Special Theory of Relativity
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Lecture - 18
Doppler Effect in Light

Hello, in our last lecture we discussed the concept of photon. We said that special theory of relativity gives us a possibility that a particular particle may have 0 rest mass and may still carry energy and momentum. In classical mechanics, we can never imagine a particle with 0 rest mass, but special theory of relativity gives us that possibility. And we discussed saying that the light can also be imagined as consisting of particles which have 0 rest mass, and they are called photon. Of course, we have a condition that if the particle has a 0 rest mass, it must travel with speed of light, and we know that light travels with speed of light. So, this is consistent in saying that light consist of particles which has photons.

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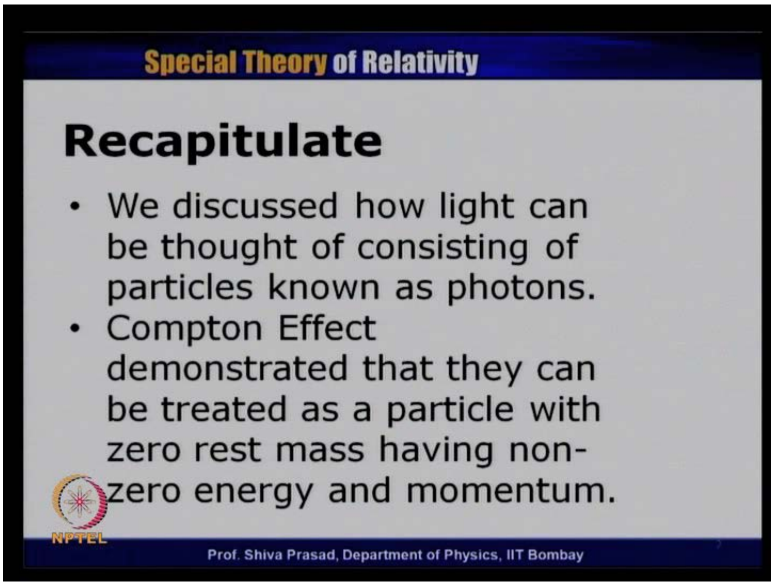

So, this is what we have said we discussed how light can be thought of consisting of particles known as photons. We also discussed compton effect in our last lecture, which was one of the experimental proof. In which we treated photon like just any particle and assumed that this gets collided or get scattered with an electron, and treated just this as like viewed in classical mechanics discussed any other collision process or scattering process, except for the fact that we utilized lit-eristic expressions.

Then we found that the frequency or the energy of the photon after scattering would get changed and we could evaluate that depending upon the angle of scattering what will be the changed wavelength or changed frequency or changed energy, then we described experimental way of determining this change and we found that the two are consistent. In that sense compt on effect is a very interesting experiment, because it provides us a proof shows a proof that light can be really thought of consisting of particles, which have 0 rest mass which carry momentum and energy like any other particle.

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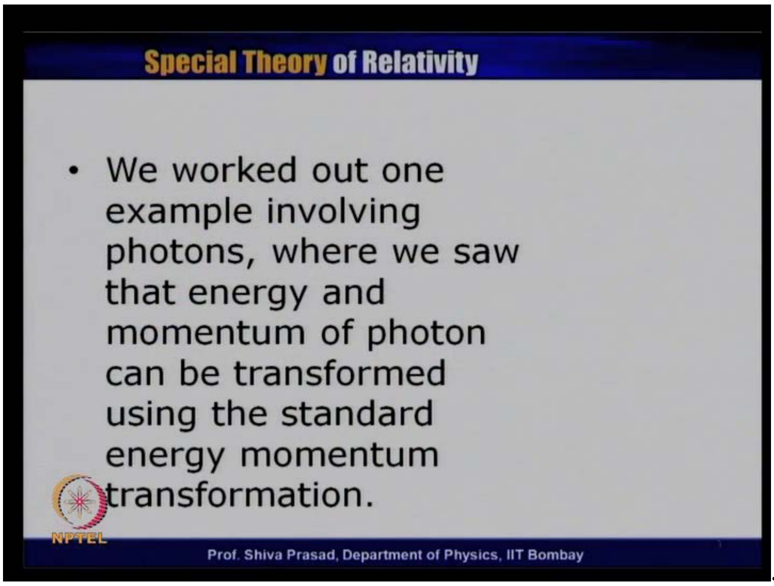

Then we worked out an example, where we saw that energy and momentum of photon can be transformed using standard energy momentum transformation. See as far as relativity is concerned there is nothing special about photon except for the fact that it has a 0 rest mass. There can be particles with different masses different rest masses similarly, photon is also a particle with 0 rest mass therefore, it does not require any other special treatment other than the fact that m naught is equal to 0. So, all the transformation expressions that we have derived earlier should also be valid for the case of photon and we dealt with an example in our last lecture, where we really saw that we can apply exactly the same type of energy momentum transformation that we can apply to any other particle we can also apply to the case of protons. Now, let us go ahead and discuss what we call as a Doppler effect?

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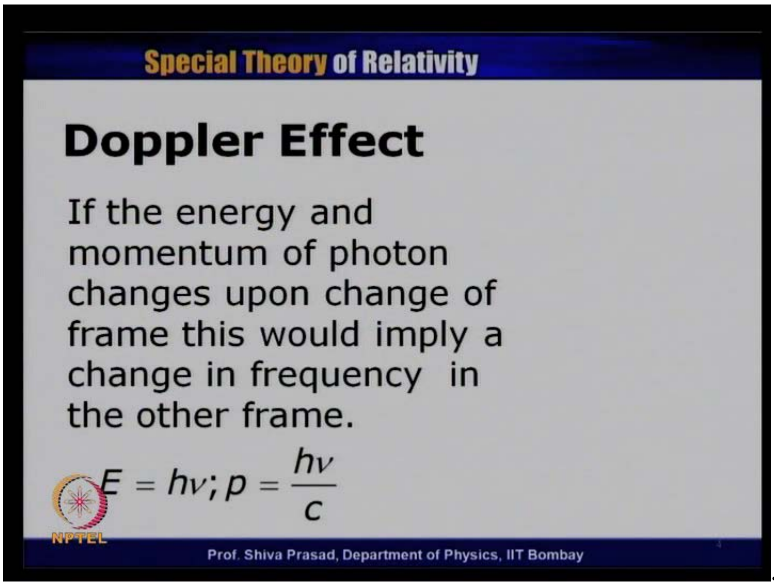

If we change a frame of reference we know that the speed of light will not change that is what we have always said, this is one of the postulates of relativity but, we have just now said that is energy and momentum will get transformed it means they would change if the frame of reference changes, and what energy and momentum is related to in the case of photon is by this expression e is equal to h nu energy is equal to h nu and momentum p is equal to h nu by c; it means, change of energy, change of momentum, implies a change of frequency; it means, if i change my frame of reference then this is equivalent of saying that the frequency of the photon has changed though this speed has not changed remember. So, its frequency has changed, and this change of frequency once we go from one frame to another frame is called Doppler effect. So, that is what we have written if the energy and momentum of the photon changes upon change of frame this would imply a change in frequency in the other frame, this is called Doppler effect.

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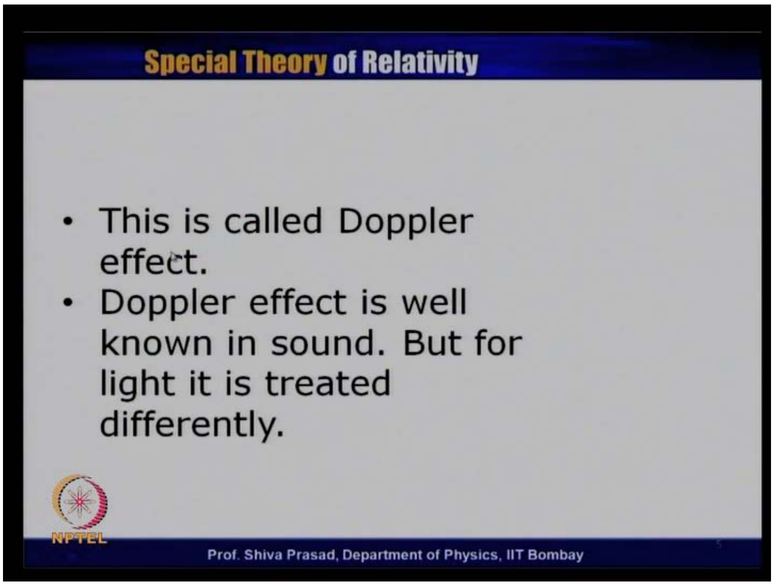

Doppler effect is very well-known in sound effect, the first example that we have always been giving of Doppler effect is a sound, which many times we see in our daily life that a train is coming towards us we find as if the frequency of these is viseling, the frequency of the visel from the train is appears to be of high pitch higher frequency was the train we find that this frequency is of lower frequency, this a well-known Doppler effect and very well explained understood by the wave theory in the classical mechanics case. So, this is similar to that but, they are certain different ways of treating it, because of the fact that I m going to describe next because of the relativistic effect the light has to be treated somewhat differently from the sound Doppler effect.

Let us just recapitulate the sound Doppler effect then we come to the actual expression of the light Doppler effect. In sound Doppler effect we get two different expressions let us take simple case, one case in which observer is stationary and the source is moving, another case in which source is stationary and observer is moving. These two cases in the case of sound are treated differently, because of the main reason that sound requires a medium to travel. So, whenever I am talking of the speed of let us say source, which is emitting sound or an observer, which is hearing that sound the motion or the speed of velocity of these source and the observer are always defined with respect to the medium, and these two expressions these two cases when observer is moving and the source is at rest and the source is moving and observer is at rest Corresponds to different cases will explain this in a very simple fashion.

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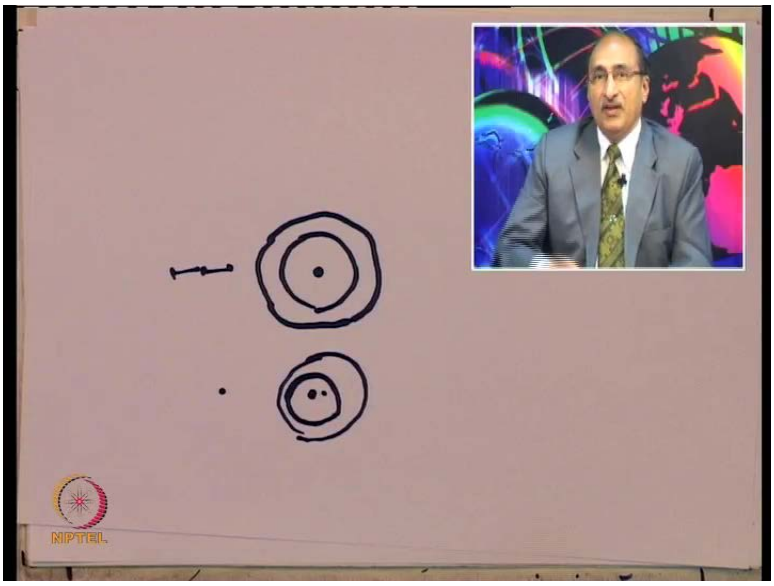

Suppose, the observer this I m sorry the source is at rest and this is the entire air medium in which is emitting sound waves then all these sound waves will move out this spherical wave front from this particular source. Now, these wave fronts these are sort of circles and I have not drawn very well. So, these are sort of circles.

Now these spherical wave fronts move in the air medium because sound requires this medium to travel; sound is essentially mechanical wave. In which this medium which requires medium this vibrations are carry forward in this particular air. If you do not have this air or any other medium we will not have will not hear the sound. So, this particular medium, which I am assuming air these waves are all generated these wave fronts are created in this particular medium. If a person is walking away or walking towards this or walking away from this then the rate at which its sort of listens at which it comes across these wave fronts is going to be different from the original cases. Now, if you have a different situation when this particular emitter of sound or the source of sound itself is travelling but, the person is stationary we will find that these particular wave fronts they themselves are created in different fashion.

Because initially when the source was here it created away from something like this and when it moves towards this particular person next time the wave fronts will be centered according to this person. Here, remember with respect to the medium, this person has travel from here to here. So, now the wave front with the circular with particular point in the center. Sorry, my figure is not very good but, essentially it means the wave the disturbance is created inside the medium that itself is different. So, the two situations are different and the expression of change of frequency that we get corresponding to these two cases that also turn out to be different.

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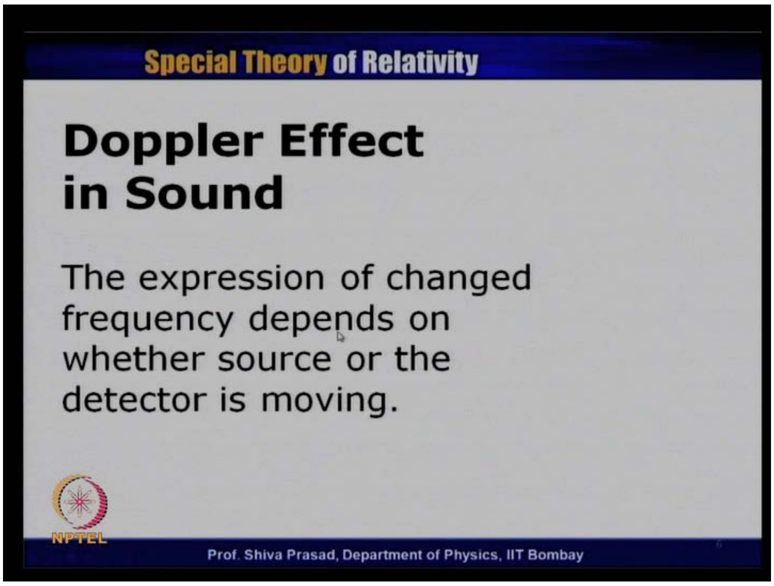

So, this is what I have written the expression of changed frequency as for Doppler effect sound is concerned depends on whether the source or the detector which is free.

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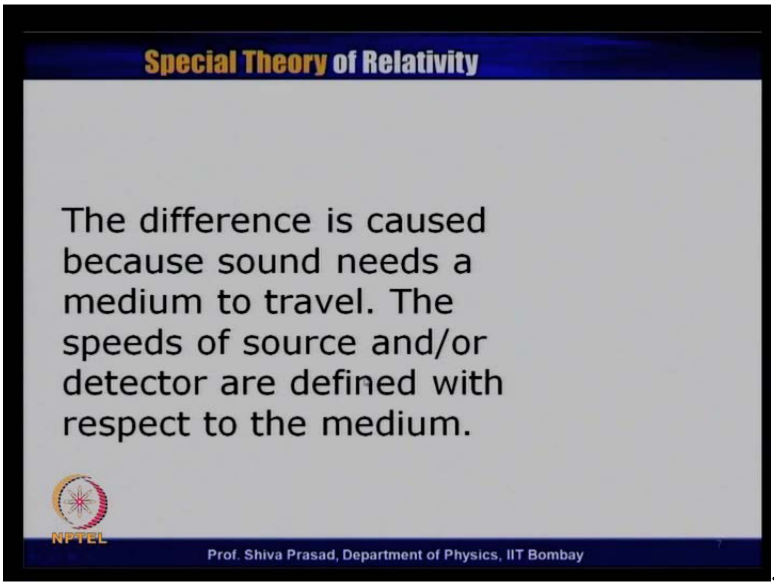

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The difference is caused because sound needs a medium to travel the speed or source and or detector are defined with respect to the medium.

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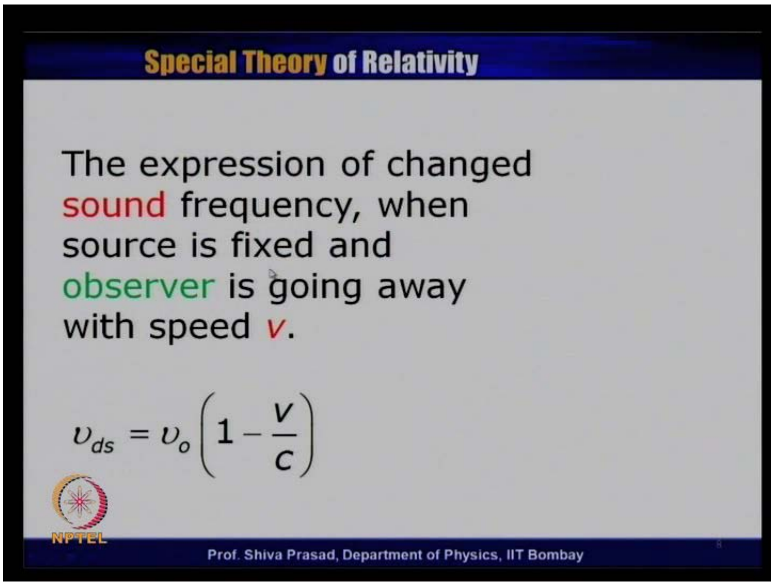

The expression of changed sound frequency when source is fixed and observer is going away with speed v is given by this, means if we have source which is fixed and this observer, which is

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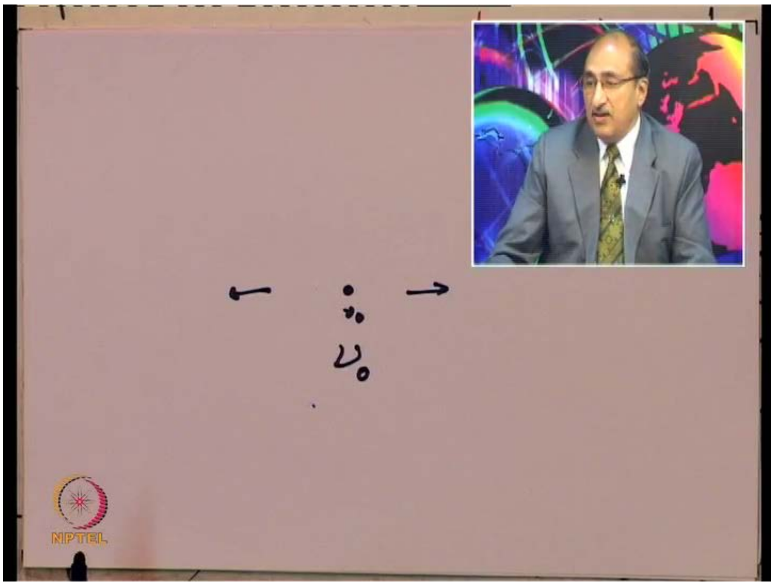

Going away from here like this or observer is going like this. If nu naught is the frequency which is being emitted by this particular source write nu naught is the frequency which is emitted by this particular source the frequency which will be heard by these two persons person’s, which are moving opposite direction their distance is increasing will be given by this expression nu d s is the changed frequency which I am calling as Doppler shifted.

Frequency is equal to nu naught which is the original frequency, which is being emitted by the source multiplied by 1 minus v upon c, where v is the speed of the observer the speed with which its going away with the number speed is related to the medium. So, that is what I have said, the expression of changed sound frequency

When source is fixed and observer is going away with a speed v is given by this is the new frequency this the frequency which will be heard by the person. On the other hand if the source is self is moving which i have said is fundamentally different from the earlier case then I will get slight different expression, and this expression will be given by nu d s, which is Doppler shifted frequency is equal to nu naught, which is the original frequency divided by multiplied by 1 divided by 1 plus v by c, where v is the speed with which the source is going away from the observer. So, this is I have written the expression of changed sound frequency when the observer is fixed and the source is going away with speed v is given by this. Now, if you are you can probably realize that the case of light has to be different, because as we have said of course,, whenever olden days we have been talking about ether medium that light requires an ether medium to travel but, special theory of relativity discarded that concept, we do not require a medium light to travel.

Therefore, we cannot create a situation when the light source is coming there is no medium in which these wave fronts are created therefore, whether the observer is moving towards source or source is moving towards observer these two things in relativity have to be treated exactly identically cannot be any difference between these two, because these two speeds all of we can talk is relative speeds between the two there is no medium in which the light is travelling unlike the sound wave. Therefore, I must get the same expression in both these cases when we are talking of Doppler shift of Light.

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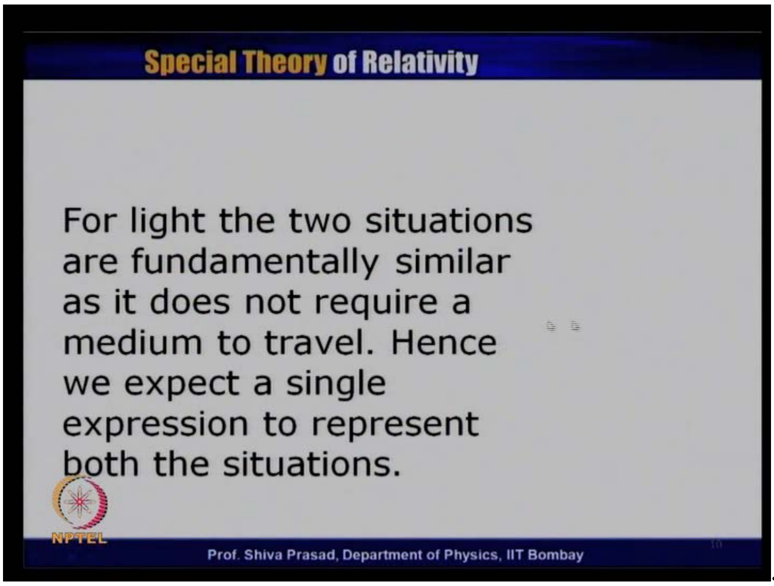

This is what I have written for light the two situations are fundamentally similar as it does not require a medium to travel. Hence we expect a single expression to represent both the situations. The two situations cannot be different I can hope you can understand the difference between a light and a sound requiring a medium to travel therefore, I can define the velocity of an observer and a source relative to that medium, while light do not does not require a medium to travel. So, if I talk of velocity and I can only talk of relative velocity between observer and the source, I cannot talk with respect to anything else because the there is no ether. So, let us first consider a case of what we call as the longitudinal Doppler effect? This is the picture, which gives what is called a longitudinal Doppler effect?

Let us first spend little bit of time understand this we get. So, this figure is essentially. Simple, we have just to fix our idea we can imagine that this source this particular person as a frame is ground frame of reference and assuming than the ground is an inertial frame of reference just to fix our idea its always simple to imagine things, and there is a train or a garage which is moving towards right with a speed or with velocity v. This particular light or this particular sorry this particular garage or this particular train emits light in backward direction.

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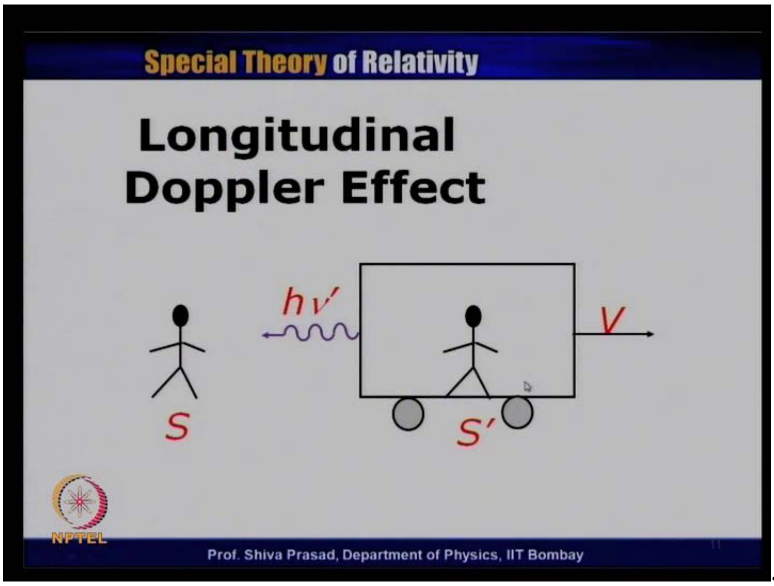

See remember the figure the way we have gone in case the light has to reach this particular observer s which is sitting on the ground it has to be emitted only this way if it is emitted this way this will not be seen by the observer as here. So, I am imagining this particular case that the motion is collinear the observer s sitting on the ground these another observer s prime, which is standing on the train which through some older somewhere shines a light in this particular direction,. So, that this particular light is seen by this particular observer. Now, when I say source frequency essentially it means frequency which is seen in the frame of reference of the source. Here, everything is contained in this particular garage train garage, which I am calling as s prime frame of reference. So, I can say that this frequency nu prime is the frequency of light as being measured by observer s prime so, its frequency which is measured s prime frame of reference or another words this h nu prime is the energy of the photon as measured in s prime frame of reference.

Now, my question is what will be the frequency which will be seen of the light? By this particular observer as and this particular changed frequency will be what we will call as Doppler shifted frequency. So, all I have done in the same problem of Doppler effect I have changed the language into the relative language or the language which we have been using of different frames. Now, if I have to find out what is the frequency of this particular light in this particular frame has essentially I have all to find out what will be the energy of this particular photon in s frame of reference? All I need is a energy momentum transformation. Once i do the transformation of energy I will know what is the energy of the photon as measured in s frame of reference remember we have said, photon is like any other particle. So, when we can apply momentum conservation then I go from s prime to s to s prime for any other particle I can equally apply also for the photon. So, my problem is very straight forward the Doppler effect in light can be treated extremely simple; simply all I have to is to apply an energy transformation.

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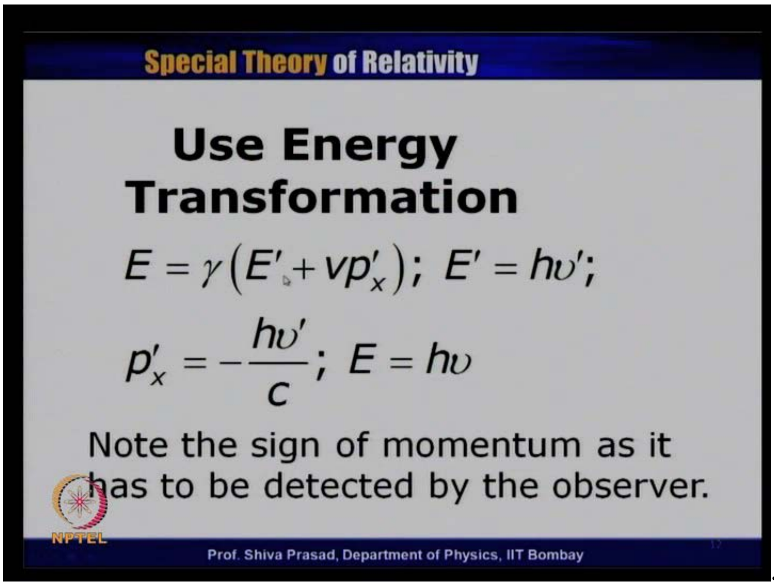

So, that is what I am doing in the next transparency. So, I have applied energy transformation, and if you remember the energy transformation I am applying a inverse transition this energy e in s frame will be given by gamma multiplied by e prime, where energy e prime is the energy s prime frame of reference plus v, which is a relative velocity between the frames and p x prime is the momentum of the particle in s prime frame of reference. Now, we have just now said, that the energy of the photon as seen in s prime frame of reference h nu prime therefore, I can write e prime is equal to h nu prime. Only thing I would like you to be attention to the fact that this particular photon has to be emitted in minus x direction. So, it reaches the observer s just look at the earlier picture.

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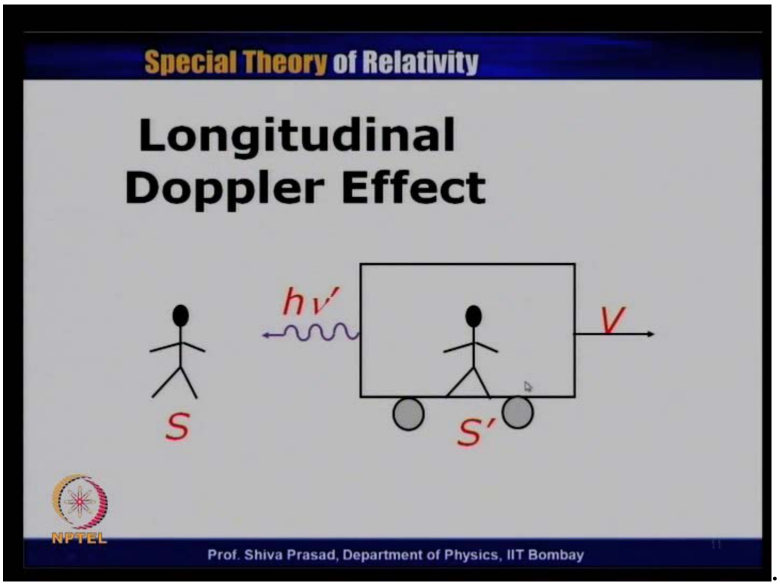

This photon has to be this is the direction of the relative motions so, this is my plus x direction this photon is being emitted in minus x direction, So, s to reach s therefore, this momentum must have a negative sign must be minus x direction.

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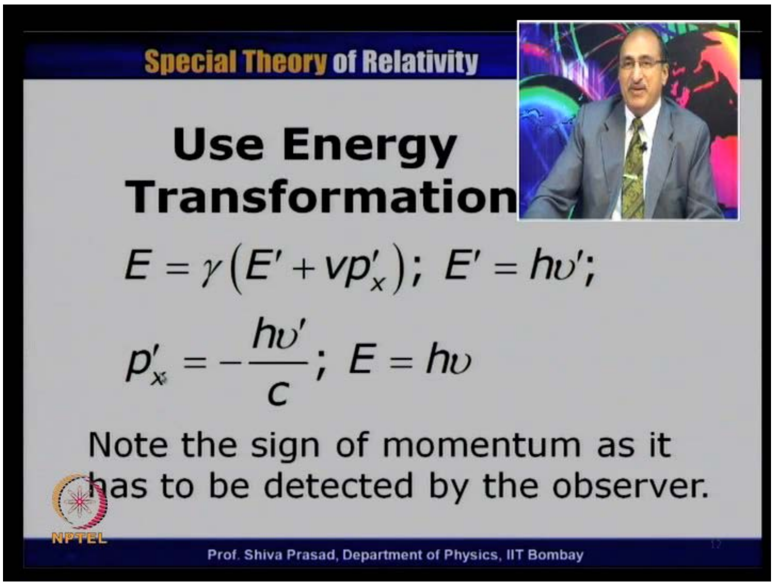

Therefore, I am writing p x prime as minus h nu prime by c this what I have done I am writing p x prime is equal to minus h nu prime by c, because if this momentum was this particular photon was travelling in a plus x direction then it will not be seen by that because we can create various types of situation the final expression will always turn out to the same this situation which I have created is the one in which this garage is moving forward and the photon is moving emitted backwards. So, that it can be seen by an observer s. So, I write p x prime is equal to minus h nu prime by c and the energy as seen by an observer s will be termed as h nu.

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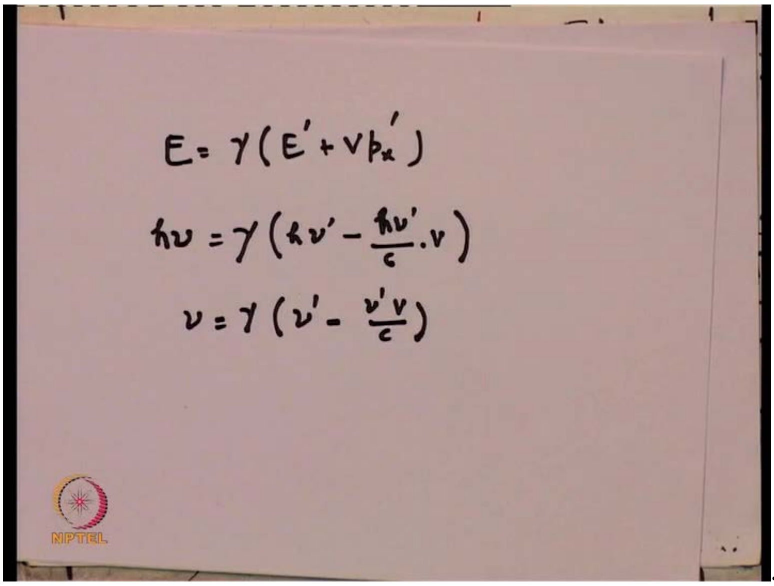

So, e is equal to h nu. In this particular expression I substitute e is equal to h nu and e prime is equal to h nu prime. So, this was my original expression e is equal to gamma e prime plus v p x prime for this e I will write h nu, which is the changed frequency this remains as gamma.

This e prime I will write as h nu prime this v is plus x direction but, this p x prime is minus x direction. So, I will put a negative sign, p prime p x prime will be given by h nu prime by c multiplied by v. Therefore, if I cancel h I will get nu is equal to gamma nu prime minus nu prime v by c. So, this is what i am writing in the next transparency. See I have just

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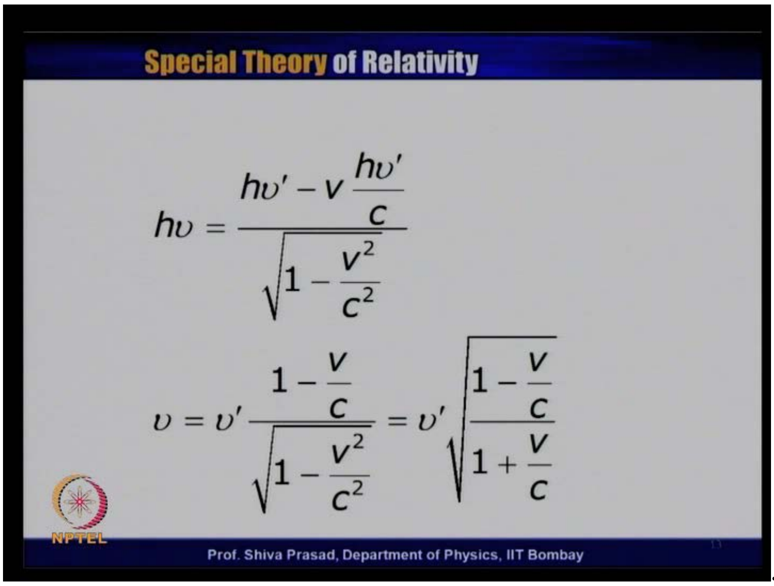

Substituted h nu is equal to h nu prime minus v h nu prime by c and this gamma have expanded written as under root 1 minus v square by c square, cancelling these h we get an expression nu is equal to nu prime, because I have taken this out. So, this will become one minus v by c divided by under root one minus v square by c square. So, this is what I can expression remember nu prime was the frequency s prime frame of reference, nu is the frequency in the s frame of reference. Now, this under root 1 minus v square by c square I can write as under root 1 minus v by c under root 1 minus one plus v by c 1 plus v by c.

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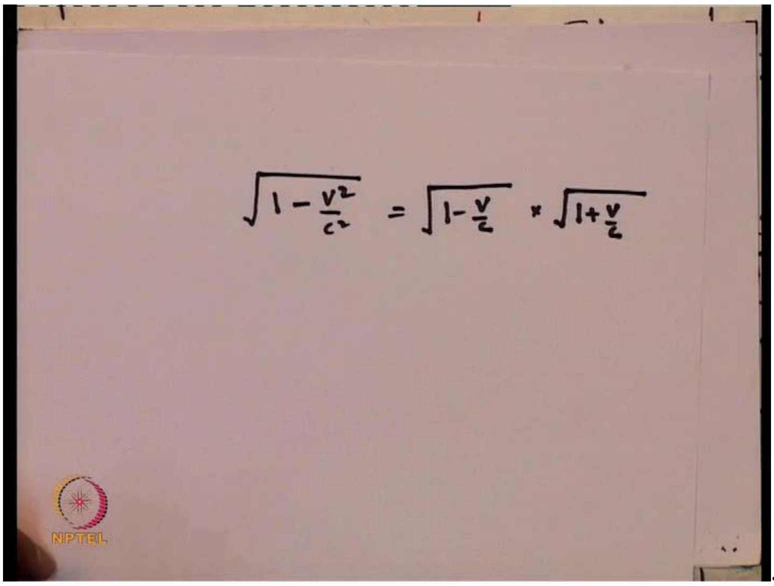

You just factorize a square minus b square is equal to a minus a plus b. Now, in the numerator we had one minus v upon c. So, this will cancel with this root of that will cancel with a root of 1 minus v by c giving with this particular expression which I have written here, nu prime nu is equal to nu prime under root one minus v by c upon under root one plus v by c. Please remember this root of this has been cancelled with under root one minus v by c under root one plus v by c still remains here and on the numerator there is an under root sign remains here. So, the new expression that I have brought is nu is equal to nu prime under root one minus v by c plus divided by one plus v by c. Now, what I would like to just mention that in Doppler effect if we try to use this slightly different type of notation. So, let us not get confuse with that particular thing and see remember in relativity language

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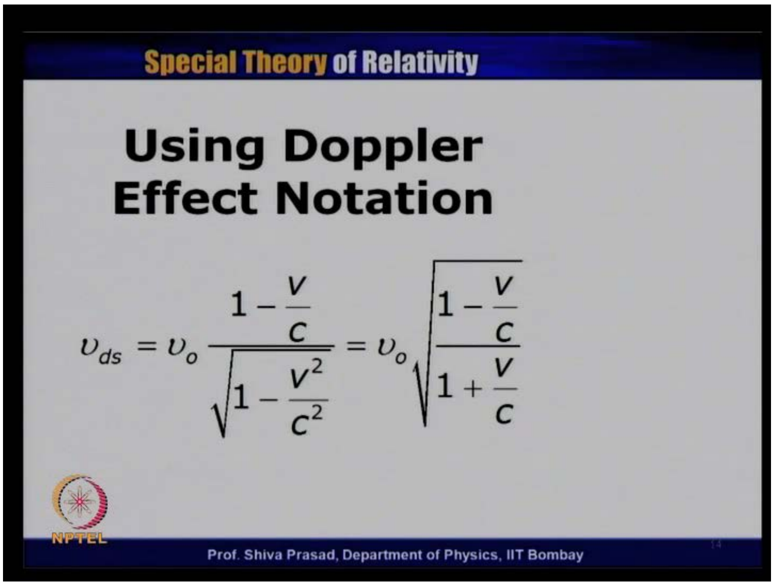

We called nu prime as frequency or h nu prime as energy measured in s prime frame of reference and nu or as a frequency or h nu as energy measured in s frame of reference but, in the language of Doppler effect the frequency, which was emitted by s prime we call that as original frequency, which we call as a nu naught which is frequency of the source and the

Frequency which we have seen can any other frame we call that frequency as the Doppler shifted frequency. So, I am just changing this particular thing; I am changing nu to nu d s and I am changing nu prime to nu naught. So, if I do that this what I call as a Doppler effect notation, I get same expression just changing this particular thing nu d s is equal to nu naught multiplied by 1 minus v upon c under root 1 minus v square by c square same expression which I wrote earlier, which is equal to nu naught under root 1 minus v by c divided by 1 plus v by c. This is the original frequency this is the Doppler shifted frequency same thing has been re-nomen clatured depending upon the way we want to talk. In Doppler effect, we talk of original frequency and the shifted frequency while in relativity we talk of s frequency in s frame that frequency in s prime frame of reference. Now, this particular expression can be simplified little bit, if we assume that the relative velocity between the frames is comparatively small in comparison to c as we given some examples: today we find this particular Doppler effect in light to be quite useful in many of the experimental techniques that we use. So, and in many of those cases the relative velocity between the frames is not that large,. So, we just approximately this particular expression under the condition that v upon c is very small in comparison to one. So, that is what I am trying to show in the next transparency. See remember my expression was one minus v by c under root this was in the numerator..

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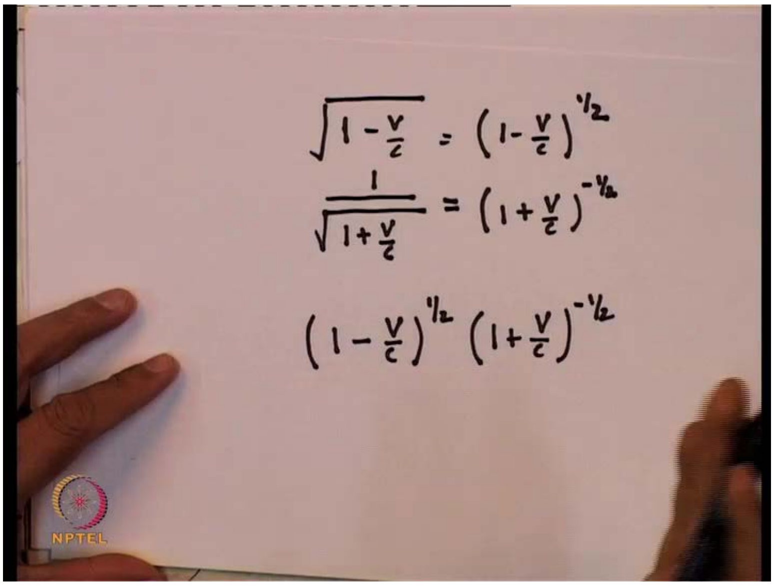

So, this can be written as 1 minus v by c to the power of half. In denominator we get an expression of 1 plus v by c this I bring to numerator this expression or let me this one divided by this can be written as one plus v by c to the power minus half, because this was in denominator this already under root. So, once I take numerator the power becomes negative. So, this I can write as 1 plus v by c to the power minus half. So, when I multiplied these two at multiply this by this expression I will get 1 minus v by c to the power half multiplied by plus v by c into the power minus half this minus sign has come call them as denominator. Now, because we are assuming that v by c is very small in comparatively v by c is very small in comparison to 1, I can expand this into binomial expression and neglect higher order terms retain only the first two terms and will get first to first approximation that in the limit of lower relatives velocity between the frames what will be the change into frequency.

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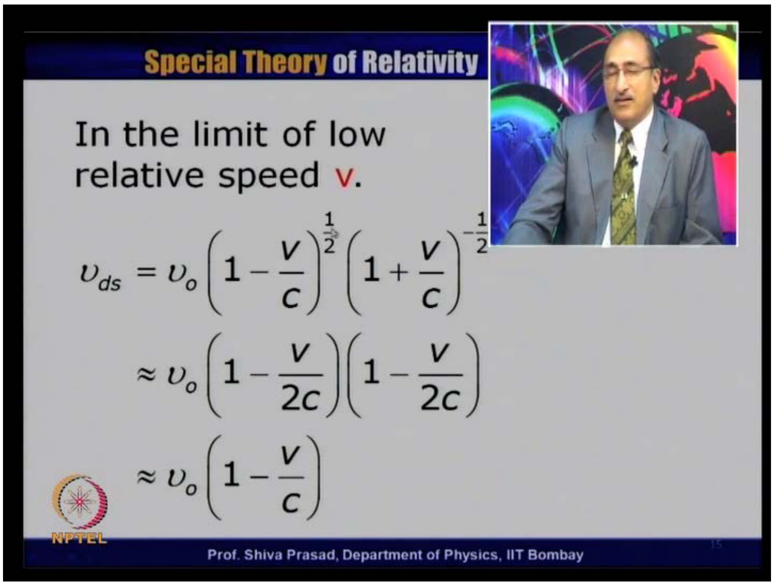

So, this what I have written this particular transparency. So, we nu d s is written as nu naught is equal to 1 minus v by c to the power half multiplied by 1 plus v by c to the power minus half, which is approximately by expanding this. So, this becomes 1 minus let us say n times n times by c n is half, which is the power. So, this becomes one minus v by two c.

Similarly, here is a plus and the power is minus half. So, this becomes one minus v by two c. So, this minus half gets multiplied here for the first term 1 plus n x 1 plus if you have to expand one plus x to the power n this, because one plus n x to the first term. So, this I cannot multiply neglects second order terms. So, one multiplied by one will give me just 1, which is this 1 when this minus v by t c gets two c gets multiplied by this one I will get minus v upon c similarly, this minus v upon two c, where it gets multiplied by this one I will get another minus v by two c when I add these two I will get one minus v by c but, just can simply work out this is very simple way just open opening up multiplying this brackets we of course, will have term v square upon 4 c square which I will neglect because this is second order term in v upon c.

So, I have find that Doppler effect effect frequency is given by approximately nu naught multiplied by 1 minus v by c which happens to be the same expression which we have derived I will not derive but, for the case of sound when we use the expression for the case when your source was fixed and train was moving. Now, there is another type of

Doppler effect which does not happen in sound, which is called transfer Doppler which is purely relative.

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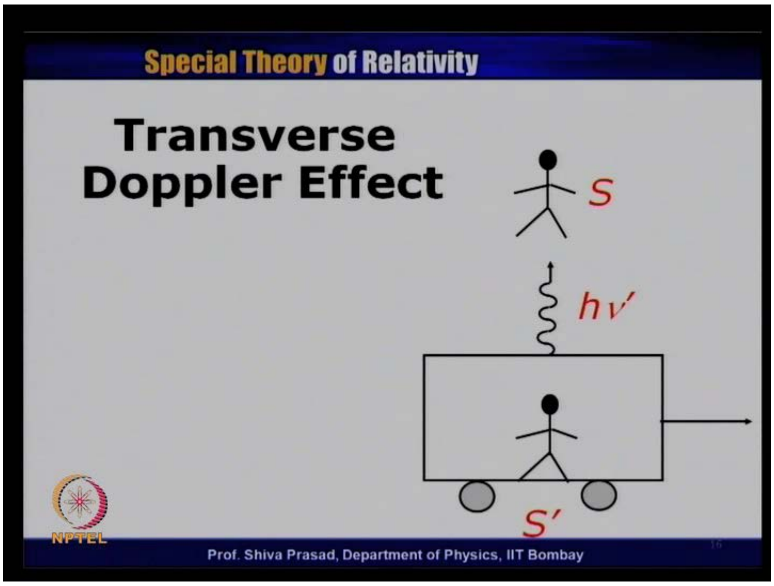

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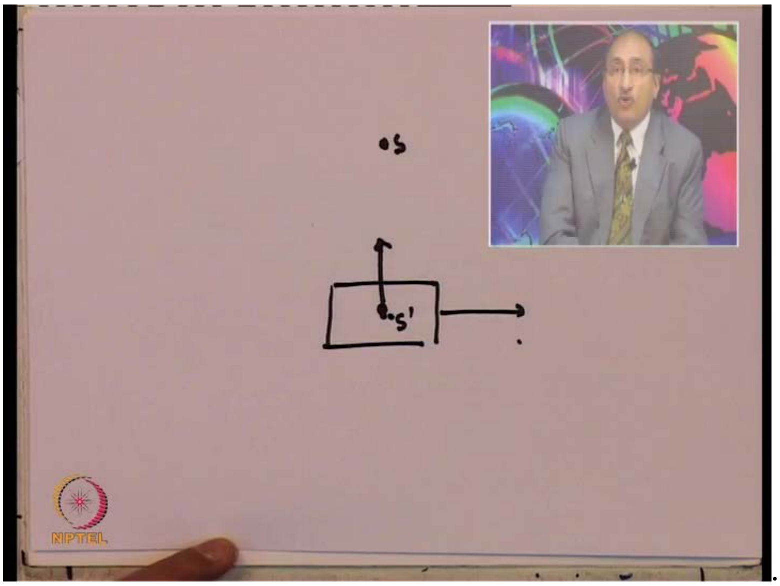

See remember in longitudinal Doppler effect we assume the source is here and train is moving just in the same line or look at the situation which is given in this particular field. Let us assume that this particular person s is standing here and this train is train tracking in front of him and this train is going like that, and this particular train is emitting sound in a direction perpendicular remember when I say perpendicular let us be careful when I say perpendicular its such that as for this particular observer s is concerned to him.

It appears that this particular its really sounds its light I am sorry that this light is travelling towards him. So, essentially it is a situation something like this that this particular train is moving like this let us just represent it by one single point here and this observer s is here and this observer s prime is here, according to s when this particular observer was just perpendicular to this particular direction of motion light is emitted towards same as seen by observer as see remember according to s prime this particular light will not really move perpendicular to this motion but, this particular observer s receives light towards him in a direction perpendicular to the direction of the motion of the train remember observer s is observing the motion of train in this particular direction and same observer as finds that light is travelling in a direction perpendicular to the motion or perpendicular to the velocity direction and it reaches the same.

So, this somewhat the same situation, which is easy now we can find out using again energy momentum transformation that if this is the frequency for this h nu prime is the energy of the photon which is emitted by this s prime observer what will be the frequency of this particular photon as will be seen by the observer s this is what we call as a transfers Doppler effect in sound we do not see such type of transfers Doppler effect is purely relativistic.

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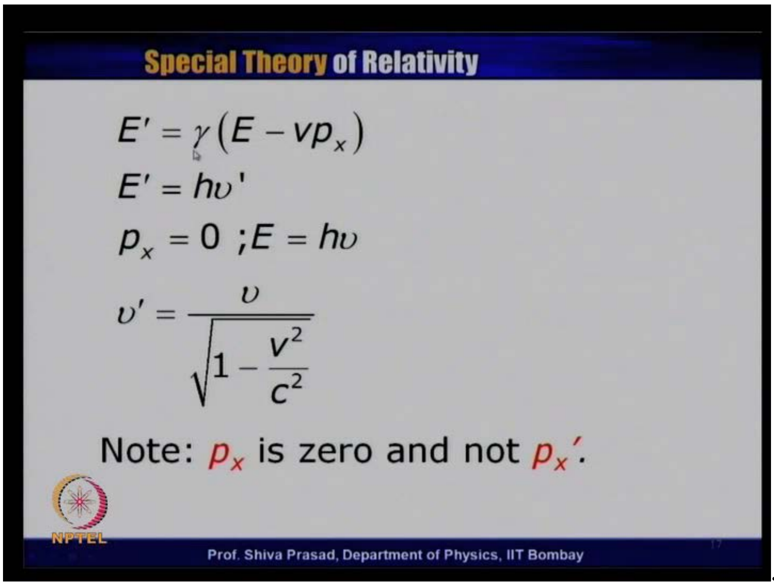

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Phenomenon and let us work it out. We use a direct energy transformation, which means e prime is equal to gamma e minus v times p x, because this is a direct transformation therefore, there is a negative sign there is no positive sign remember I have used direct transformation because I know that p x has to be 0, because remember x is the direction of relative velocity and according to an observer in s frame this particular photon is moving purely in y or minus y direction perpendicular to x direction therefore, p x is 0 let me come back to this particular figure which I have drawn here in this particular paper.

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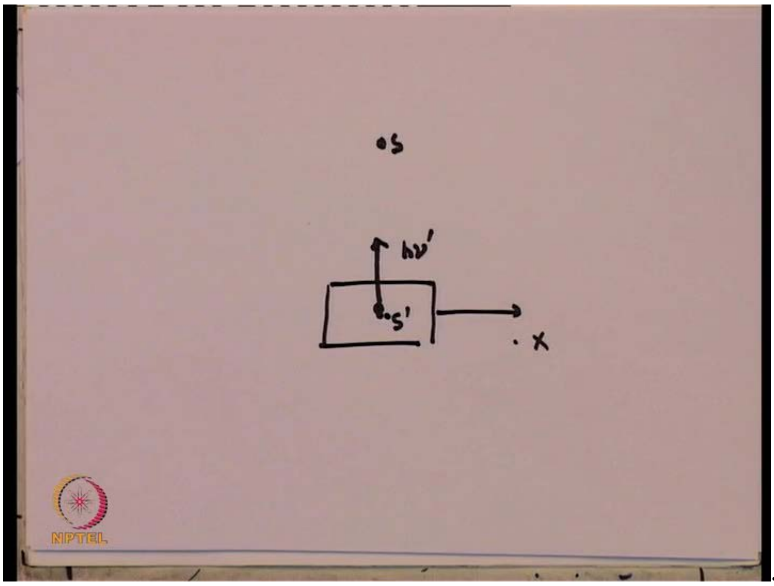

Here there is a s which is this motion is in this particular direction. So, this is by x direction now according to this observer s this photon is travelling in this particular direction. Therefore, according to observer s the x component of the momentum of the photon is 0. Now, in s prime we faced through a photon towards I am using the word through if it has to emit a photon in this particular direction this particular photon because this is moving relative to s would not be exactly in a direction perpendicular to it.

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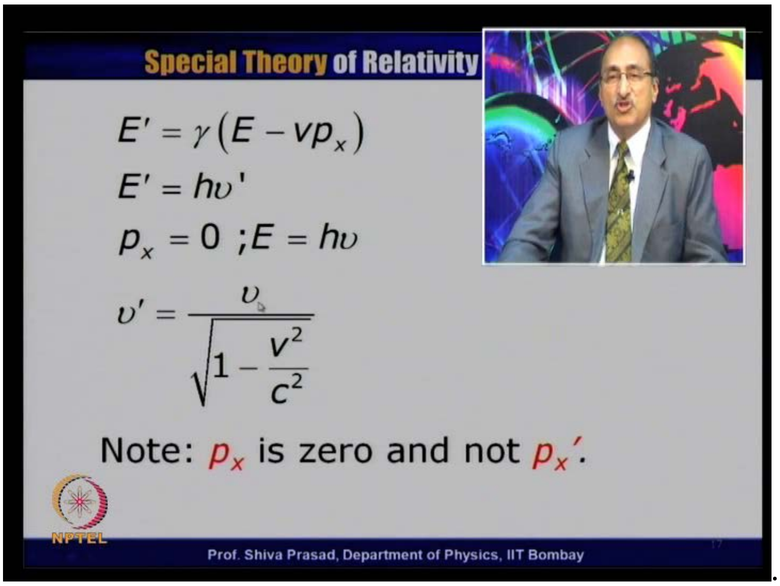

So, let us not get confused it s not p x prime which is 0 but, its p x which is 0. So, e prime is equal to gamma e minus v p x and with p x is equal to 0 of course,, using exactly the same way e prime is equal to h nu prime and e is equal to h nu. So, I put for e prime h nu prime and for e is equal to h nu p x is 0. So, I just get nu prime is equal to gamma nu again I change these two by relativistic notation my Doppler effect notation this nu prime was original frequency nu naught this nu is Doppler shifted frequency.

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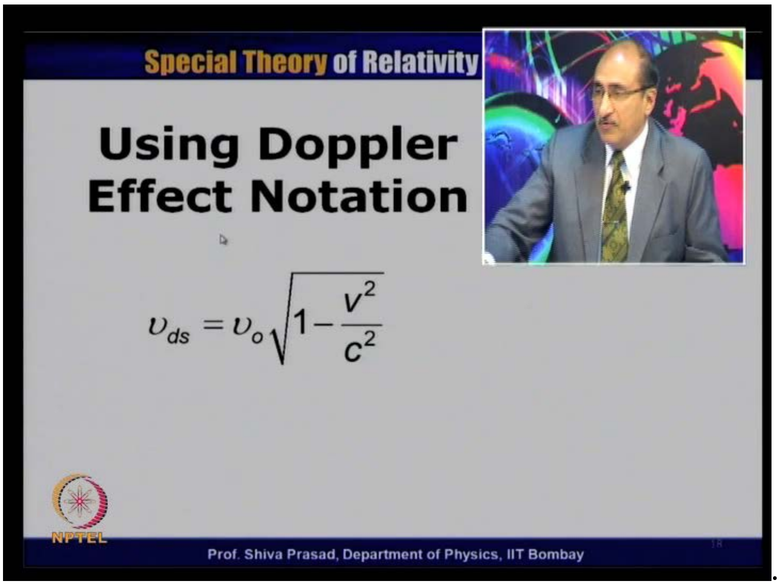

Because this is being observed by observer s I will call this as nu d s this I will call as nu naught this what I get the expression nu d s is equal to nu naught under root 1 minus v square by c square. So, this is as I said, a purely relativistic effect called Transfers Doppler effect. If you commence if you look at the expression that we have obtained for the case of light for example, for longitudinal Doppler's effect we can very easily see that, if the light source is moving towards the observer its frequency will go up if it is moving away its frequency will go down remember the expression that we had found out was nu d s is equal to nu naught 1 minus v by c.

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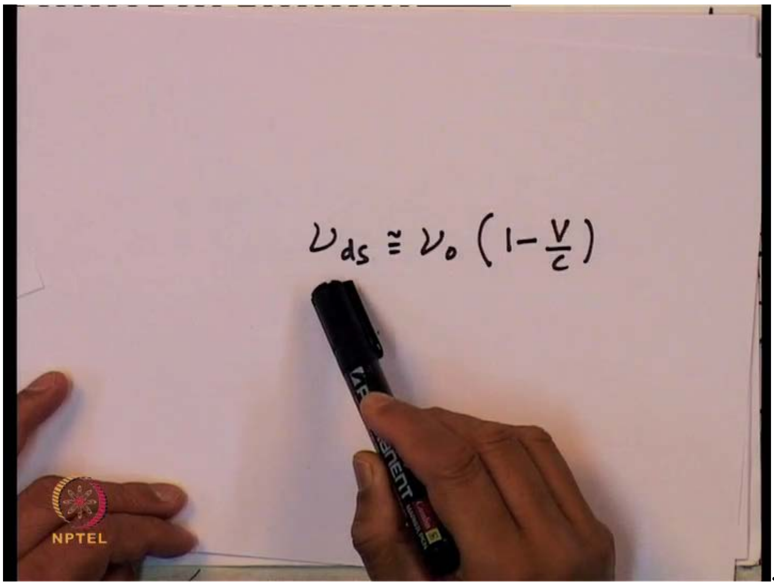

We have shown this particular expression approximately we put it approximately. Now, clearly if v is positive it means remember we have taken the motion to be in minus x direction means if the source is moving away I am sorry if the source is moving away from the observer.

Then this particular case one minus v by c will be smaller than one and therefore, nu d s will be smaller than nu naught therefore, if the train is going away from the observer we find the Doppler shifted frequency to be smaller. If it is moving towards him then this v has to be changed sign this will become one plus v by c and in that case nu d s will be larger. So, this is in that sense similar what we have we generally see in the case of sound the expression is also very similar to the sound. So, there is nothing very surprising about it. Only thing which I want to tell you that as far as the transfers

Doppler effect is concerned in this particular case the change frequency will always turn out to be smaller. Now, let us do some comments for the Doppler effect. In the case of longitudinal Doppler effect what we found that in the limit of v less than c the expression is similar to the sound Doppler effect,. So, it is quite clear like the normal sound Doppler effect if the source is moving towards the observer the frequency of the light will appear to be increased.

Similarly, if the source is moving away from the observer the frequency of the sound will be appearing to be less. So, this is what have written as the expression nu d s is approximately equal to n naught multiplied by 1 minus v by c, which is as we have set expression what has been similar to observer in the case of sound for of course,, this specific case and therefore, we expect essentially the result be identical that when a v is a positive nu d s is smaller than nu naught when v is negative it means the source is moving towards the observer then in this particular case this nu naught gets multiplied by factor larger than one and nu d s turns out to be larger than nu naught. So, light we see exactly the similar effect but, in the case of transfers Doppler effect will always see a reduced frequency the observed frequency is always lower than the source frequency.

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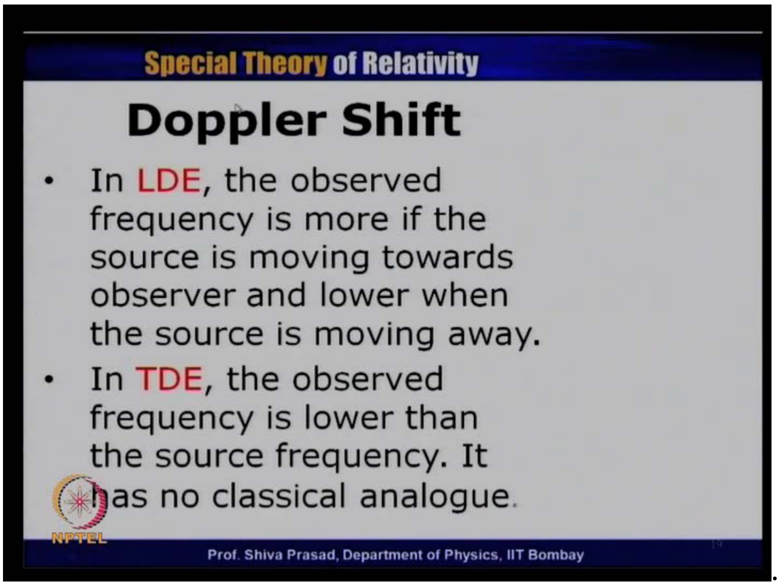

This is what I have written here in l d e means longitudinal Doppler effect the observed frequency is more if the source is moving towards observer and lower when the source is moving away. In transfers Doppler effect the observed frequency is lower in the source frequency as we have said it has no classical analogous. I would like to just mention here is that the transfers Doppler effect can also be derived or can also be thought to be as a cause of time valuation.

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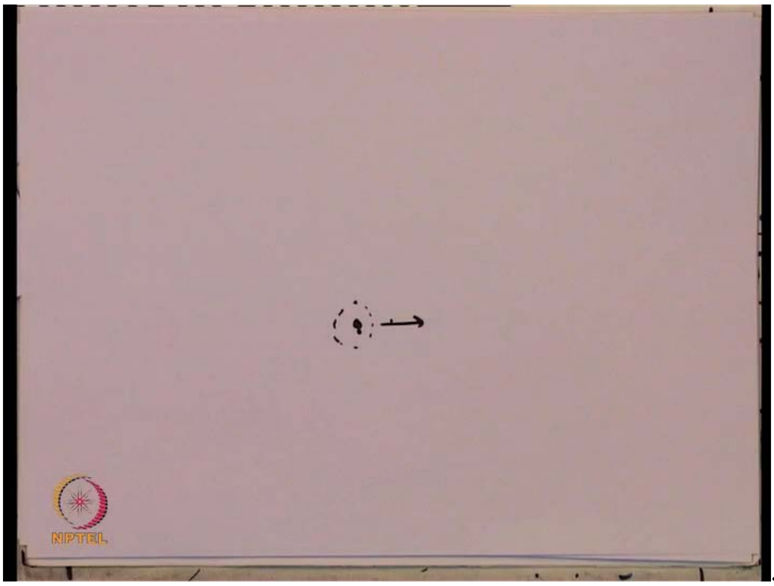

Let us see how, if this is a carrier this draw a carrier or just draw a point source to be more specific and this point source is moving in particular direction. Now, this particular source is emitting light we can always think or we know that light is electro-magnetic wave. So, when we say in a wave there is a maximum there is a minimum no there is a maximum amp we know displacement we call is a in the mechanical wave here in terms of electro-magnetic wave this will be a point, where we have magnetic field, which is highest or electric field which is highest or whatever you want to talk about it when there is a maximum and there is a minimum.

So, displacement is maximum and minimum that is what the wave is light. Now, difference between two consequent maximum one after the other. So, this is a time difference between these two is not we call it as time period in a classical wave. Now, if the wave is being emitted its being emitted here so, it is goes like this spherical wave fronts goes like this there is another way we emitted,. So, again its spherical wave front goes away from this. Now, both these the time difference between emission of the first maximum and the second maximum that time difference both are being measured by this frame of reference when I am measuring the frequency of time period in s prime frame of reference, and because these are being measured in s frame itself because that is the source which is emitting there the light therefore, when its emitting first maximum the second maximum the time difference between these two is proper time difference, because both these time the the time difference between these two is being measured only in s prime frame of reference.

So, therefore, the t prime or the time period in s prime frame of reference is a proper frame of reference therefore, if is a proper time interval sorry therefore, an observer of aground will find the time interval between these two events emission of first maximum second maximum will be dilated, because in that frame of reference when the first emission was there it started from here the second emission according to s prime occurred when this particular source moved away. So, first maximum I was emitted from one particular point this second maximum was emitted from a different point,. So, these two events emission of first maximum emission of second event maximum according to s did not occur at the same point by that time the source has been away but, in the first frame of reference both of them occurred at the same place therefore, time interval between these two emissions of maximum is proper in s prime frame of reference. Therefore, in s frame of reference this will appear to be dilated, this what I have written consider transfers Doppler effect,

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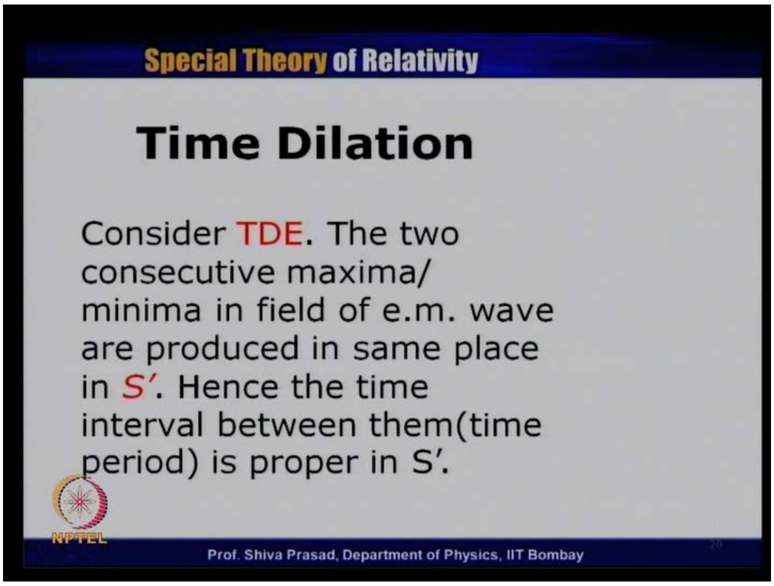

The two consequent consecutive maximum minimum in the field of electro-magnetic wave are produced at same place in s prime. If the time interval between them, which we call as time period its proper its prime frame of reference it is to be s prime frame of reference and therefore, in s frame it may be dilated. So, this is what is a picture I had said its coming from here.

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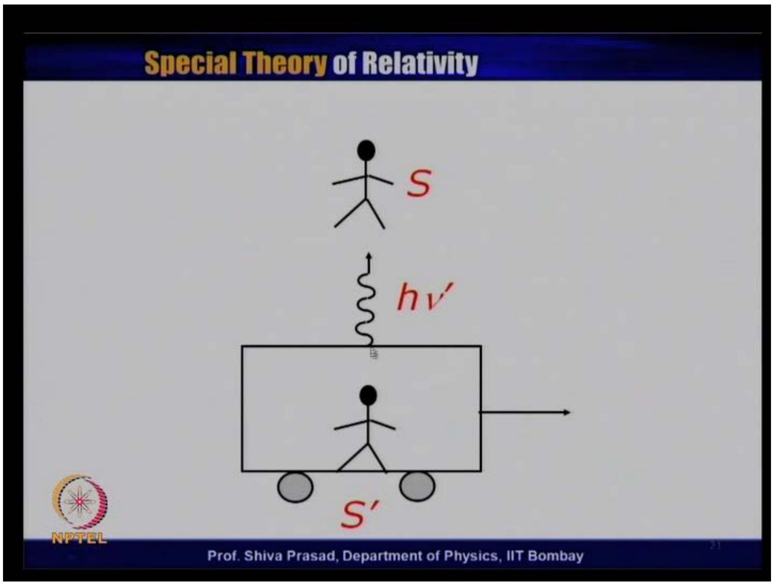

So, when first maximum was emitted and the second maximum was emitted by this particular person according to s prime he was at the same position but, according to s when first was emitted was here and second was emitted are they the man has moved away therefore, these two events did not occur in the same place therefore, in s frame the time interval will be dilated. So, using time dilation we can write that time interval or time period in between these two emissions in s frame will be equal to gamma time t prime and as we know by wave theory this time period is inversely proportional to one divided by frequency. So, this t I can write as one upon frequency,

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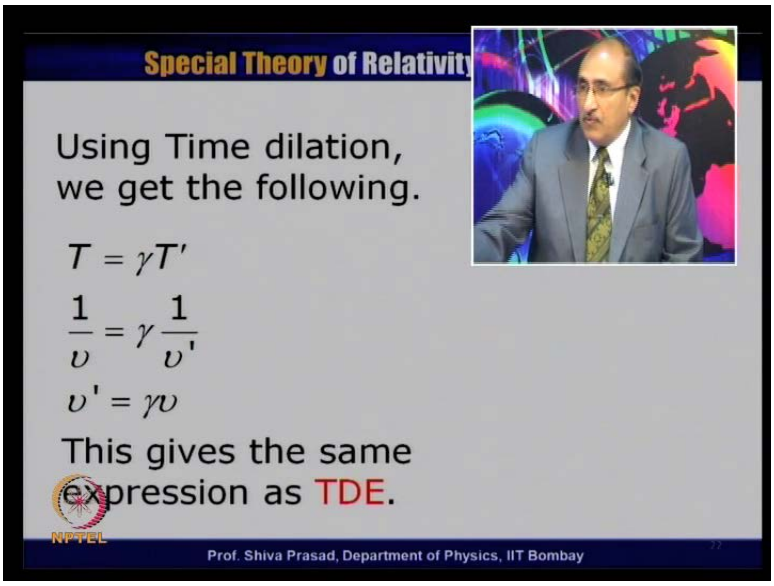

This t prime I can divide it by as 1 minus 1 divided by nu prime, because this is a frequency in s prime frame of reference its t prime it is is also an s prime frame of reference, which gives me exactly same as nu prime is equal to gamma nu exactly the same expression, which we have earlier found out by using energy transformations. So, what I am trying to say is that transfers Doppler effect can be thought as arising due to time dilation phenomena and because there is classically no time dilation therefore, classically we do not have any transfers.

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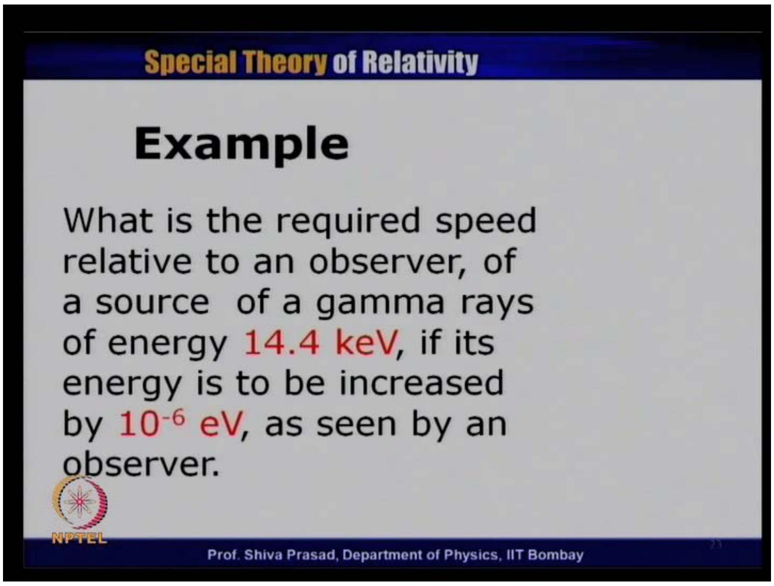

Doppler effect, let us say one or two examples and try to at the end of the lecture we try to give some idea how the Doppler effect can be used let us first take very simple example, the question is what is the required speed relative to an observer? If a source of gamma ray of energy 14.4 k e v we say k e v means k volts one thousand volts; it means, 14.4 multiplied by thousand electron volts if its energy is to be increased by ten to power minus six electron volts we just like to we would appreciate the numbers.

This is 14.4 k e v 14.4 multiplied by thousand electron volt and this is 10 to the power minus 6 electron volt with these small change you know line or you know many, because this order of thousand electron volt and this is 10 to the power minus 6 electron volt. So, I want to change sending or emitting electro-magnetic waves gamma rays 14.4 k e v, and I want to change simple very very slightly of the order lines of order minus line you know of that order I want to shift its energy by 10 to the power minus 6 electron volt, then I want to apply I give it to Doppler shift, what is the speed data I would require? So, that if we can see we increased by 10 to the power minus 6 very small very tiny amount,. So, as we have seen the frequency has to be increased; it means this source must move towards the observer.

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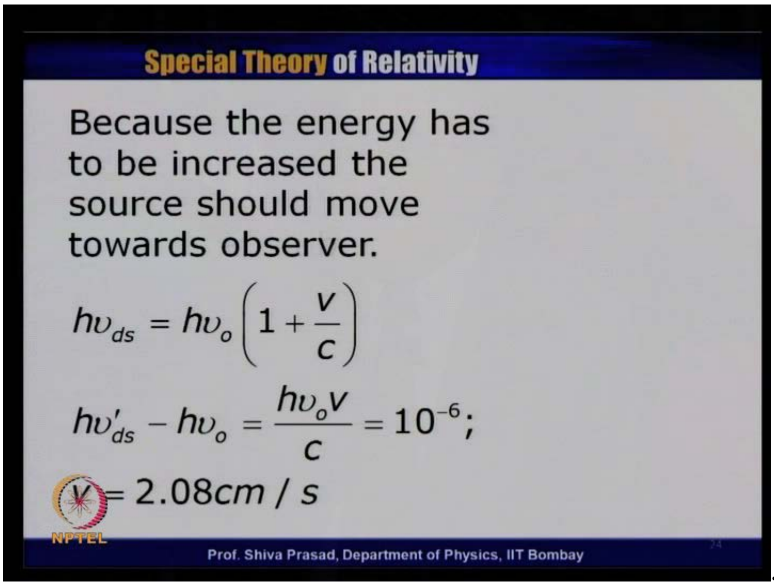

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Therefore this particular source must come towards you person who is observing this frequency. So, this you can see as am sitting here i want to observe it increased by ten to the power minus six electron volt therefore, the source which is emitting this particular gamma rays must come towards me, and as we were seen that this speed is definitely not very small in comparison to c,. So, I will use the expression that we just now derived for the case of v much smaller than c.

So, this what I have written because the energy has to be increased the source should move towards it should not be absorber should be observer then I use that particular expression h nu I have multiplied by h nu d s must be equal to h u naught into one plus v by c. Here I have used plus expression, because I already increased the frequency therefore, v has to be taken as minus v. So, sign has to be changed alright put the expression this h nu naught I take on the left hand side this becomes nu prime d s minus h nu naught h u equal to h nu naught multiplied by v by c.

This change I want 10 to the power minus 6 electron volt. So, I will put 10 to the power minus 6 here this I know its 14.4 k e v v by c I can calculate this speed, which is turning out to be 2.08 centimeter per second. These are very reasonable speed you know very easily calculating manipulated in the laboratory can be observed in the laboratory therefore, Doppler effect provides you a facility by which you can shift the energy of sources by very very small amount and therefore, as we have seen just now it can be used for some very great studies which are generally not possible by any other mis to resolve that type of small what used to call in spectroscopy hyperfine fields structure. Here the energy levels are different very very by small amount that can be done by you know by Doppler shifting the source the the gamma rays source in source case the gamma rays and 14.4 k e v by the way is a very standard source, which people use very often when cobalt 57 decays to iron 57 emits a radiation of 14.4 kilo electron volt.

Now, these are certain implications and that implications I would like to discuss today what is emission and absorption? Let us as go back to atomic physics and let us go to very very simple thing we do know want to talk it is not a force on atomic molecular physics we do not want to talk too much in details about the spectroscopy. Let us just assume that we are doing something like a hydrogen atom or something like that, which we know very well when there is a transition takes place its emits a photon of course,, this concept of transition is track is much later but, they were let us let us try to see the implication of this particular concept of the photon and also the Doppler effect as far as these are concerned. Suppose, there is a transition then as a result of transition the photon is emitted we know that this the way photon is emitted in atomic theory.

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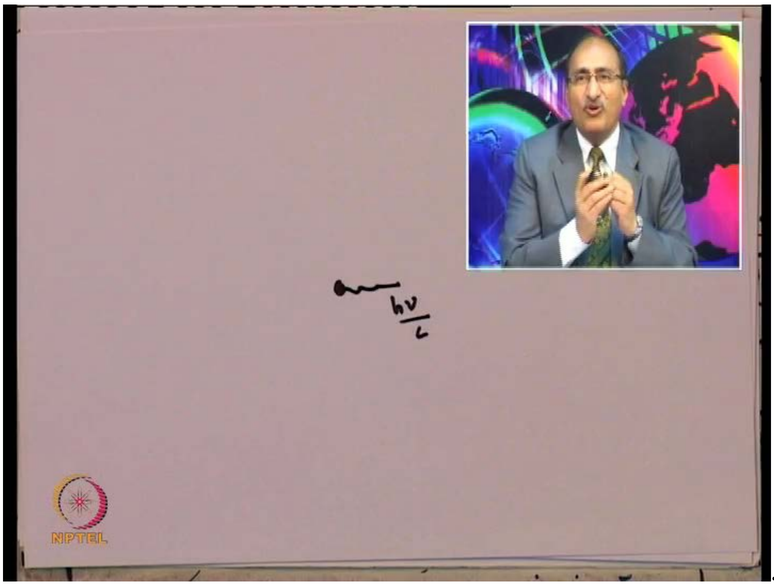

Now, because we know that photon has a momentum if we strictly speaking whenever we are writing an expression for finding out the energy of the photon as a result of certain transition, this momentum must also be conserved. Which essentially means that if we have let us say an atom which undergoes a transition and emits a photon and let us assume that this particular atom was at rest in a given frame of reference this this photon has a momentum, which is given by h nu by c, because initial momentum was 0 therefore, final momentum also has to be 0; it means, this atom must required back in this atom requires back then this will also gains some energy some kinetic energy and if it gains some kinetic energy that kinetic energy also has to be supplied by the transition energy, because once the transition has taken place that energy has to be partly utilized for creating the recall energy and partly for giving energy to the photon; it means, the energy of the photon will turn out to be slightly smaller than the transition energy available to it.

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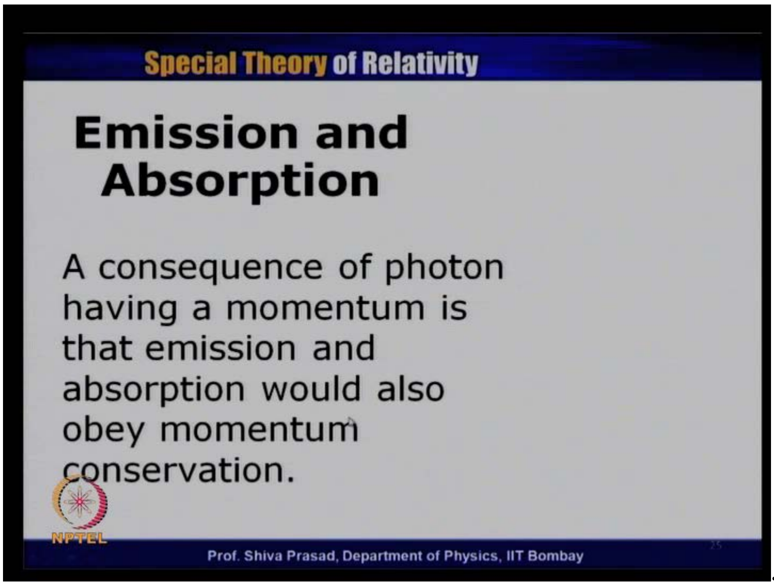

This what I have written a consequence of photon having a momentum is that emission and absorption also obey momentum of conservation this is one as to be realized.

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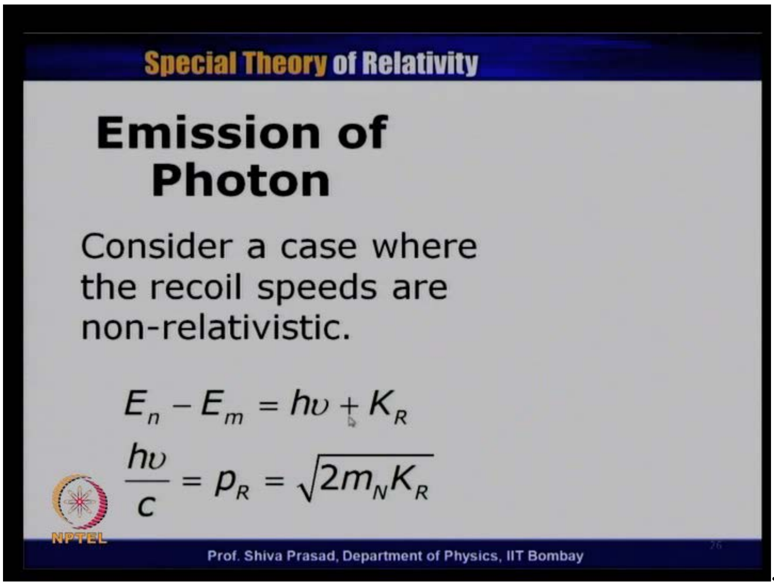

Let us consider a case of emission of a photon and let us assume that all require speed of non-relativistic let us not consider a very high energy transitions at the moment therefore, we can use partly the classical expressions of energy momentum kinetic energy and momentum relationship. So, let us suppose there is transition which takes place from n street to m street energy corresponding to this particular level was e n corresponding to this level was e m,. So, this is a total energy available to me as a result of transition. This energy we now used for two in two ways it will give energy to the photon and also it will give recoil energy to the atom we of course,, I must write e n minus e m is equal to h nu plus k r where k r is the kinetic energy of recoil, and I must get a second equation which is the momentum conservation, which is h nu by c is equal to momentum of recoil.

Of course, this momentum and kinetic energy in the classical case are related by very simple expression k is equal to v square by two m and let us assume that m n is the mass of atom, which I am calling mass of nucleus or mass of atom whatever you want to call it it depends on what generally nucleus most heavy that is why I have written m n. So, p r can be written as under root two m n into k r. Now, I can substitute this k r by writing this p r k by two m.

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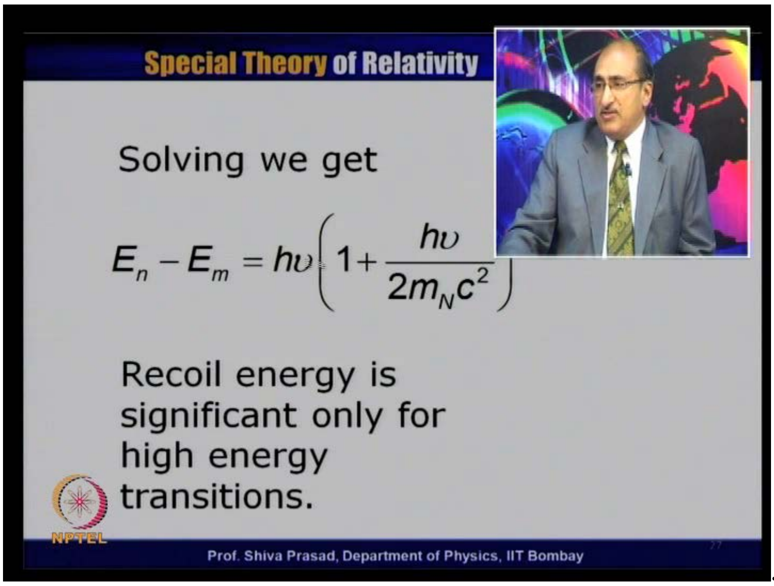

N and therefore, we can find out an expression for the frequency of the emitted photon, which I have written. So, this is the expression, which actually is a quadratic nation. So, there is h nu is here h nu square and in case of you have to solve this particular quadratic equation to get correct expression of h nu but, we must realize that this h nu will be slightly smaller than e n minus e m of course,, I must also see that generally if we take for some hydrogen atom case the energy that we are talking of the order the ground state energy of hydrogen atom is minus 13.6 electron volt atom of ten electron volt, Well m c square of case of photon, which is nucleus one of the lightest nucleus is of the order 940 m e v.

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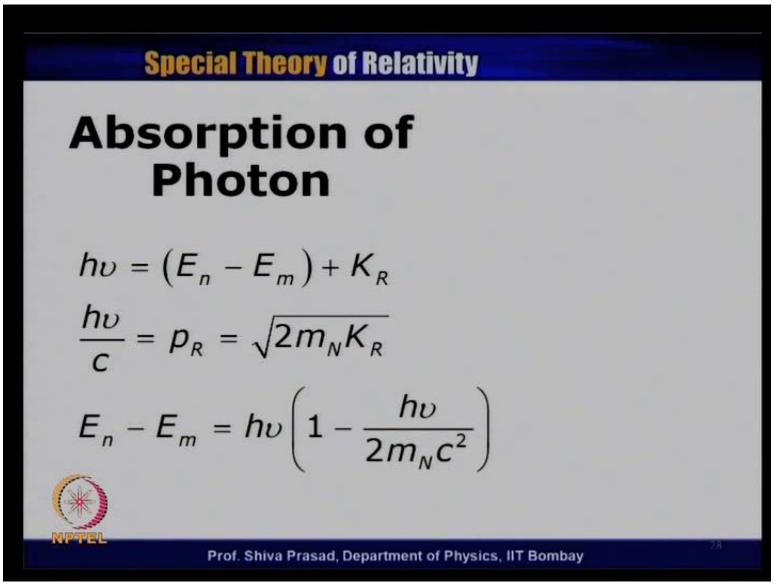

So, 10 power 9 electron volt. So, this expression is extremely negligible therefore, most of the time we neglect recoil specially, when we are talking of some hydrogen atom but, we are talking of some nuclear transitions in that particular the recoil can be significant and therefore, we may we must account for this particular thing similarly, if we are talking of transition of photon if a photon has to absorb it has to not only supply energy for the atom to go to higher energy state but, this atom this photon when is absorbed the atom will start moving front because momentum has to be cancelled, because initially there is a non-0 momentum once photon is absorbed the same momentum must carry y an atom therefore, the expression will become h nu is equal to e n minus e m, Plus k r exactly.

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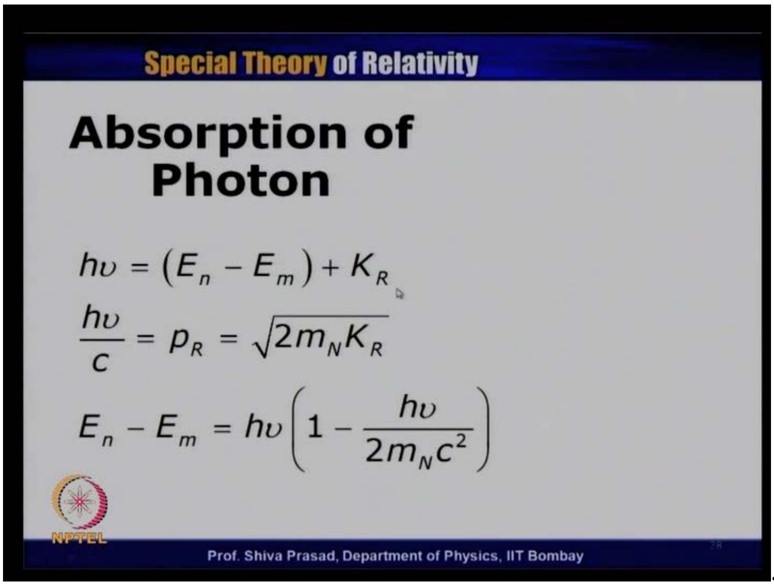

In a same fashion h nu upon c will be equal to p r and this particular equation will get e n minus e m is equal to h nu divided by 1 minus h nu upon two m n c by c square.

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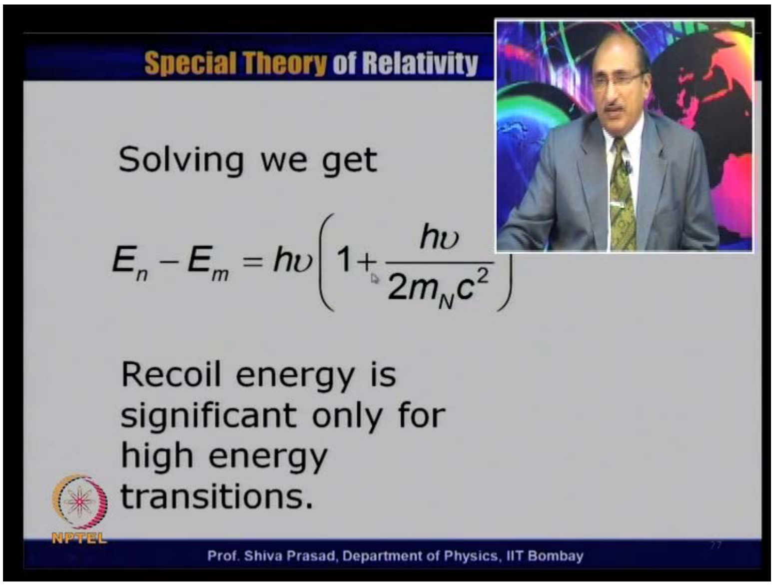

So, c here there was a plus expression here there was a smaller expression the e n minus e m will be slightly larger than e n minus e m. Now, my question is that what we call as a resonant absorption is it possible that we come across a situation when a photon is emitted as a transition a particular transition and a photon is used for causing similar inverse transition in some other atom; it means, a photon has been created when photon has gone from n x state to m x state now this photon has come out now is being absorbed exactly the similar atom and you want this particular atom to go from m state to n state. It possible we have just now said that in the first case the energy of the photon will be slightly smaller than the transition energy in the later case we require the energy to slightly larger than e n minus e m. See first quantum mechanics also gives you that there is all the photons have a particular bandwidth you cannot have a perfect monochromatic photon.

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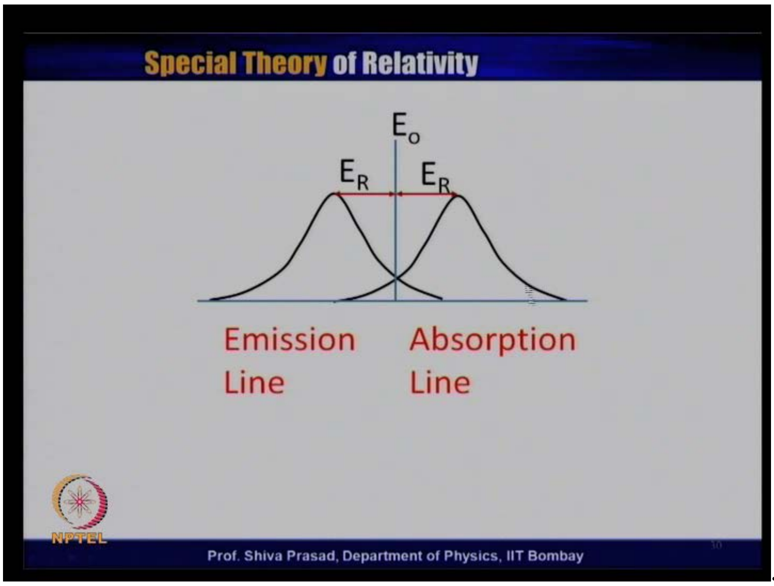

So, it is. So, happens that if the line width turns out to be larger than the difference between the between these recoil energies of that order. Now, we can sort of resonant absorption can still take place but, in the case of many nuclear transitions it is not possible to do that. So, for example, if this is your emission line, which has this is e naught is e n minus e m and this is absorption line they have finite land widths and this is, because of the recoil this is because the differences because of the recoil and then this recoil energy this is very large comparison to this land width its hardly a resonant absorption taking place, on that is way comes to Doppler effect can the recoil energy.

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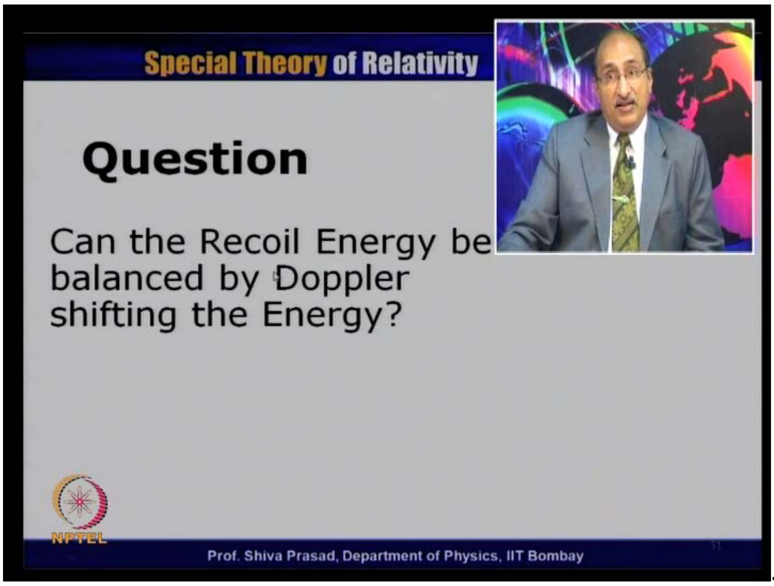

Whatever the recoil energy whatever is the recoil energy can be compensated by providing a slight amount of Doppler shifting can I increase the energy or can I decrease the energy of the photon by giving the sources certain amount of energy because of the Doppler shift can be shifted.

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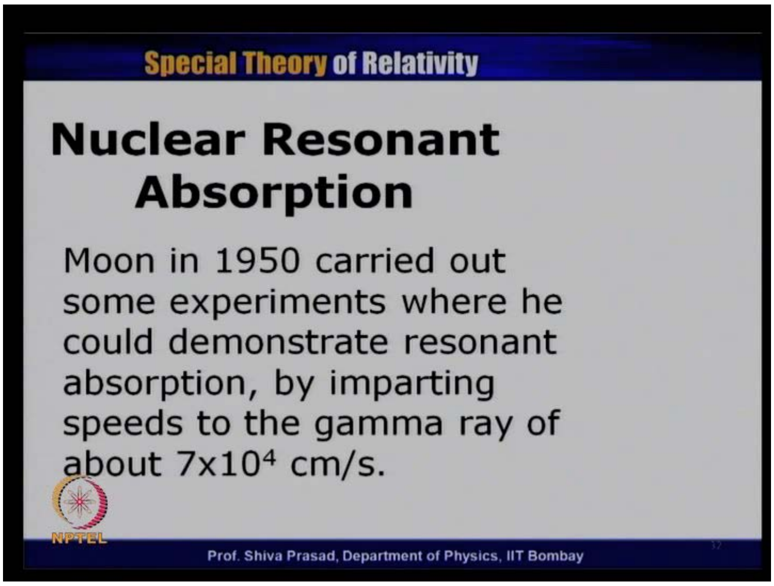

In 1950 moon was a scientist who carried out some experiments and where he could demonstrate the resonant absorption by imparting the speeds to the gamma ray of about seven into 10 to the power 4 centimeters per second remember. We are talking of gamma ray, which are energies not that 13.6 electron we have not talk earlier or found. So, the in this particular case it required energies out of the 7.10 to the power 4 centimeters per second and the he could observe a resonant of absorption in situation in which it was not possible.

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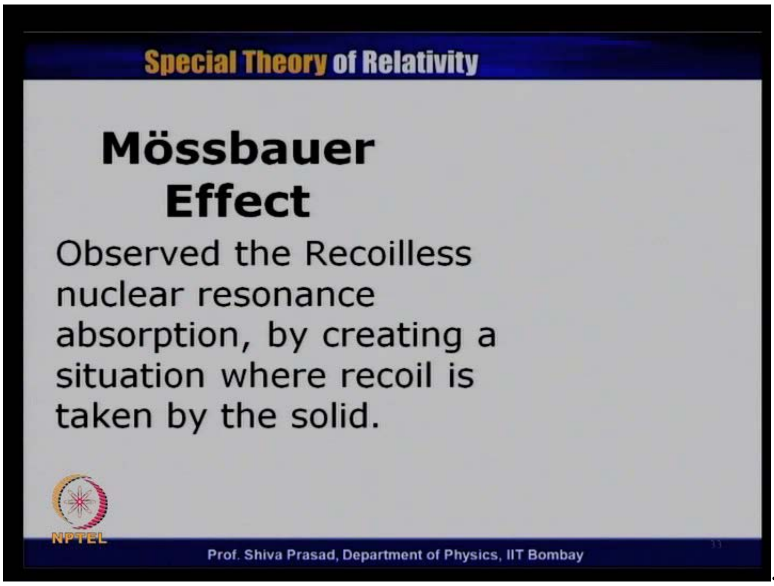

By this particular case resonant absorption is interesting. See what happened in much later time most observed that he can really certain situation he can create I mean that certain ways in which he can create situation in which the total recoil is taken by the solid and total recoil can essentially be neglect can be can be made essentially recoilless and when he observed this particular thing he tries to call this particular thing we can see could see recoilless emission then what is a why we are talking recoilless. So, much? The idea is that in that particular case in the case of recoilless emission all these lines, which are which have high per field structure in which you know find that the lined the energy differences are extremely small they can always studied by slowing applying a Doppler shift giving energy of the order millimeter per second or centimeter per second and therefore, can we can scan all those energy level diagrams even though the energy sources have high energy and gap between these energy level is extremely fine but, is still I can measure by shifting the energy slowly varying the energy in photon by very small amount by giving it a speed and therefore, these photons can get Doppler shifted.

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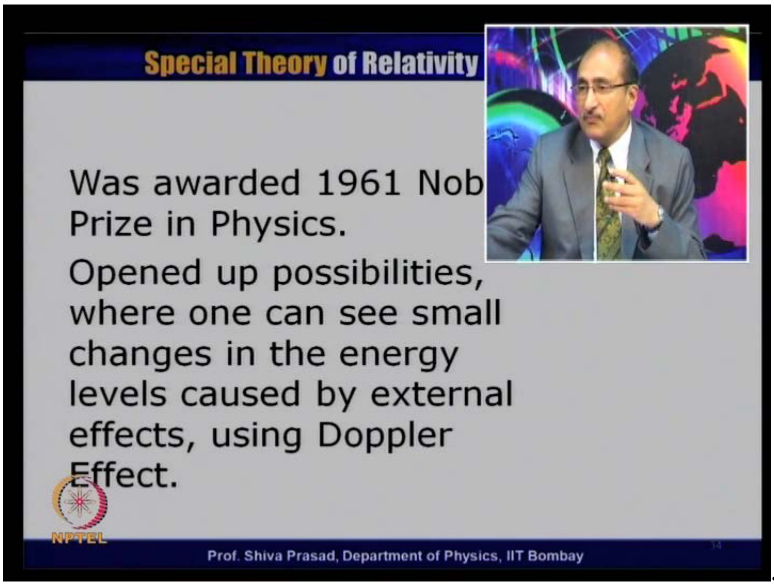

So, Mossbauer was awarded nineteen sixty one noble prize in physics and to opened up possibilities, where one can see small changes in the energy levels caused by external effect using Doppler effect. Lastly see speaking I will give quickly an example which is very interesting is called laser cooling of a gas.

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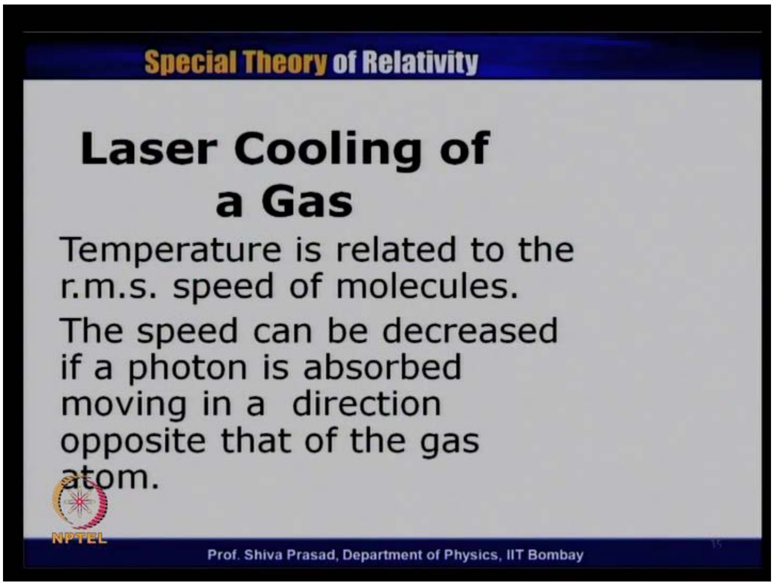

Normally lasers or always thought for heating something normally use laser to or cutting something or building something it is never thought that a laser can also used to cool something but, it is interesting that laser can be used to cool gases let us see how? See we realize that when we talk of a gas the temperature of the gas is related to the root means square values of the speeds of the molecules. If the root means square speeds is high the temperature is high if they are smaller then the temperature is smaller. So, if by sum means I can reduce the r m speed root minutes per square speed of the gas molecules the gas molecule. Now, let us see how Doppler effect helps in particular case? Let us we see the particular situation take a very very specific case.

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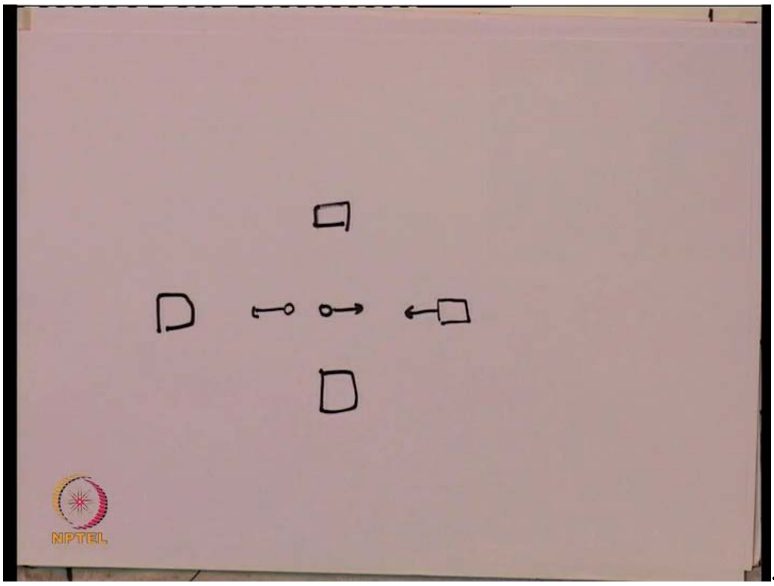

Let us imagine that this particular thing is a source of laser its emitting from in this particular direction and there is a gas molecule, which is moving towards it. If it moves towards it and if I adjust the such a fashion that for a given transition the energy whatever is the energy required the energy of the photon is slightly smaller than that so, energy the photons which are emitted have a slightly smaller energy than the transition energy. Then what will happen than this particular atom in its frame this particular photon is moving towards him this particular atom will fine than the energy of this photon is slightly larger and if it becomes equal to the transition energy this photon will get absorbed by this particular atom, and as a result of this absorption the photon of this momentum of this photon will be transferred to this atom and this photon is moving in this direction this atom will slow down its momentum will go down. Now, if an atom was moving not in this particular direction.

In that particular direction in that particular case it will not get absorbed, because according to this particular person in this particular frame the energy of the photon will not equal to the transition energy which happens in this particular case therefore, it will selectively absorb photons only which are coming towards it, and therefore, all the atoms which are moving in this particular direction there is speeds are going to be slow down. Now, I want speed of the atoms is go slow down not one direction but, in all the directions. So, what I do I create once force of laser photons here and this generated by laser another source here, third source here, fourth source here, fifth on the top sixth at the bottom all moving towards gas. So, if a photon is moving in this particular ay gas atom is moving this particular direction it is a speed will slow down by the absorption for this photon if it is moving in this particular direction, this speed of this will reduce by absorption on this photon.

If it is moving in this particular direction is speed of this particular thing will get reduce, because an absorption go down from this particular direction and overall the speed of this can be reduce. So, let us just quickly read whatever i have written temperature is related to the r m s speed of the molecules; the speed can be decreased if a photon is absorbed moving in a direction opposite to that of the gas atom.

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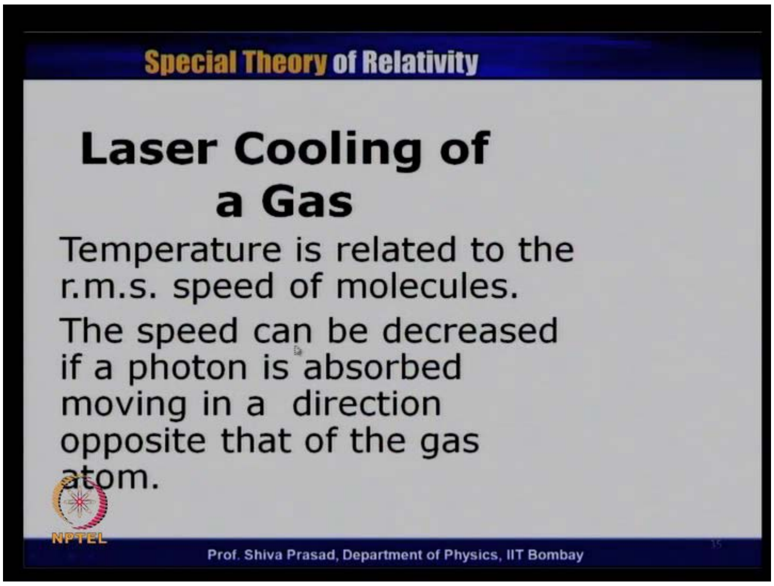

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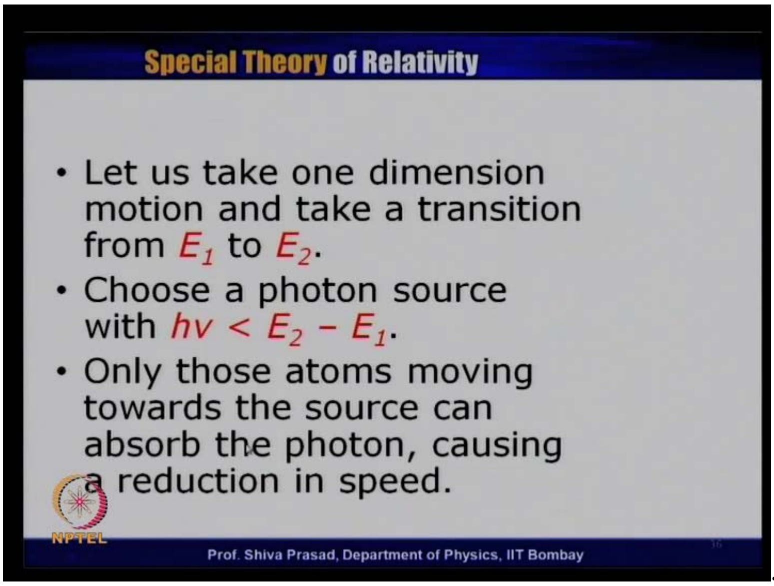

Let us take one dimension motion and take the transition from e 1 to e 2 expressive transition choose a photon source with h nu less than e 2 minus e 1 only those atoms moving towards the source can absorb the photon causing the reduction in speed, this is what is called the system is called six laser is called optical molasses create a situation in all six directions.

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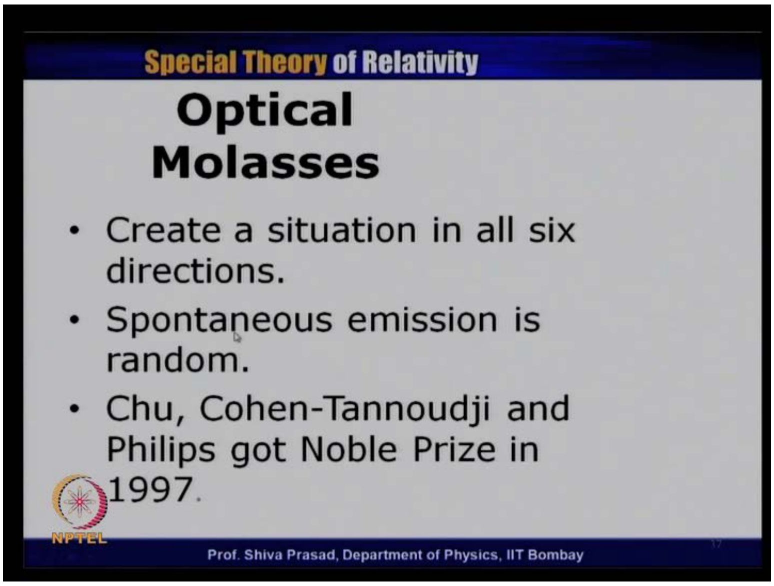

I was also say that when the photon is absorbed that also will emitted back this you call as a spontaneous emission but, this is spontaneous is under mean all the direction therefore, it increase certain sort of temperature but, we can show that overall the goes will go down Chu Cohen-Tannoudji and Philip got noble prize in nineteen ninety seven for the laser cooling and related experiments.

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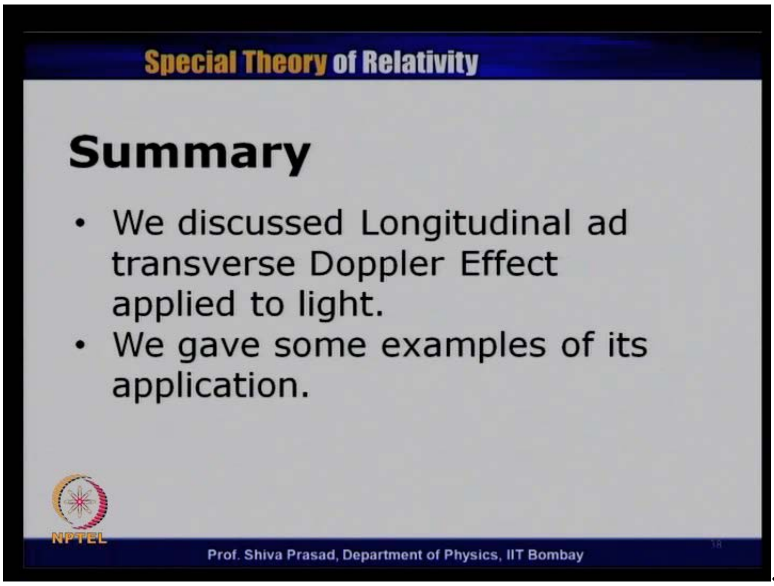

Now, I will summarize of whatever discussion today’s lecture, we discussed longitudinal and transverse Doppler effect as applied to the light and we gave some examples and its applications.

Thank you.