Special Theory of Relativity
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Lecture - 19
Example in C – Frame

In our last lecture, we had discussed the Doppler effect in light. As we had said that Doppler effect is sound is well-known, we visualize many times in our daily life. But in light, it is quite is like different treatment and probably not so much apparently known. We also discussed a special Doppler effect, which you call as a transfer Doppler effect, which is normally it is not have any classical law will not be (( )) sound for that matter. These two aspects we discussed then we gave some flavor of some experiments in which Doppler effect is used.

Just to give an idea that you know the Doppler effect that we have dop has certain usefulness as far as certain part of physics is concerned effect. And even in medical science Doppler effect is used quite effectively. So, I think Doppler effect is finding more and more uses, but what we did in last lecture just to give some idea about the Doppler effect. Today we would like to go to a slightly different concept basically it is for problem solving.

We define what we call as a center of frame of reference. Normally in classical mechanics when we are not dealing with relativity, center of mass is always described in a fashion, such that we have a standard definition of a center of mass how to find it out. And if we change our frame of reference to that center of mass of a system of particles, then we call that particular frame of reference as center of mass frame of reference. The advantage of dealing with center of mass frame of reference is that, if there is no external force with the system center of mass becomes an automatic inertial frame of reference.

Because if there is no external force to a system of particles, the center of mass must be at rest or must moving with constant velocity as seen from any other special frame of reference. So, if you move our self to the center of mass automatically we are landing up ourselves into an inertial frame of reference. Now, we slightly modify this particular definition as we have discussed earlier.

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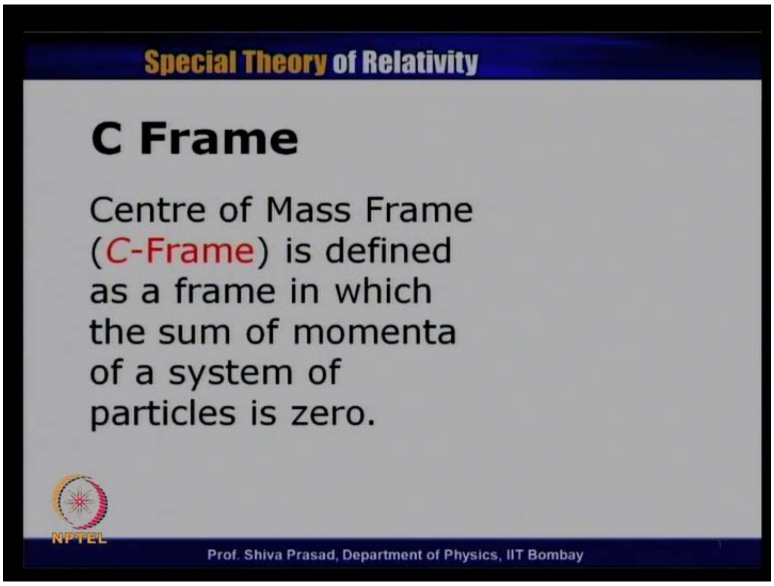

And we say that center of mass frame of reference is that frame of reference in which the sum of the momentum of all the particles is 0, this center of mass frame of reference is occasionally referred as C frame. So, what we have said that center of mass frame is a frame in which the sum of momentum of a system of particle is zero. Normally as we have already said that there is a set of particles which considered is a system in which we are interested in. So, there could be a force inter in between the particles, it means a force being applied by one particle on any other particle that we call as internal force, and if f force is applied by a particle, which is not part of the system that is what we call as external force.

So, I repeat that if we find that some of the moment, if we frame of reference in which the sum of momentum of all the particles of the system is 0 that is what we call as C frame or center of mass frame. Similarly, we can define or we generally call a L frame as a laboratory frame of reference. This frame of reference is a frame in which you have describe your problem or you describe your experiment or in which you were performed the experiment.

This frame of reference could be a C frame or center of mass frame of reference or could be any other inertial frame of reference. So, I may describe a problem my problem in a frame of reference which does not happen to be center of mass frame of reference or I could choose to describe the problem only in a center of mass frame of reference. So, laboratory frame of reference is generally a frame of reference in which we have described my problem or my experiment.

Now, why we talk specifically of C frame specially relativity will see that many problems become very simple if we work it out in the C frame. Though the problem may not have given to you C frame it might have been given in some other frame, but still if you transform the problem into C frame the problem that is how to be simpler. So, in today's lecture we will be essentially discussing some problems in which we will find the use of C frame to be convenient frame and problem easy frame.

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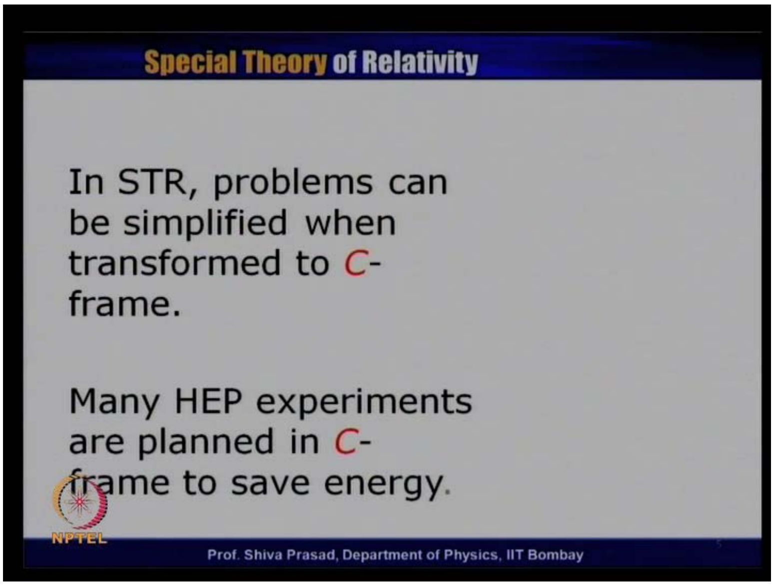

So, this is what I have written. In STR which is special theory of relativity problems can be simplified when transformed to C frame. I would also like to mention that when we actually perform many HEP, HEP is stands for high energy physics, if you perform experiment like collision experiments, scatter experiments in high energy physics. Many times we prefer would design this experiment in C frame of reference in center mass frame of reference.

So, that overall there is energy which will be seen little later that you know energy that is turns to be in smaller C frame of reference. Basically, because you do not have spend energy to accelerated to mass frame of reference. Effectively means that if you have an particular particle and you want to design an experiment, which another particle comes and colloids it. In that case this frame of reference is not a center of mass frame of reference because the net momentum is not zero.

So, you rather like to have an experiment in which both the particle as come together and colloids such that center mass. So, that it become in a center of mass frame of reference or the net initial momentum is zero. So, let us come to one particular problem which is actually simple problem, but it illustrates the fact that how it can become simple if it is solved in C frame. And it is rather difficult to solve in L frame even though the problem appears to be very simple.

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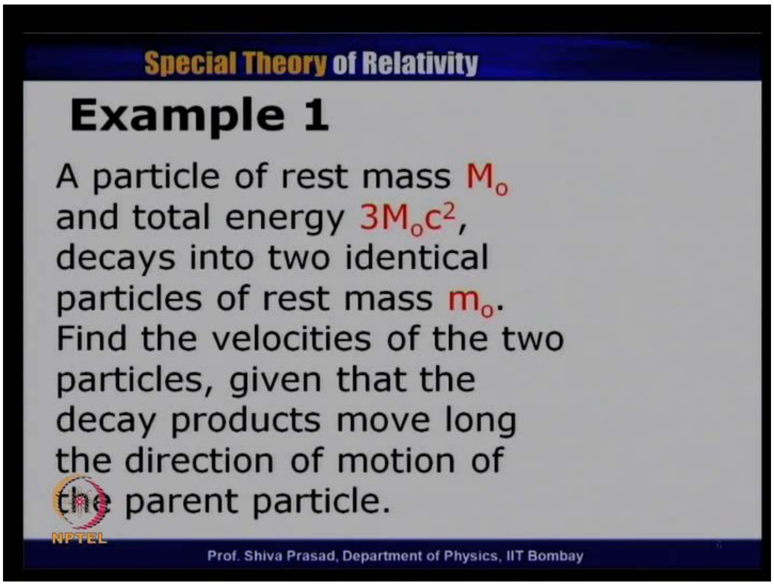

Let us look at the problem; there is a particle of rest mass capital M naught, which has a total energy of 3 M naught c square. Now, we must be clear that when we are talking of total relativistic energy, it means it includes rest mass energy. So, the total energy which means kinetic energy plus rest mass energy is 3 M naught c square. This particular particle decays into two identical particles. So, this particular particle goes away and eventually decays into two particles each one of them have a rest mass of M naught. So, their identical particles whatever might be the particles may not be a relativistic sort of experiment just for understanding the problem.

So, in decays into two masses two particles which both of them which have a rest mass of M naught. Now, I have to find out what will be the velocity of these particular particles, when they have decayed. Given the fact that the decay product, it means the both the particles, move along the direction of the motion of the present of the parent particle. Let me just read again, a particle of rest mass capital M naught and total energy 3 M naught c square, decays into two identical particles of rest mass small M naught. Find the velocities of the two particles given that the decay products move along the direction of the motion of their parent particle.

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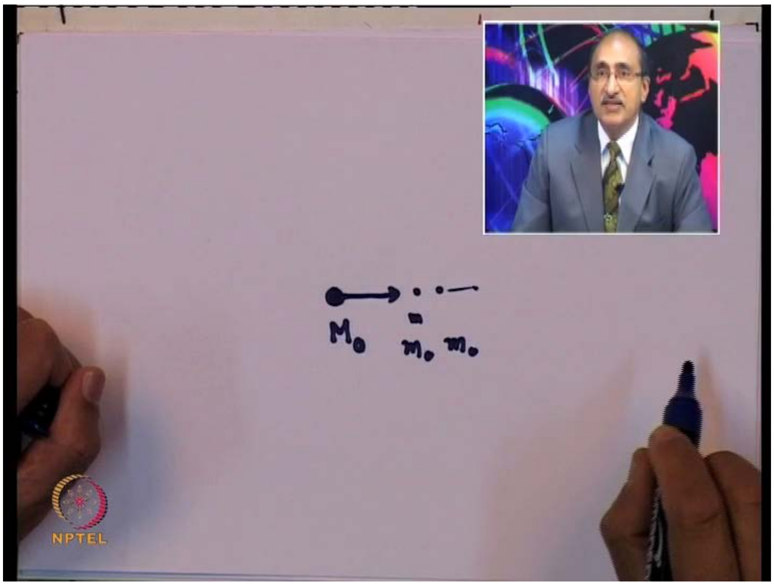

If I want to draw into a small and simple picture essentially it means that, this is a particle which is of mass rest mass capital M naught. This is moving in a given frame and eventually decays into two particles two small particles. Each one of them has rest mass M naught; obviously, because it has to conserve energy and conserve momentum. Therefore, this particular particle also be moving what has been told that both these particles move in this same direction as this particular particles, it means the this also moving in this direction this direction or opposite direction.

But the total motion is confined only to one dimension which we can call for an example direction that does not matter what direction. But everything the problem is just a single dimension problem a one dimensional problem in which everything has been described. On a traditional classical mechanics problem where we have to conserve energy and momentum this problem would have been absolutely simple. But we will see that if we try to work out this problem this particular fashion.

You see that there are certain complications and it is not easy to work out this particular problem, until we go to a different frame of reference which is the center of mass frame of reference. Let us look at this particular problem, first try to let us attempt to solve only in the L frame the laboratory frame the weight has been described. We will discuss what are the difficulties which comes across.

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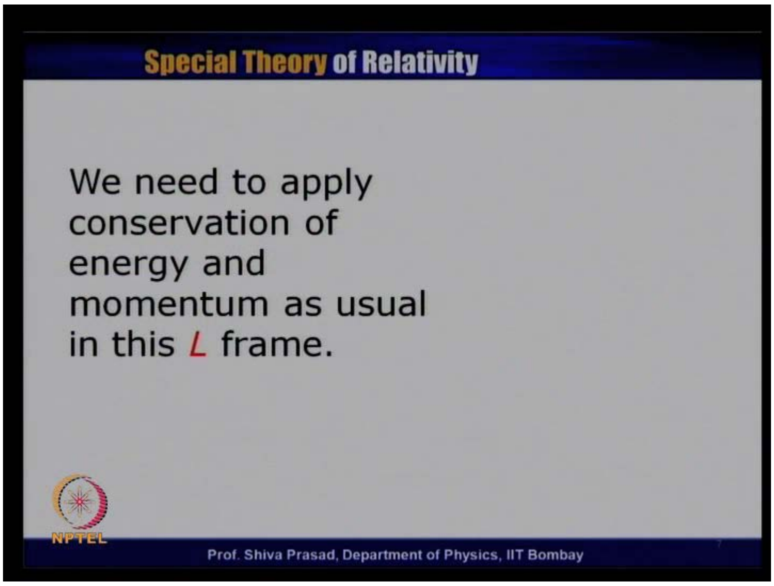

So, as we said the idea is that we need to apply the conservation of energy and conservation of momentum in this particular frame. Energy has been given because there is only one particular particle initially, so energy has been given s 3 M naught c square. So, I have to find out this energy and we have to find out corresponding what is a momentum of particular particle. Once we find the momentum, I know the initial energy, I know the initial momentum it decays into two particles. We have to write initial energy is equal to the sum of the energy of the resultant particles. Similarly, initial momentum vector is equal to the initial the final momentum that is what we have to do.

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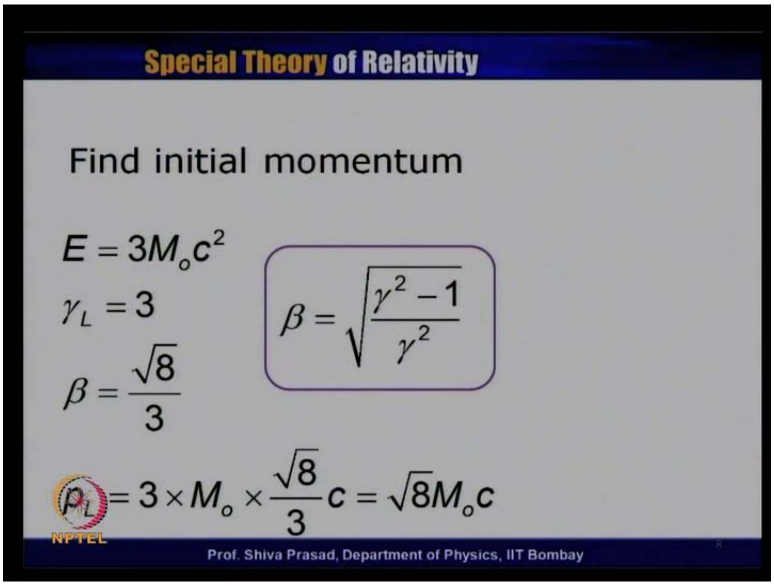

So, let us first find out the initial momentum, we have been given that E is equal to 3 M naught c square. And if you remember what we have said that the energy is given by gamma M naught c square, where M naught is a rest mass of any particular particle.

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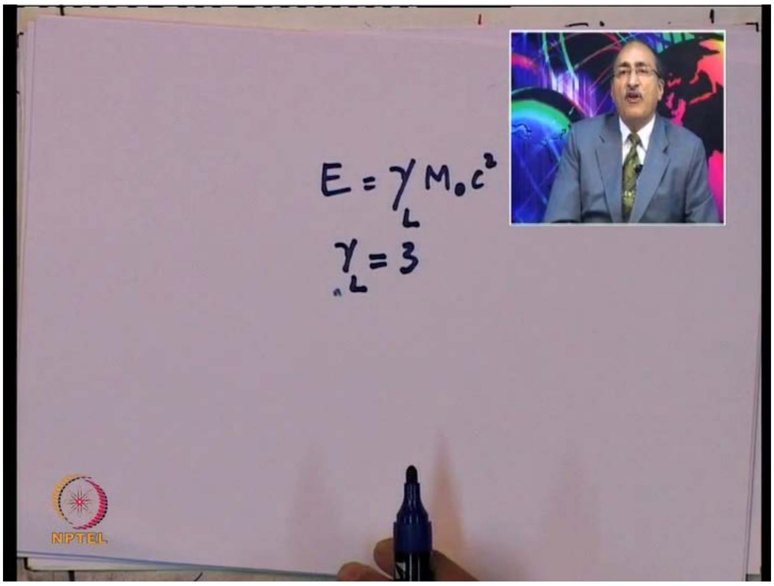

And in this particular case because the energy has been given as 3 M naught c square, it is quite clear that gamma is equal to 3, only thing because this has been described in laboratory frame of reference. So, let us write a subscript L to make it clear that this gamma that we are talking or the velocity that I will be talking on which this particular gamma depends. It is actually the velocity in the L frame or in the laboratory frame.

Now, once we have this particular gamma L, I can always find out what is velocity of this particular particle. And once I find out the velocity of the particle I can always find out the momentum of the particle. Of course, I can also use direct energy momentum relationship that also I will use just to show that we get the same result. Let us first try to find out the velocity value using this gamma L.

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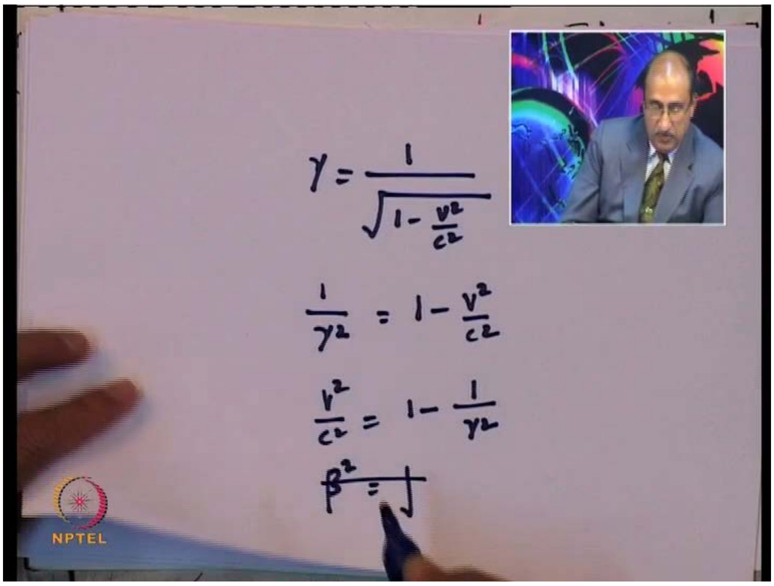

If you remember the value of gamma the expression for gamma is given as one upon under root 1 minus v square by c square, sometimes we are using v, sometimes we assume. But I should be clear now when we use v when we use u. Now, I want to solve it for v upon c, I want to know if I want to have I know the value of gamma I want to find out the value of v by c. So, let me square it and take inverse of it, if I square and take inverse of it I will get one upon gamma square, square it under this goes away.

Take inverse, so 1 minus v square upon c square comes on the numerator, this becomes 1 minus v square by c square. So, I can solve this particular equation. So, this becomes v square by c square is equal to 1 minus 1 by gamma square, which means v by c square is what we call as beta square in our notation.

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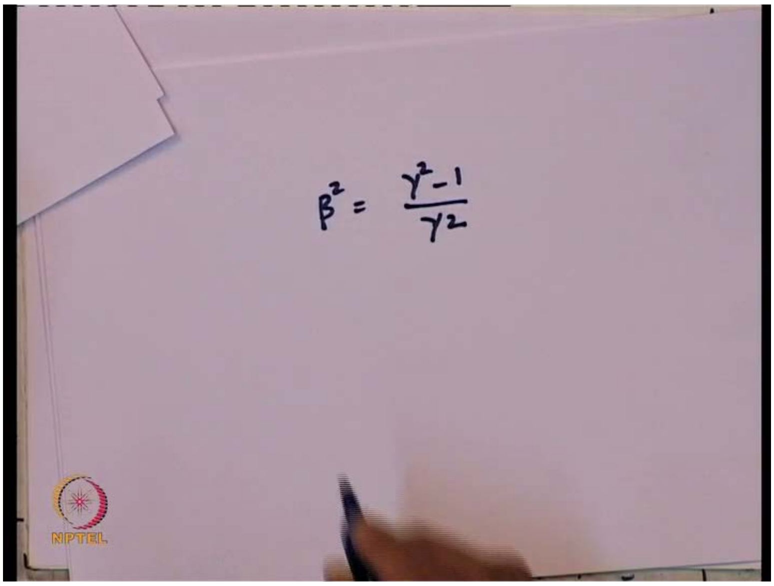

So, I can write this as beta square as sorry there is no do it here, beta square is equal to gamma square minus 1 upon gamma square. So, this is the expression which I have written this particular transparency that beta is equal to under root gamma square minus 1 upon gamma square. So, this expression gives me the value of beta which is v by c, using the value of gamma that we find, in the present case gamma is 3. So, I substitute it here this becomes three square minus 1, this becomes under root 8 divided by gamma square which is 9, this 9 can be taken out of the under root sign. So, this becomes under root 8 by 3. So, this p this beta is under root 8 by 3, which is v by c.

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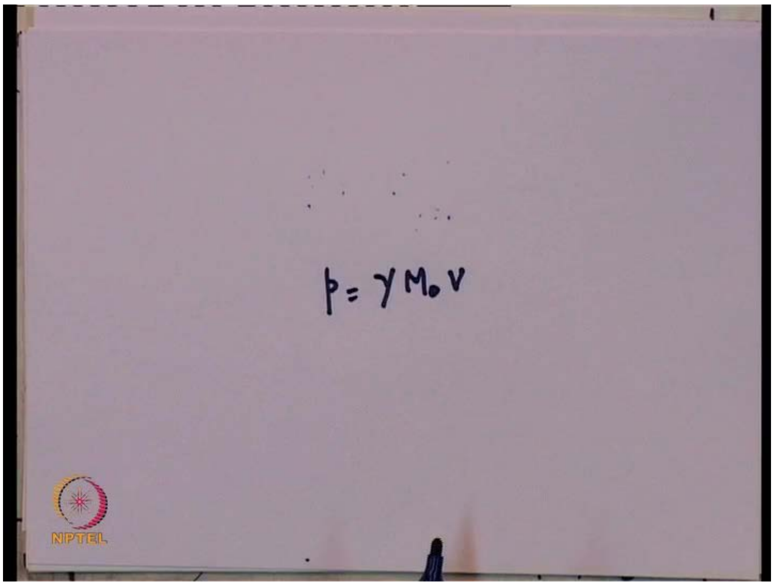

Now, once I know the speed I can always find out what will be the momentum of the particle. And the momentum of the particle as we know is given by the momentum is equal to gamma M naught v. We know gamma is equal to 3, M naught is M naught and v is under root 8 by 3, c we have just not found out. So, I have substituted this in this particular expression. So, p L, L is again because I am describing this laboratory frame is equal to 3 multiplied by M naught multiplied by root 8 by 3 c, which gives me root 8 M naught c.

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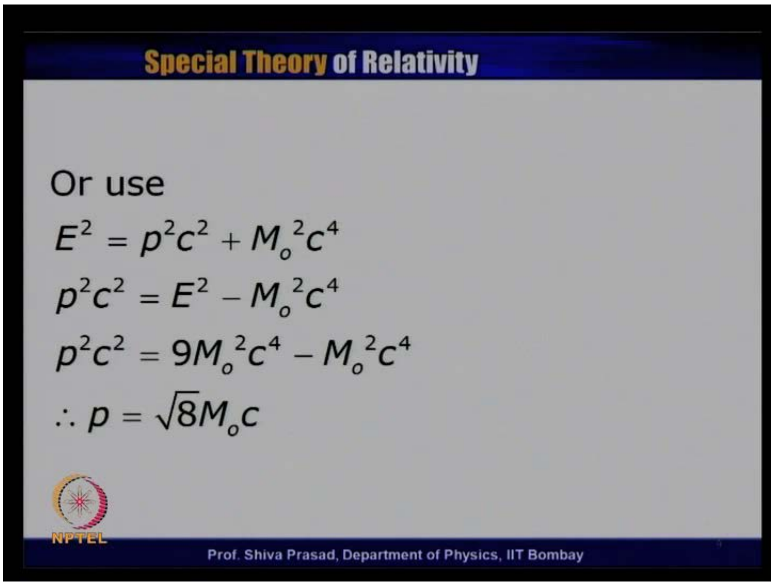

Force as I said I could have found out this particular expression also by using directly the energy momentum relationship, which is E square is equal to p square c square plus M naught square c to the power 4 this is very commonly used relationship. So, from this particular thing I have to evaluate momentum I know the energy. So, p square c square will turn out to equal to be E square minus I take this on the other side, minus M naught square c to the power 4, E being 3 M naught c square, E square will become 9 M naught square c to the power 4.

So, I substitute it here, I have to subtract this expression minus M naught square c to the power 4. So, this becomes 8 M naught square c to the power 4, c square I will cancel out and p I will get under root 8 M naught c. So, what we have done so far I know the energy the initial energy of the system which happens to be just one particular particle, which now eventually decays into two particles. So, this particular particle has a total energy of 3 M naught c square and it has a total momentum of under root 8 M naught c.

So, once this particular particle decays into two particles, the total energy must remain same. Similarly, the total momentum must remain same because this purely one dimensional problem. So, I have not taken the x axis and y axis because just described. I have not described it in the vector form or I need that this particular value of momentum must be conserved.

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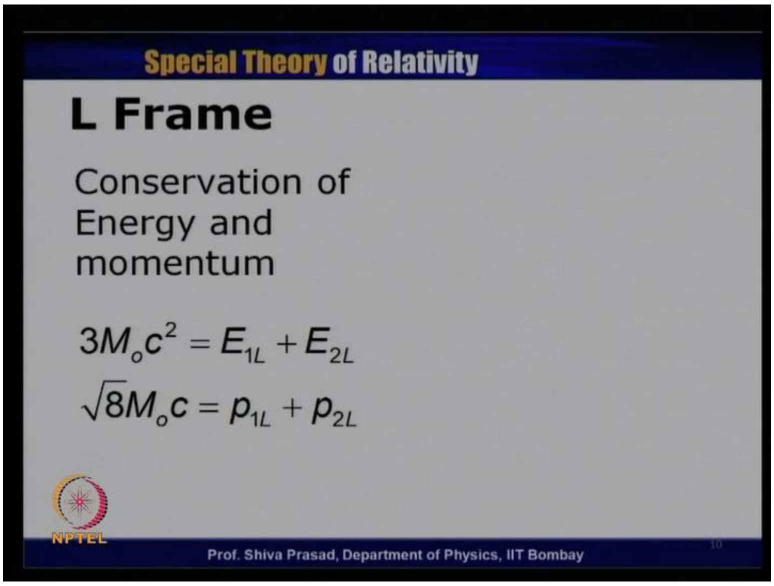

So, I will be able to write the following equations which I have written in the next transparency. First we have found out what is the energy and the momentum of the original particle, what we have to do is to write the energy and momentum conservation equations. We have seen it has been given in the problem that the initial energy is of is equal to 3 M naught c square, there is only one particular particle so there is only one particular particle system.

Once they are two particles, which have resulted out of the decay of this particular particle if E 1 L and E 2 L are the energies, then 3 M naught c square must be equal to E 1 L plus E 2 L. Now, corresponding to 3 M naught c square I have seen that the value of the momentum is under root 8 M naught c remember this is original particle, so I have used capital M naught here.

This must be equal to the sum of the momentum of the two particles we have been told that the particles move along the same direction its original direction. So, I can write just in a scalar form, I do not have to use vector form because as we have said this is purely one dimensional problem. So, this must be equal to p 1 L plus p 2 L, where p 1 L is the momentum of the first particle, p 2 L is the momentum of the second particle. As I have said E 1 L is the energy of the first particle E 2 L is the energy of the second particle.

In a traditional classical mechanics this would been a very very simple trivial dissemble problem, all you have to do is to express this energy into momentum in the form of momentum or this momentum get form of energy. And just to solve this equation you have to one knows and two equations to solve it, this one dimension is simple. Mainly because in traditional classical mechanics in which there are no relativity, the relationship between E and p are purely simple. Of course, in that case rest mass what we try to do is the conserve the kinetic energy, it not bother about that yes of course. Now, in this particular case what will find out that energy and momentum relationships are little more involved, which has been written in the next transparency.

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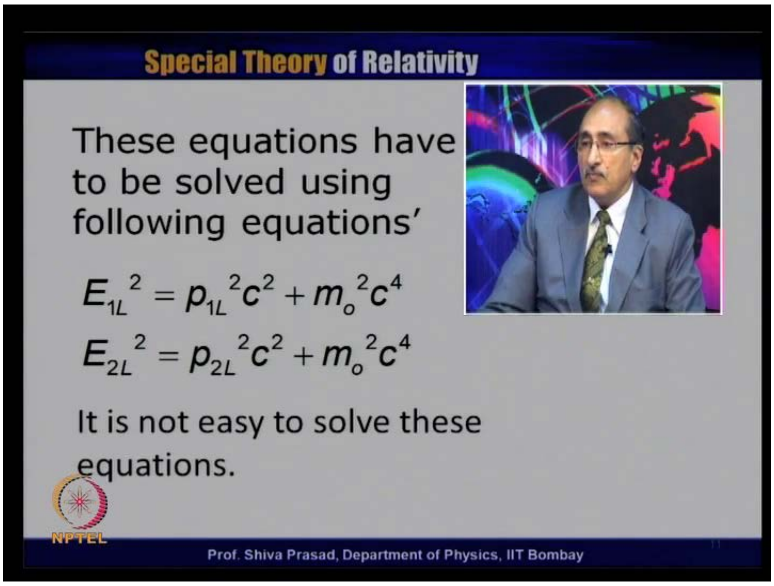

So, the energy of the first particle is related to the momentum of the first particle by this particular expression, E 1 L square is equal to p 1 L square c square plus m naught square c to the power 4. Remember I am using small m not because the energy and momentum of the particle that I am going to relate has rest mass of small m naught. This is not the original particle, the original particle has decayed\ the resultant particle, which I have got which has rest mass of M naught. And originality in this particular problem the two particles have the same rest mass of this M naught that is what I have given. So, I am using small m naught here in both these expressions.

Exactly similar equation I write for as a relationship between energy and momentum of the second particle, E 2 L square is equal to p 2 L square c square plus m naught square c to the power 4. Remained that this L has written just to make sure that I am talking of laboratory frame of reference. As we will see that this is not easy to solve this equations because if I try to express this particular equation in that particular form, let us say let us try to express energy in terms of the momentum. Then I have to take under root of this particular expression to find out what this E 1 L.

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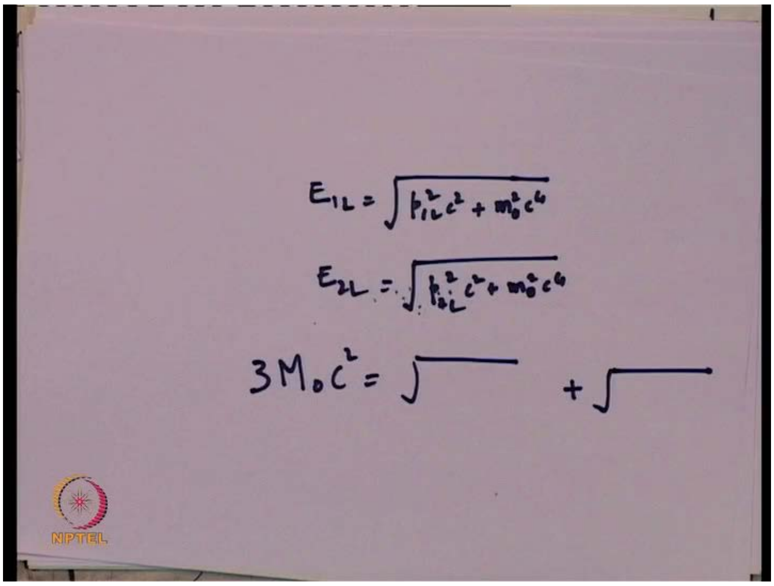

So, I must write E 1 L is equal to under root of p 1 L square c square plus m naught square c to the power 4. Similarly, for E 2 L I have to write under root p 2 L square c square plus m naught square c to the power 4. When I write in the conservation of energy expression, I have to write M naught c square is equal to this plus this so this becomes this under root plus this under root. If you can solve this equation of course, you will get a energy and momentum in fact, will get p 1 L and p 2 L.

Taking the other two other equation also to consideration writing p 1 L also in terms of the initial momentum I will be able to solve it. But as you can see because of the presence of these square roots etcetera these are not very straight forwarding equations to solve. On the other hand this equation becomes fairly this problem becomes fairly simple if I go to the center of mass frame of reference. So now, let us attempt to solve this particular problem in center of mass frame of reference of course, here the problem fairly simple because there is only originally one particle.

So, if I have to find out what is the velocity of the center of mass it is going to the velocity of the particle the initial particle that I am talking about. So, I go to a frame of reference in which the initial particle is at rest and because in this frame the particle is at rest the momentum is 0, there is only one particle. So, that automatically becomes the C frame or the center of mass frame of reference. So, let us describe let us go to a frame of reference in which the original incident particle or un-decayed particle will not be incident, but un-decayed particle was at rest.

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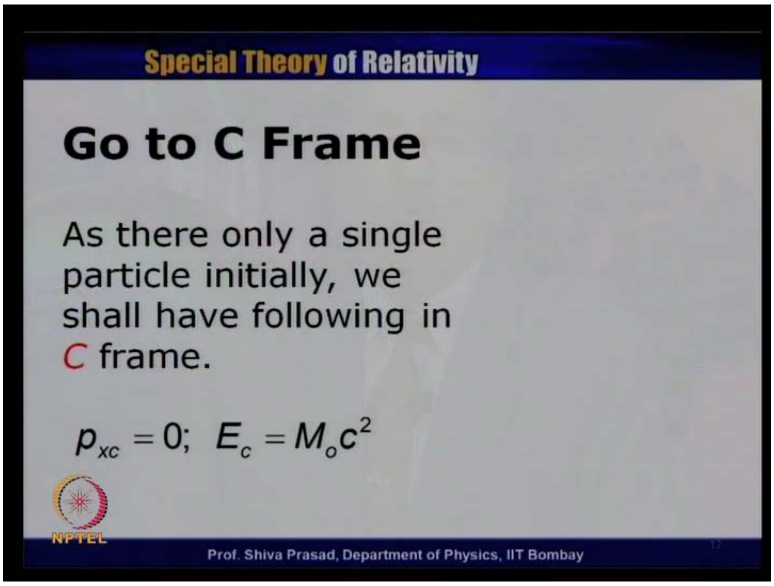

So, this what I have said, let us go to C frame as there is only one single particle we shall have the following conditions. I know very clearly because if I have gone to the frame of reference of that particle itself the particle if not moving in that frame. Therefore, its total energy has to only the rest mass energy that cannot be any other energy therefore, the total energy C frame will have to be just m naught c square. And of course, if the speed of the particle is 0, then momentum is also 0. So, therefore, the C frame the initial value of momentum is zero which has to be if it is to be called as C frame and the total initial energy is just capital M naught c square.

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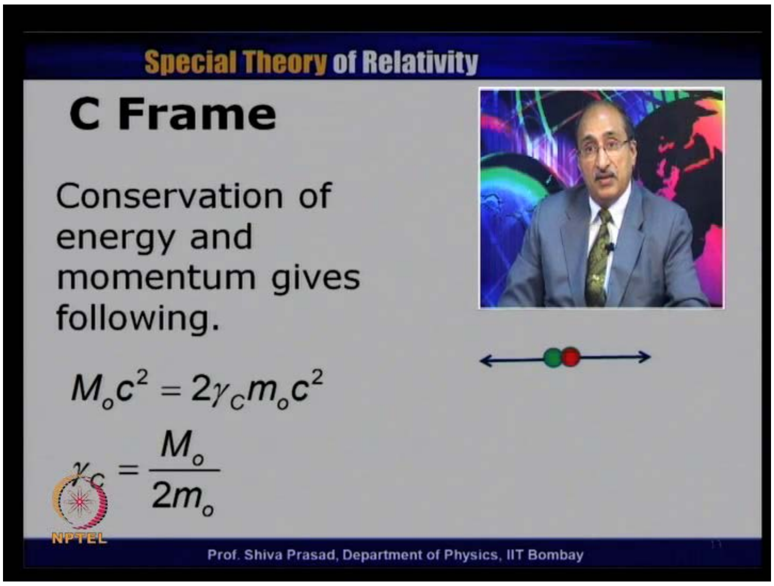

Now, let us try to see if I have to conserve energy and momentum in this particular frame of reference, remember because the rest mass of the two particles that I am getting as a result of decay happened to be same. The only way these particular particles can decay is when they move back-to-back, it means one particle goes in this particular fashion, another particle of course, goes in this particular fashion. That is only way that they can conserve and make momentum 0, because remember initial the momentum is 0.

So, the final momentum also has to be 0, if both the particles moves in the same direction there cannot be momentum cannot be 0, we have been told that a only one line. So, there is only possibility is that one goes in this particular fashion, another goes in this particular fashion. And because their rest mass is happened to be same therefore, there energy has to be equally shared because in order to conserve momentum both must have same momentum obviously. And energy and momentum relationship depends on their rest masses, their rest masses being same in forces that their energy is also same. Let me just explain this particular particle little bit more.

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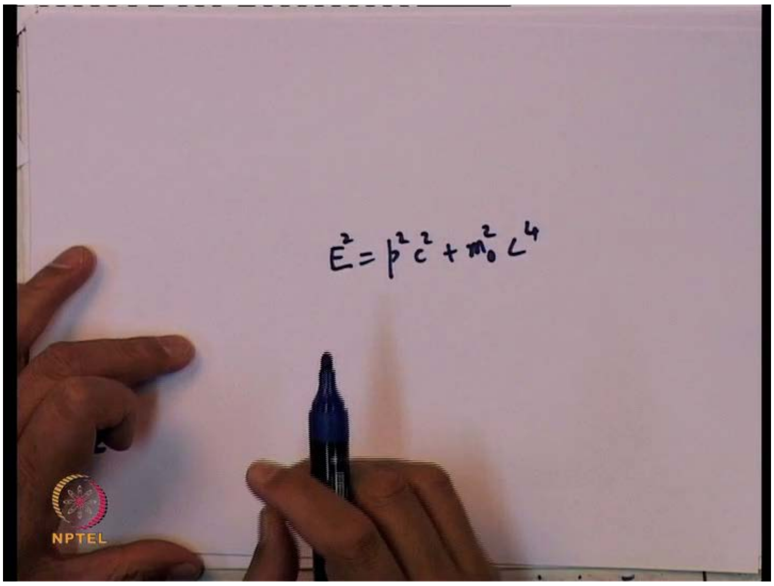

So, we have E square is equal to p square c square plus m naught square c to the power 4. If first particle p's is same as the second particle, m naught is same as a second particle, then E of that particle also has to be same as second particle. Therefore, the two particles in the center of mass frame of reference must have same energy as well as the same momentum. Same momentum because the initial momentum was 0, so final momentum also has to be 0 and energy have to be same because the rest mass is also happened to be same.

Therefore, we can see that the problem has to be simple, it means each one of the particles must be having the energy of half m naught c square. So, this is what I have written here that M naught c square should be equal to 2, this is I can write as gamma c m naught c square. So, this m naught c square these are the total energies have to be equally shared.

And this energy of each particle can always be written as we just now have said gamma M naught c square and because this center of mass frame of reference, so I am writing gamma c. So, each particle must be having the same energy this two must be equal to the total initial energy in this frame, which is M naught c square. So, this is what I have written M naught square M naught c square is equal to 2 gamma c m, small m naught c square. It immediately gives me the value of gamma c, which is c square cancels here M naught divided by 2 small m naught. Once we have found out the value of gamma c I can immediately find out what will be this p's of this particular particle, in this particular frame of reference by using the expression which you have just written earlier.

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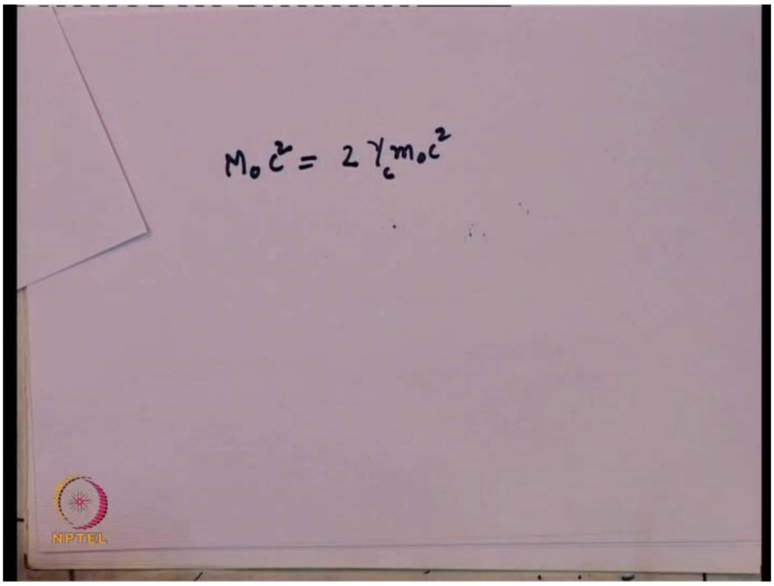

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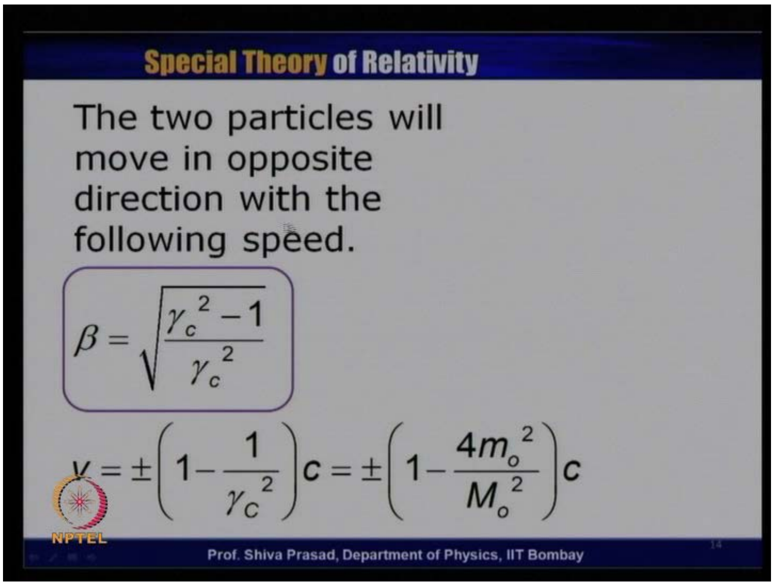

So, the two particles will move in a opposite direction with the following speed, this expression is same expression which we just now I have worked out little bit earlier. And if I substitute the value of gamma c which I have obtained, I will get that this speeds of the particle will be like this, by will given by this expression. One moving in plus direction another moving in minus direction, you can call it x direction one moving in plus x direction another moving in minus x direction. So, these are the two velocities.

But what we have found out are the velocities in C frame of reference, if I have to find out 11 laboratory frame of reference, all I have to do is simple velocity transformation. I have to know what is the velocity of the C frame, which by the way we have found out earlier. Once we know the velocity of that particular frame take care of proper science and you can transform and obtain the values of speeds or some velocities in laboratory frame of reference. So, we transform into C frame then brought it back to L frame, but remember the problems very simple, we do not have deal with all those under roots and time to work out and trained to solve those equations problems essentially very simple.

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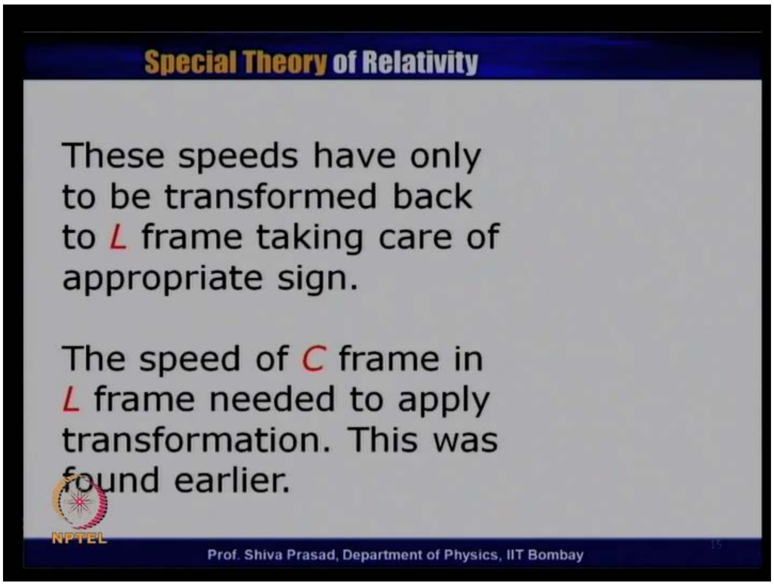

So, these I have I have written these speeds have only to be transformed back to L frame taking care of appropriate sign. The speed of C frame in L frame needed to apply transformation this was already found out earlier, if you remember earlier, the earlier case which we have done here. We already found out that beta is equal to under root 8 by 3, so from that we can found out p is equal to under root 8 by 3 c. Once we know the relative velocity between the frames we know the velocities of the particles and C frame moves a velocity transformation bring it back to laboratory frame. So, will not work it out further I think this work go out roots simply.

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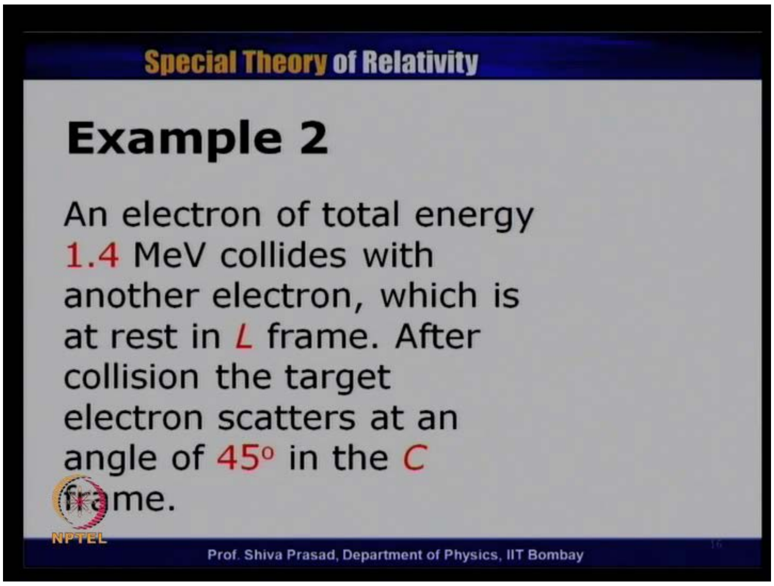

Now, let us go to a slightly more involved problem it is not really involved it is slightly more in the sense that instead of one initial particle we have two particles and things are not really one dimension equation. So, let us read the problem, an electron of total energy 1.4 MeV, again when we say total energy it means it is root less mass energy is total energy is 1.4 MeV. M is stands about 6, electron volt collides with another electron, which is at rest in L frame. So, in laboratory frame of reference there is one electron which is at rest, another electron comes and hits it. Colloids it or it can scatter or gets scattered whatever you want to call it that is what important.

After the collision the target electron, which means the electron which is originally at rest is found to get scattered at an angle of 45 degrees, but not in the laboratory frame, but in the C frame in the center of mass frame of reference. So, the problem involves both laboratory frame and C frame. So, in that sense the problem is little more involved. So, after the collision the target electron which has originally the electron which is at rest is found to get scattered at an angle of 45 degree in C frame.

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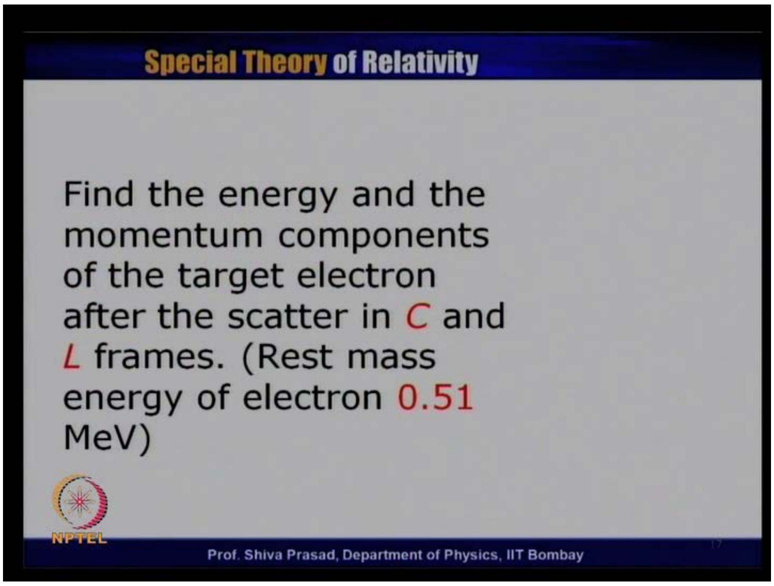

So, from the nature of the problem itself, particular problem itself is cleared that we have to give talking about C frame. Find the energy and the momentum components of target electron after the scatter in S and C in C and L frames. So, after this scatter after the scattering has taken place what are the momentum and energies of target electron in the frames of course, rest mass energy of the electron has been given as 0.51 MeV. So, I think the problem is clear that this one particular electron, which is being hit by an another electron, one electron is at rest, another electron comes and hits here hits it.

The electron which was at rest is found to move it an angle of 45 degrees in center of mass frame of reference, k of because when you are talking of 45 degrees. It means it no longer a one dimension it has to be you have to work out in two-dimension. And then you have to find out what will be the energy and momentum of the particle, work out for the both the particles necessary or target electron in laboratory frame of reference as well as C frame of reference.

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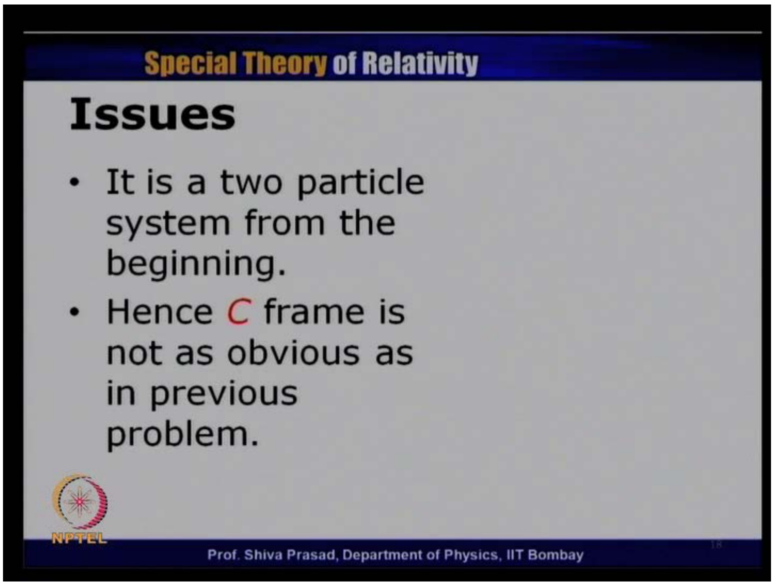

What are the issues in this thing; first of all it is a two particle system from the beginning as we have said lets no longer a one particle system. So, like an one particle is very easy to find out what is center of mass frame of reference, because you have to just go to the reference frame of reference of that particle itself in which this particle has to be at rest. So, here it is not so obvious, but we have to work it out.

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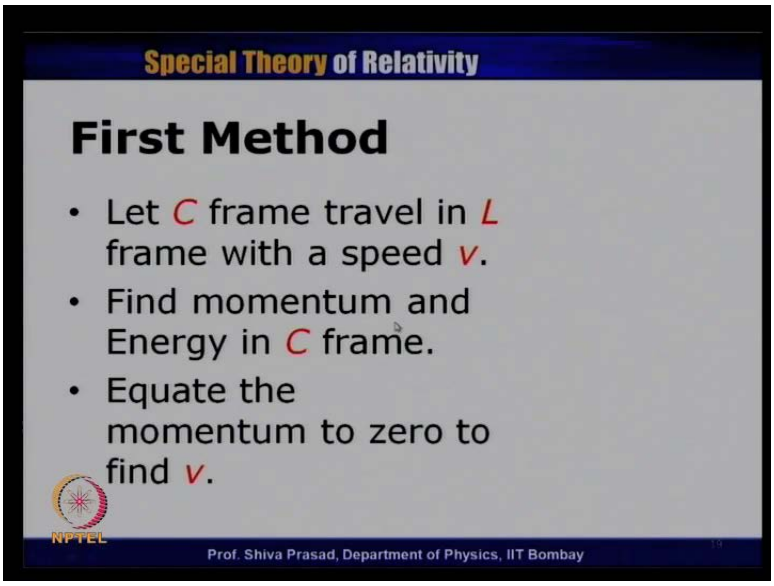

The first method which is slightly I mean it is straight forward method, but little more longer method in fact will give you a comparatively more straight forward method. Let us assume that C frame travels in the laboratory frame of reference with a speed v. Find the momentum and energy on both the particles in the C frames. So, assume could in an arbitrary frame of reference, which moves relative to L frame with a speed v. Then find the momentum and energy on both the particles in the C frame, then take the sum of the momentum put it equal to 0. Because, I am looking for that particular frame of reference in which the sum of momentum is 0.

So, once I put I take the momentum of first particle and momentum of second particle in a frame of reference, which is moving with a speed v relative to L. And equate this particular momentum sum of the momentum to 0 and solve for v that will be the velocity of the center of mass frame. So, this is very straight forward standard method of finding out even if we had n particles that is what I am doing principle. If I want to do in most simple fashion then find out the momentum go to a particular frame of reference with moves with a speed v.

Transform all the momentum to that particular frame of reference, takes summation of momentum put it equal to zero. That velocity that will obtain will be the velocity of the center of mass frame of reference. So, let us first work it out that that way, before we go to more involved little more trickier way, but at simple method.

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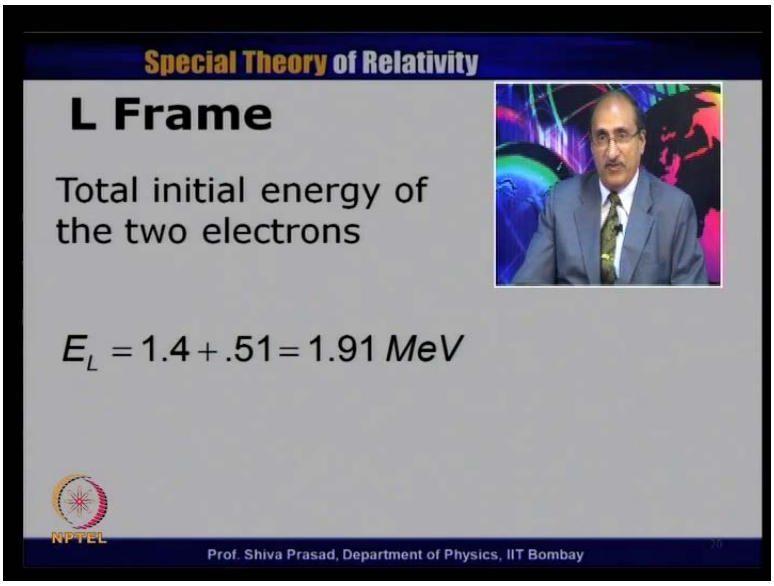

So, let us first find out the total initial energy of the two electrons, as we have seen that one electron is at rest in the laboratory frame. So, I am doing this particular problem first in L frame. So, that is 0.15 electron MeV and the other electron, which is at which is coming and hitting it that has a total energy 1.4 MeV. So, total energy in the laboratory frame is 1.91 MeV. So, this is the total energy which is available to me in the laboratory frame of reference.

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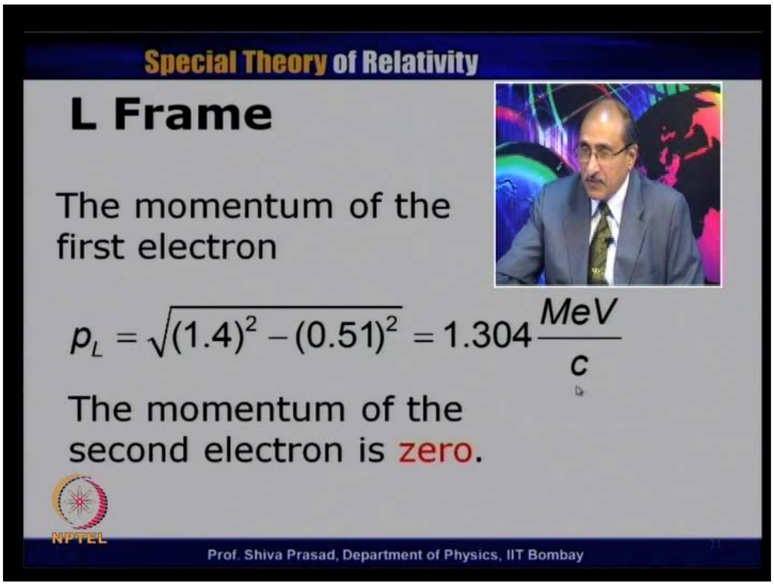

Let us work out on momentum. Second particle is at rest and the target particle is at rest in the laboratory frame of reference. So obviously, this is momentum is zero or we have to work out the momentum of the other particle which is coming and hitting it here, find out what is the momentum of that particular particle. I know the energy of that particle which is 1.4 MeV, I use the same standard expression E square is equal to p square c square plus m naught square c to the power 4. M naught square c to the power 4 will just 0.51 square, 0.51 MeV square, so this is what I have written 0.51 MeV square.

So, this is p square c square plus M naught square c to the power 4 and this was E square which is the total energy. So, I am solving for momentum like we have did for earlier problem. So, this becomes 1.4 square minus 0.51 square, only thing which I have done I have taken a special units here. So, that this particular normal issue had been p L c, but I have taken units of MeV by c. So, this value of c I did not writing here. So, these this very conventional unit I would traditional units sorry, in which we express momentum in the units of MeV by c energy by c.

So, energy we express in MeV or g E v whatever depending upon the type of problem and express momentum in MeV by c or g V by c. So, the momentum of this particular particle is one point we yes work it out sort it 1.304 MeV by c of course, the momentum the second electron is 0 that is what we have just now said.

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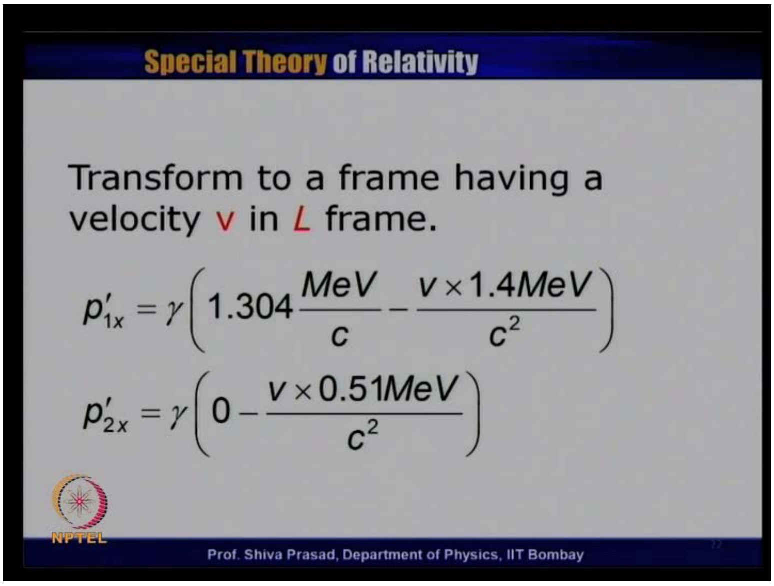

Now, transform it; go to a frame of reference which is moving with a speed v in the L-frame, I use momentum transformation. Momentum transformation standard, gamma, what is the original momentum p x, p 1 x prime is equal to gamma p x minus v e by c square. This is the momentum transformation equation momentum, we have just find out 1.304. I am writing the unit to make it clear minus v is the speed relative speed between the frames the energy is 1.4 MeV divided by c square.

For the second particle the initial momentum 0. So, gamma multiplied by 0 into 0 minus 0, v into its energy is only is rest mass energy. So, v multiplied by 0.51 MeV divided by c square. In fact, you just take the sum of these two put it equal to 0 solve for v. That will give me immediately the velocity of the center of mass frame of reference. That is what I have written for C frame E 1 x prime plus p 2 x prime is equal to 0.

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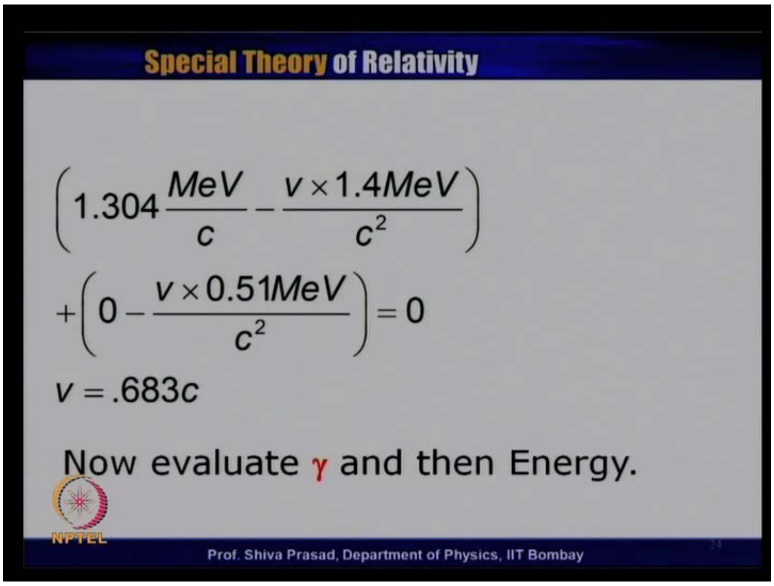

So, just sum these things and work out for v you will get v is equal to 0.683 c from this particular way of working it out. And then once you know the velocity you can find out gamma, once you have found out gamma then you can transform the energy and find out what is the energy of these particles in the center of mass frame of reference. Then center of mass frame of reference initial some momentum is anyway 0, energy momentum conservation and you will be able to solve the problem.

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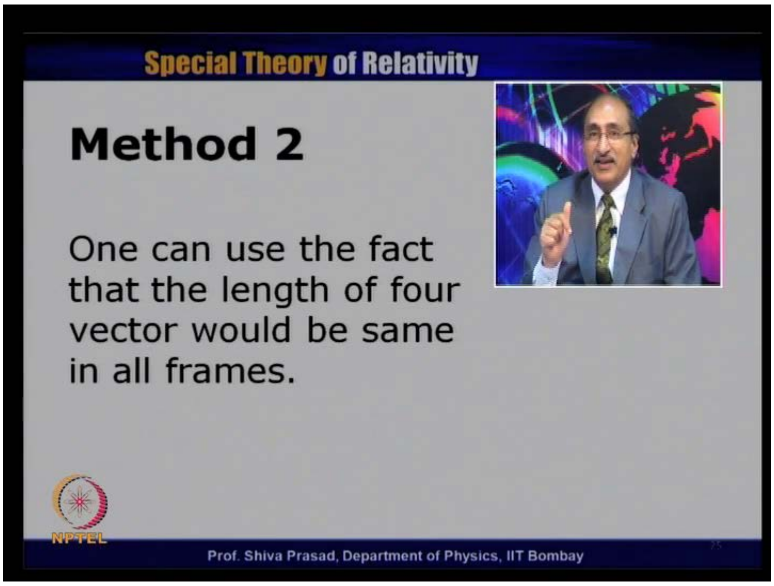

But as I told that is slightly quicker method to work it out by using the convention of the 4 vectors. So, let us adopt the method two of working out this particular problem going to center of mass frame of reference by using the concept of four vectors. As we will see that this is slightly tricky, but it is much more simpler.

So, this is what I have said method two, one can use the fact that the length of four vector would be same in all the frames. We have discussed it earlier, when we describe the concept of four vectors that the length of four vectors is what we call as a four scalar. Thus we change the frame of reference the individual components of the four vector may change, but its length is unchanged that is a four scalar. So, once I go from one frame one initial frame to another initial frame of reference length is going to be same.

So, what I can do now calculate the length of energy momentum four vector in the laboratory frame of reference for the system of the particles. And I know that this particular length is going to be same in the center of mass frame of reference and I know in the center of mass frame of reference summation of momentum would be 0. So, it will have session of only four terms of energy if you remember the four terms of the energy momentum four vectors, where p 1, p 2, p 3 and rather p x, p y and p z and fourth term is i E upon c. So, fourth term depends on energy, let us just work it out and see how similar this particular problem becomes.

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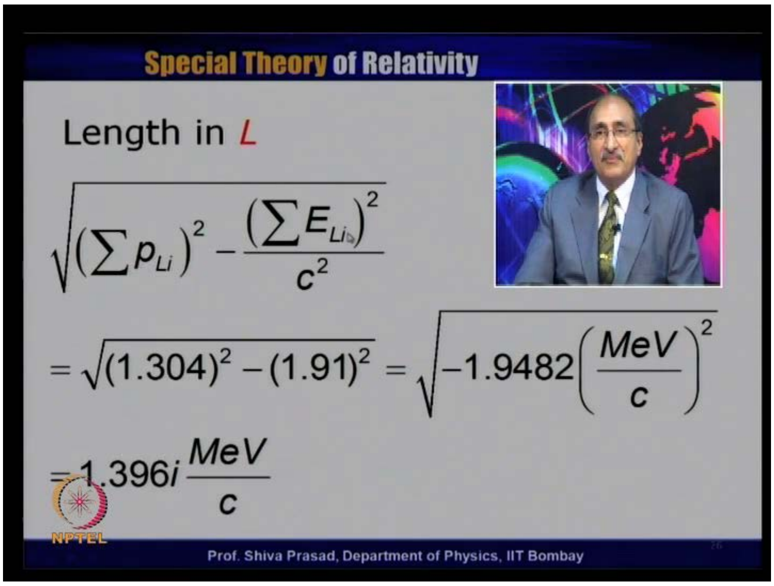

So, first let us write the length of the four vector for this summation for the system of the particle. So, first three terms you are taking the length. So, it will E 1 square plus E 2 square plus E 3 square plus a 4 square, first three terms are only the momentum terms.

So, essentially it means the first three terms will give me a summation of summation of p L I square where L is the momentum the laboratory frame of reference. Some over the all particles this case they are only two particles.

So, what I will do? I will add the momentum of the two particles and this particular term will be obtained from that. Similarly, this particular term depends on the summation of the energy of two particles because I am applying this particular I am calculating the length of energy momentum four vector for the system of the particles, for two particles together, not for the individual particles. So, when I am writing for the two particles together this summation will be the sum of the energies of two particles; first particle has the energy of 1.4 MeV, the second particle is at rest, it has only rest mass energy which is 0.51 MeV.

So, summation of p L i will be only 1.91. sum of first energy of first particle plus sum of energy sum of energy first particle and the second particle, first particle having energy of 1.4, second particle having energy 0.51. Similarly, here summation of the momentum magnitude of the momentum for the first particle, which is 1.4 and for other particle is 0. So, this becomes 1. Sorry, I am sorry… The momentum of the first particle is 1.304 for the second particle is 0. So, just because 1.304 square.

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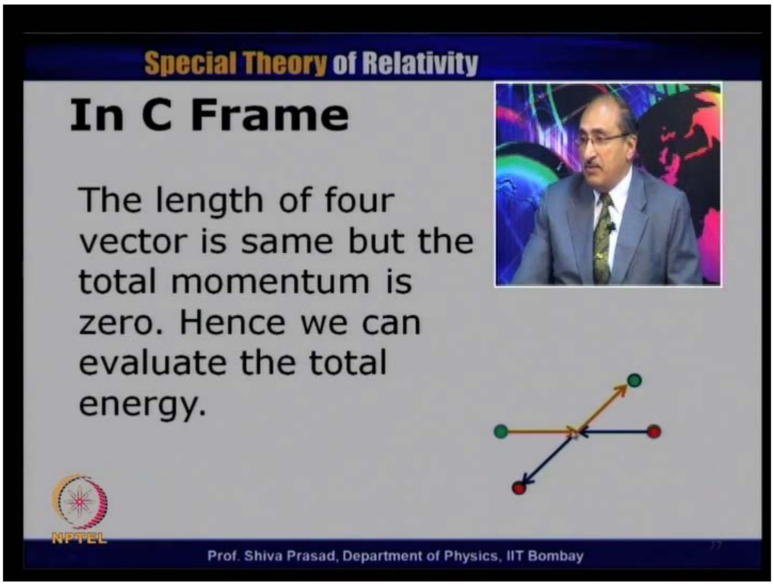

I calculate this number, this turns out to be an imaginary number, the length of four vector can always be an imaginary, we have discussed this problems earlier. So, this will be given by 1.396 i MeV by c. Now, I know if I go to any other frame of reference this length is for the same system of particle is going to remain same. But if I go to a specific frame which is called center of mass frame of reference in that summation of momentum will be zero, it means this particular term would be 0. So, if I want to write in center of mass frame of reference, this term is 0 and only the energy terms will be present.

So, it is much easier to find out the energy in the center of mass frame of reference, let us see how? First let us realize that because the two particles that I am talking are 1 electrons. So, what will happen in the center of mass frame of reference; one particle comes like this, another particle comes like this, both are electrons. They have to come in opposite direction because they have to make the total momentum 0, so their momentum must be in opposite direction. Now, they can scattered one of the electron goes this way, another electron goes this way.

We have been told that this angle 45 degree, this has been given in the problem that is angle is 45 degrees, all right? Now, the only way because initial momentum same as the final momentum. So, this is the term goes this way making an angle of 45 degree, the other electron has to go opposite back to back just like in earlier case, except for the fact that is not in the same line. But now, tilted at an angle of 45 degree, but otherwise this electron also to go to back to back because the rest mass of this electron is same, this is electron. So, rest mass is same therefore, the energy has to be same and the momentum is also same. So, once we realize that this is what is going to happen, then I realize that this particular energy, must be same as this particular energy as we have discussed earlier.

Now, what would summation of the what will be length of the four vector in C frame, summation of p i is 0. It means it must be minus summation of E c i square by c square, when the symbol c here, to demonstrate that this is centre mass frame of reference. So, this particular thing because the E c is 0 some of the I am applying again I am reminding I am applying from the system of the particle and for the system of the particle is centre of mass frame of reference summation E is 0.

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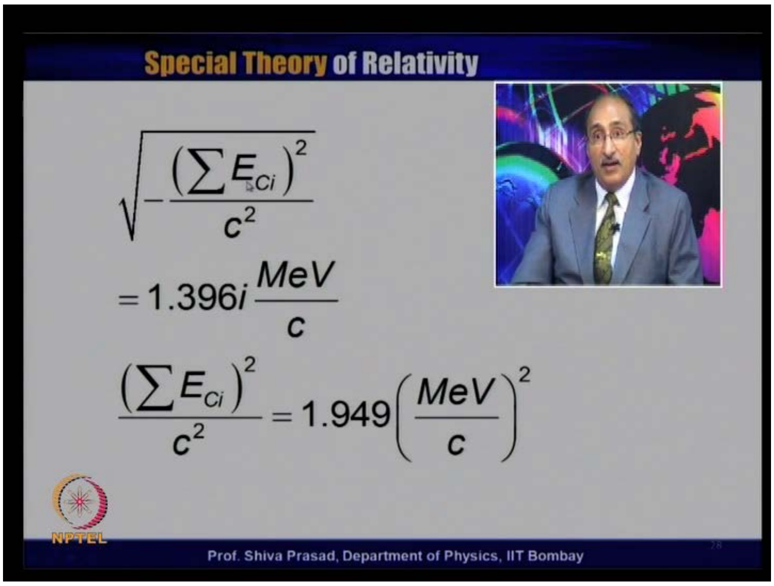

So, it contains only one term which is this and this must have exactly the same value as we have done earlier, because the length is going to be same in this frame for the same system of particles. So, I just evaluate this particular thing and in peaceful I get summation of E c i and I realize I take under root. And this is sum of the two energies and because energies are going to be shared equally by the two electrons. So, each one of them will have an energy of 1.396 divided by 2. Let me repeat this length of the four vector has only one term which is here which contains the submission of energy of two electrons. The submission of the energy of the two electrons and with the energy of both equal of two electrons, it means E c i is just… By this particular summation is just the double of the energy of the individual electron. So, this one particular 1.396 if I divide divided by 2 and I will get the energy of the two electrons.

This is what I have written the energy of the two scattered electrons of the magnitude of the momentum are same. We therefore, get E c is just as 1.396 divided by 2 is equal to 0.698 MeV. So, each of the electron will have this much energy because of the symmetry involved in the centre of mass frame of reference. The two particles have to go back to back and both being the same particles with same same rest mass have to have the same energy and of course, they have to have a same momentum.

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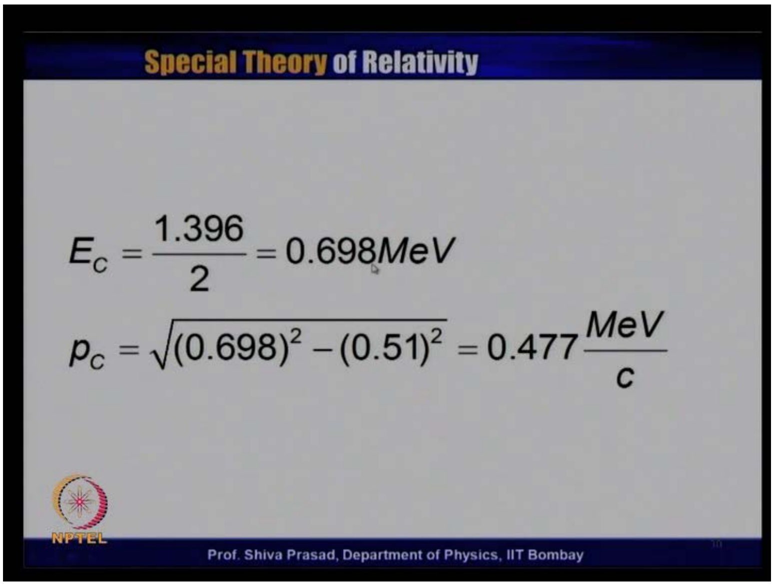

I can calculate what will be momentum of the particle by just using exactly the same relationship. I will get that the momentum of the individual particles individual electrons will be 0.477 MeV by c in C frame of reference this centre of mass frame of reference, I found out energy and momentum. Now, is C frame will have laboratory frame of reference make a transformation. Again you have to find out the speed of C frame with respect to L frame of course, if you have adopted the second method we have so not obtained the speed on the first method we found out, the speed of the centre of mass frame of reference.

But in this particular method we have not yet found out, but we can now find out because in this particular C frame the energy of the electron is 6.98 MeV, while its rest mass energy was 0.51 MeV. So, the gamma will be 0.698 divided by 0.51, once I know gamma I can calculate what will be the speed.

This what I have done of course, before that let me let me also tell that we have to calculate x component and the y component of the momentum. Because now, make an angle of 45 degrees and because it is an angle of 45 degrees. So, x component and y component will be just whatever is the momentum divide by root 2 cos 45 or sine 45 both are under 1 upon root 2. So, these are the x component and the y component of the momentum and I must you know we should be clear, that we have to transform not just one momentum, we have to transform x component of momentum and y component of momentum and also the energy of the particle to L frame.

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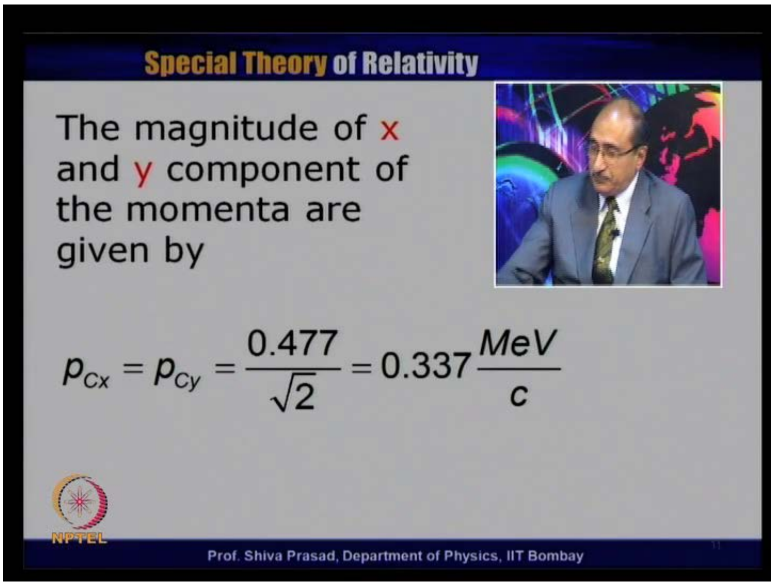

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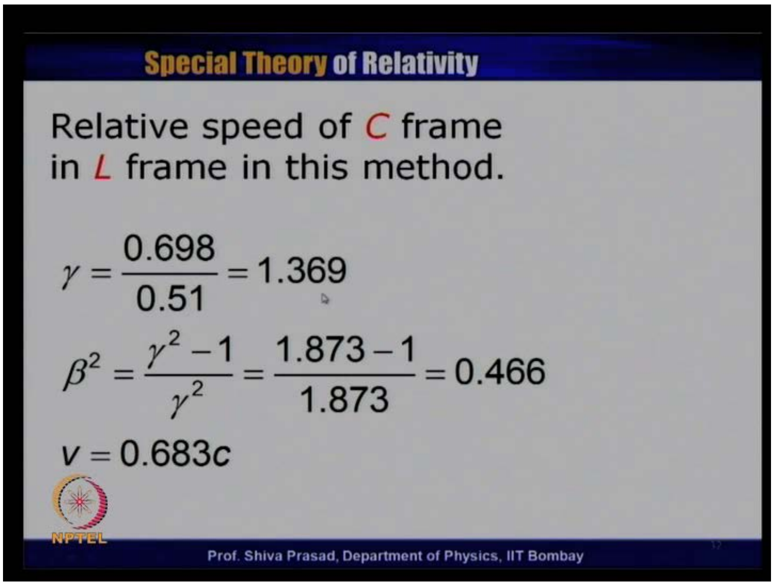

So, as I said relative speed of c can be obtained by 0.698 divided by 0.51, which gives me the value of gamma is equal to 1.369. And I can calculate the beta as we have done using the expression gamma square minus 1 divided by gamma square, which gives me same value, which I obtained by the first method which is 0.683 c. I know the velocity I know p x, I know p y, I know E the centre of mass frame of reference, applied transformation equations go back to laboratory frame of reference. You know all the energy all the momentum back to laboratory frame of reference if you are not very sure you have done mat everything correctly. Again apply conservation of energy and momentum in that frame they have to be obeyed in that frame of reference if you have done any mistake.

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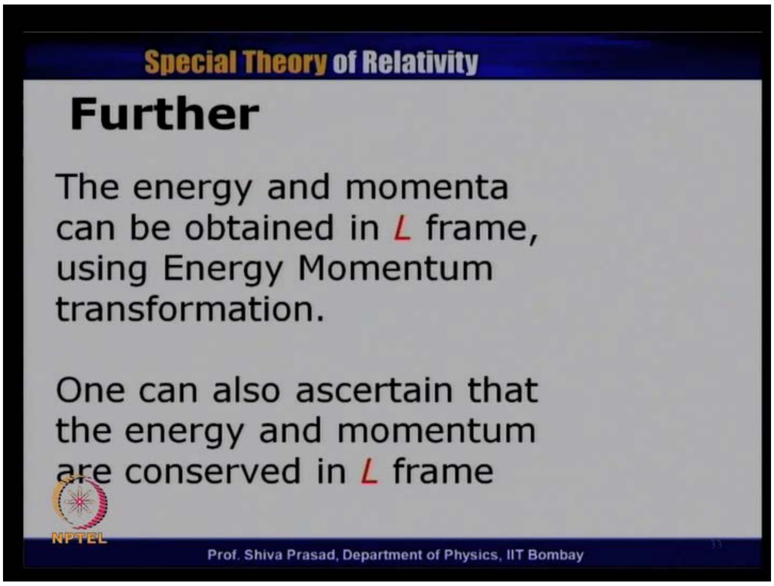

So, that is what I have said the energy and momentum can be obtained in laboratory frame using energy momentum transformation standard transformation, we have been using those equations earlier. One can also assert in that the energy and momentum are also conserved in that L frame which they have to, but that is not a part of problem to show that. But if one wants to be show to see that I have done no mistake, this is the way we can calculate. I just wanted to point you out one particular thing that, once we did when described the experiment on centre of mass frame of reference the energy that we got was much less, in the laboratory frame of reference it was 1.4 plus 0.51 MeV. While in the case of the centre of mass frame of reference is 1.396 MeV.

So, just let us state this particular fact this if we have to decide this particular experiment, then in the centre of mass frame of reference the energy that is being used only 1.396 MeV, while in the laboratory frame of reference its 1.91 MeV. So, this is what sort of illustrates what I have been dealing earlier, which I told earlier that many times it is simple it is energetic favorable to design the experiment centre of mass frame of reference. So, do not spend energy in accelerating or letting this centre of mass (( )). Now, let us take one more problem in which also we are using the concept of four vectors and the concept of centre of mass frame of reference, this is also very simple problem, but involve many more particles.

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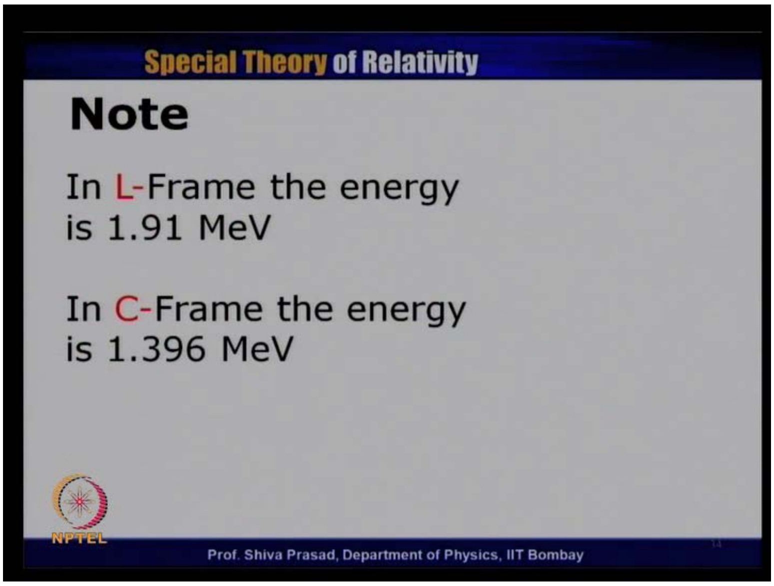

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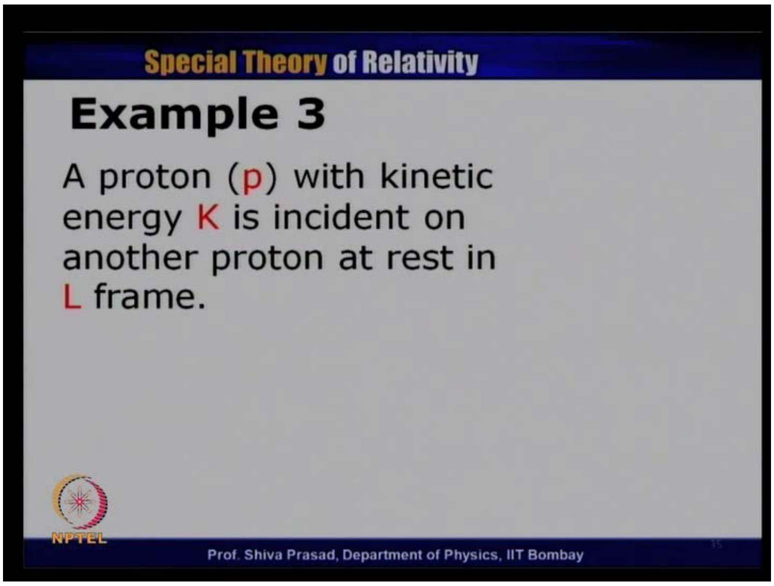

So, problem is as follows; that there is a proton p which has a kinetic energy K, this problem has been given in terms of kinetic energy. Most of the time there problems are given in terms of total energy, in this it happens to be given in terms of kinetic energy, no issues we know how to convert it, no problem. We also know the expression between energy, kinetic energy and the momentum, so we can use that expression in. So, we have a proton it has a kinetic energy K and this is incident on another proton, which is rest in the laboratory frame of reference. And even if I would not accept L frame, I would have realized that this one is centre of mass frame of reference. Because, one particle at rest, another particle is moving, so obviously, the centre of mass cannot be at rest. So, this obviously, not a centre of mass frame of reference.

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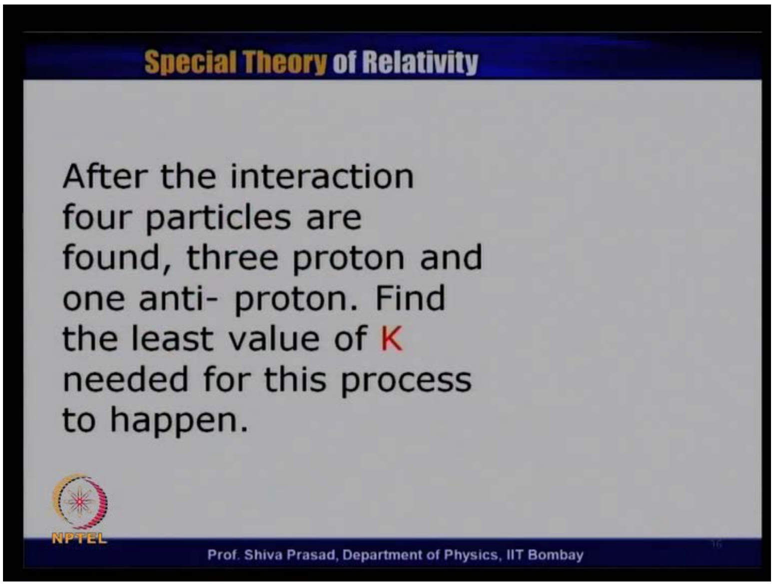

Now, this proton when it incidents on the another proton results into four different particles. After the interaction four particles are formed not two particles as in the earlier case. The four particles we have found, three of which them are proton and the fourth one is an anti-proton. Just to you sure anti-proton is has a same mass as a proton same rest mass as a proton. But it has a charge which is negative to it, it has a charge which is negative, protons are positively charged particles, while anti-proton is a negatively charged particle. So, you get three protons and one anti-proton.

Now, question is little more tricky; what should be the least value of kinetic energy K? What should be least kinetic energy that you must supply to that incident proton? So, that this particular reaction becomes possible. See from two protons you are generating four particles; three of which are protons and one of them is anti-proton. So obviously, if you at the both the protons were at rest you could have not done it, because the energy that was available to you is only two M naught c square, where M naught is a rest mass of the proton.

You are getting four particles, each one of which has a rest mass of M naught as mass of rest mass of proton it was not possible. Obviously, we require certain amount of energy, you cannot just make of conservation of energy, because momentum also has to be conserved. But if you go back to the center of mass frame of reference there certain amount of easiness. Because, in centre of mass frame of reference I am very clear initial momentum is 0 and in that particular frame of reference, to a require least amount of energy is that I can create all 4 protons at rest.

If I think in terms of centre of mass frame of reference I have 2 protons, which are coming back-to-back like this and hitting it. They have to have equal momentum because this is centre of mass frame of reference they have the same rest mass. Now, if you are giving them too much of energy you have 4 protons, which may move anywhere else, 4 protons plus 3 protons plus 1 anti-proton, 4 particles. But if I want this particular thing to have least amount of energy I can have such a velocity, such that these 4 particles are all at rest. And because all the four particles are at rest then momentum is still 0.

So, this is what I am trying to explain I have written the transparency the least energy required would be when all the four particles are created at rest. But that is only possible in the C frame of reference, because it is only in that particular frame of reference that the momentum 0. In laboratory frame the two particles, the four particles not be created at rest, because the momentum will not be conserved. Because, initial momentum is non-zero, it is only in C frame that the initial momentum is 0. So, you can imagine that all the four particles are created at rest. So, we go to center of mass frame of reference.

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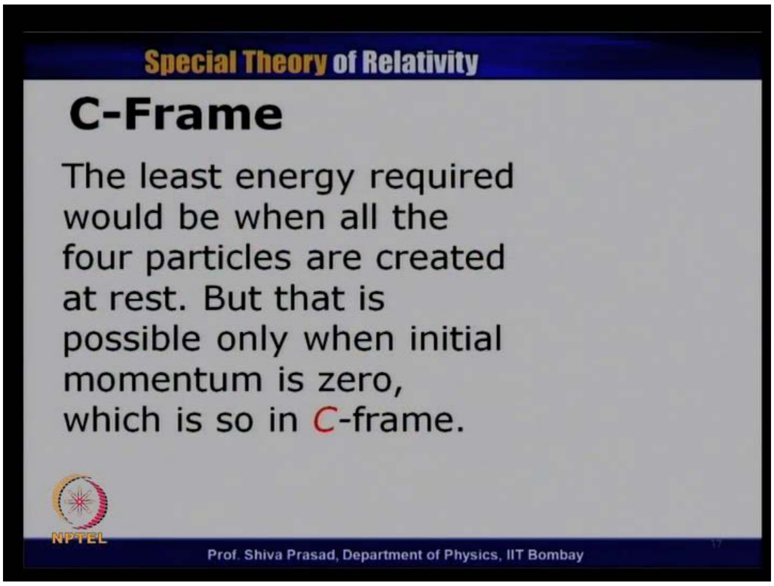

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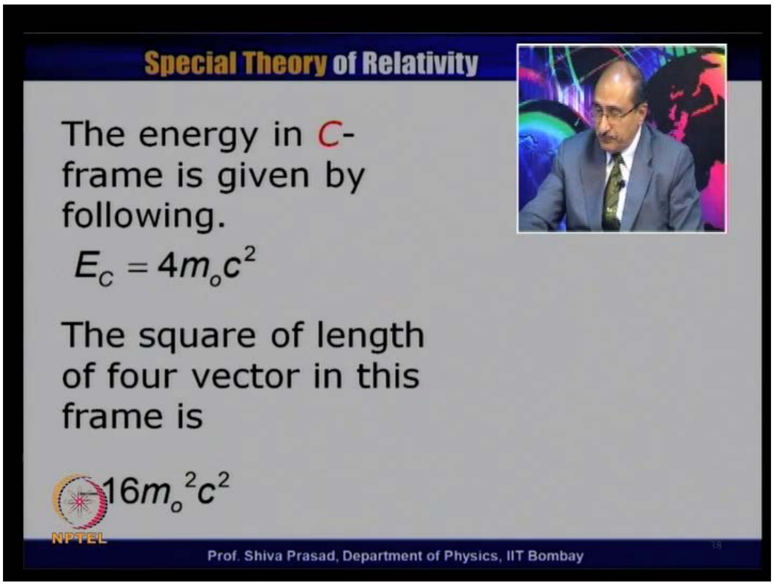

The energy is C frame corresponding to the case when all four of them are at rest must be equal to 4 m naught c square, where m naught is the rest mass of the proton. Because, each four all the four particles are at the same rest mass m naught and they are not moving, because I want the least amount of energy. So, the total energy which will be generated in the centre of mass frame of reference has to be 4 m naught c square. Therefore, the square of the length of the four vector this frame, because this momentum 0 is minus E square by c square.

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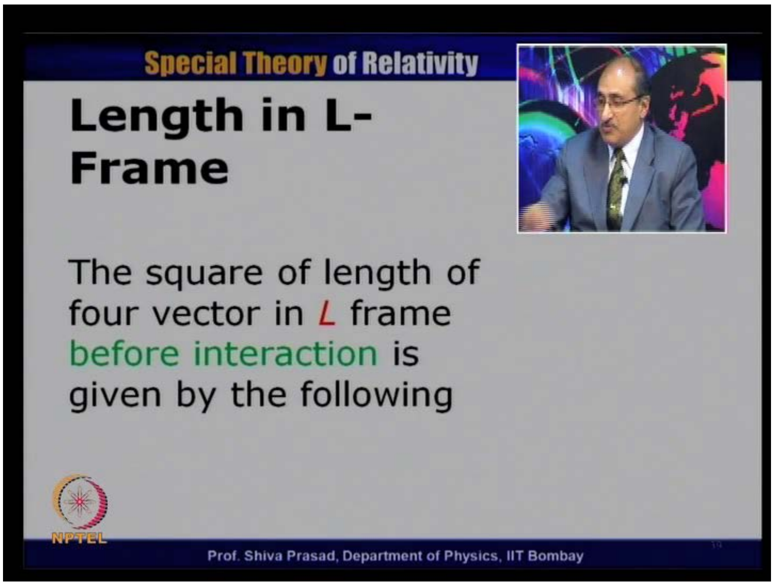

So, it must be minus 16 m naught square c square. So, this will be the length of the four vector in the centre of mass frame of reference after the reaction is over when we have got four particles. But what will be the length in the laboratory frame of reference, before the interaction we have momentum which is not equal to 0, I can find out what is a momentum, there is only one particle which is moving. So, I can find out what will be the total momentum of that particular particle moving its energy. I will know the energy of the first particle, I know the energy of the second particle. I can find out the length of the four vector just like I did in the previous problem.

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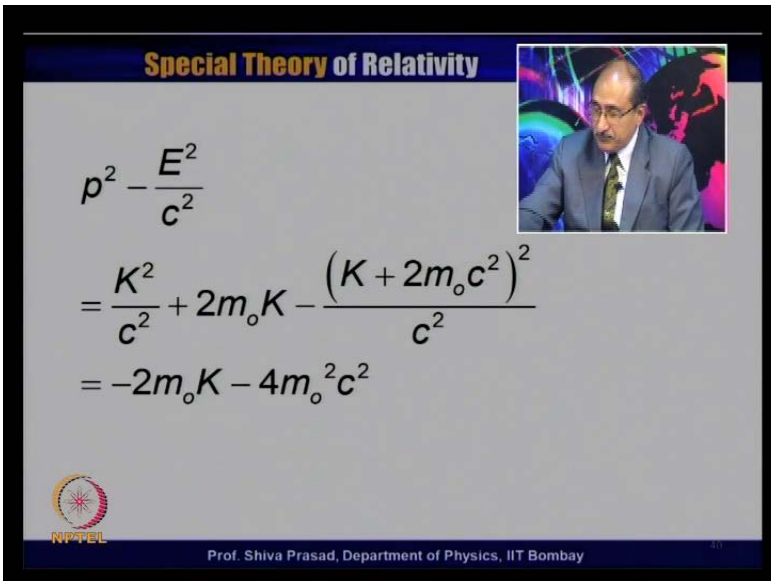

Because, there is only one term I have not written summation here, here of course, I have write the totally, not put the summation sign makes things simple. So, this p square will be given by K square by c square plus 2 m naught K, this is the standard expression relating momentum to the kinetic energy (( )), what we did earlier. The total energy, energy of the first particle which is coming with the speed is kinetic energy plus rest mass energy. So, K plus m naught c square there is another proton, which is at rest which has energy of just m naught c square.

So, total energy is K plus 2 m naught c square, this is summation of E, I have not written summation sign here, this is the total energy of the four vector. So, that the length of the four vector in laboratory frame of reference is minus 2 m naught K, minus 4 m naught square c square. Now, this length of four vector after collision has to be or after the interaction has to be same in the length frame in the laboratory frame of reference and has also be described in the in the center of mass frame of reference.

So, what I do I take the length of the four vector of the system of the particles before interaction in L frame and equate it to the length of the four vector after reaction in C frame. So, this is what I am doing I am taking before interaction the length laboratory frame and equating it to the length of the four vector, in after the reaction in C frame, which I know is minus 16 m naught square c to the power 4, if I want the least energy.

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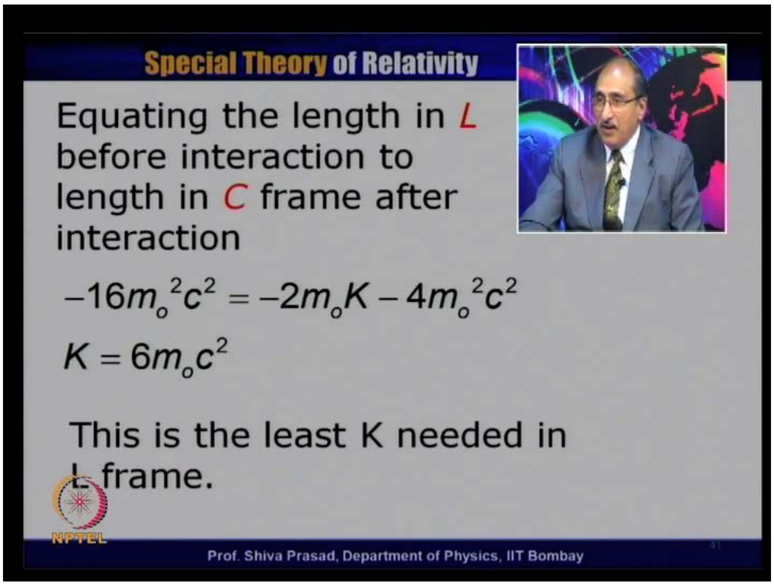

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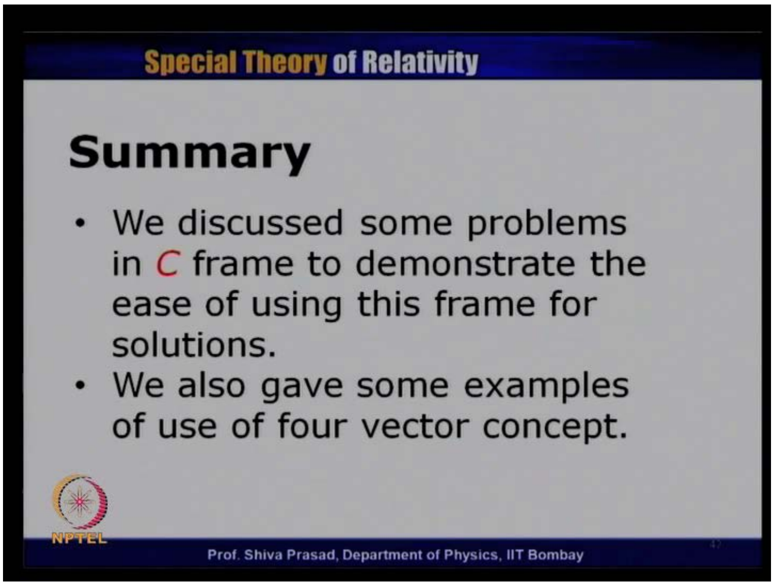

This is what I have written equating the length in L before interaction to the length in C frame after interaction. I must have minus 16 m naught square c to the power m naught square c square, given by this particular expression, I solve this K terms out to be equal to 6 m naught c square. So, this is the least k needed in laboratory frame. So, in laboratory frame of reference this particular particle must have kinetic energy of at least 6 m naught c square in order that reaction becomes possible. In summary we have discussed some problems in center of mass frame of reference and demonstrated how it becomes easy to solve these problems and we also gave some examples of the use of four vector concept.

Thank you.