I take this definition, I have to take this particular quantity take the derivative of this quantity also. So, is a function in which, where you appears twice therefore, I must differentiate by parts in order to get the actual expression for force applicable for special theory of relativity, which have done the next transparency.

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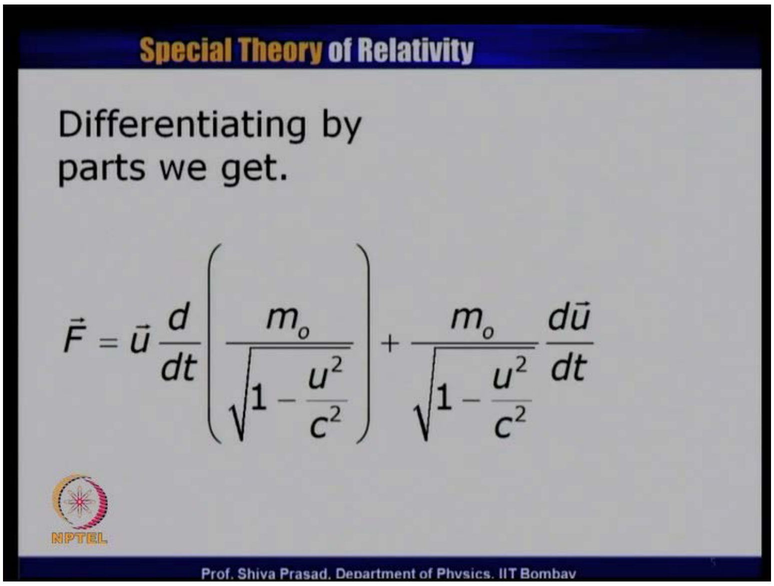

So, we (( )) differentiate by parts, as we have seen, the force that we have written was force is equal to d dt of m naught u under root 1 minus u square by c square.

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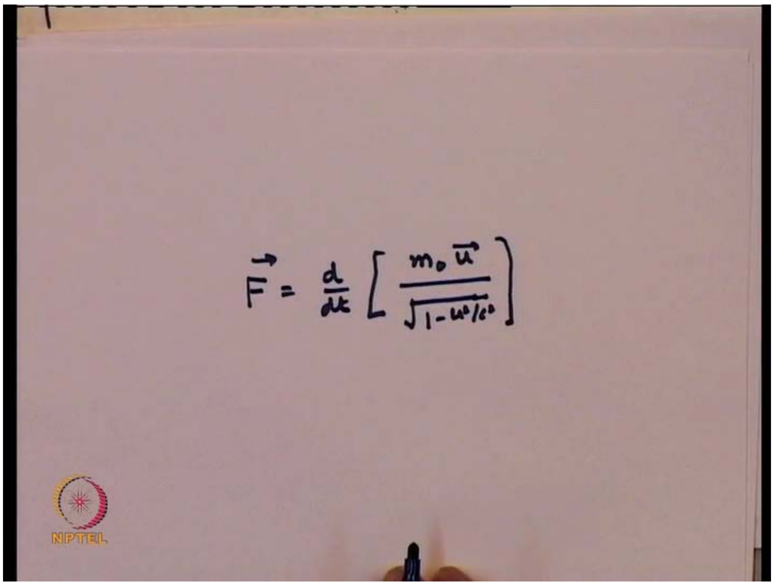


So, I am trying to differentiate by part. First I take u, take the derivative of this quantity, plus I take this quantity multiplied by derivative of u, this is what I have written this particular transparency. So, u multiplied by d dt of m naught under root 1 minus u square by c square plus m naught divided by under root 1 minus u square by c square multiplied by du dt is just simple differentiation by parts. Now let us little more closely at this particular equation and try to see whether I can make meaning out of it.

So, next transparency that should I am trying to do. I am trying to look at this particular first term, which is d dt of under root 1 minus u square by c square, this m naught which are appearing outside I have brought it inside, I multiplied by c square and also divided by c square. The idea is that, I can express this in terms of the new definition of energy because I know this particular factor multiplied by m naught, multiplied by c square can be written as the energy of the particle, total relativistic energy of the particle which is defined as gamma u m naught c square.

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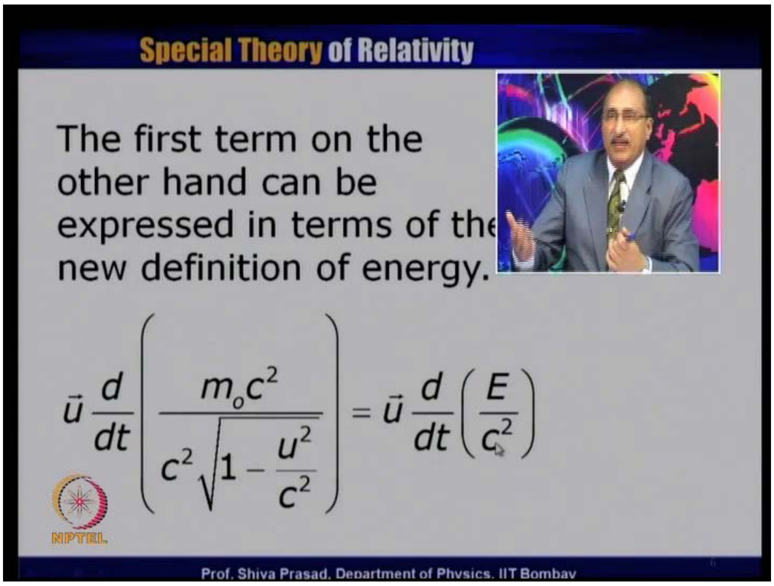


So, this is my gamma u m naught c square. So, this whole quantity can be written as the total energy of the particle divided by c square. So, the first term in that particular integration by part, can be written as u multiplied by d dt of E by c square, where E is the total relativistic energy of the particle.

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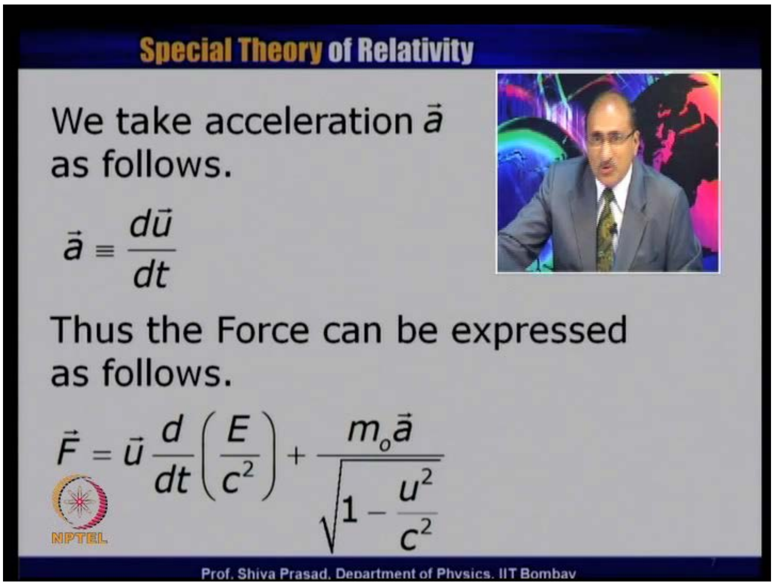


So, just expressing in a different form. Now du dt we know that can be written as the acceleration, if you are given in a frame, if you see a particle moving in the rate of change of velocity of that particular particle in this frame, obviously defined as the acceleration. So, a has to be expressed is du dt which means that the force now can be expressed as u multiplied by d dt of E by c square, which we have just now discussed, plus du dt have written as acceleration of the particle. So, m naught a divided by under root one minus u square by c square.

So, very strange we find that with this new definition of force, force is given in terms of summation of two quantities in which only one of the quantity here depends on acceleration. Here the first quantity depend actually on the time rate of the total energy of the particle, which is in total contrast with the classical mechanics. So, let us look at some of these things, let first discuss this at this particular equation and try to see what modification I have to make in terms of our understanding in relativity, about the force which is quite different from the classical mechanics, this is what I have summarize in the next three points.

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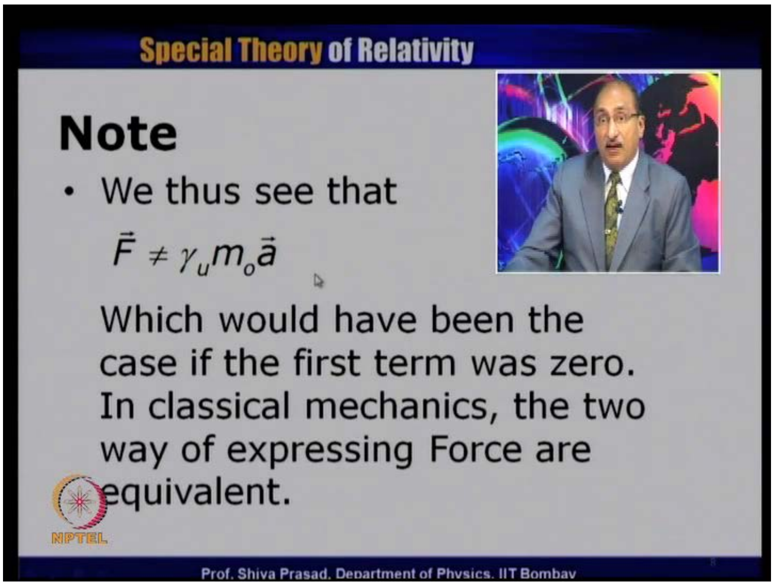


First in that we realize that f naught is not given, f is not given by gamma u m a. Had I taken other definition of force equal to mass into acceleration, then probably I would have had a tendency of writing force is equal to gamma u times m naught a, but as you have said that relativity we do not take that particular definition. But take the definition, of rate of change of momentum which has causes an additional term in the force. Therefore, it is very clear that in this case force cannot be written as gamma u m m naught a in general, unless off course of first term 0.

If you remember the two terms, here this is the gamma u m naught a. This term is an additional, if this term happens is to be 0; that is a very different case. Otherwise it is always sum of two terms, and therefore of is not return in terms of just gamma u m naught a, remember has ambition earlier just in the beginning of this lecture, in classical mechanics these two definitions are equivalent. I could have written force equal to mass into acceleration or I could have written force as rate of change of momentum. Here I take only force as rate of change of momentum and not in general F is equal to m a, which I might have a tendency see that, because I am depends on speed of the particle relativity.

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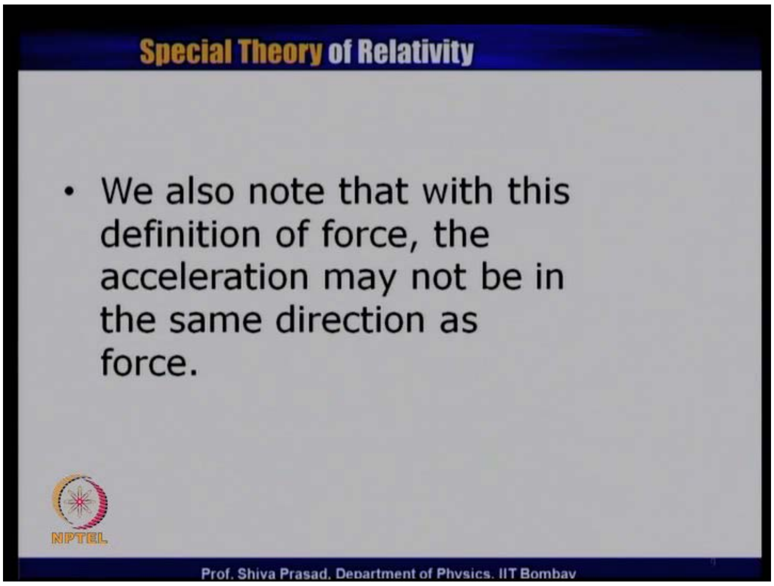


So, just replace m by know gamma u m naught is does an for there. The second thing which is also very interesting is that acceleration may not be in same direction as force is look, here this force is the vector quantity. Acceleration is the vector quantity. So, u is the velocity of the particle. Now we can always have a situation, in which the particle is in of moving in a particular direction and acceleration of the particle is in different direction or we apply a force which is in a different direction. Therefore, which is not in the same direction as the direction of the velocity of the particle.

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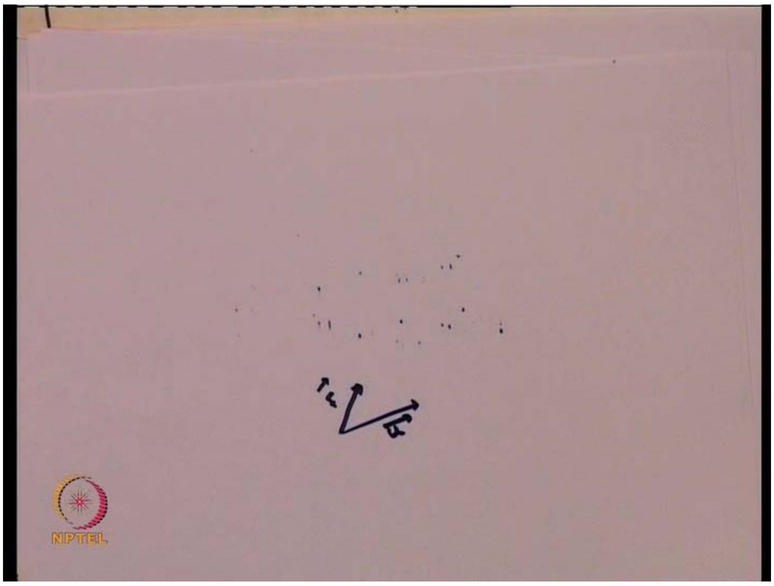

For example, a particle could be moving this way, going this way and I may to decide to applying force in this particular direction. Not classical mechanics could not made any difference. If the force was in this directions, the acceleration could also be in the same direction because alpha is equal to m a. But in this particular case, because the force also depends on the direction of u therefore, the force will no longer be exactly in the direction of (( )) acceleration in will not be exactly in the direction of F. It is also depend on u, which is an total contrast with the classical mechanics, where force was just in the same direction as acceleration.

Here force will produce an acceleration, which also depends on the direction of the speed of the particle in that frame, this is what I have written. We also note that with this definition of force the acceleration may not be in the same direction as the force. Force is the another third thing which I would like to mention about this particular equation, is that if force is constant it need not mean that acceleration is constant because it depends the first term depends on u, in fact the second term also there is the gamma u. So, in peaceful as speed of the particle changes, the same force may cause different acceleration while in the classical mechanics, because mass for velocity independent, so if your force remain same acceleration will always remain same. But here, acceleration will keep on changing because first we apply force the velocity will keep on changing and depending upon the velocity the acceleration velocity will keep on changing, this is totally contrast in the classical mechanics, of course we can understand if you have to say that, our speed of light is our ultimate limit and the particle will never able be able to cross the speed of light.

It means first we are applying the force and as the particle is trying to approach, the speed of light, will obviously expect that it cannot accelerating this velocity cannot change that much. So, the acceleration will not be that high must we are approaching to the velocity of light. So, therefore, in that sense from redistrict concept we can understand it, but from in classical mechanics it is total contrast.

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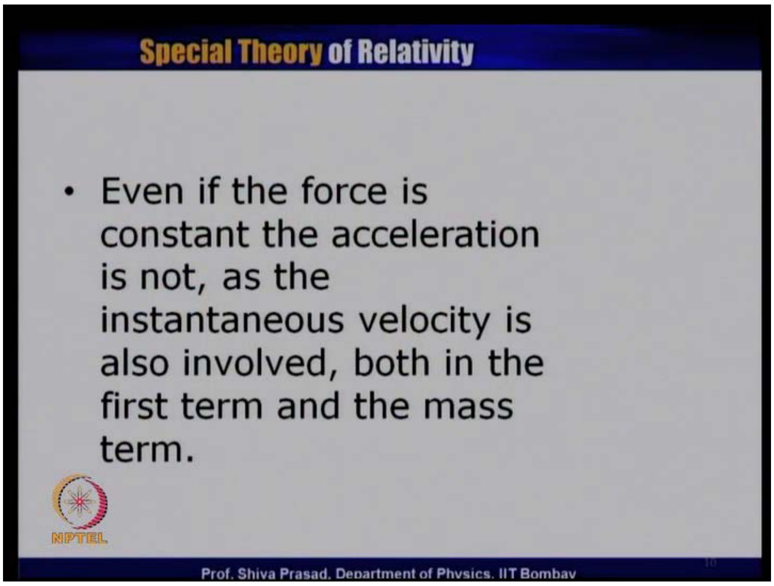


So, this what I have written even if the force is constant, the acceleration is not, as the instantaneous velocity is also involved both in the first term and the mass term. So, both the terms involve velocity. Therefore, you not have in this. If the force remains constant as the function of time, it does not mean that the acceleration also remain constant as a function of time.

Now let us look little more closely at the second term, the first term, the term which involve u or which involve the rate of change of energy whether we can give a little more physical meaning to the rate of change of energy on the term d dt.

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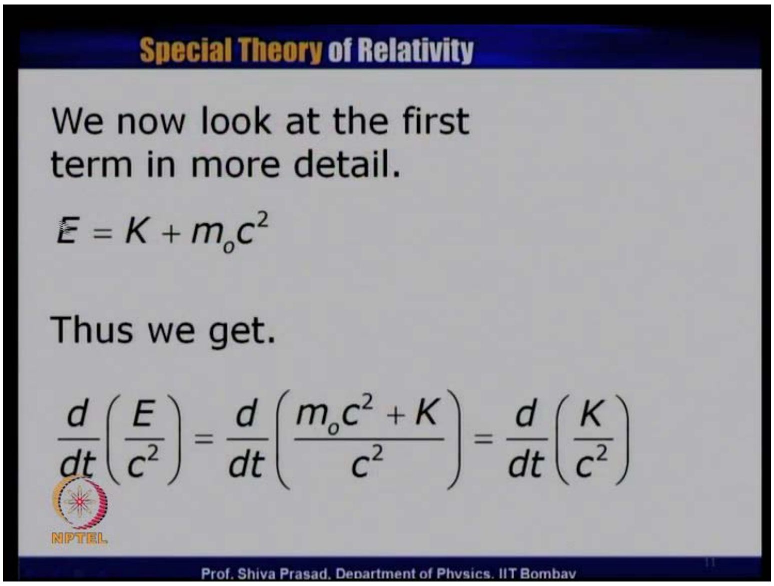


Now, (( )) we have said that this total relativistic energy E can be written as sum of kinetic energy plus the rest mass energy m naught c square. The rest mass energy we have discuss was a total relativistic concept. So, m naught c square is the rest mass energy of the particle plus the kinetic energy of the particle will give me the total relativistic energy of the particle.

So, if I just look at this time derivate of E upon c square, I can write this as time derivative of m naught c square plus K just substituting for E, K plus m naught c square. So, this what I have just substituted here, divided by c square. Of course the first term will give me d dt of m naught and m naught is the rest mass of the particle, which of course not expected to change as the function of time.

Therefore, this particular derivative will give me 0 therefore, this term can be simply written as d dt of K upon c square, which is what I have written here. So, d dt of E by c square can also be written as d dt of K upon c square because E and K differ only by the rest mass energy and rest mass energy cannot change as the function of time.

Now, let us look go back to your classical mechanics and try to see, how this particular K or rate of change of K is interpreted in classical mechanics. If one remembers the work energy theorem, one would realize that kinetic energy is results because of a work done by a given force.

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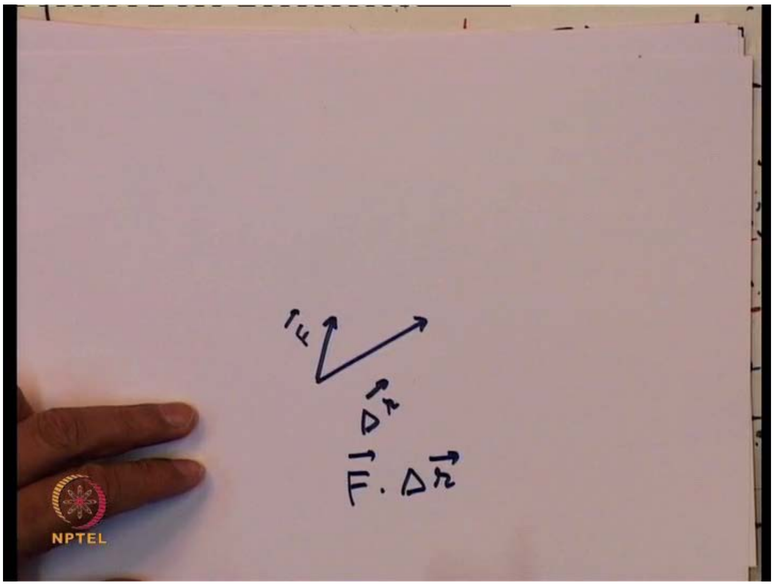


Let us look at the other transparency, as you have said (( )) suppose there is a particle which is moving in a given direction and we apply a force then if in a given time delta t, this particular particle moves by distance delta r and we take a dot product of F and delta r. This is what is called work done by the force and this by work energy theorem should be equal to the change in the kinetic energy.

So, if the particle was moving without any force, of course is constant, the kinetic energy has to remain constant. It will not change. Once we apply a force, once the force is applied, whatever is the displacement in a given time delta t. If this displacement is delta r the F dot delta r F dot delta r is called the work done by the force and this is equal to the change in the kinetic energy because now under the application of force, the particles kinetic energy would change and the change in the kinetic energy in the same time delta t will be given by F dot delta r this is well known classical result which is called work energy theorem.

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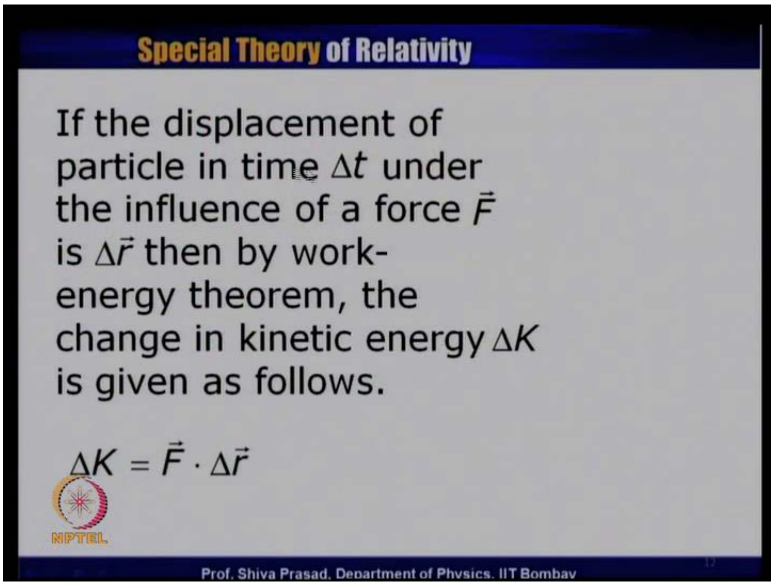


This what I have written, if the displacement of the particle in time delta t under the influence of a force F is delta r, often the particle could be moving even for we are applying force, it does not change thing. As we have said, in general assume, F need not be in the same direction, then by work energy theorem the change in kinetic energy delta K is given as delta k is equal to F delta r. So, I can divide this by delta t both the sides and assume that delta t tends to zero. So, these deltas can be change to derivative, differentiates. So, that is what I will be moving next.

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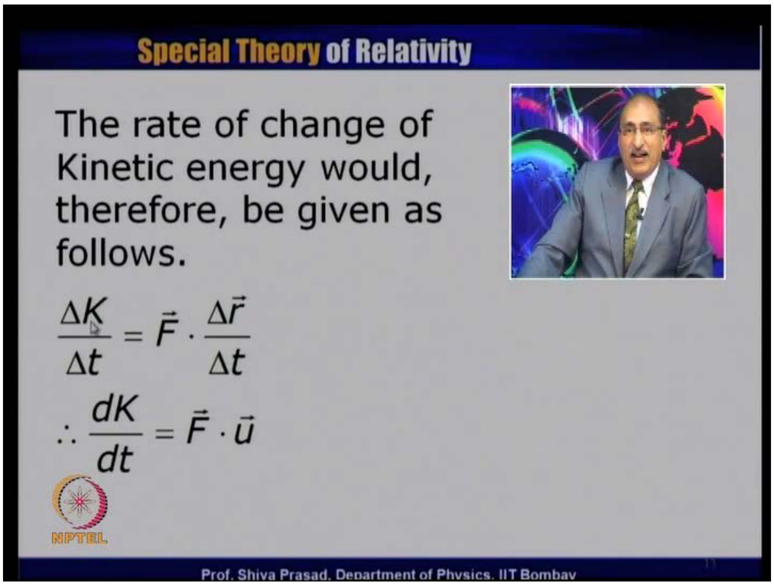

The rate of kinetic change of kinetic energy therefore, would be given by delta K divided by delta t, which I can write as, F dot delta r by delta t. Delta r by delta t is the total displacement of the particle time delta t divided by delta t therefore, this will give the velocity. So, this is the velocity of the particle.

So, delta r delta t can be written as the velocity of the particle and if I make delta t tends to 0, I can take this as instantaneous velocity of the particle. This is why I am writing, F dot u and this deltas, I am replacing by d because I am assuming that delta t is tending to 0. I can write this as dK dt is equal to F dot u.

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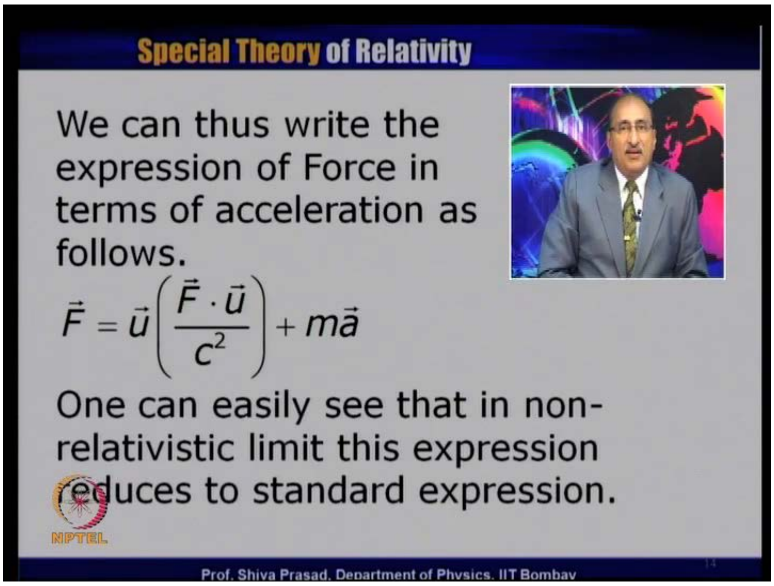


Now, therefore, the expression of force can be written more clearly, the way we wanted to write it, containing two term, the first term has been reinterpretated in terms of F dot u. So, it will be written as F is equal to u multiplied by F dot u divided by c square plus m into a, where of course m is gamma u m naught that is a member. So, this m is also speed dependent or the velocity dependent mass, it is gamma u m naught.

So, this is the way we write, we can thus write the expression of force in terms of acceleration, as F is equal to u F dot u divided by c square plus m a. Of course, one can see that in non relativistic limit, this expression reduces to standard expression. In non-relativistic limit, this m can be assumed as m naught because gamma u will be very close to 1, because u is very small in comparison to c, gamma u is very, very close to 1.

Therefore, m will be equal to m naught and if u is very small in comparison to c there is a c square in denominator, these values will be very small in comparison to c square.

Therefore, even the first term will be neglect and therefore, F can be written as F is equal to m naught a, the way we are used to writing classical mechanics. Therefore, we can see that it really reduces so its standard expression, in the non relativistic limit. Now let us look at some special cases, in which the force happens to be in the same direction as acceleration, which we have just now said. Because of this complicated equation that we have I would have called a complicated, in which there u was involved in there is a u involved here. There is u involved that means there is work done by the force involved, then only we get force equal to mass into acceleration.

So, you have two terms, now what are the cases in which force and acceleration could be in the same direction that can happen in two cases. One of the cases will be, where this term 0. This term happens to be 0 then F dot u for example, when F dot u 0 it means force is applied perpendicular to the direction of this speed, discontinues velocity then this term will be 0. Then F will be given equal to mass into acceleration. F and a will be in the same direction.

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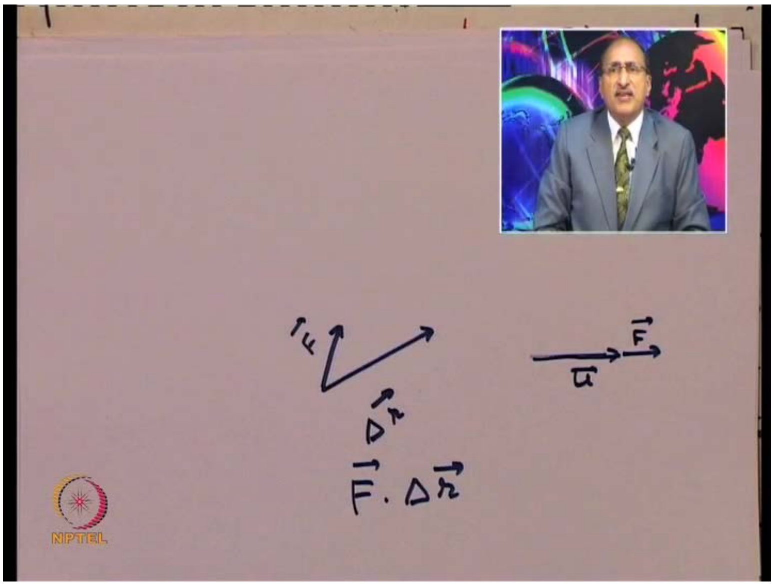


There is a second special case, where we have only one dimensional motion, it means the particle is moving in the particular direction you are applying assoils force in the same direction. So, particle is speed changes also in the same direction, all the motion is confined in the x direction.

So, instead of for example, in this particular case which have written here I take u direction to be this and force direction is also in the same way. So, the motion of the particle is in the same direction, is only a one dimension motion. The particle layout motion will keeps on moving in the straight line. If its keeps on moving in straight line then in that case, this vector u and this vector a is in the same direction. Therefore, force will also be in the same direction as acceleration.

So, the two special cases where I will see that force and the acceleration are having the same direction. Let us discuss these two cases these, two cases are interesting and are of course, comparitively (( )).

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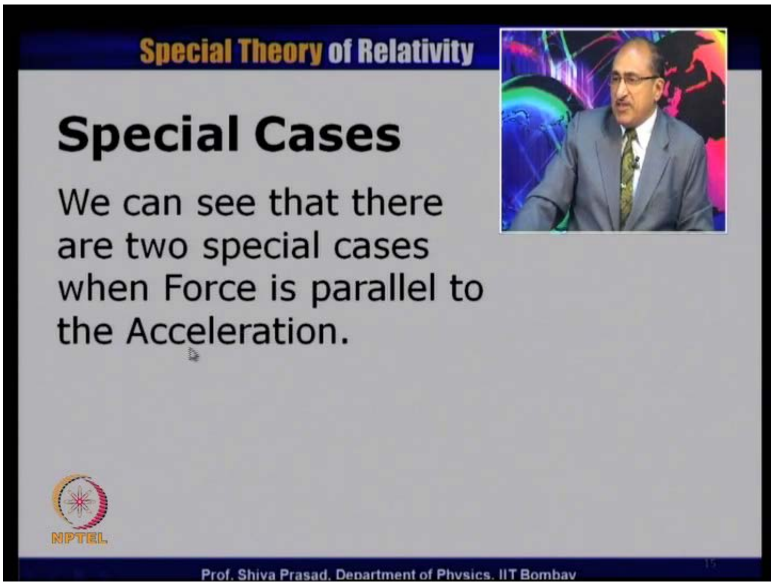


So, what I have said we can see that the two special cases when force is parallel to the acceleration. Let us take case one, if the force is always perpendicular to the instantaneous direction of velocity, of course once we apply the force the velocity will change, magnitude in general, both in magnitude and direction or at least one of them that has to change. But if it always happens that force is always perpendicular to the direction of velocity, that F dot u will always be equal to 0, that has we have said the first term will become 0 and then will have a simple case of F is equal to m a.

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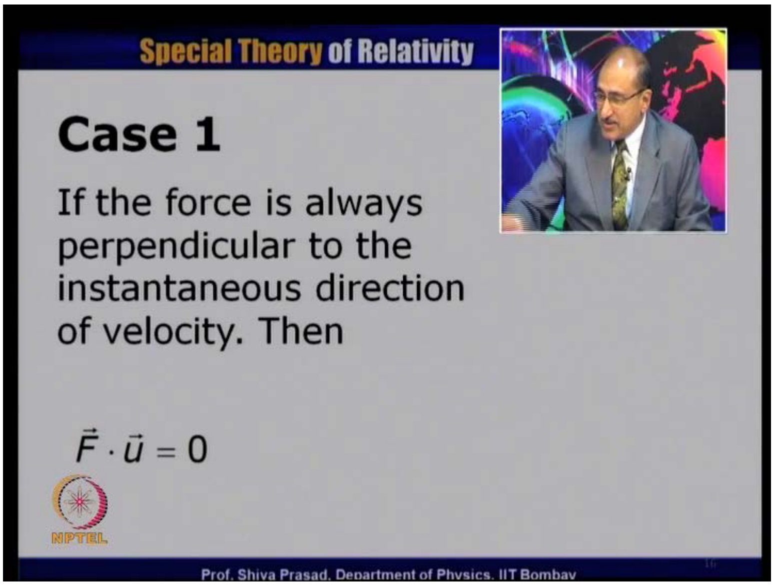


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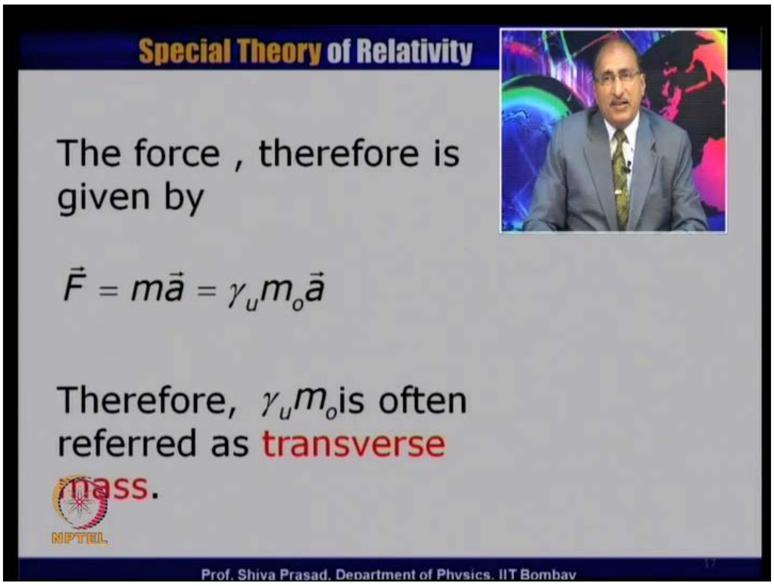


The force will be given by F is equal to m a, of course m is we have said, is a velocity dependent mass. So, it must be written as gamma u times m naught a. So, this is would be the case or which we thought that was could have been one of the definition of force. In principle with this definition of force, is applicable only when force is perpendicular to the direction of u, in that particular case F is actually in deep given by gamma u m naught a.

So, some times because this gamma u m naught is called transverse mass because this will be the mass of the particle. Now we can write the force equal to mass into acceleration with this mass gamma u m naught, only when the force is applied perpendicular to the direction of velocity of the particle. Therefore, many times its termed as, a transverse mass. Now, we must we wondering, whether we have a situation if you remember electronic magnetic theory, we know that this situation is called quite common. If there is a charge particle in the magnetic field then the force on that particular particle is always given by u cross B e u cross B.

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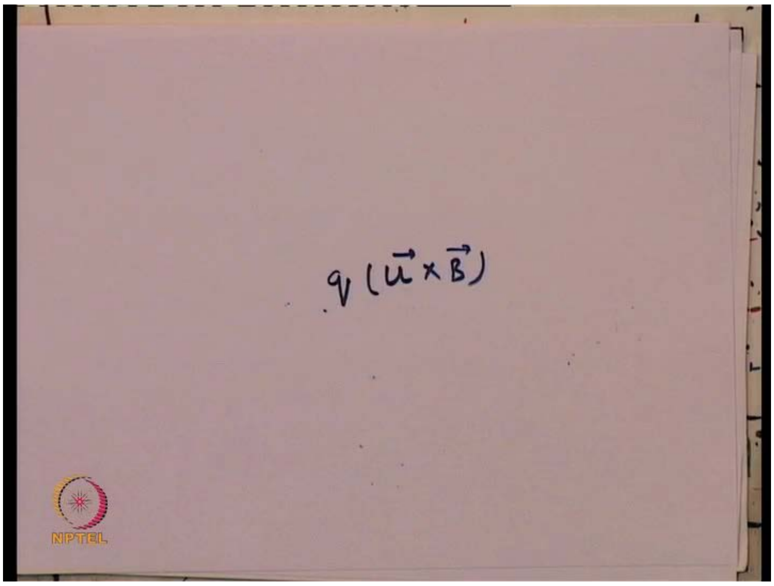


Let us say q u cross B is a force on the charge q, if it is moving with a velocity u and because this is the cross product u cross B, it means this force will always be perpendicular to u. So, this magnetic force will always be perpendicular to the direction of u.

So, let us just take a simple example of this particular case, when a charge particle is under the influence of the magnetic field, just give as an example of the case in which the force is actually in the same direction as acceleration.

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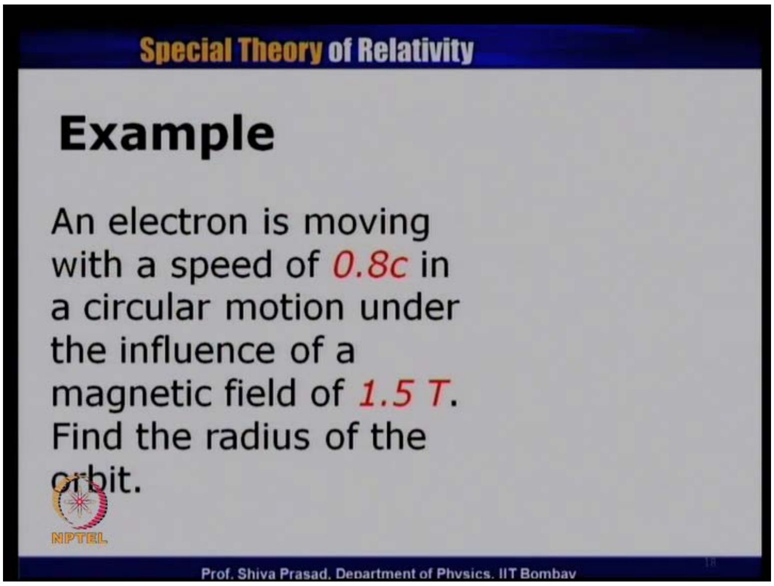


So, this is the example which have taken, an electron is moving with a speed of 0.8 c. In a circular motion under the influence of a magnetic field of 1.5 tesla, we have to find the radius of the orbit. This case as I said, simple because force and the acceleration under the same direction.

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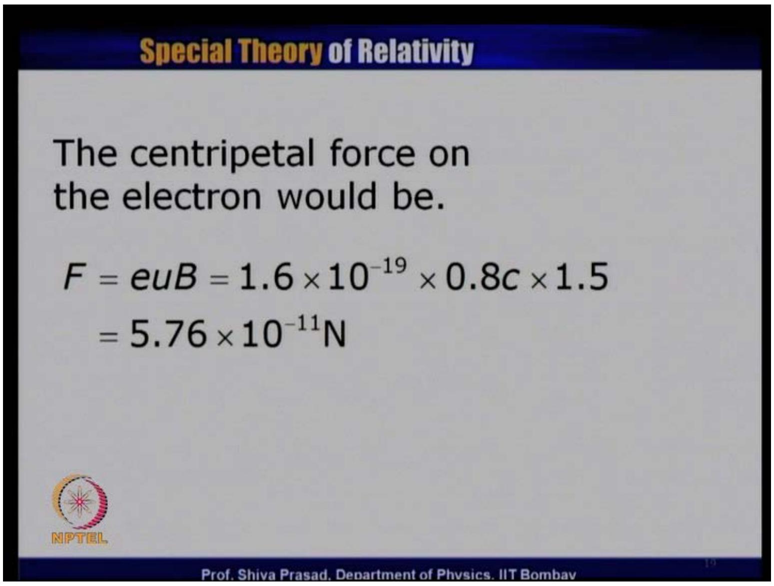


So, let us first find the force. Force will be given by q u cross B as we have just now said, because this is an electron. So, q is equal to the just charge of an electron which is e and because u and B are perpendicular each other and this is cross product. So, u cross B will just give you u multiplied by B. Because of u magnitude of u multiplied by magnitude of B, multiplied by sign of the angle between the two, sign of the angle being 90 degrees, this give you one.

Therefore, the magnitude of the force will be euB I am putting the value of e after taking the approximate value 1.6 into 10 power minus 19 coulomb. u has been given, it was as 0.8 c. Multiplied by the magnetic field which has been given as 1.5 tesla. This gives me that the net force on the charge will be 5.76 into 10 to the power minus 11 newton. Of course, this will always will directed towards the in a direction perpendicular to the direction of the velocity, which actually will provide the centripetal force for this particular electron to move in a circle.

I mean in a case, a particular particle is moving in the circle, there has to be a centripetal force and if I assume that the speed of the particle is constant then there has to be constant centripetal force acting towards the centre, always towards the centre of the circle. It is a well known classical results. On this particular case, I can find out the acceleration because my expression of force into (( )) force relation between force and acceleration is comparatively simple, and acceleration will be just given by force divided by the transverse mass, which is gamma u m naught. We know for the case and we have done many problems earlier, there for the case when u is equal to 0.8 c, then gamma u turns out to be 5 by 3.

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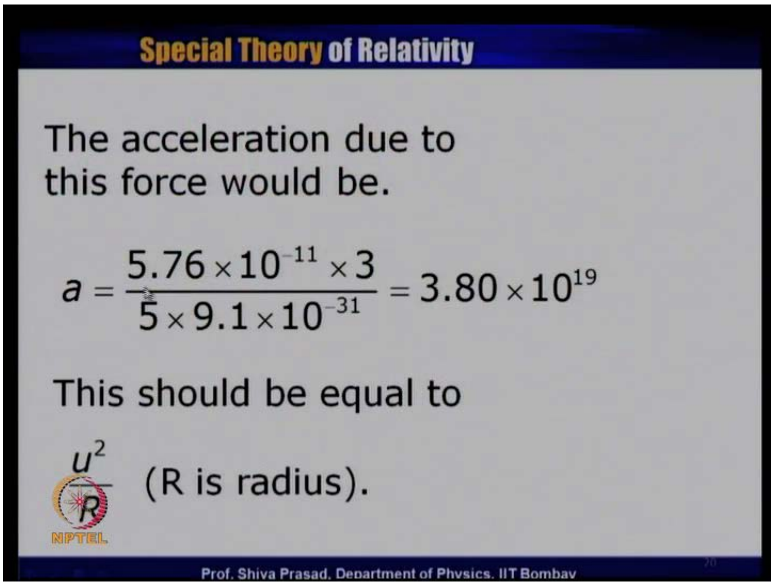

So, this 5 by 3 have used here. This is the value of the force which we have just now found out in previous transparency is 5.76 into 10 power minus 11 Newton. So, this is what I have written here, divided by m naught, rest mass of the particle. I am taking approximately the mass of the electron as 9.1 into 10 power minus 31. Of course as I say this is the gamma u, 5 by 3 which is here.

If I take this, this acceleration turns out to be 3.80 into 10 power 19 in cgi (( )) in si units. Now we know from classical mechanics that if a particle has to move in a circle, there has to be centripetal, there has to be real force because this acceleration amount of this particular particle is equal to u square by R assuming of course u to be constant, and the magnitude of u to be constant, direction is not constant is always changing. So, velocity is always changing, but this speed is constant.

So, this acceleration, in this case particular particle this particular electron has to move in a circle. The total acceleration of the particle must be equal to u square by R, where R is the radius of the circle. I have just now found out, what is the acceleration of the particle. So, I can substitute it to equal to u square by R. I know u, I can always find out what will be the radius if electron has to be move in a circle.

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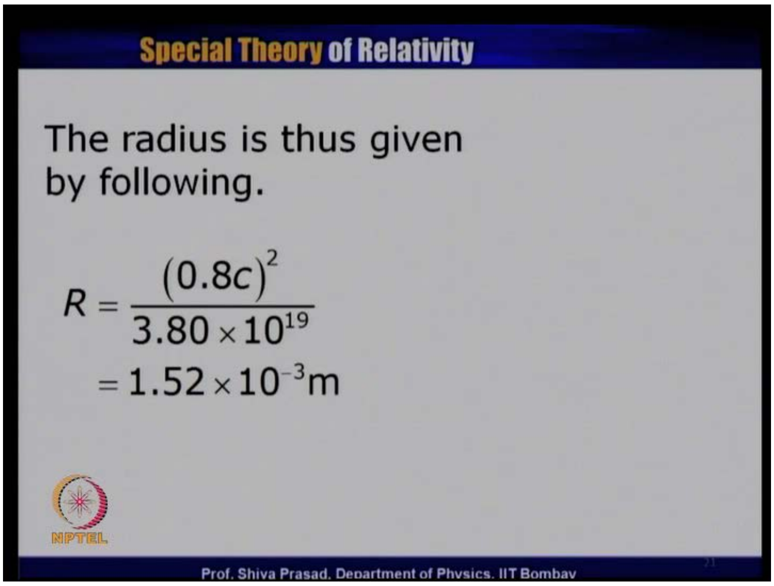


The radius turns out to be u square which is 0.8 c square divided by this 3.8 into 10 power 19, which we have obtain earlier as an acceleration. So, this must be the radius as we have seen this comparatively simple case, where we found out that acceleration is in the same direction as the force and only difference which we have to make to taken into account in relativity is, instead of using m naught have use gamma u (( )). Now let us consider the second case that is the case in which there is a straight line motion, this also very interesting case. So, let us discuss this particular case little bit more in detail.

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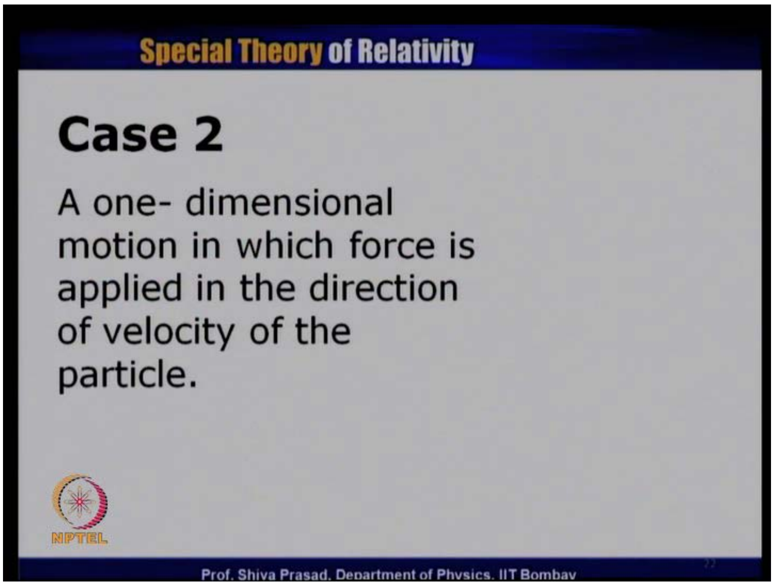


This is my case two, a one-dimensional motion in which force is applied in the direction of the velocity of the particle.

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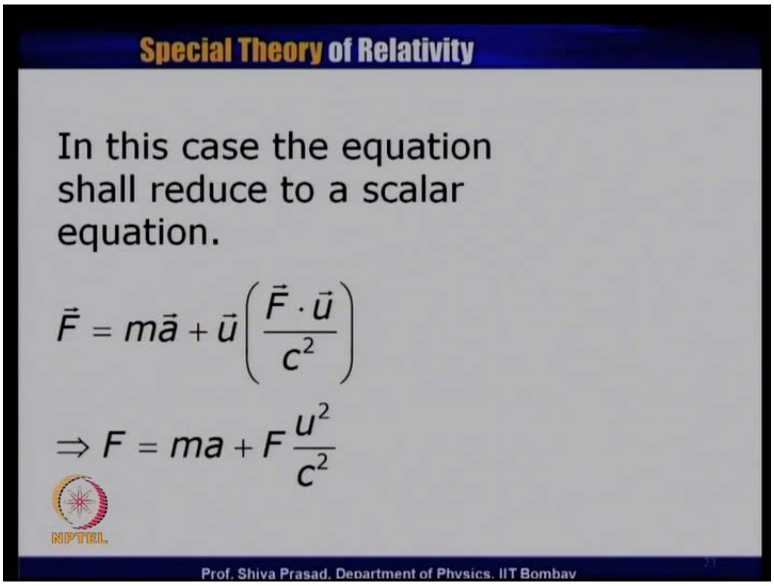

What happens in this particular case, this was my expression of course (( )) to rearrange the first term which was there, I have put it second term just to make it looks very simple.

So, I have put F is equal to m a plus u into F dot u divided by c square, this was my original relationship between the force and acceleration. (( )) everything is in the same direction. So, I need not write vectors F dot u, because F is the same direction as u and F dot u will be F, Multiplied by (( )) magnitude of F multiplied by magnitude of u multiplied by cosine of the angle between the two, and because angle between them, between the two 0 between the two vectors is 0. Therefore, that is equal to 1.

So, these becomes magnitude of F multiplied by magnitude of u. Everything is the same direction so this I can just write F multiplied by u square of course divided by c square and this a can also write without vector sign because I realize that everything is in same direction. There is only one particular motion, the only one particular time direction.

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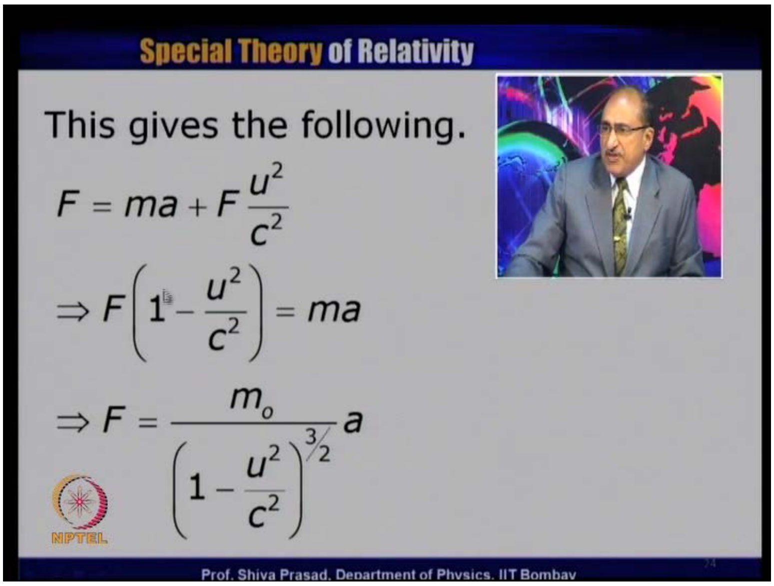


So, I can write this equation in a scalar form by writing F is equal to m a plus F multiplied by u square by c square. Now we can find out the relationship between the force and the acceleration because I can take this force term on the left hand side and find out in this particular case, what will be the relationship between the force and the acceleration. In case, there is a motion just in a single line in case of one dimensional motion, it is very simple. Let us just put in the second transparency, that is what I have put.

So, this is the expression which had written earlier F is equal to m a plus F divided by F multiplied by u square by c square. This term is taken on the left hand side, I take F common. I get F multiplied by 1 minus u square by c square, must be equal to m a. This particular quantity here I can divided by here remember this m is also gamma u m naught.

So, therefore, this I can write as F is equal to m naught and this is a gamma u which was one upon under root 1 minus u square by c square and there is another factor of 1 minus u square by c square which is coming from this here. So, when this is divided remember here this m naught here for m. So, m naught and the additional gamma u have incorporated here and this becomes now 1 minus u square by c square to the power 3 by 2.

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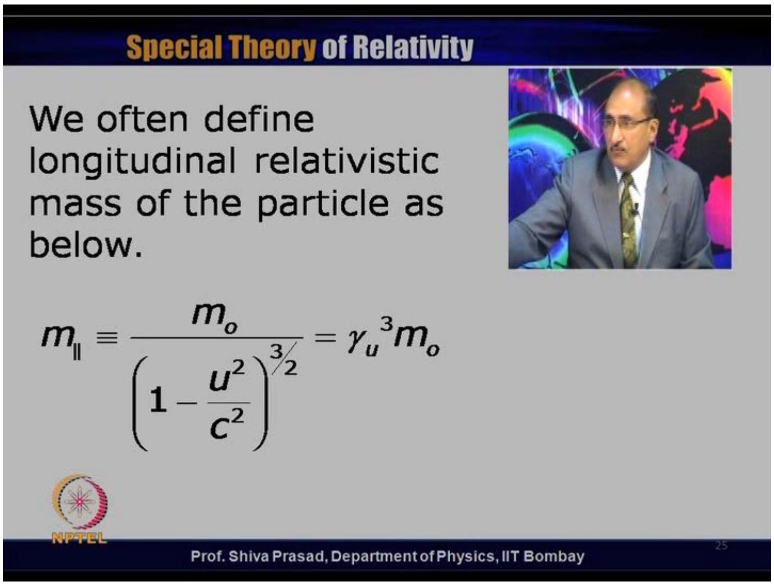


Very interesting result, because it says that in this particular case force is related to acceleration. Force is in the same direction as acceleration, but the mass of the particle, the way mass will behave will be m naught multiplied divided by 1 minus u square by c square to the power 3 by 2. Its gamma q gamma u q while in the case of transverse motion, I am in (( )) the force was perpendicular to u. It was just gamma u, but in the case of one dimensional motion its gamma u q. So, this is generally called as a longitudinal mass.

So, we often define longitudinal relativistic mass of the particle as m naught divided by 1 minus u square by c square to the power 3 by 2, which is equal to gamma u cube m naught. So, with the motion the particle is an one direction the mass turn out to be gamma u cube times m naught. As far as relationship between the force and acceleration is concerned it is been very, very clear, when the relationship between the force and acceleration is concerned this is way you should write immediately.

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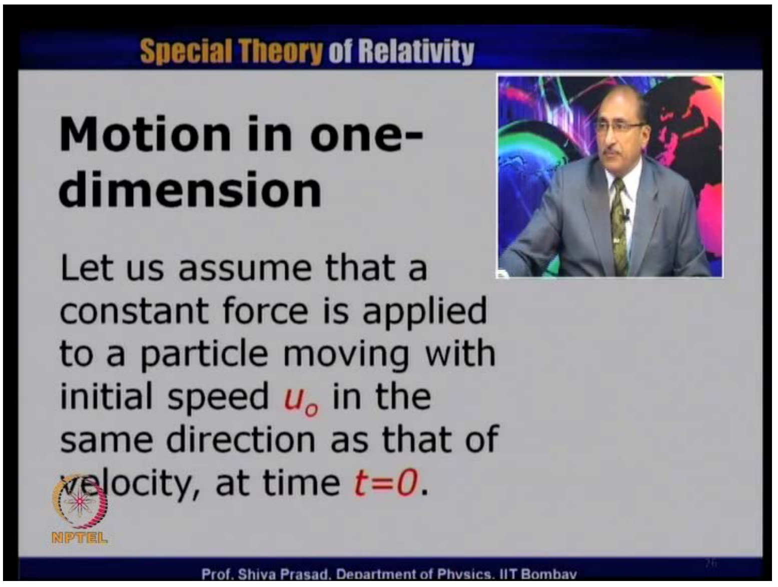


Now, let us look little bit more carefully we have lot of what we call as kinematic equations in the classical mechanics, we are if you know the initial velocity you can find out the velocity final velocity if you know the acceleration at a given time, you can find out the distance etcetera, etcetera. So, let us look at some of these equations and try to get little bit more insight of, what is happening when a force is applied in a one dimensional case on a particular particle.

So, let us assume that a constant force is applied to a particle which is moving initially with a speed of u naught, in the same direction as that of velocity. With this exactly the case which we have just now said, that the particle is moving in this particular direction with the velocity u naught and a time t equal to 0, I apply a force which is also in the same direction F.

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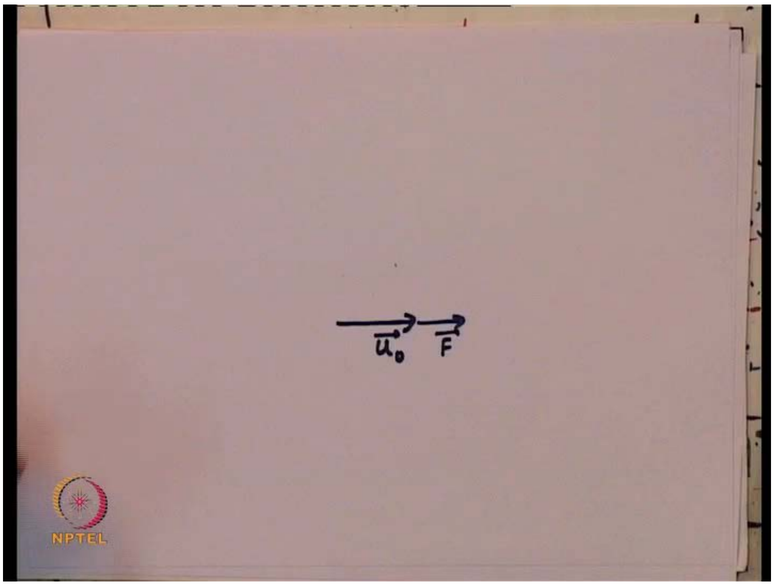

So, by motion what were all the change in the motion (( )) the same direction. So, everything within the same direction, so this is what is the case? My question is that (( )) I applied the force for a given time what happens to its velocity.

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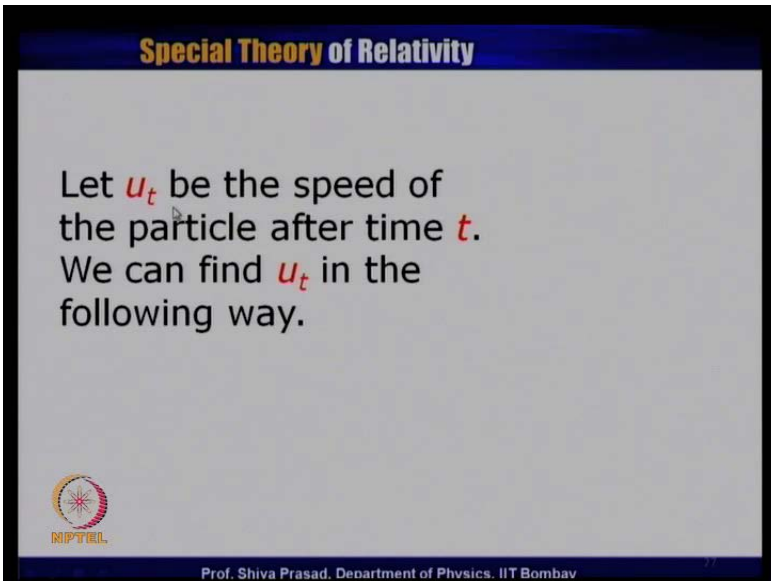


So, let us assume that after time t, we applied we leads to let us assume the force is constant. A constant force is being applied. I am interested in finding out what happens after time t. So, let us assume of course the velocity of the particle will change, direction will not change because force and the velocity are in this same direction, but its magnitude will change. So, let us try to find out or let you let after time t the speed of the particle or velocity, whatever you have to call it, because it any one dimensional with the speed u t be the speed after time t. How do I find u t? I have to integrate this equation.

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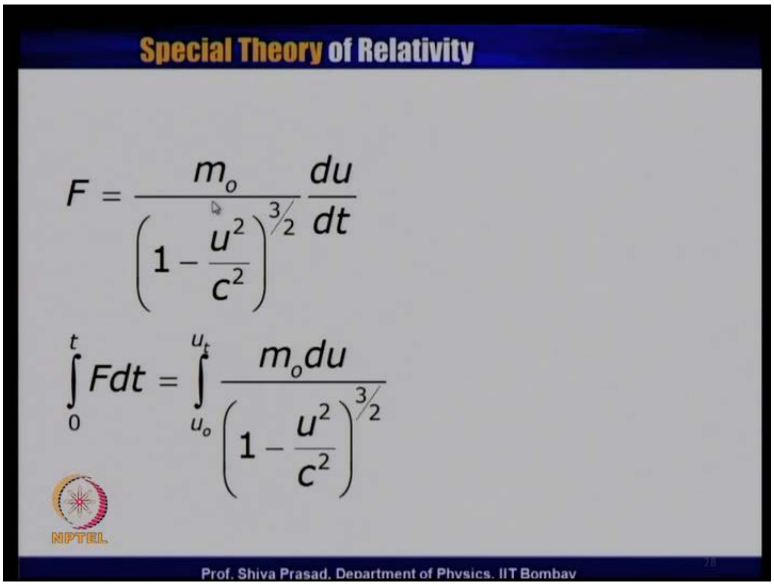


This was my force equation, acceleration have written as du dt rate of change of speed or rate of change of velocity as acceleration. This is just have written as gamma u cube m naught. This dt I will take on this side, du I let it remain in this side and integrate.

This is the way we derive those kinematic equations, standard kinematic equation in classical mechanics. We just assume force should be constant, live force equal to mass into acceleration, then integrate and then obtain various (( )) equation what happens that the speed of the particle at a given time t, what happens to this displacement of the particle at a given time t, we are doing exactly in the same thing except that our expression have not become little more difficult.

So, this is what I have to integrate in order to find out, what will be u t at a given time t. Therefore, I am just integrating it integral from 0 to t and this is this is the initial velocity u naught, this is the final velocity after time t, u t m naught du 1 upon u square 1 minus u square by c square to power 3 by 2, just integrating this expression is just written here. Now in order to integrate let me not use this definite integral, let me use still indefinite integral. It is comparatively easy. Then I will put the limits later. So, this is what I am doing next, I am taking the indefinite integral of this equation of the right hand side. Let me solve the integral in the indefinite form then I will put the limits later.

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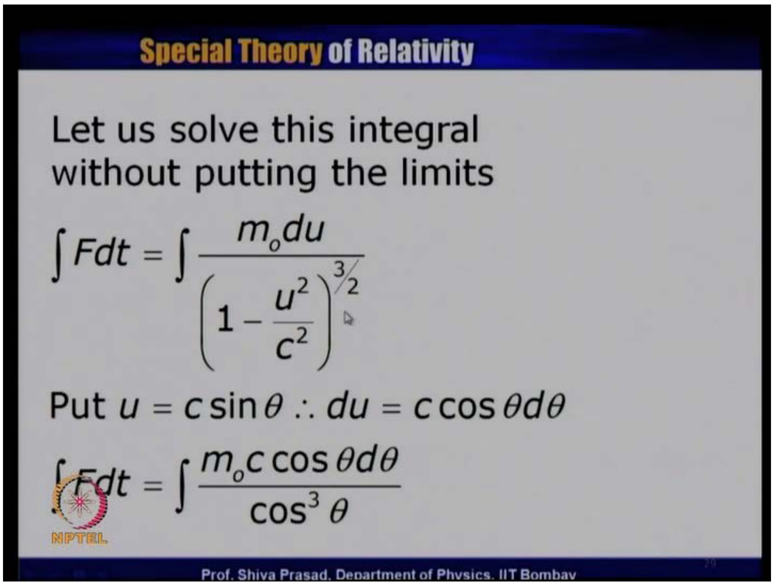


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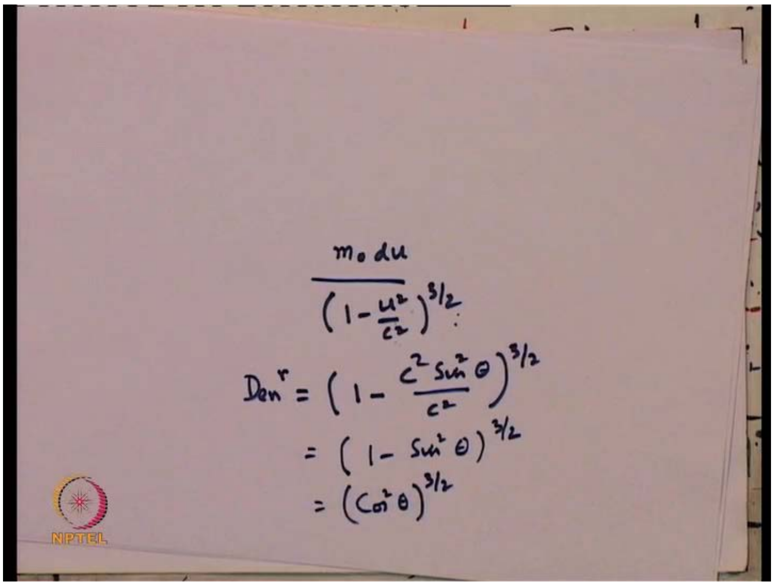


So, let us solve the integral without putting the limits. So, I will just put integral F dt of integral m naught du 1 minus u square by c square to power 3 by 2. There is the standard substitution to solve this equation, we put u is equal to c sine theta then we take the differentiation we get du is equal to c cos theta d theta. This du is now being replaced by c cos theta d theta, it is very simple integral. For this u square, I am putting c sine theta. Let me just write it here.

In the denominator we had one, we had m naught du and here we had 1 minus u square by c square to the power 3 by 2. If I put u is equal to c sine theta so this becomes, I am just looking at the denominator. 1 minus c square sine square theta by c square to the power 3 by 2, this c square will cancel. So, I will get 1 minus sin square theta to the power 3 by 2. 1 minus sin square theta is cos square theta.

So, this becomes cos square theta to the power 3 by 2. This under root and this half goes away, this becomes cos cube theta. So, this is what I have written here. Integral of F dt is equal to m naught for du as I have said, we have written c cos theta d theta divided by cos cube theta, this c remains here, this cos cube theta will cancel with one of these powers. So, this will come cos square theta and eventually you will get 1 upon cos square theta which is sec square theta d theta.

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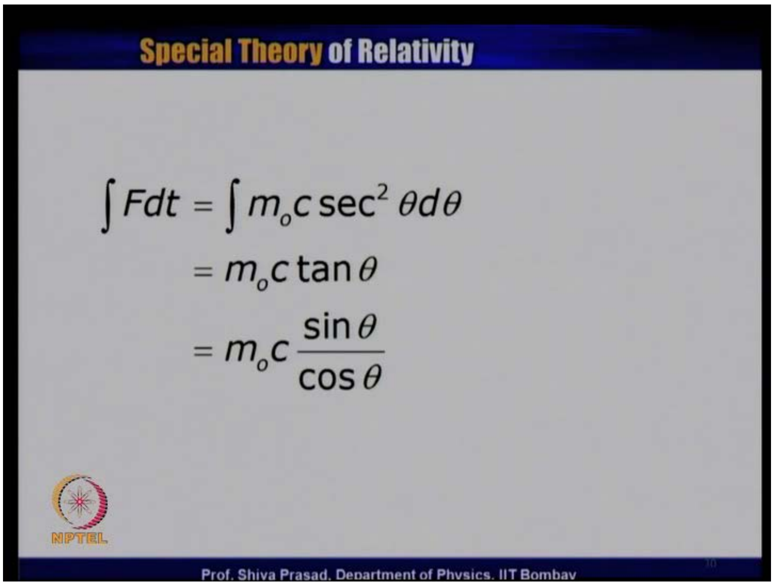


So, this equation becomes integral of F dt is equal to integral of m naught c sec square theta d theta. We know that, sec square theta when integrated it becomes tan theta. So, this equation just becomes m naught c tan theta. This tan theta, I know sin theta divided by cos theta, I would like to express this thing back in terms of u. So, what I will do I have written anyway c sin theta as u. So, this I can put as c sin theta as u, this cos theta I will express as under root 1 minus sine square theta and for sin theta again I will put u by c.

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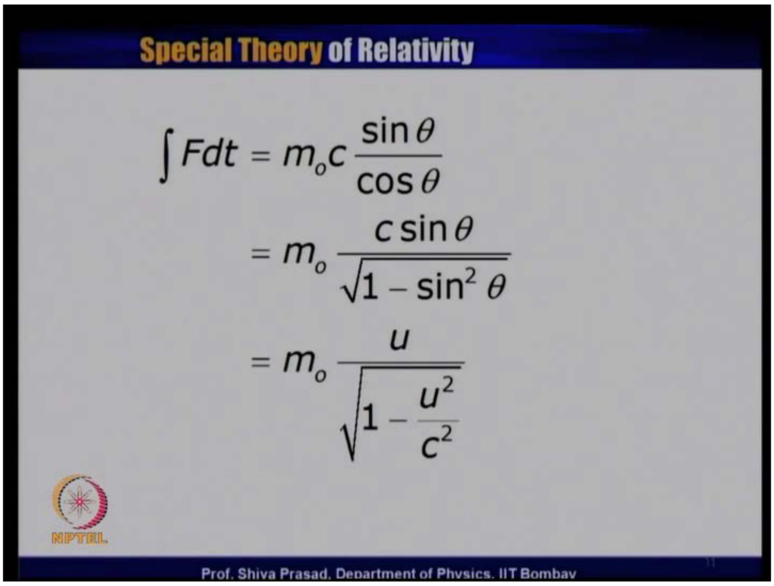


This is what I have done in next transparency. m naught c sin theta by cos theta, sin theta as we have said is this c sin theta is u, which have written as u, this cos theta have written as under root 1 minus sin square theta, because c sin theta is equal to u. Therefore, sin theta will be u by c. So, for sin square theta itl becomes u square by c square.

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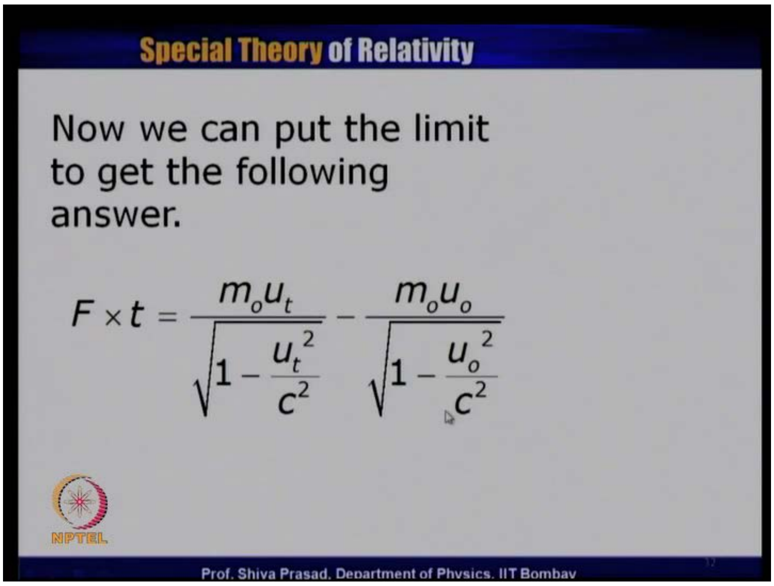

So, this integral F dot dt gives me m naught u divided by under root 1 minus u square by c square. So, this is what I get as the result of integration. Now we can put the limit, say what limits were from u naught to u t. So, I take this same expression, I first put u t here I put u t here minus there I put the limit u 0, which is u 0 here, u 0 here. This is what I am putting here.

So, F multiplied by t is m naught u t divided by under root 1 minus u t square by c square minus m naught u naught divided by 1 minus u naught square by (( )) c square. So, this is what I will find that, for in this equation is not in a simple form to express u t that of course that we can move later. But this definitely gives, that if time if force is applied for time t this gives the relationship between u t and u naught.

Before, I try to derive this particular relationship which probably will do in the later lecture. But let us try to look at this particular equation and try to see the implication of this particular equation. So, for that we have taken an example, consider a one dimensional motion which we have been considering in this particular case.

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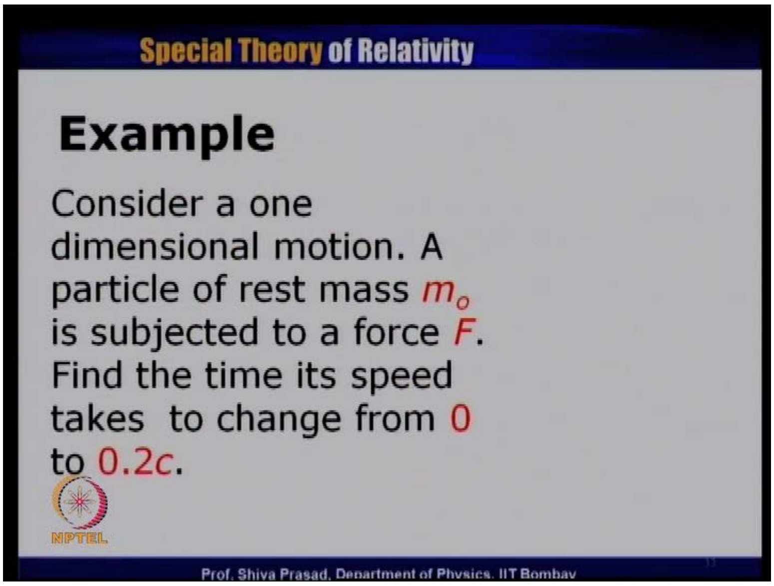


Let us assume that there is a particle of rest mass m naught, which is subjected to a constant force F. First find the time its speed takes to change from 0 to 0.2 c. I have put this particular problem in a fashion, so that I can use this equation directly without actually working out for u t. So, initial velocity has been given, which is 0. Particle is starts from rest and final velocity has been given as 0.2 c and there is the particular force F, whatever might be the magnitude of the force and its rest mass is m naught.

I am interested in finding out how much time it will take, which will depend on the force and m naught. But I am giving the I am interested in finding out the answer, intervals of F and m naught because I want to really look how the time value will vary, when the speed really approaching the speed of light.

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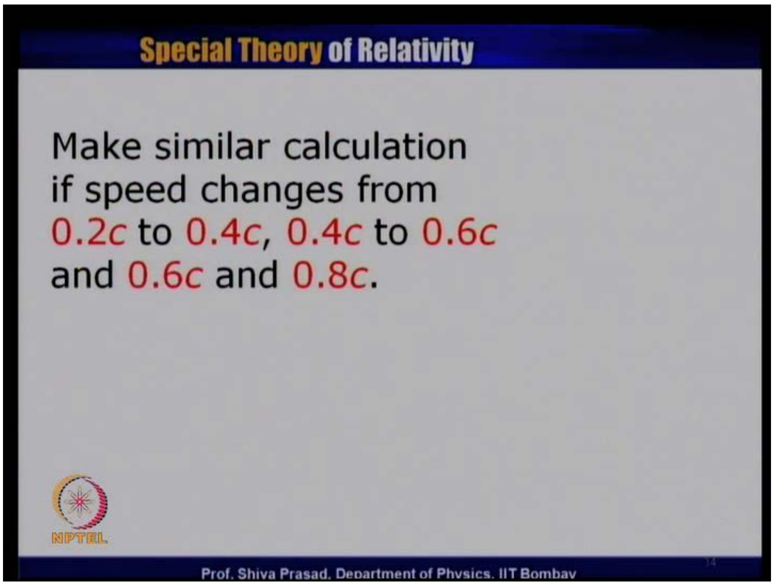


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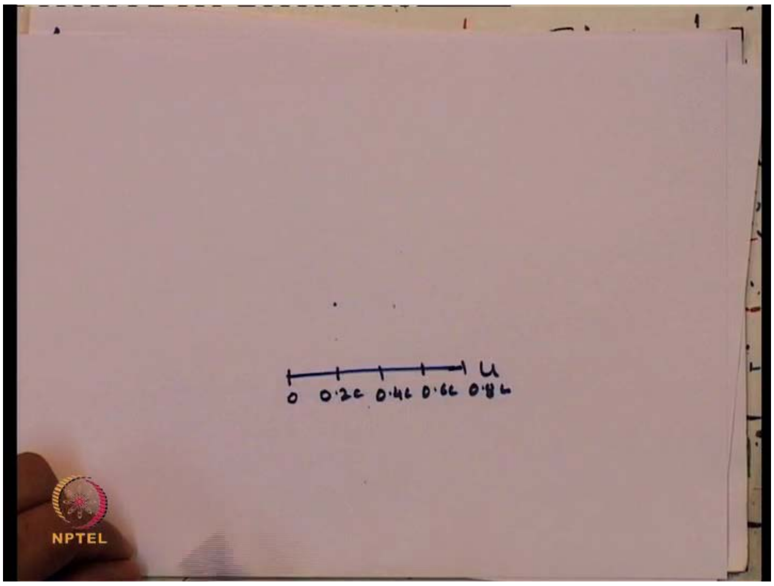

Now, we also like to make a similar calculation if the speed changes from 0.2 c to 0.4 c and then 0.4 c to 0.6 c and then 0.6 c to 0.8 c means, interval speed interval is same, it is 0.2 c. So, you will go for 0 to 0.2 c. So, if we take plot speed, go from 0 to 0.2 c then 0.2 c to 0.4 c, 0.4 c to 0.6 c and then 0.6 c to 0.8 c. So, velocity intervals are same. Force is same. I am interested in finding out the time, that particle will take to go from here to here, go from here to here, go from here to here, go from here to here, next what is the (( )) question.

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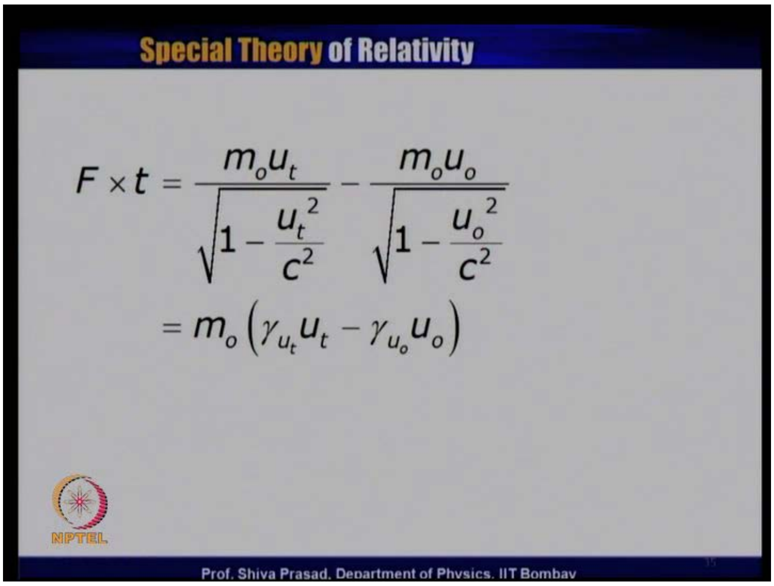


This is my question, force multiplied by time is equal to m naught u t divided by 1 under root 1 minus u t square by c square minus m naught u naught divided by under root 1 minus u naught square by c square. This particular quantity m naught divided by 1 minus (( )) one divided by under root 1 minus u t square by c square. I am writing as gamma u t (( )) one at time t, one at time t is equal to 0.

So, gamma corresponding to this speed is gamma u t. gamma corresponding to the initial speed is gamma u not. So, the same expression I have written as m naught multiplied by gamma u t, u t minus gamma u naught. m naught has been taken common, u t is here. This particular under root one upon this particular thing has been gamma u t, this m naught has been taken out here, this u naught is present here. This 1 divided by 1 minus u naught square by c square has been taken as gamma u naught. So, this is my expression.

So, for the first case I am starting with 0 initial velocity. So, this term is 0. All I have to do, should take gamma u t multiplied by u t. u t is 0.2 c for the first case, what I have to find out? what is the gamma corresponding to this speed of 0.2 c? it means I will take this as 1 upon under root 1 minus 0.2 square. I will substitute in this particular expression and obtain what will be the time taken of course, this time will be this own quantity divided by force. But as I said I want the answer in terms of both m naught and the force.

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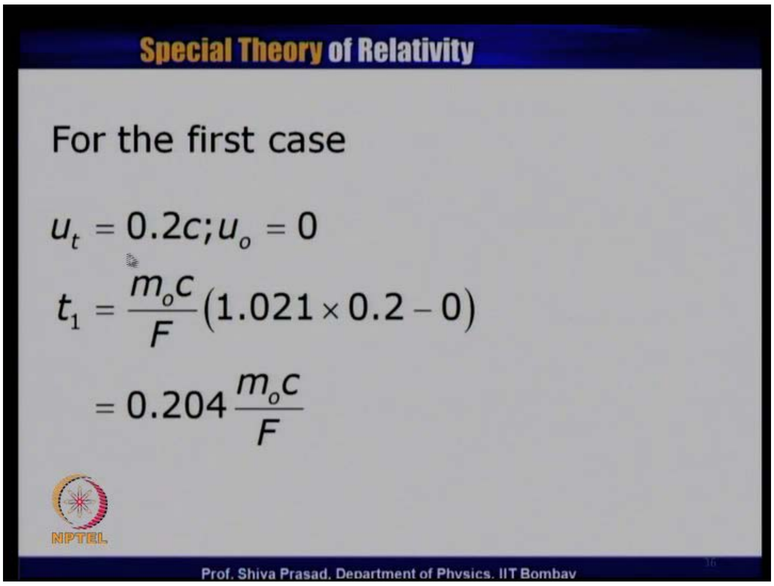


So, as we said for the first case u t is equal to 0.2 c 0 u naught is equal to 0. So, time which have set t 1 is equal to m naught c divided by F, multiplied by 1.1 which is gamma gamma u t which is corresponding to 0.2, happens to 1.021. It makes only 2 percentage change and (( )) even we are at the speed of 0.2 c multiplied by 0.2 of c have been taken out here and the second term is 0. So, this gives me 0.204 m naught c bar F.

So, the time taken for the particle to reach its speed of 0 2 c will be given by 0.204 m naught c upon a (( )). This factor will be common in all other my subsequence results. I am only will be comparing this particular factor.

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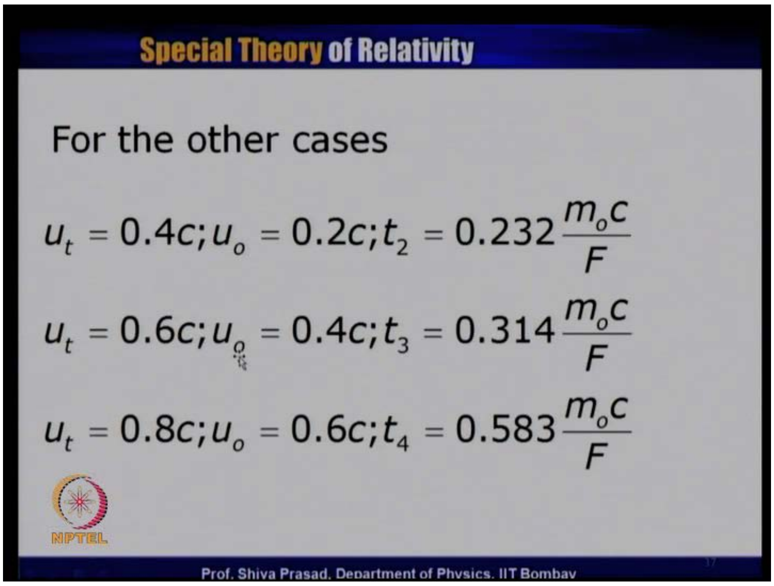


Now, for the other case let us take u t is equal to 0.4 c and u naught is equal to 0.2 c. I put exactly in the same expression, force now I have to take this factor as well as this factor into consideration, we just substitute those numbers, it is very, very simple.

State forward calculation, if I take that number we get 0.232 m naught c by F. If my initial speed was 0.4 c and the final speed for 0.6 c, then my this I have written as t 2, this is t 3 for third case, is 0.314 m naught c divided by F. If my initial speed was 0.6 c final speed is 0.8 c, after time t 4 which is 0.583 m naught c by F.

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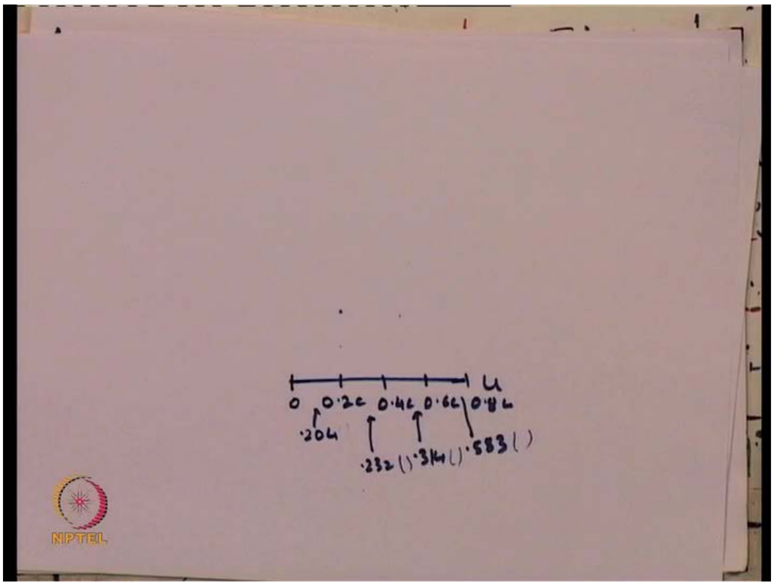

So, as we can see that with whatever the factor, if we go between 0 to 0.2, the value which I get is 0.204. In this interval I get 0.232, in this interval I get 0.314, in this interval I get 0.583. So, there is the speed interval is same, this force is same. But this first half of the particle who travel the same speed interval 0 to reach between the 0 to from 0 to 0.2 c in 0.204 multiplied by whatever is this factor. Between 0.2 c and 0.4 c it takes larger time between 0.4 c and 0.6 c still takes larger time and between the 0.6 c and 0.8 c it still takes a larger time.

So, yes the particle is approaching this speed of light. We are seeing that cover the same speed that a to reach, to make the same changes in the speed to make the same changes in the speed, you require larger and larger time. That is expected because now you are approaching the speed of light and as we know that we cannot no really reach the speed of light (( )).

So, as we are approaching here and here with the same force you will find that particle is taking larger and larger time to make same change in the velocities, this is what the interesting result that we see from this.

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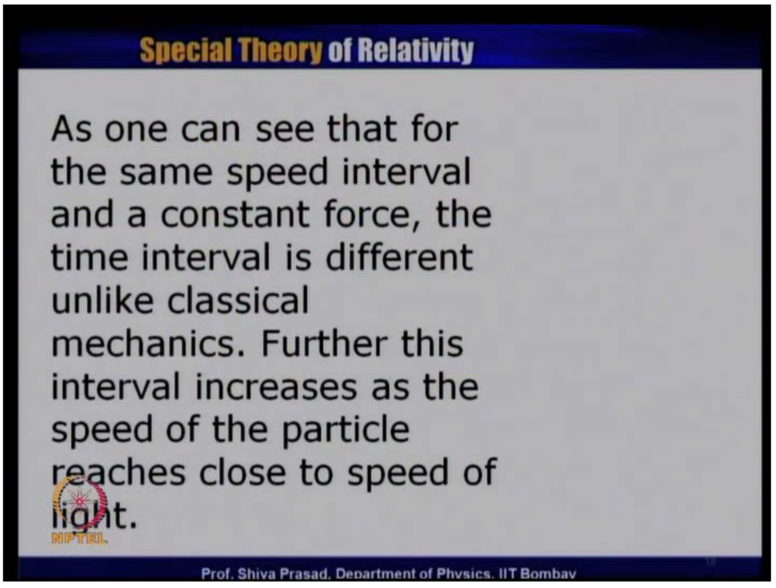


So, this what we have said, as one can see that for the same speed interval and a constant force, the time interval is different unlike classical mechanics. In classical mechanics, if you remember, this was standard expression in kinematic equation.

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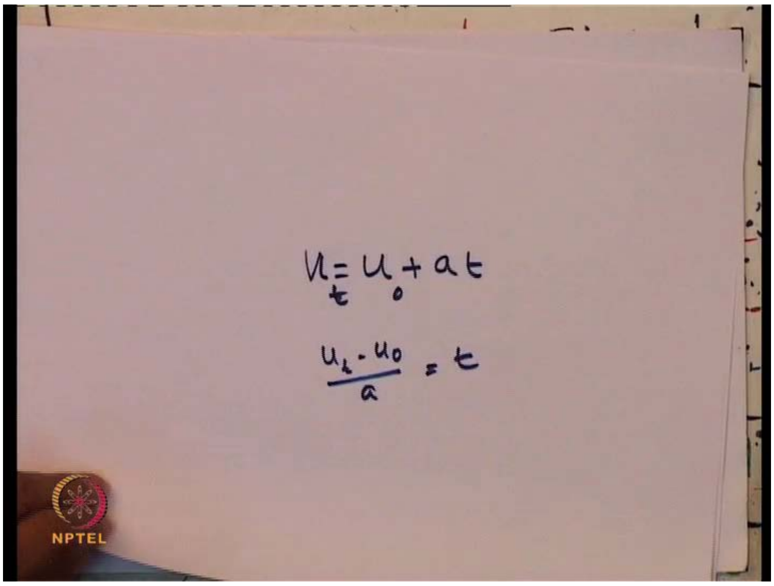


We write v is equal to u plus a t, where v is should be the final speed. This I would call as, u t minus u 0 plus a t using our normally literature. So, u t minus u naught divided by a will be time and this a will always be constant in the classical mechanics, if the force is constant because mass remains constant in that case. Therefore, the same interval will be covered, same velocity interval will be covered in the same time. While here is does not happen. That is what I have said, that the time interval is different unlike classical mechanics. Further this interval increases as the speed of the particle reaches close to speed of light.

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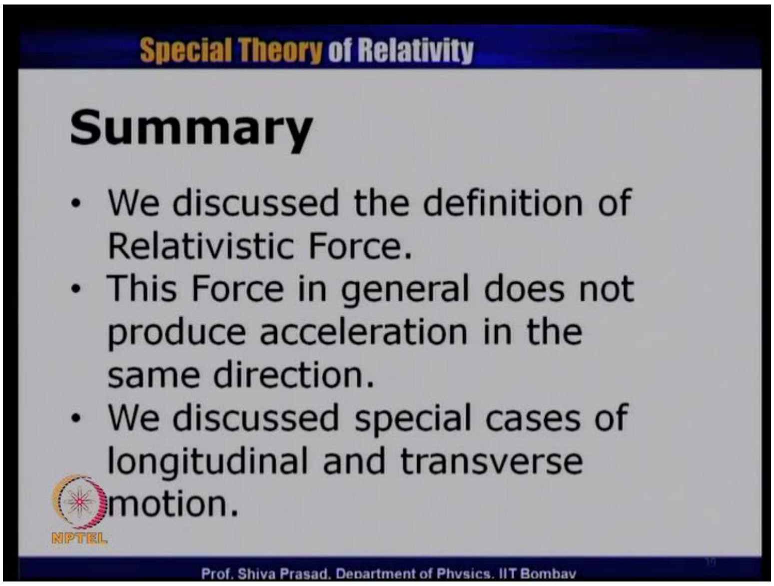

Now, I like to summarize what ever we have discussed. We discussed the definition of relativistic force. We have said that this force in general does not produce acceleration in the same direction. We discussed special cases of longitudinal and transverse motion, I am calling longitudinal transverse motion not sure with this as good way of saying, what I mean I think should be clear when the force is applied in the direction perpendicular to you and when there is a one dimensional motion we consider these two special cases in which the force happen to be in the same direction as acceleration.

Thank you.