Special Theory of Relativity
Prof. Shiva Prasad
Department of Physic
Indian Institute of Technology, Bombay

Lecture - 17
Zero Rest Mass Particle and Photon

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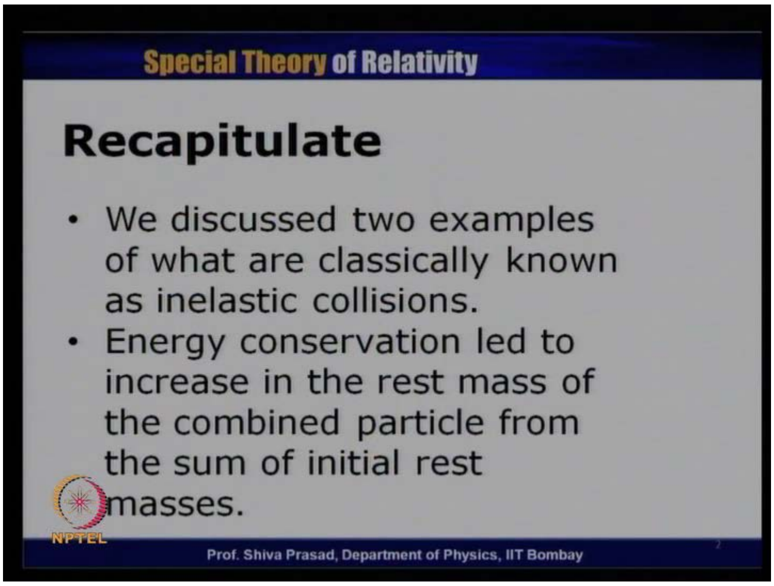

So, let us start with from whatever we had discussed last time. We had discussed two examples of what in classical mechanics are known as inelastic collisions. In classical mechanics, we are used do not conserving the mechanical energy at that particular moment of time used to just conserve momentum then such types of collisions were discussed. However, we had discussed in our last lecture that in relativity, we have to also consider conservation of energy along with conservation of momentum.

Hence we have to apply conservation of momentum, which eventually led to increase in the rest mass of the combined particle from this sum of the initial rest masses. So, if you have two particles, which collide with respect to each other, and then they get stuck to each other and then if this is m 1 and this is m 2, then the combined mass becomes larger than m 1 plus m 2. This is because we are conserving energy and we say that effectively this type of energy is actually never lost, and whatever was the energy probably has been converted to some different form of energy and not really mechanical energy. So, this is these are the type of examples that we discussed last time, when we said that even in the classical inelastic collision we will have to conserve the total relativistic energy. Now, let us go ahead.

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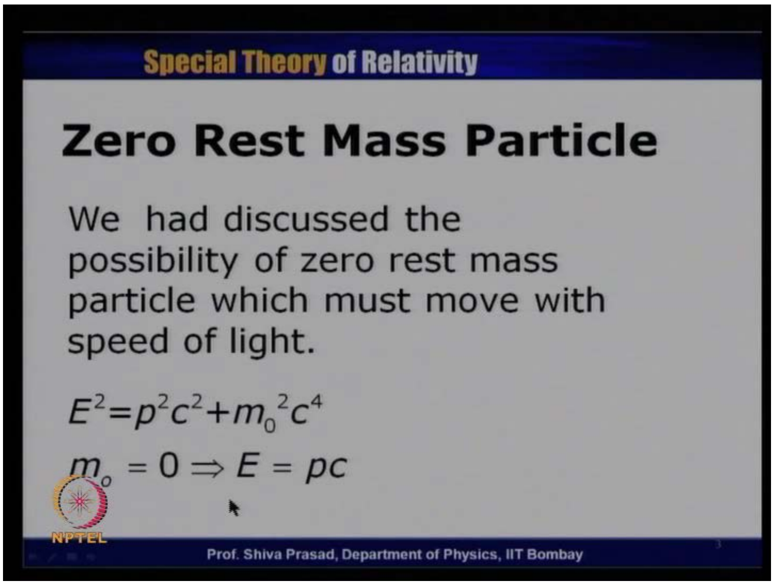

You know in our last lecture, we had introduced a concept of zero rest mass particle. So, let us go little bit ahead with this concept of 0 rest mass particle and try to discuss in today's lecture little more about this zero rest mass particle. See, we had said at that particular time, that it is possible that this particular particle could have a finite or non zero energy and a non zero momentum. But in that case, this particular particle must move with speed of light. If it so happens, you know that this is the standard expression for energy to momentum. This is the total relativistic energy E square is equal to p square c square plus m naught square c to the power 4.

So, if m naught becomes 0, this relationship just reduces to E is equal to p c. It means, for a particle with 0 rest mass, the energy and the momentum would be related by such simple equation as E is equal to p c. Of course, this particular particle must be traveling with the speed of light.

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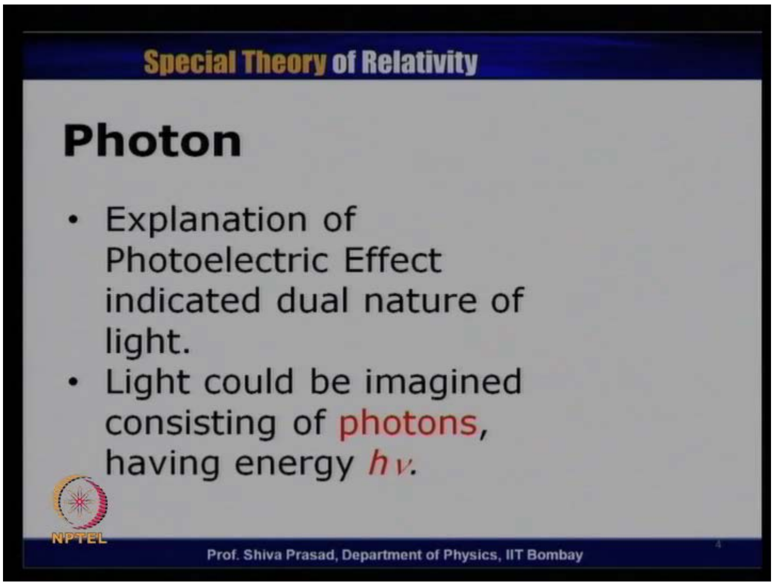

Now, let us introduce the concept of photon. It was originally in the explanation of photoelectric effect, that the concept of photon was evolved. It was realized at that particular time, that the type of frequency dependence of the energy of the electrons, which are released as a result of photo electric effect, can be explained only if you assume that light is particle line. So, this is exactly that we discuss in photoelectric effect experiment, that light has a dual nature. It could show as a particle and it could also show as a wave. So, photon was essentially, the idea of photon was created with the particle concept of light in mind and that was necessary to explain the photoelectric effect and this particular particle was considered to be having energy of h nu, where nu is the frequency of the wave that we have been taking about.

So, if there is an electromagnetic wave or there is a light wave with frequency nu, we can imagine that this particular light consists of photons and each one of them as energy h nu. Of course, this particular photon travels with a speed of light. Therefore, this photon qualifies to be called a particle with 0 rest mass. So, this is what I have written. Explanation of photoelectric effect indicated dual nature of light and light could be imagined as consisting of photons, having energy h nu. Now, we have just seen the relationship E is equal to pc. So, we can find out what will be the energy of the, what will be the momentum of the photon.

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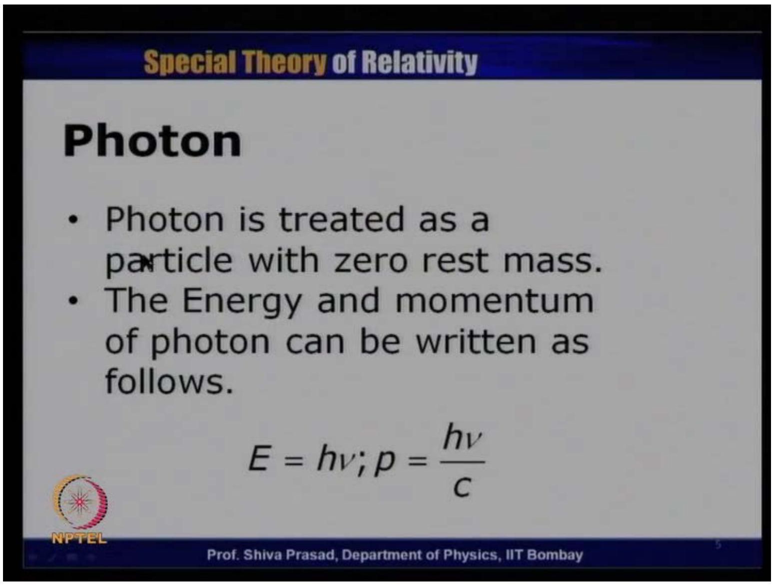

So, this is what I have return in the next transparency. Photon is treated as a particle with 0 rest mass. The energy and momentum can be related by this particular expression because e we have just known seen from the explanation of photo electric effect experiment, that e is to be given a value equal to h nu and p must be equal to E by c from the relativistic expression that we have earlier considered. For a 0 rest mass particle, it means the momentum of the photon must be given by h nu by c.

So, whatever we are saying about special theory of relativity is correct and the wave we have invoked, the 0 rest mass particle is correct. In that particular case, a particle like photon must have a momentum also which is equal to h nu by c. It was generally known, even in earlier cases that light carries momentum. But here, we sort of get involved with rather newer concept, when I treat particle photon just like any other particle traveling with the speed of light and having energy h nu and momentum of h nu by c. Actually, in the photoelectric effect experiment, it is not really possible to test the momentum of the photon, because actually the photoelectric effect as to be observed when the electron is bound inside an atom. Therefore, quite a bit of (( )) energy taken by the atom or by the solid and therefore, it is not possible to really test that the photon has a momentum.

In fact, in photoelectric effect experiment, we only apply conservation of energy. However, there is another experiment, which we call as Compton Effect. In which, it is possible to really check and experimentally verify that this idea photon is realistic idea as far as particle of 0 rest mass is concerned. It can really be treated with the particle energy equal to h nu and a momentum equal to h nu by c. So, let us just describe the Compton Effect experiment.

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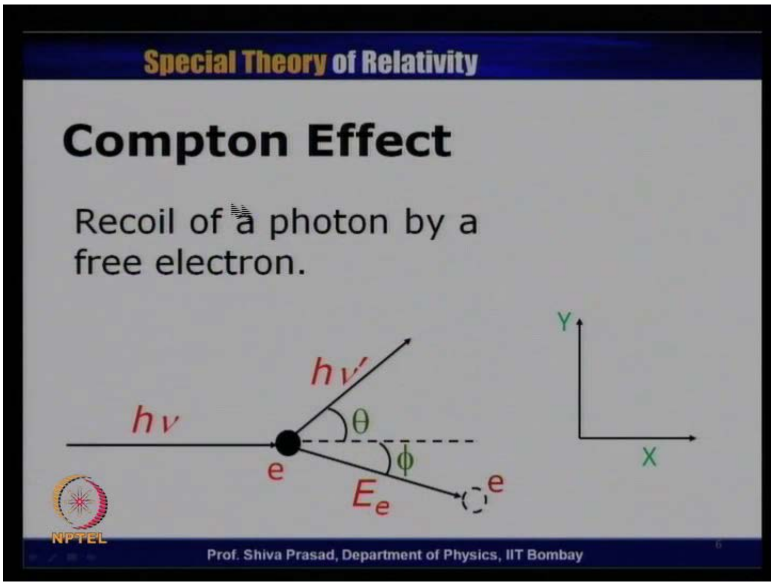

So, Compton Effect is actually recoil of a photon by a free electron. So, let us assume that we have an electron e, which is at rest and there is a photon, which comes here with energy h nu. This photon gets scattered and eventually comes out in a different direction. This direction makes an angle theta with a initial direction of the motion of photon and it comes out with different energy, the changed energy h nu prime. As the result of this scattering, this electron scatters; goes other way at an angle of phi from the incident direction of the photon with an energy e. So, this is just like a simple classical scattering problem, which we are required to use that one ball coming and another ball coming and hitting and one ball moving at an angle and another ball requireling, except that the particle that we are talking here, one is photon and another is electron.

So, what we want to do, we will treat photon as really a particle of energy h nu and a momentum of h nu by c and would like to apply conservation of energy and momentum to this problem, like we have been applying for any other particle that we have discussed earlier. Let us see what result that we get. So, what I will do in the next transparency, I will write the energy conservation and the momentum conservation because that is what we are supposed to apply. Now, as far energy is concerned, the initial energy before scattering was the energy of the photon. It does not have a rest mass energy, because m naught is 0. So, m naught c square is also 0. So, the total energy of the photon is h nu.

This particular electron, which was at rest and only the rest mass energy was equal to m naught c square. So, the total emission energy will be given by h nu plus m naught c square. After scattering, you have a photon, which goes this way and it comes out with a reduced energy or a different energy. It means a reduced energy, because part of the energy will be taken by the electron. So, this particular h nu prime is the new energy of photon; changed energy of the photons. So, photons which comes out as the energy of h nu prime and the electron, now also gets sudden amount of kinetic energy. Let us assume that the total relativistic energy is E e. So, the final energy after this scattering must be equal to h nu prime plus E e.

So, this is what I will be calling as the conservation of energy. So, I will get one equation for conservation of energy. I have to also apply conservation of momentum. When I have apply conservation of momentum, momentum means a vector quantity, I have to consider both x and y direction. I have shown this as the x direction, which is the direction of the initial motion of the photon and perpendicular to it is the y direction and I have assumed that the scattering as taken place in x y plane.

So, I have to conserve not only the x component of momentum, but also the y component of the momentum. So, we realize that as far this particular particle is concerned, the photon is concerned, the initial momentum was h nu by c. We have just now discussed that. This electron was at rest. So, its momentum was 0. This particular photon has momentum only in the x direction. Therefore, the x direction initial momentum is h nu by c and the final momentum in the x direction will be the momentum of this component of the momentum of this photon along x direction and the component of the momentum of the electron along this particular direction.

In the y direction, which is this direction, there was no initial momentum. So, final momentum should also be 0. It means, if I take the component of this particular momentum along the y direction and component of this momentum of this electron along the y direction, the two must balanced out. So, these are the equations that I am going to write. Only thing I will write, the momentum of the electron as p subscript, all of you, p subscript e.

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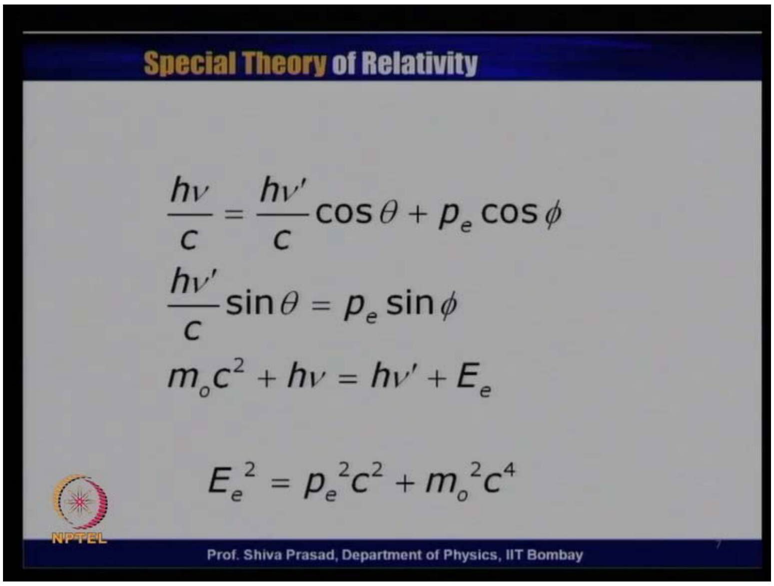

So, let us now write all the three equations, which I am putting in the next transparency. Let me just show here my picture to make it clear.

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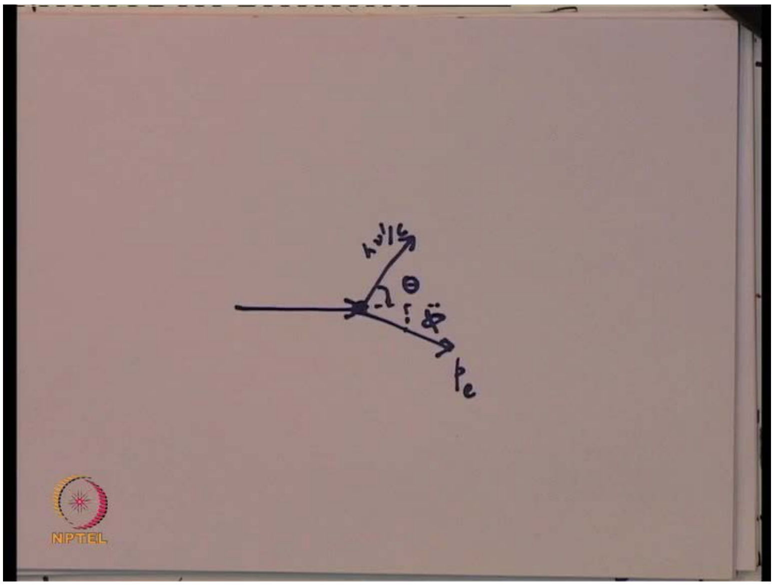

This was my initial photon. This was my electron. This was going in this particular direction. Make an angle theta and this was electron, which was going in this direction making an angle of phi. So, my first equation says, which is the conservation of momentum along the x direction, and this is the initial momentum of the photon because initial momentum was only along the x direction, which is h nu by c. Now, momentum of the photon, which has been scattered, is h nu prime by c. This is h nu prime by c.

So, if I take the component along the x direction, it becomes h nu prime by c cos theta. Similarly, the momentum of this electron, let us assume is p e. So, along the x direction, its component will be p e cos phi. So, this is what I have written in this particular transparency. h nu by c is equal to h nu prime by c cos theta plus p e cos phi. Now, we take the momentum along the y direction. If I take momentum along the y direction, this momentum must cancel out by this particular momentum. The momentum of the photon, outgoing photon in this particular direction is h nu prime by c sin theta. Momentum of this electron is p and in the y direction, it will be p e sin phi. phi expect that h nu prime by c sin theta must be equal to p e sin phi.

So, this is the second equation, which I have written here. h nu prime by c sin theta is equal to p e sin phi. This is of course, we have discussed is conservation of energy, which is initial energy, rest mass energy of the electron plus the initial energy of the photon is equal to outgoing energy of the photon plus the total relativistic energy of the electron, which here of course is rest mass energy plus kinetic energy. Of course, we are using symbol p here and e, they are not independent variables, because energy is always related to momentum. Therefore, we have a further equation, which relates energy to the momentum, which is written as E e square is equal to p e square c square plus m naught square c to the power 4, standard relationship between energy and momentum.

Now, we have work out with these equations and try to find out what will be this scattered frequency or energy of the photon or energy of this scattered photon, as you will be discussing a little later in the experiment. It is not always very easy to know the parameters of the electron that is scattered. It is somewhat easy experimentally to calculate the energy of the photon, whether it is incoming photon or outgoing photon.

So, what will try to do, out of these equations, we will try to eliminate those things which are having a relationship with the electron. So, first thing that I will like to relate, remember phi was the angle in which the electron was scattered. So, there is the cos phi here. This is the sin phi here. So, I will first like to eliminate this phi, which is rather easy to do it because this p e cos phi, I can take this particular thing to the left hand side. So, this right hand side will become just p e cos phi. Here, on the right hand side, it is p e sin phi. I square and add the two equations. If I do that, I will get here p e square cos square phi plus p e square sin square phi, because cos square phi plus sine square phi is equal to 1. So, I will just get p e square. So, that is the equation which I am writing in the next transparency.

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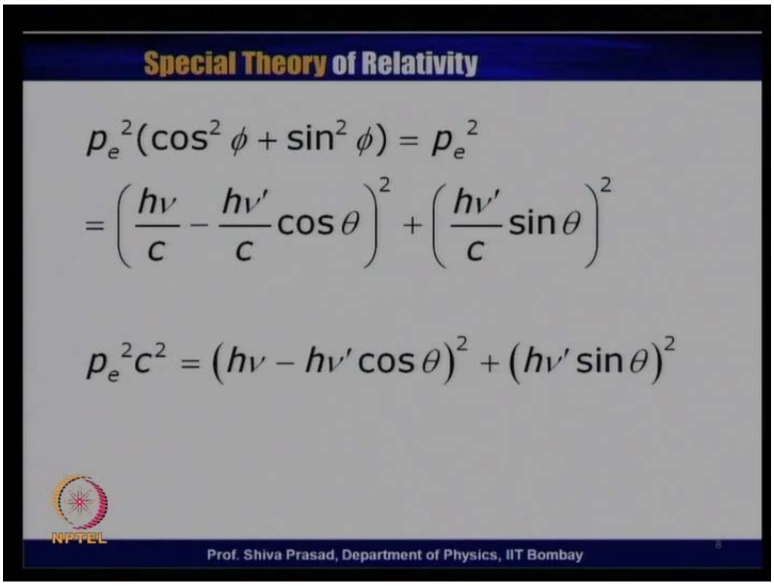

So, I am writing p e square is equal to cos square phi plus sin square phi. It should be equal to p e square. This was the term which I have taken in the first equation from the right hand side to the left hand side. Let me just go back. This is h nu by c and this was h nu prime by c cos theta. So, I have taken this to the left hand side. This is what is written here. h nu by c minus h nu prime by c cos theta. Of course, I have squared it because I have squared it and added it. In the second equation on the left hand side, we had just this term h nu prime by c sine theta. So, I have square and added.

So, this is what I have got after using the first equations and eliminating phi. Now, just to make this equation look somewhat simpler, what I have done, I have multiplied by c square from the both the sides. So, if I multiply by c square here, I will get p e square c square, which appears here, because I have multiplied by c square and this is already a square here. So, inside it will get multiplied by just c.

So, when I have multiplied this by c, I will just get h nu here. When I multiply by c, I will just get h nu prime here. When I multiply by c here, I will get h nu prime. So, I get this equation as p e square c square is equal to h nu minus h nu prime cos theta square plus h nu prime sin theta square. Let us open this equation and try to see whether we can still simplify. So, we have to just the squares and again try to manipulate the terms.

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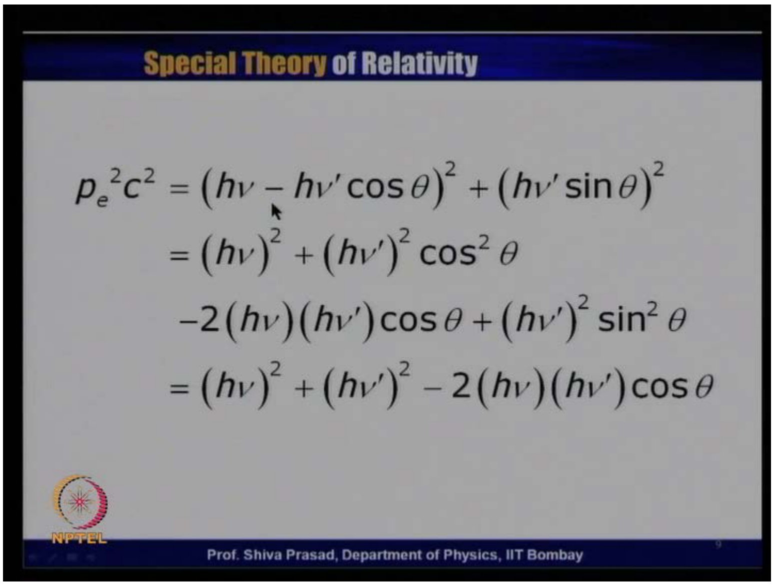

This is what I have done here. Same equation, which have written from the last transparency, h nu minus h nu prime cos theta square plus h nu prime sin theta square. So, I will first explain this term. So, I will get a square plus b square minus 2 a b. So, a square is h nu square, which I have put h nu square and b square is h nu prime cos theta square. So, it becomes h nu prime square cos square theta minus 2 a b. So, minus 2 into h nu into h nu prime cos theta. So, this becomes the expansion of the first term.

The second term is just h nu prime square sin square theta, which I have kept it like that. We realize here that I have h nu prime square cos square theta and there is h nu prime square sin square theta. So, when I take these two terms into consideration together, I will get h nu prime square multiplied by sin square theta plus cos square theta, which again gives me 1. So, these two terms together will just leave me h nu prime square.

So, I go to the next step. This h nu square, I just put it as h nu square. These two terms gives me h nu prime square. So, this I put as h nu prime square and this term, I keep it as it is, which is minus 2 h nu into h nu prime cos theta. So, this is what I have got after elimination of the 5 term relationship between momentum and this particular theta corresponded to where the energies of the incident photon and the outgoing photon. This p e square c square, I can also evaluate from the conservation of energy.

As I said, our idea is to eventually eliminate anything which has to go with electron. Here, we have the momentum of the electron here. On the right hand side, there is nothing which is relating to electron. This is related to the incident photon. This relates to the outgoing photon and similarly, these two terms. So, I would like to get eventually rid of this particular term p e also and there is a easy way because I know that e square is equal to p square c square plus m naught square c to the power 4. So, what I will write is, I will write this p square c square in terms of the energy of the electron. So, that is what I am doing the next transparency.

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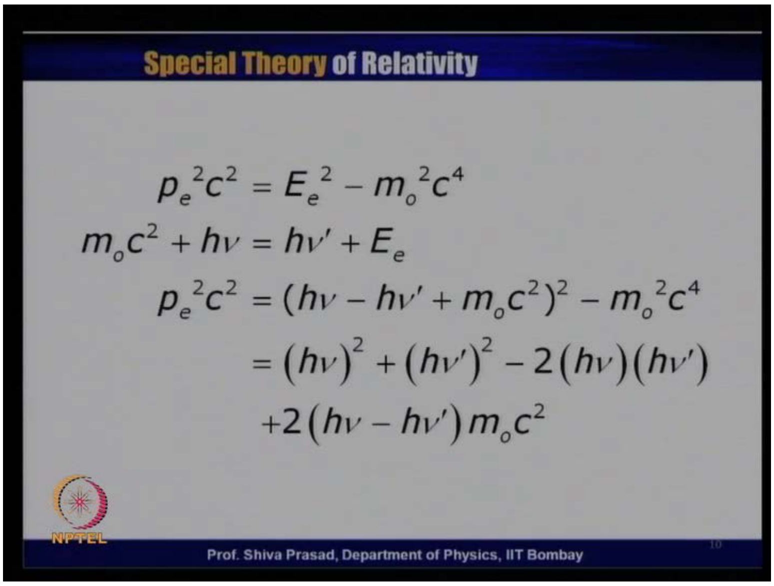

So, this was the energy and momentum relationship for the electron. I have just taken p e square c square on the left hand side and written this as E e square minus m naught square c to the power 4. Now, this energy I have taken from the energy conservation, which I have written earlier. This was my energy conservation equation, m naught c square plus h nu is equal to h nu prime plus E e. So, from this I can extract E e. This E e will be m naught square c square plus h nu minus h nu prime.

So, I know what is E. So, I will substitute this E e here and then I will get on the right hand side everything in terms of h nu. This p e square c square I will eliminate from the first equation, which I have just now written and therefore, I will get rate of energy of the electron as well as the momentum of the electron. So, let me write this particular equation, which I have written here. For E e as, just now seen I have written, from this equation h nu plus m naught c square minus h nu prime, I just slightly reorganized these terms, minus m naught square c to the power 4, which was here, which I just plot here.

Now, expand this particular equation. Now, we have a plus b plus c square. So, we have to take a square plus b square plus c square plus 2 a b plus 2 b c plus 2, all those things. So, let us do it. So, let me just take the square of this particular quantity. I will take h nu square, which is here. I take the square of this quantity, which is h nu prime square, which is here. Remember it will be positive sign here. Then there will be a square of m naught c square and will become m naught square c to the power 4 and this m naught square c to the power 4 will cancel with this m naught square c to the power 4, so I not writing neither this term not that.

So, I have written only these two terms out of these square terms. Now I have to write 2 a b. When I write 2 a b, I write this as h 2 h nu in to h nu prime to m naught c square, which I have written here. Sorry, 2 h nu into h nu prime, this is a and this is b. So, 2 h nu in to h nu prime because there is a negative sign here, this becomes minus 2 h nu into h nu prime. Now, this m naught c square has to be multiplied by this with a factor of 2 and this also has to be multiplied by this as a factor of 2. So, I have combined these two terms and I have written here as plus 2 h nu minus h nu prime taken them together multiplied by m naught c square. So, I have just expanded this particular term here and using this particular thing, I get this as the value of p e square c square.

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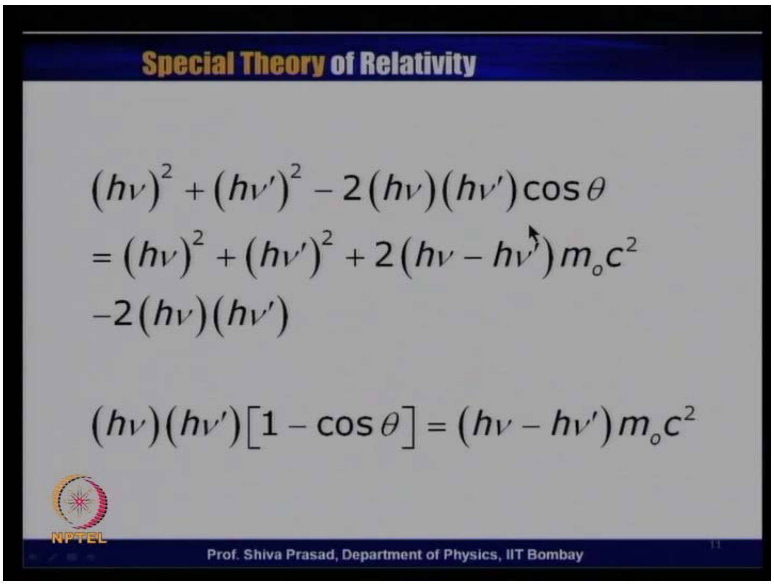

Let us see whether we can simplify it further. This is what I have written earlier, h nu square plus h nu prime square plus 2 h nu minus h nu prime m naught c square minus 2 h nu h nu prime. This I am equating with p e square c square. which I had obtain earlier for elimination from the conservation of momentum equation, which is you remember was h nu square plus h nu prime square minus 2 h nu h nu prime cos theta. We just go back to the transparency. This is what we have written earlier, p e square c square is equal to h nu square plus h nu prime square minus 2 h nu h nu prime cos theta. This equation was obtained by eliminating phi from the first two equations of the inner momentum conservation. The second p e square c square, I have got by applying energy conservation.

So, I can equate this right hand side to that right hand side, which I have done in this particular transparency. So, h nu prime h nu square plus h nu prime square minus 2 h nu h nu prime cos theta from momentum conservation, and this is from the energy conservation, which I have written. You can see very easily that I can simplify this equation further because h nu square will cancel. This h nu square and this h nu prime will cancel with this h nu prime square. Here, what I will be getting, these two factors will also cancel and if I slightly organize and bring this particular term on the left hand side, what you have is, two is already canceled here, so you will get h nu into h nu prime. This being on the left hand side will give me 1 minus cos theta on the left hand side and only this term on the right hand side, which is h nu minus h nu prime m naught c square.

So, this is an equation, which must be obeyed if whatever I am saying is correct, which gives me the relationship between the frequency or the energy of the outgoing photon in terms of the energy or the frequency of the incoming photon and the angle of the scattering, which is theta. This equation can be further simplified, if you write in terms of lambda. Let us try to work out this particular example in a slightly a simpler fashion.

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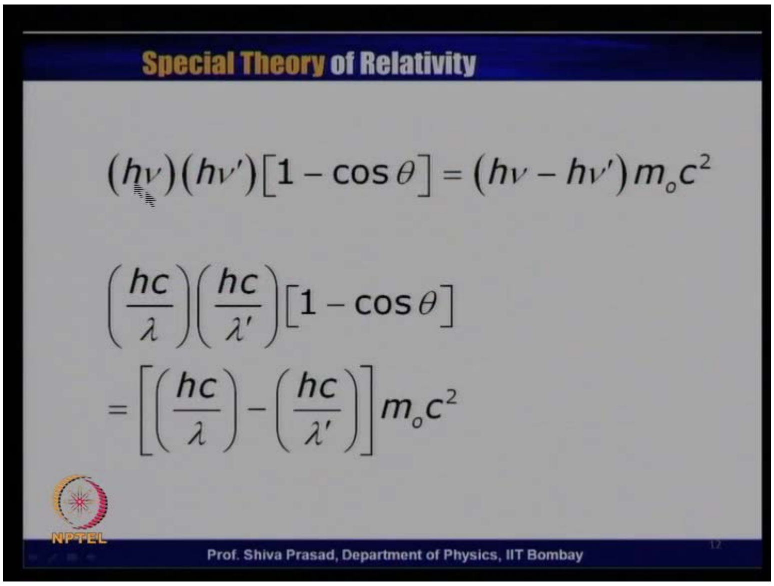

This is what I have written earlier in the last transparency. Now, I express everything in terms of lambda. The frequency of the incoming photon can always be written as c divided by lambda because frequency multiplied by lambda must be equal to speed and a speed of photon is c. Therefore, I can write nu h c by lambda, which is the wave length of the incoming photon. h nu prime, I can write again as h c by lambda prime, because nu prime is equal to c by lambda prime. So, this is what I have written c by lambda prime.

Remember, in general, we will find that nu prime has to be h nu prime and has to be smaller. It means lambda prime has to be larger, into 1 minus cos theta, which is the same term. Here also, I express nu h c by lambda and nu prime as c by lambda prime, writing this equation in this particular fashion. So, this h nu has been written as h c by lambda and this h nu prime has been written as h c by lambda prime.

So, I can cancels lot of h c's. You can see there is an h c here and there is an h c here. There are 2 h c's here. So, probably one of them will remain and will cancel one of the h c's. There is another c here and there is m naught c square. So, there will be one c which will be going away and the equation will turn out to be fairly simple. This was my equation which I had written here earlier.

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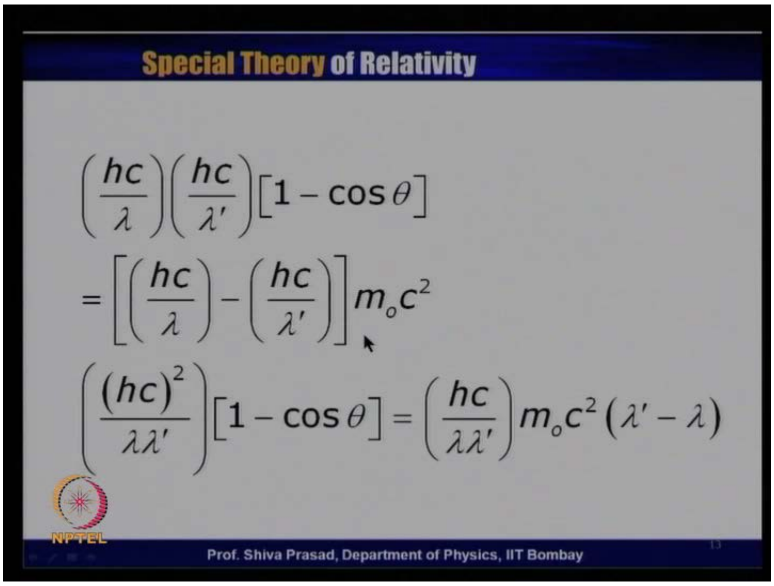

So, I get h c square here because there two h c's here. This is lambda into lambda prime, which I have written here. I take h c and I just use this particular thing and simplify this by taking h c out. So, it will become lambda prime minus lambda. So, this is lambda prime minus lambda and lambda lambda prime in denominator, which I have written in denominator, multiplied by m naught c square, which is this equation.

Now, I start cancelling the term. I write this on the left hand side now, lambda prime minus lambda and this on the right hand side. This h c will cancel with this h c. So, there will be one h remaining here and one c remaining here and there will be m naught c square. So, one of the c will cancel with c. So, there will be one h remaining here and now, the right hand side, there will be just a m naught c, which will be remaining here.

So, the equation which I get is as follows. Just I am trying to write in a different fashion. This, on right hand side, I get the standard well known Compton Effect equation, which says lambda prime minus lambda is equal to h upon m naught c multiplied by 1 minus cos theta. It means, whatever I am trying to say, this scattered photon will have an increased lambda and that difference between the original lambda and the increased lambda will be given by h upon m naught c 1 minus cos theta. Cos theta being always smaller or at the most equal to 1.

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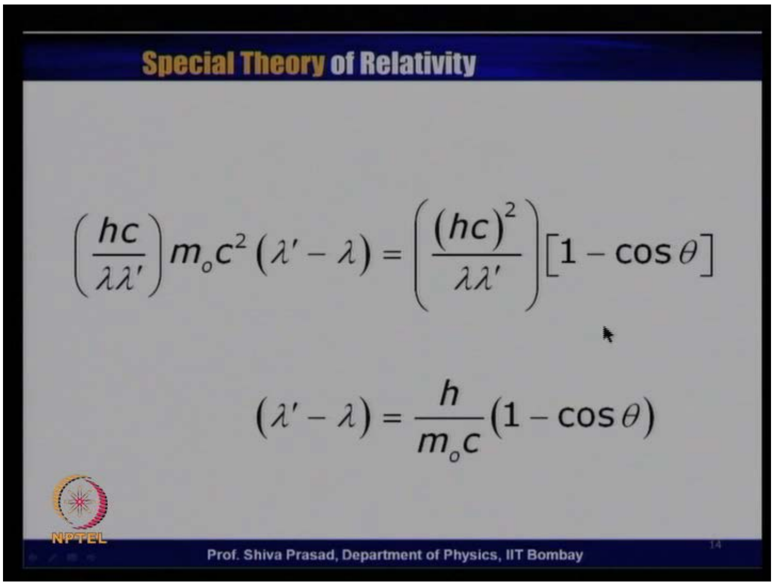

Either lambda prime minus lambda will be equal to 0 or will always be positive as I had expected, because part of the energy of the photon must have gone to the electron. Therefore, the photon, which is coming out, must come out with somewhat reduced energy. It means, reduced frequency and therefore, larger lambda. So, this is the standard Compton Effect expression.

Now, question is that, can I experimentally verify it? If I really experimentally verify it, then I have sort of shown that my idea of photon as the 0 rest mass particle with energy h nu and momentum h nu by c has a meaning and probably, as something which we can take forward. Now, it is very easy to say that you have a free electron and photon comes and hits here. But, where do I get when I want to perform an experiment, a free electron? It is not easy to get a free electron and perform this particular experiment, when you really want to experiment.

So, generally in a Compton Effect experiment, we use a metallic foil. When I use a metallic material, in metals they are supposed to be a very large number of nearly free electrons. I am mean, I am strictly saying they are not really free, but to a reasonable approximation, I can treat them as more or less free. The other problem is that these electrons which are somewhat free and they may not all be addressed, but does not matter. If we have to perform an experiment, we have to see that.

So, what we do, we take a metal and allow electromagnetic wave. In fact, as you will be seeing, that this h upon m naught c turns out to be extremely small. Therefore, the change in the lambdas' is very small. It is in the order of one tenth of next term, which is very small. So, normally when we perform this experiment, we use gamma waves.

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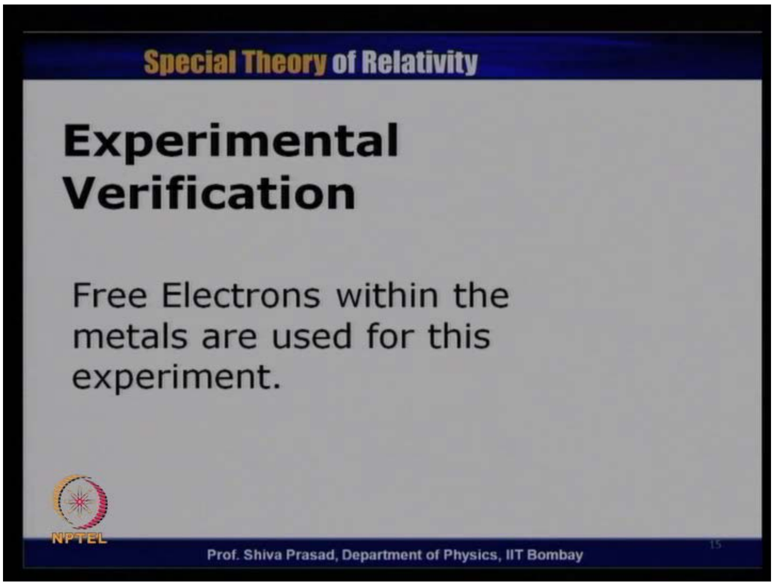

So, this is the way we perform the experiment. We use free electrons within the metals for the experiment and experiment is something like here.

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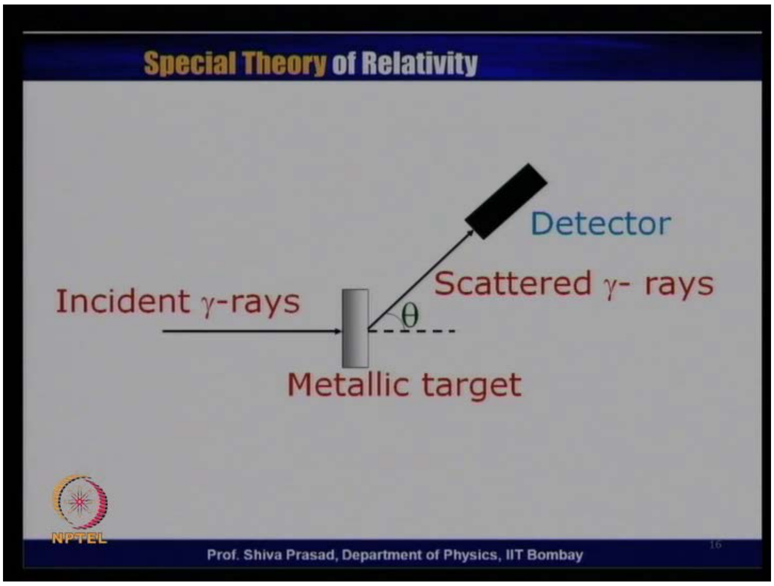

We have a metallic target. There is an incident gamma ray, which comes and hits here and we have a detector, which detects gamma rays. It is possible to obtain detectors, which can detect and which can count, which can tell how many photons have come at a particular angle.

So, if my photon beam is initially coming into this direction and I am collecting my photons in this particular direction, by counting the number of photons reaching the detector. I can measure their frequency, which is also easy to measure. Then I know all these photons must have come at an after scattering at an angle theta. Of course, this theta I can change and keep on verifying my experiment.

So, this is what normally is done. That you allow this particular incident gamma ray to incident on metallic target and you have detector fixed at a particular theta. Collect the photons and measure their energy. Keep on changing theta and keep on going to different theta and at every angle, you keep on measuring the number of photons and keep on finding out there energies when we do this particular experiment, we get something like this. The curve is somewhat like this.

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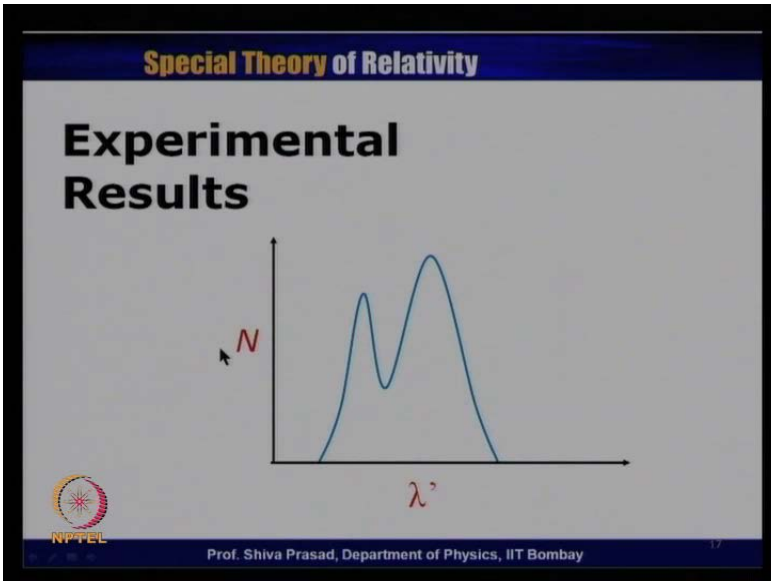

So, if we take the lambda prime, which is the frequency that wavelength that we measure and find out, how many number of photons are coming with that particular wave length approximately. Now, in fact you take a range between a lambda lambda plus t lambda. But, let us not go into those details. We just take N verses lambda prime curve. You find two peaks; one at larger lambda and another at smaller lambda. This peak is generally having much larger width. Now, this original peak corresponds essentially to the original wave length lambda, which was present there. This particular maximum that is indeed corresponding to the lambda prime, that is expected out of the Compton scattering.

So, you do except and you do find a experimental verification. Of course, there is a line width, but we know that there is a natural line width, which is always present. But here, the line width seems to broaden especially in this particular case. Also, we find that the original wavelength is also present. Now, this can be explained as follows. When the photons are coming and being incident on the atoms of the metal, not every photon will get scattered only from the free electron. Some of these photons we also get scattered from the atoms. If they get scattered from the atom, remember in the Compton Effect experiment, what was appearing was h upon m naught c.

So, this m naught for the atom is going be very very large. In fact, you know had it not been an electron, but any other particle, this expression would still be alright. Except that, mass has to be different. Mass that you have to use, has to be of the mass of the particle that is scattering photon. If that happens to be an atom, this m naught is going to be extremely large. Therefore, lambda prime minus lambda is going to be an extremely small, which is essentially negligible. Therefore, you will not find any measurable change if the photons gets scattered from the atom. Therefore, you do also find the original lambda, even though you are measuring or you collecting photons at an angle theta, because the original wavelength is also present, because these are the photons which have got scattered from the atoms and not just from the electron.

Also, we see a larger line width and this particular line width is expected because electrons are not really at rest. They are moving with different type of velocities and therefore, there has to be some amount of distribution of the photon energy that you are measuring. Therefore, this particular width is being explained because of larger number of distribution of velocities of these electrons that you see in the metal. But, basic crux of this issue is that, you do really see that this particular experiment is just does prove that it is possible to get lambda prime minus lambda is equal to h upon a m naught c 1 minus cos theta. This particular peak does satisfy that equation. Therefore, we can treat photon really as a particle with energy h nu and a momentum h nu prime by c.

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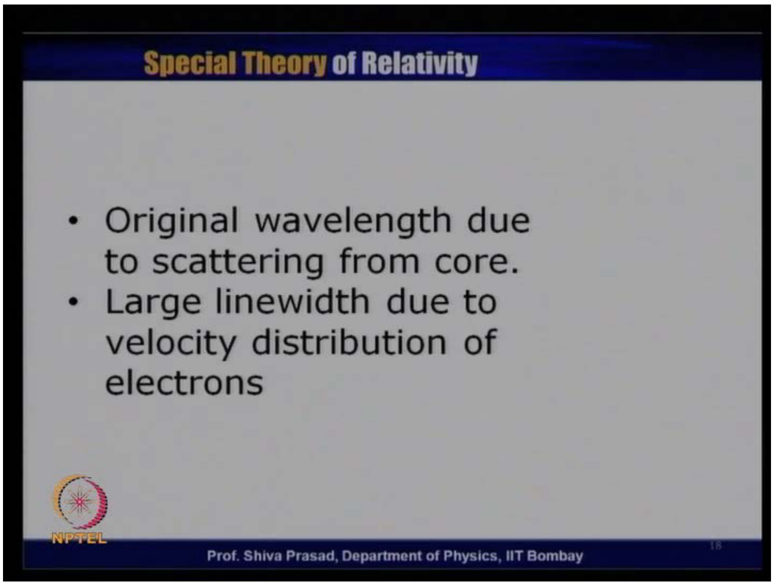

So, this I have sort of written, the calculation of the Compton Effect experiment. The original wavelength is due to this scattering from core and large line width due to velocity distribution of electrons. Now, first we have come to these issues of photons. Let us try to discuss one or two problems based on the photon. All that we have said now is that photon is like, in relativity, is like any other particle. Only thing special about the photon is that, its rest mass is 0. Otherwise, as far as conservation of energy and momentum are concerned, they can be treated just like any other particle.

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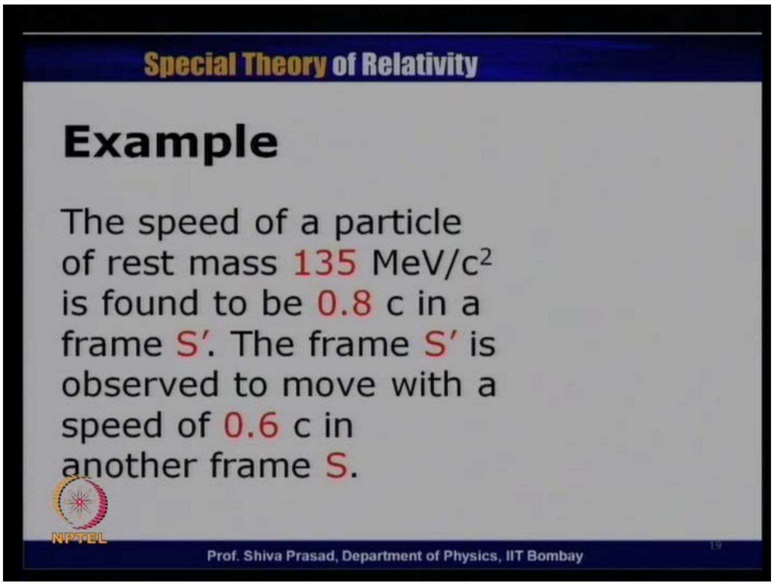

So, let us take one particular example, in which there are particles involving are also involving photons. So, this is my example. So, example is little bigger. So, let us try to slowly understand this particular example, before we try to work it out. So, there is a one particular particle, which has a rest mass of 135 MeV by c square. As I have a told you earlier, that rest mass is expressed in the units of energy. So, we can write this as 135 MeV. But, actually it means that, the rest mass is 135 m e v by c square is m naught c square, which is 135 MeV.

So, we will write in this particular expression, the energy always in the unit of MeV by c square. So, it has a rest mass of 135 MeV by c square. I am sorry, I will write mass in the units of MeV by c square. It is found to be 0.8 c. The speed of this particular particle is 0.8 c in a frame S prime. Normally, we have been giving in the frame S, but here, problem is slightly different.

So, this particular speed has been given in a different frame S prime and the speed of this particular particle as measured in S prime 0.8 c and this S prime itself moves relative to S with is speed of 0.6 c. So, we have a frame S. In this frame, S prime moves with the speed of 0.8 c I am sorry, S prime moves with a speed of 0.6 c and in this particular frame, S prime frame, a particle moves with a speed of 0.8 c.

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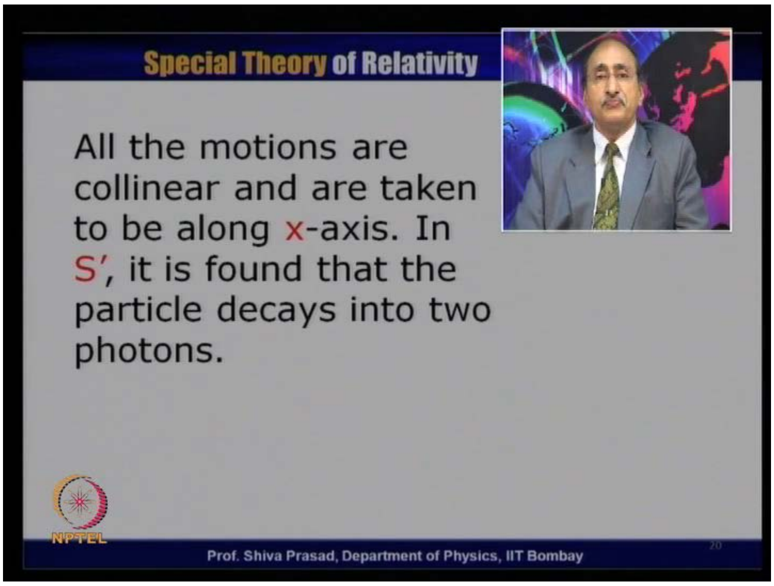

Now, next part of the problem. All the motions are collinear and are taken to be along x direction. So, we assume that everything is moving in the line. In S prime, it is found that the particle decays into two photons. So, what is found that, of course, if taken to S prime, it is also be taken s, but the final things are all given in the frame S prime. So, what we find that this particular particle decays into two photons. So, particle is no longer existing, but it has now become two photons. Two photons have emerged and the particle disappears, so to say.

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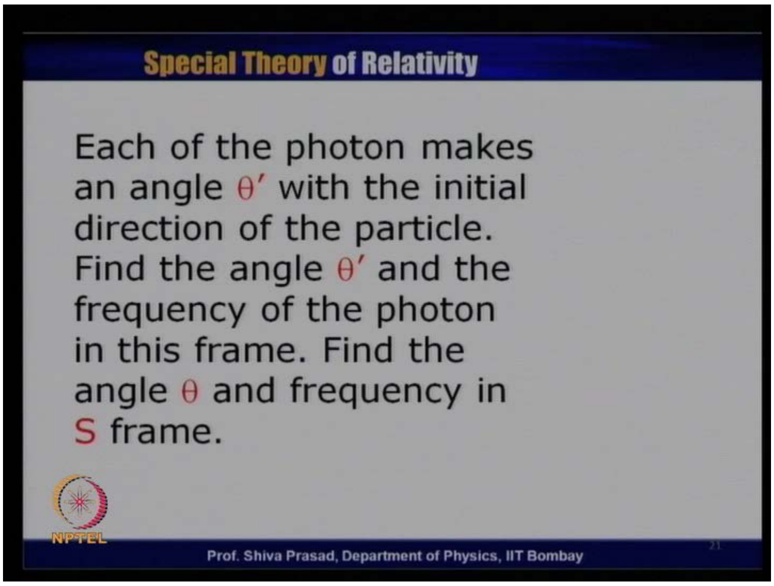

Now, it is found that in this particular frame of reference S prime, each particle makes an angle theta prime with the initial direction of the particle motion. Now, question is, find the angle theta prime and the frequency of photon in this particular frame. In fact, instead of frequency, I can just find the energy. Once I know the energy, I can always find out the frequency. Find the angle theta and the frequency also in the S frame. Though by experiment has been described in S prime frame, but I have also to find out these corresponding quantities in a different frame, S frame.

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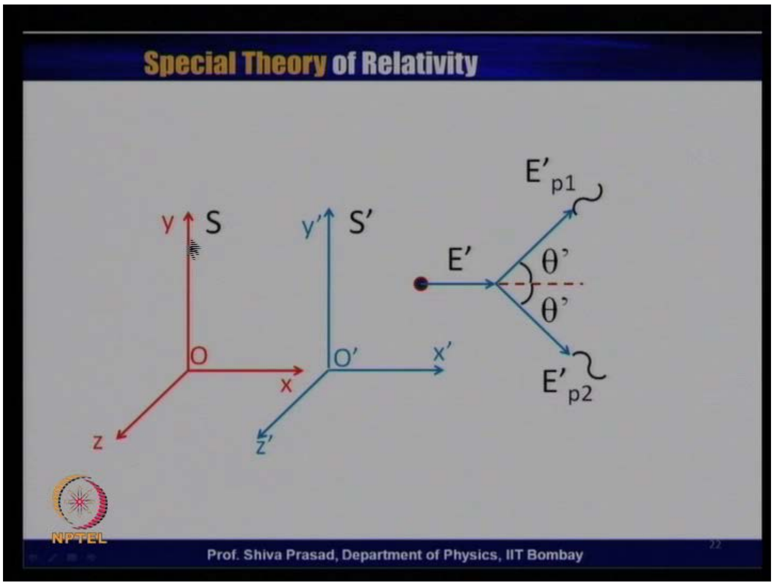

So, this is my picture, which I have shown here. This was my S frame. This was my S prime frame. S prime moves a relative to S with a speed of 0.6 c. I have shown everything in blue here, just to match with this particular frame of reference. So, as far as the observer sitting in S prime is concerned, that particular observer, observes that this particular particle suddenly disappears in two photons come and they make equal angle with respect to the initial direction of the motion of the particle. This angle is theta prime. This is theta prime. This is theta prime with respect to the original direction of the motion of the particle.

Let us assume that this photon has the energy of E prime p 1. Let us assume that this particular photon as energy of E prime p 2. Question is, what is the frequency of the photon and of the photons and what is this angle theta prime? What has been given to this particular problem is only that this angle and this angle are same. Having now discussed conservation of energy and momentum and discussed the fact that photon to be treated like any other particle, this problem is like any other simple problem of collision or scattering, in which all I have to do is to apply conservation of energy and momentum, but relativistic equations have to be used. That is all. So, because the problem has been given in S prime, so let us try to first find out before collision or before this particular decays occurred, let us not use the word collision, the decays occur, what was the energy and what was the momentum. So, everything is being written in the S prime frame of reference. Once I write in S prime frame of reference, the particle speed is 0.8 c.

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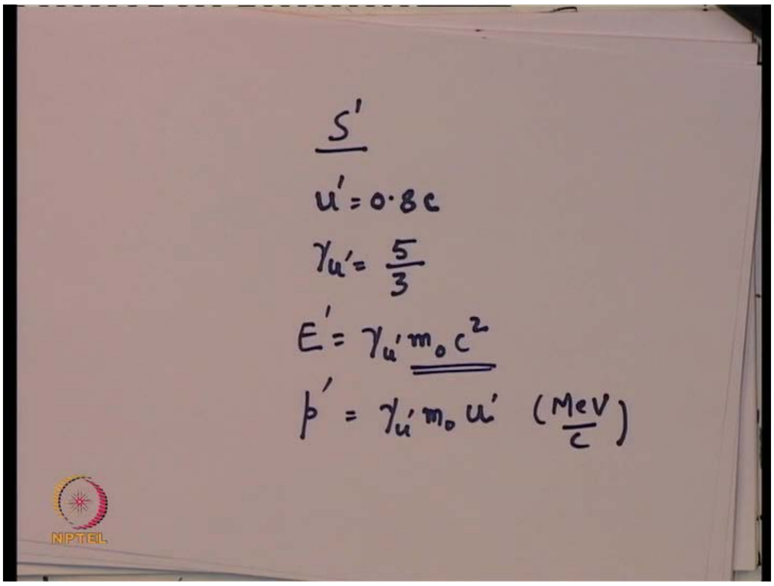

So, if I write S prime frame, the speed of the particle is 0.8 c. Using this particular u, I can calculate what will be the value of gamma u. As you have seen earlier, gamma u corresponding to 0.8 c always gives out to be a (( )) comparatively a cleaner number, which is 5 by 3. Once I know gamma u, I can find out what is the energy of this particular particle because energy of the particle E is given by gamma u m naught c square and m naught c square, I have been given is 135 MeV. Similarly, I can find out what is the momentum of the particle, which happens to be only along the x direction, so I am not putting the vector sign, is equal to gamma u m naught u.

So, this is what I have written in this particular transparency. Of course, because this is S prime frame of reference, so I must use e prime. Here also, I must use u prime to be more precise because everything is being observed. Here also should be gamma u prime because everything is being observed in the S prime frame of reference.

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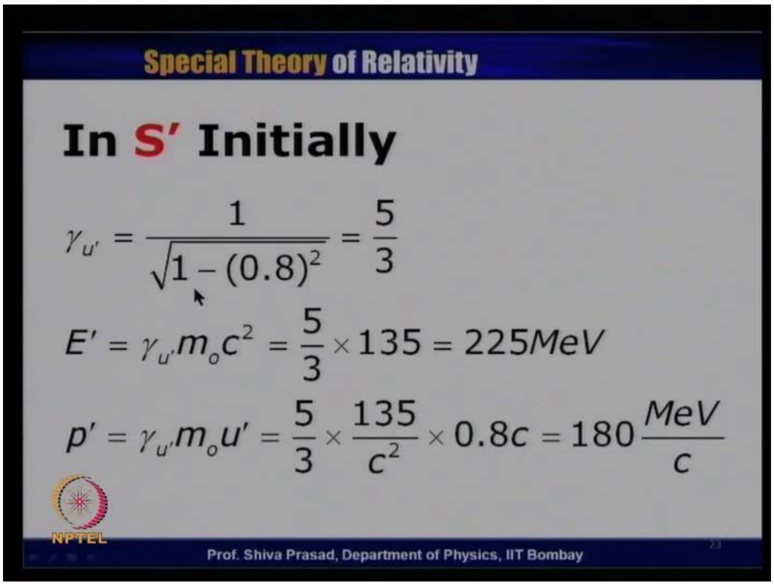

So, let us look at this particular transparency I have written gamma u prime is equal to 5 by 3 using a value of u prime as 0.8 c. E prime is equal to gamma u prime m naught c square, which is 5 by 3 into 135, which is equal to 225 MeV. We just calculated this number. P prime is gamma u prime m naught u prime 5 by 3. This particular value of m naught u prime I have, and this particular value of u prime is 0.8 c. For m naught, I have used the value 135 by c square because m naught c square was 135 MeV. Now, what I have done here, I would like to express the momentum in the unit of MeV by c.

So, one of the c, I have retained here. One c cancels from this. So, there has to be 1one c in the denominator because there is the c square here and there is a c. So, I cancel only one of the c. Another c, I have written in the units.

So, I have to just calculate this 5 by 3 multiplied by 135 multiplied by 0.8 and this c, I am just taking as a part of the unit. So, if I calculate this number, I get 180 MeV by c. So, basically I have used these equations, which I have written here in this particular paper because this is m nu prime m nu prime gamma u prime, substituting these values, only thing expressing this particular quantity in the units of MeV by c. So, this is what happens in S prime before this decay occurred.

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Now, let us see what happens after the decay occurs. After the decay occurs, then what you are getting are two photons. One going this way and another going this way making an angle theta prime with respect to each other. So, like before, like in the Compton Effect experiment, I have to conserve initial momentum to the final x component of the momentum, and I have to balance the y component of the momentum. That is what I am doing in the next transparency.

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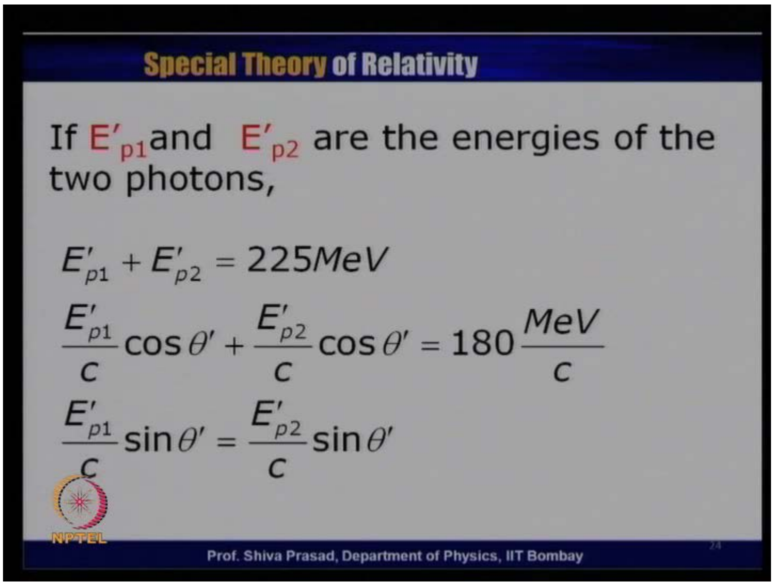

If E prime p 1 and E prime p 2 are the energies of the two photons, the sum of the two energies must be equal to the initial energy that is available to me. That we have just now seen is 225 MeV. So, this is equal to 225 MeV. Along the x direction, I have to take this is the energy. So, momentum will be this divided by c, because this is the photon, which is the 0 rest mass. This is for the first photon. x component must be cos theta, just like Compton Effect experiment; cos theta prime rather. For, the second photon, you must have E prime p 2 divided by c cos theta prime. This must be equal to the initial momentum because the initial momentum was only along the x direction, which was 180 MeV by c. As you can see here, it was 180 MeV by c.

So, I am writing this as 180 MeV by c. In the y component of the momentum in the y direction, there was no momentum initially. So, final momentum must also cancel out. Therefore, E prime p 1 by c sin theta prime must be equal to E prime p 2 by c sin theta prime because these thetas are same. Sin thetas have to be same and c of course, have to be same. So, I get E prime p 1 is equal to E prime p 2. It means both the photons must have the same energy and if they have the same energy, I know their sum is 225 MeV.

So, the energy of each of the photon would be 225 divided by 2. So, I get immediately the energy of the individual photons. Now, I know the energy of the photon, which I can substitute in this particular expression. I can find out what is theta because this theta is same. These two are same. So, I will get 2 into whatever this energy by c cos theta is equal to 180 MeV by c. So, I can immediately find out what is cos theta, which I am giving in the next transparency.

So, as I said that E 1 E prime p 1 must be equal to E prime p 2, which is 225 divided by 2, which turns out to be equal to 112.5 MeV and cos theta prime in the right hand side, there was 180. So, this is equal to 225 by 2, but two the two terms are equal. So, this becomes 225. So, cos theta prime, this c cancel with this c. So, your cos theta prime just becomes 108 divided by 225, which is written here, from which you get theta prime is equal to 36.87 degree.

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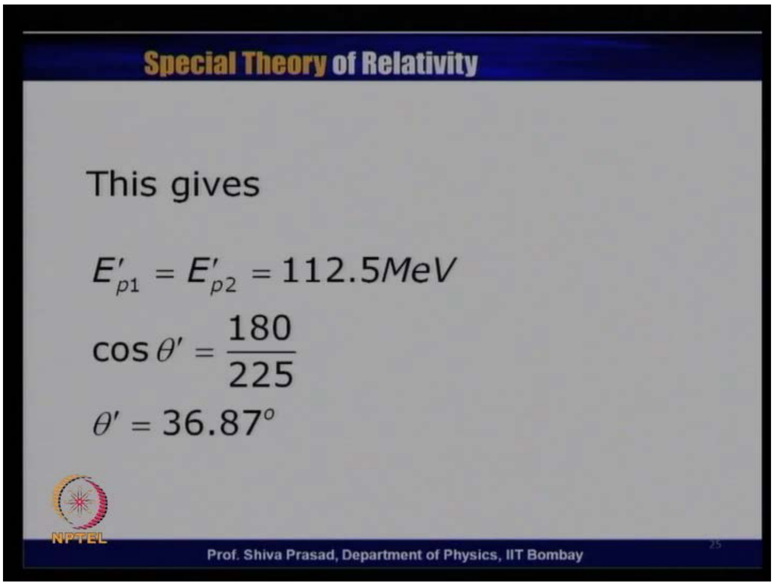

So, approximately at an angle of 37 degree, these photons will get scattered from the incident direction of the motion of the particle. Now, the question is that, we have to find the same information in S frame of reference. What I can do, I have to use energy momentum transformation. I go back to S frame, find out the energy of the particle. I am doing exactly the same thing as I have done here. So, that is what I am doing in the next transparency. I go back to S frame from S prime frame of reference.

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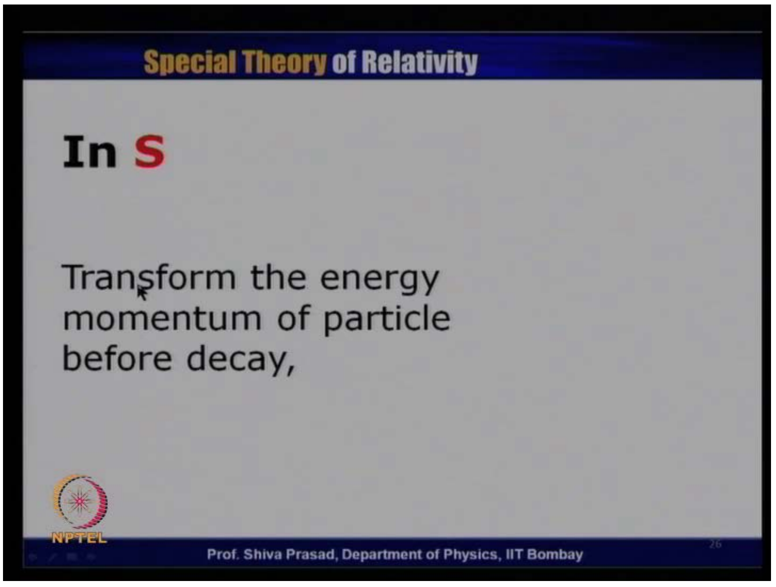

So in S, transform the energy momentum of the particle before decay. Because I am going from S prime to S, I we will be using inverse transformation. That is what is being shown in the next transparency.

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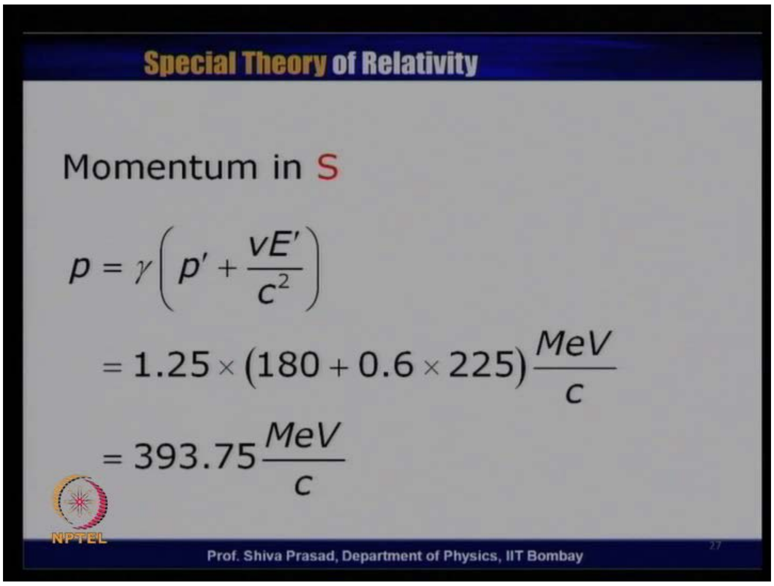

If you remember, this was the equation of the momentum transformation and I have put plus sign because I am talking of inverse transformation going from S prime frame to S frame. So, p will be equal to gamma p prime plus v E prime by c square. So, I know what is p prime of the particle before it decays and I know the energy of the particle before it decays. I substitute in this particular expression, and I should be able to get the momentum of the particle, before it decays in S frame. Of course, gamma that I have to use, is between S and S prime frame and their relative velocity is 0.6 c and not 0.8 c and corresponding to 0.6 c, we have seen many times earlier, gamma turns out to be equal to 1.25.

So, this gamma is 1.25. Momentum p prime was 180 MeV by c. So, this unit I am retaining here plus v, which is 0.6 c. So, there is a 0.6 and there has to be; one c will cancel with this and another c will come out in this unit. This is 225 MeV. So this, everything becomes in the unit of 225 MeV by c. So, if I calculate this thing, I get 393.75 MeV by c.

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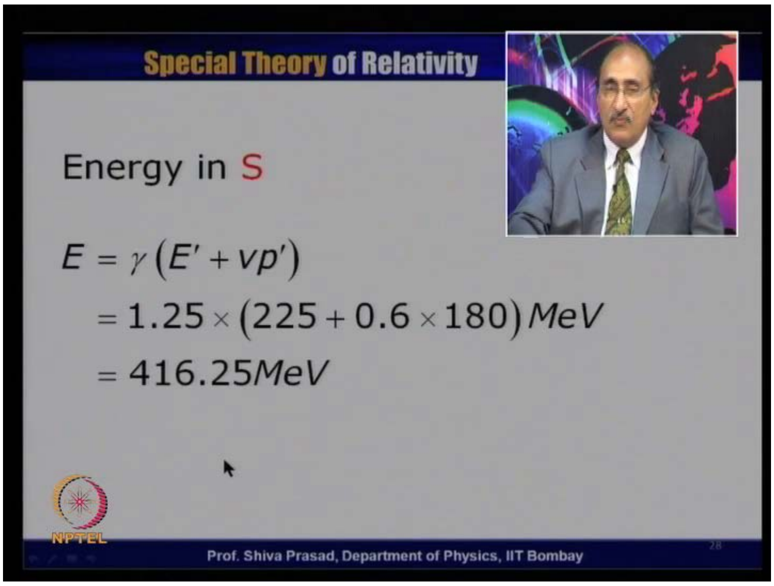

So, this is the initial momentum of the photon, sorry, momentum of the original particle before it decays in S frame. Similarly, I can calculate the energy. For energy, I have to use energy transformation. Again I am using a plus sign, because I am talking of a inverse transformation. So, E will be equal to gamma E prime plus v p prime. Gamma is 1.25. E prime was 225, p prime was 180 MeV by c and there was 0.6 c, which is the speed. So, this c will cancel with this c and everything becomes in the unit of MeV. So, I get 1.25 multiplied by 225 plus 0.6 multiplied by 180 in the unit of MeV. So, by energy, if you calculate this number, turns out to be equal to 416.25 MeV.

So now, problem is exactly similar of what we have discussed earlier. Except that, now the initial particle energy is 416.125 MeV and its initial momentum is 393.75 MeV by c. Let me apply exactly the same way, transformation conservation of energy and momentum and get the new angle, the angle in S frame.

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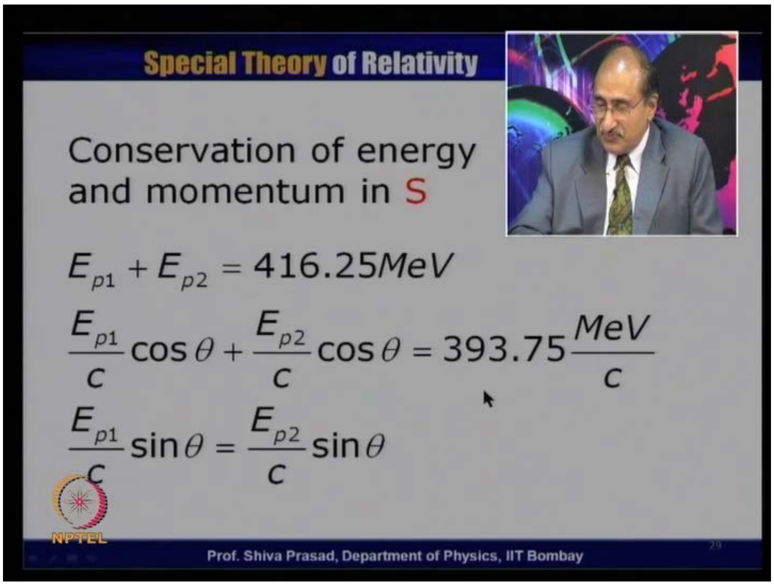

So, that is what is being done here. Just exactly the same equations that I had written in the S prime frame. Except that, instead of prime, I have removed those primes and the numbers that I am using are those numbers which corresponds to the energy and momentum in S frame. As far as energy is concerned in S frame, it is 416.25. So, this I have written here. As far as the momentum is concerned, it is 393.75, which I have written here. I am not using prime because everything is in S frame. Otherwise, all the equations are exactly identical. Once I write this particular equations, exactly the same way I will solve, I will get E p 1 is equal to E p 2 and therefore, the energy will turn out to be half of the energy. I substitute it back and get the new cos theta.

So, I get just E p 1 is equal to E p 2 and that number divided by 2, we get 208.125 MeV and cos theta turns out to be 393.75 divided b 416.25, which is equal to 18.92. Now, we can verify these numbers. In fact, what we have done, we have used transformation before the particle could decay. I could have even used the transformation after I have obtained the photons because their energy will also be transformed. Their energy and momentum will also to be transformed, when I going from S prime to S frame because for us photon is like any other particle. So, I can use exactly the same transform equation even for photon. So, that is what I am going to do next and show that I get the same result.

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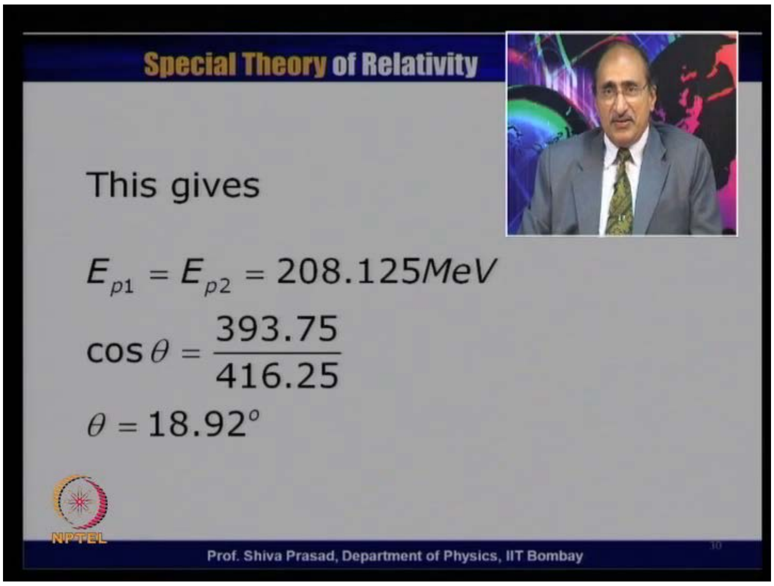

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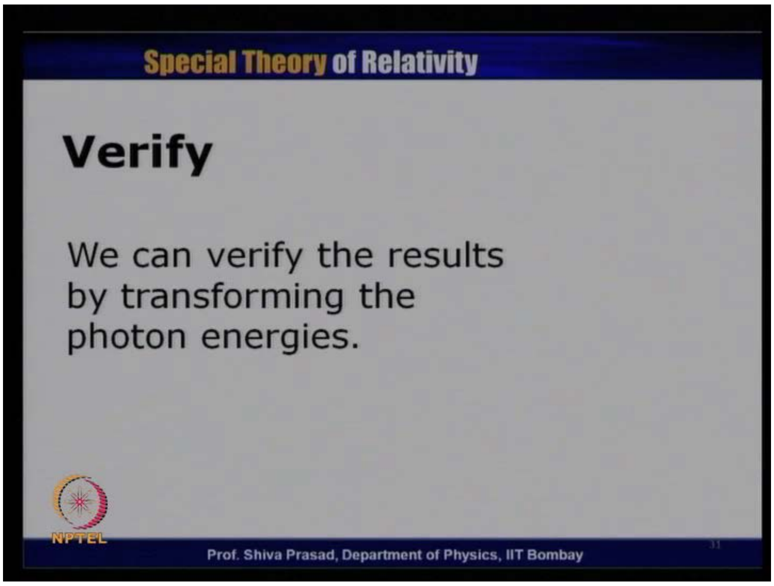

So, I said verify. We can find verify the results by transforming the photon energies and also momentum.

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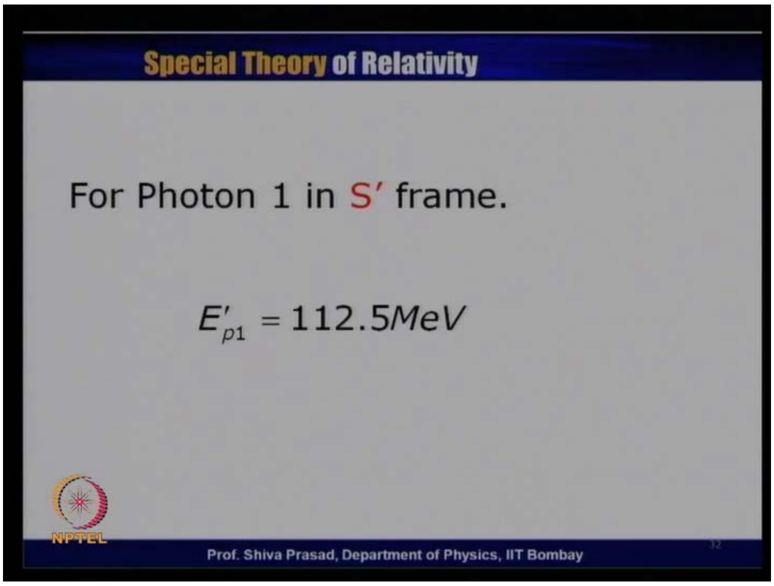

So, let us look at photon 1 in S prime frame of reference. This is what I had got. Its energy is 112.5. This is what we calculated earlier. Now, let us calculate its momentum.

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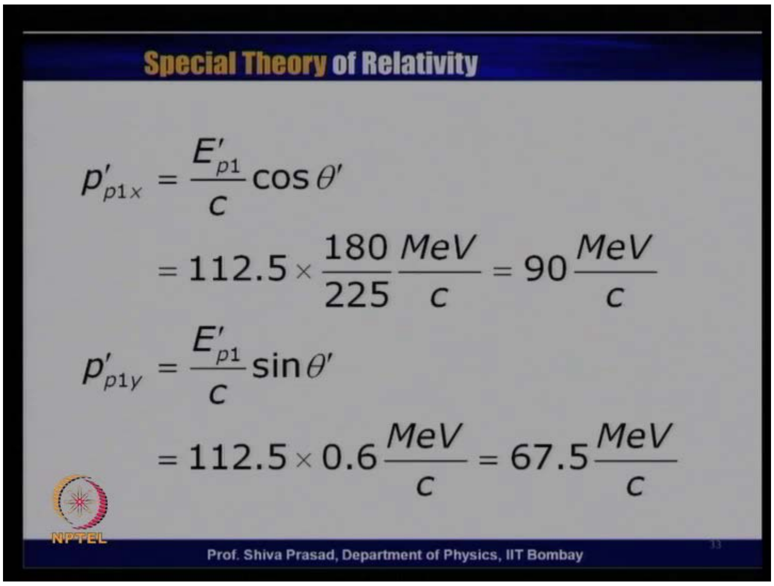

This particular particle was moving at an angle theta prime for incident direction. So, if I take the S component of the momentum; this was the momentum. total momentum of the particle of the photon multiplied by cos theta prime, which I had already calculated as 180 by 225. This was the energy. This c I take in the unit of, the momentum, MeV by c. So, the x component of the momentum, if you calculate this number, it turns out to be 90

MeV by c. The y component of the momentum is this momentum multiplied by sin theta prime. If I do that particular thing, I take this particular value is 112.5 and take sin theta prime. This turns out to be 0.6 and I get this number as 67.5 MeV by c. So, I have found out what is x component of the momentum, and what is the y component of the momentum. Now, transform this thing back to S frame.

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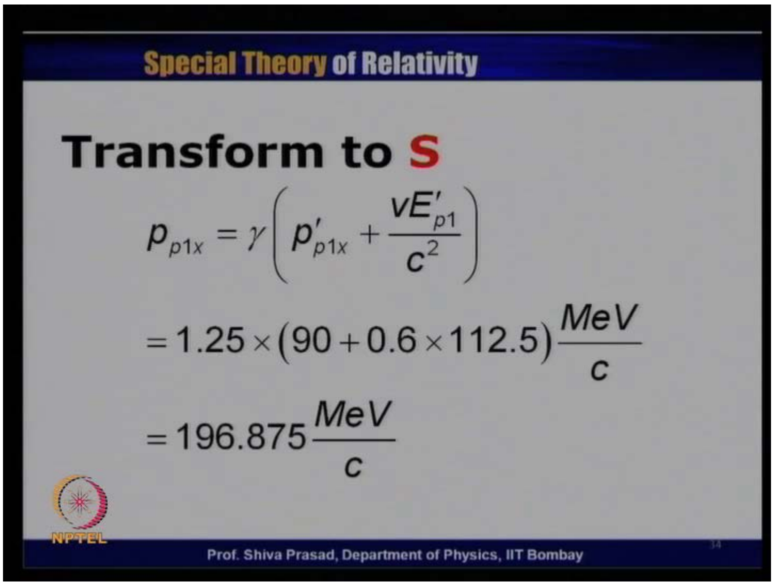

So, I calculate p 1 x, which is the momentum of this photon, x component of the momentum of the photon in S frame of reference. Your same transformation equation. There is no difference. Photon is just like any other particle. So, I write gamma times p prime for the first particle, x component plus v E for the first particle is divided by c square. Gamma has to be 1.25 because I am transforming from S frame to S. Exactly similar equations. This momentum is 90.6 multiplied by these 0.6 multiplied by 112.5. You calculate this number and you get 196.875 MeV by c.

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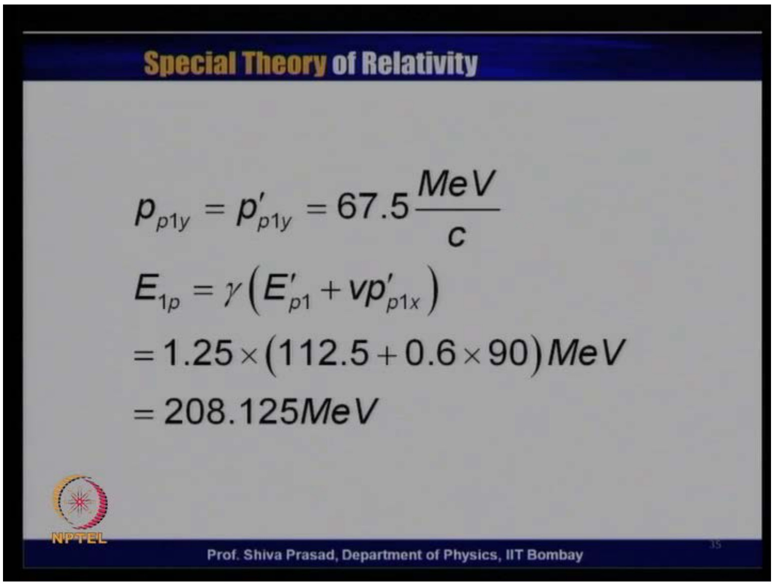

Now, as far as y component of momentum is concerned, there is no problem because they are same. So, I just write p 1 y is equal to p prime p 1 y, which is 67.5 MeV by c. Of course, I have taken the photon which is the positive component in the y direction. If you take the other photon, that will have the exactly the same value, but will have a negative component in the y direction, if the photon was coming down rather than going up. So otherwise, this particular expression will be valid; this number is valid for both these photons.

Now, I conserve and transform the energy of the photon, If I transform the energy of the photon, I get energy of the photon in S frame as gamma multiplied by the energy of the photon in S prime frame plus v multiplied by the x component of the momentum of the photon in S prime frame of reference. So, this gamma as we have seen is corresponding to a value of 0.6 c, which is the speed between S and S prime frame. I substitute here 1.25. E prime p 1, we know is 112.5. This V is 0.6 and this was 90, this is 0.6 c and this is p prime was in the unit of 90 MeV by c. So, this c cancels out and I get everything in the unit of MeV.

So, if I calculate the energy of the photon in S frame of reference, this will turn out to be 208.125. Now, I have to find out the angle in S frame. That I can find out because I know the y component of the momentum and I know the x component of the momentum, which is here.

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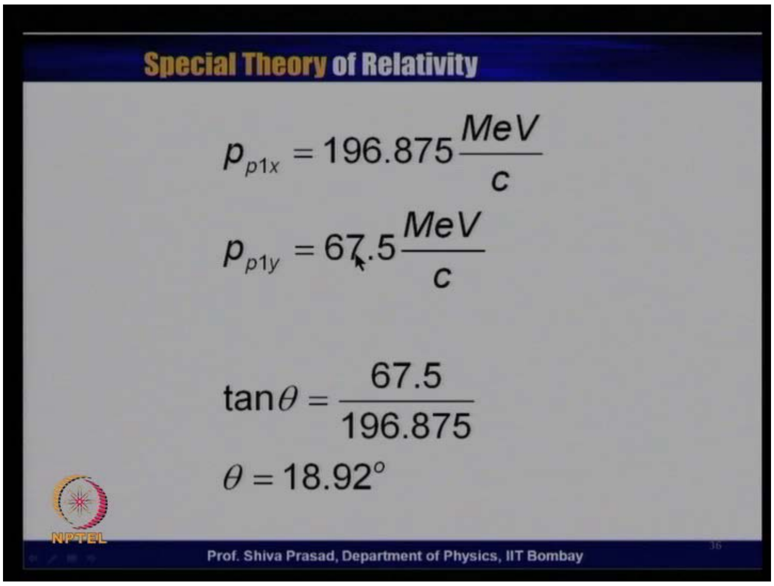

So, that is what I am doing in this particular transparency. This is the value of x component of the momentum. This is the value of the y component of the momentum. I just take tan theta, this divided by this gives me the value of theta, which I get exactly the same as 18.92. Of course, I have also found out the energy of the photon in S frame and what has been asked in the problem is the frequency. But, as I have said that, if you know the energy, I can always find out the frequency by dividing it by h.

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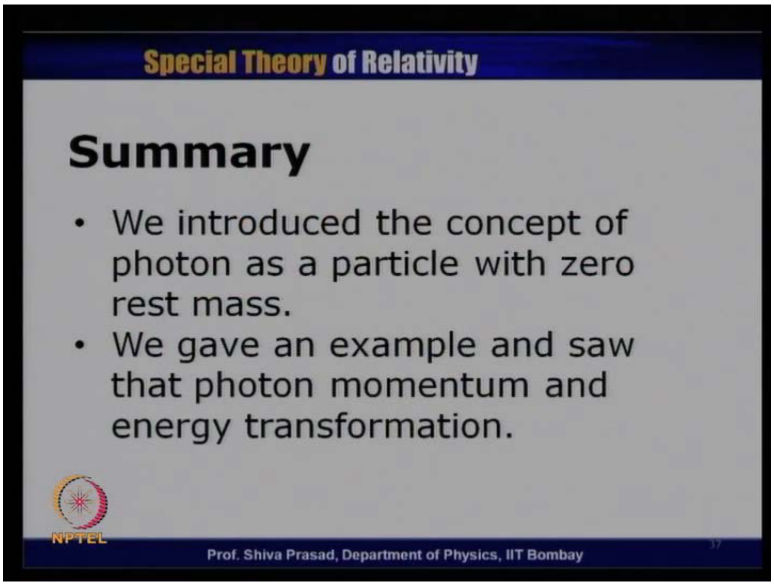

So essentially, the problem gets solved. The basic idea of this particular problem is just to tell that, like we are applying energy momentum transformation for any other particle, I can apply also for the case of the photon. So, in the end, I will summarize whatever we have discussed. We had introduced the concept of photon as a particle with 0 rest mass. We gave an example and saw that photon momentum and energy transformation, how they have been applied.

Thank you.