Special Theory of Relativity
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Lecture - 16
Relook at Collision Problems

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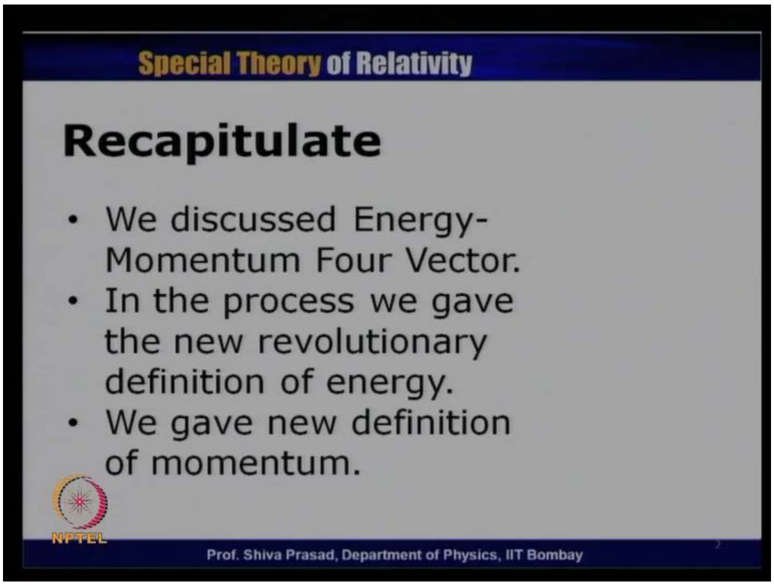

Hello. Let us recapitulate what we did in our last lecture. We had an idea of looking at the conservation laws, which are obeyed in all the inertial frames as per the postulate of special theory of relativity. So, we were looking for a new definition of momentum, which could satisfy that particular criterion namely, that is momentum is conserved in a given frame; it should be conserved in all the frames. When we were looking at that particular aspect, we realized that the conservation of momentum critically depends on the fourth component of what we originally called as momentum four vector.

Then we tried to reinterpret that particular fourth component and invoked that, that is to be called as the energy of the particle; which was a totally revolutionary and totally different concept of energy, which does not have a classical analogue. So, this is what we discussed and we said that this particular four vector is now re-termed as momentum energy four vector. So, this is what is recapitulation?

We discussed the energy-momentum four vector. Then in this process, we gave a new revolutionary definition of energy; we did not have any classical analogue. And of course, as a consequence, we arrived at a new definition of momentum. So, we had a new definition of momentum; we had a new definition of energy. Of course, the definition of momentum in the classical limit meaning that, the particle speed is much smaller than speed of light, will reduce to the classical definition of momentum. But, the same will not be true for the energy definition, because energy was a totally different concept altogether.

We were said that, if a particle is at rest, that particular particle also has some energy, which we call as rest mass energy. And, the kinetic energy we defined as only the additional amount of energy gained by the particle when the particle starts moving. So, this was totally a revolutionary concept. And, this had such a great amount of impact even on a common public that, people started almost naming Einstein by E is equal to m c square; E is equal to m c square became synonymous with Einstein. People even know, who may not be understanding physics; they are still aware the glamorous relation of E is equal to m c square.

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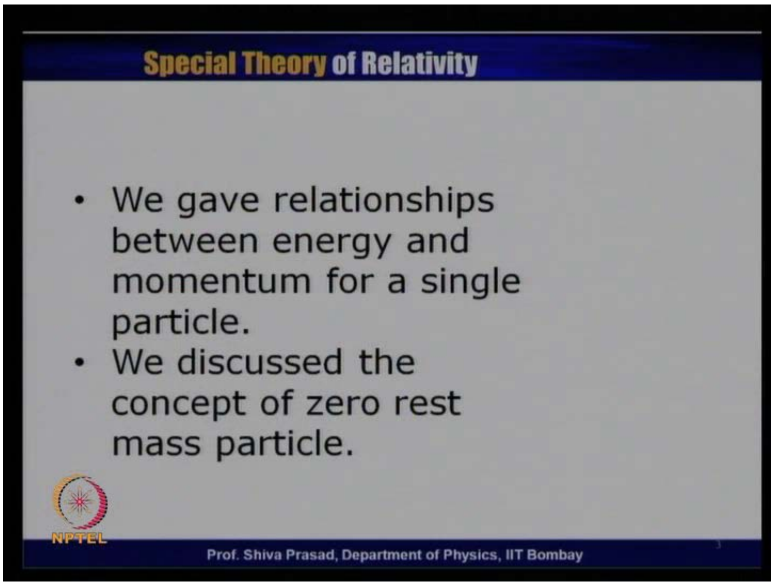

Then we gave the relation between the energy and a momentum of a single particle, because once we have to solve the problems, we have to start looking at the relationship between the energy and the momentum. And, some of the problems we are going to describe today. Then we also discuss a totally new concept, which was not existent in the classical mechanics, because in classical mechanics, a particular particle has to have some finite mass. Here we considered, we imagined a particular particle, which could have a zero rest mass; and that particular particle could also have energy and momentum. The only condition is that, in that particular case, the particle must move with the speed of light. So, this was again a new concept, which came out from this special theory of relativity of a particle with zero rest mass.

Now, today, what we will be doing; we will try to work out some of the problems about the collision; and specifically, we will revisit the earlier problem of the collision, which we had started with. As a consequence of which we had decided that the conservation of momentum will not be valid in all the frames of reference unless we redefined the momentum. That is the way we had started if you remember, how we lead to new definition of momentum.

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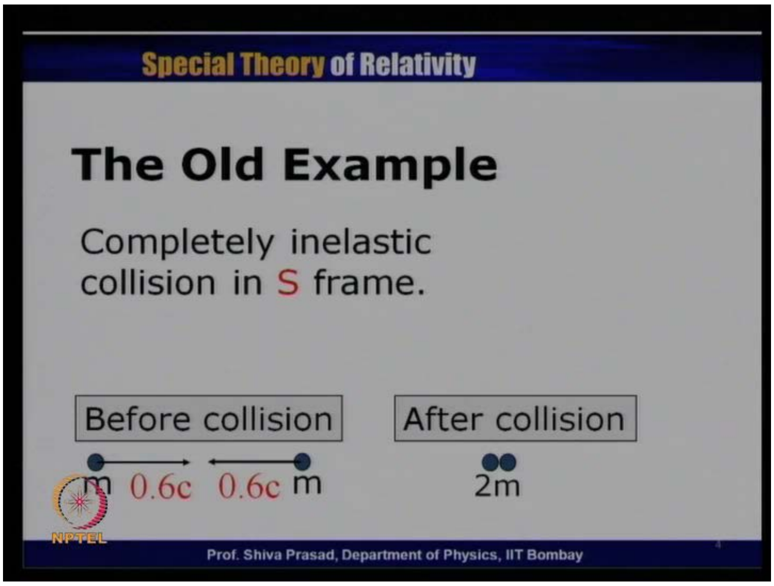

So, let us look at this particular old example, which is here. Just to remind you that, what we had discussed in our earlier case that, there was one particular particle m, which was moving to the right-hand side, which we can call as a plus x direction with a speed of 0.6 c. There is another particle, which is also of mass m, which is moving to the left-hand side with the speed of 0.6 c. These two particles collide with each other and eventually gets stuck to each other. So, after collision, you have a combination of the two masses. If we remember the example, which we had given earlier; and that time we did not have a new concept of rest mass; we did not have a concept of new energy. At that time, we had just talked about one mass, because mass was sort of that sense, essentially sort of universal. So, we had said in that particular case that, of course, in this particular example, when we look at this particular frame of reference; then we do find that, momentum is conserved, because here the initial momentum is 0; and after the collision, the final momentum is also 0. Therefore, momentum was conserved.

Then, we transformed the frame of reference and went to the frame of reference of this particular particle. It means we assumed another frame of reference in which the particle is at rest or the frame of reference is moving with a speed of 0.6 c along the plus x direction. Then we applied the velocity transformation that we had derived on the basis of Laurens transformation and tried to find out the speeds of these particles in that particular particles frame of reference, which we called as S prime frame of reference. Then we discussed and we found out that, the momentum was not conserved in S prime. That is where we have arrived at this particular conclusion that, momentum needs to be redefined.

Now, let us look at this particular problem again in the light of the new information that we have got regarding momentum and energy. So, what I would sort of rephrase this particular problem now that, we will assume that, this particular particle – the mass m that I am talking is m naught, which is the rest mass of the particle. So, there is one particular particle, which has a rest mass m naught, which is moving to the right with the speed of 0.6 c. There is another particular particle, which has a rest mass m, which is moving to the left-hand side with the speed of 0.6 c. Let us not talk about this particular thing. Remember this particular picture I have taken just from that old example. In fact, this picture would probably require modification once we have described this particular problem. This is the way we had described it earlier.

Now, let us look into the new definitions and try to work out the same thing and try to really convince ourselves that, if momentum is conserved in one frame, it has to be conserved in the other frame also; which we did not in the earlier case when we did not have the knowledge of new definitions of energy and new definition of momentum. So, initially, what will do, we will look at this particular frame only for here this particular collision has been described. So, I assume that, there is a particle with mass – rest mass m naught, which is moving to the right with a speed of 0.6 c; and another particle with rest mass m naught moving to the left with a speed of 6 c. And, let us try to write the new values of the momentum that I will obtain from this particular case.

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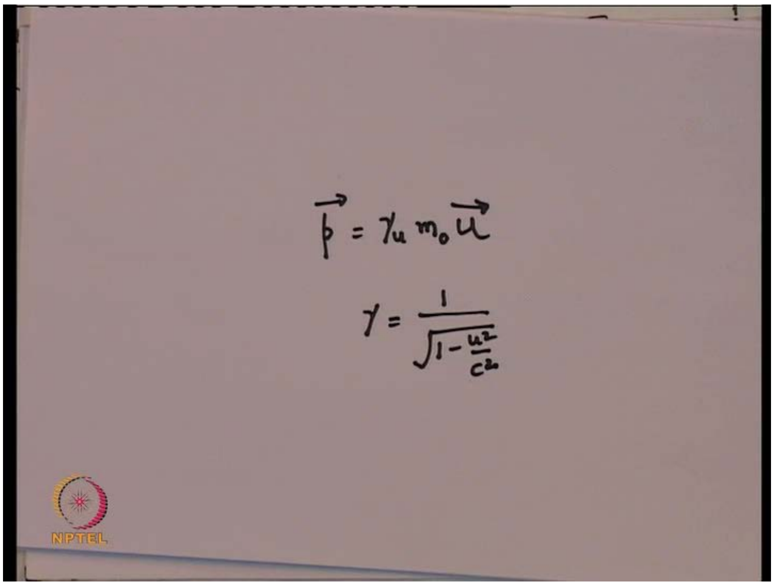

Now, if we remember, the new definition of momentum was p is equal to gamma u m naught u. So, basically, differs from the classical definition, because there is also a gamma u vector. And, this gamma u as we know, is related to the particle velocity, which is given by under root 1 minus u square by c square. So, here we have a collision of two particles; and each is of rest mass m naught. So, what I will be doing; first, let me try to calculate the momentum of the first particle and then I will calculate the momentum of the second particle as given in this particular frame of reference.

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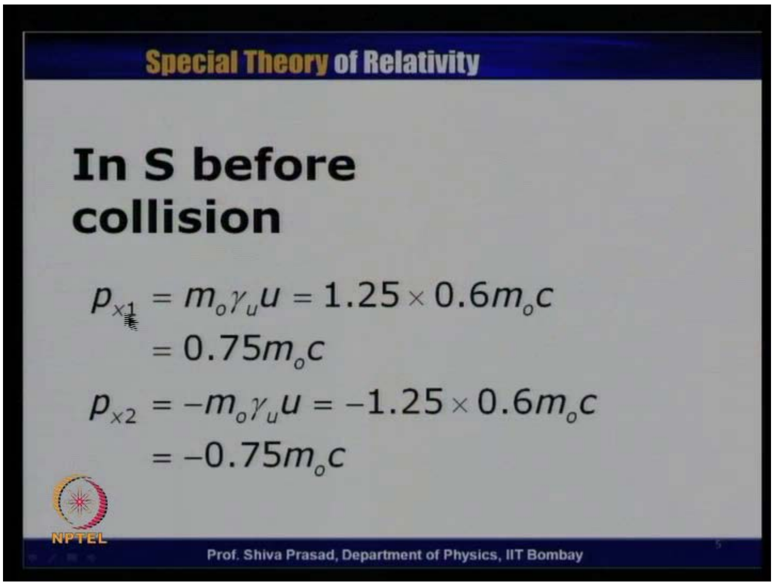

So, this is what I have written here. So, this is the momentum of the first particle, because it is particle. So, I have written 1. And, this is the particle, which is moving to the right with the speed of 0.6 c. Now, this is the expression, which again I have just written; which is m naught gamma u times u, which is the new definition of momentum. (( )) let me calculate gamma u. As we have just now seen, gamma u is equal to 1 upon under root 1 minus u square by c square. And, if I substitute the value of u is equal to 0.6 c in this particular expression, we know that, this expression would lead to a value of gamma u is equal to 1.25; u is equal to 0.6 c is one the speeds, which leads to a rather simple result, a simple answer for gamma u. So, gamma u will turn out to be 1.25. So, that is what I have written here. This is the value of gamma u, which is 1.25. I have slightly reorganized this equation to express momentum in the units of m naught c. So, this speed u is 0.6 c and this is m naught. So, this becomes 1.25 multiplied by 0.6 m naught c. If I simplify it, this becomes 0.75 m naught c.

Now, the second particle is also moving with the same speed of 0.6 c. But, it is moving in the minus x direction. Therefore, this momentum has to be negative. Momentum is a vector quantity; it has to be negative, because sign is opposite to it. So, we use the exactly the same expression minus m naught gamma u u. Again gamma u is 1.25 and speed is 0.6 c and the m naught. So, I get same numerical value, but with a different sign. So, particle – second particle's momentum is minus 0.75 m naught c. Of course, the y component and z component we need not bother, because in that particular direction, the momentum is anyways 0. We are assuming that, the motion is only along the x-direction. So, this is what I have found out that, are the initial momentum of the particle number 1 and particle number 2 before collision. And, as you can see that, magnitude wise, both of them are same; only their signs are different. So, if I add these two momenta, these two momentum would lead me a 0 value, which we had expected even earlier that, the initial momentum of the two particles, sum of the momenta of the particles would be 0.

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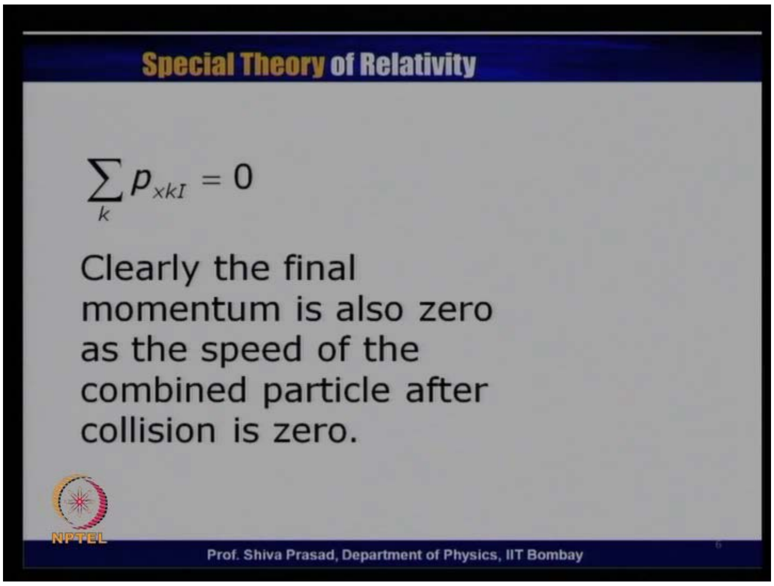

So, this is what I have written in the next transparency that, p x k; where, k of course, gets added up for the two particles from 1 to 2, is equal to 0. And, this I symbol refers to the fact that, this is the initial momentum, which is before the collision. So, clearly, (( )) the initial momentum and final momentum obviously 0, because if I look back at this particular equation, p is equal to gamma u m naught u. And, if u has to be 0, then of course, p has to be 0. And, because in this particular frame it has been given that, after the particle get stuck, they come to a rest. Therefore, this u must be 0 in that particular frame. Therefore, p is also 0. So, clearly, the momentum after the collision is also 0. Therefore, the momentum is conserved. This is what we had expected that, momentum should be conserved in this particular process.

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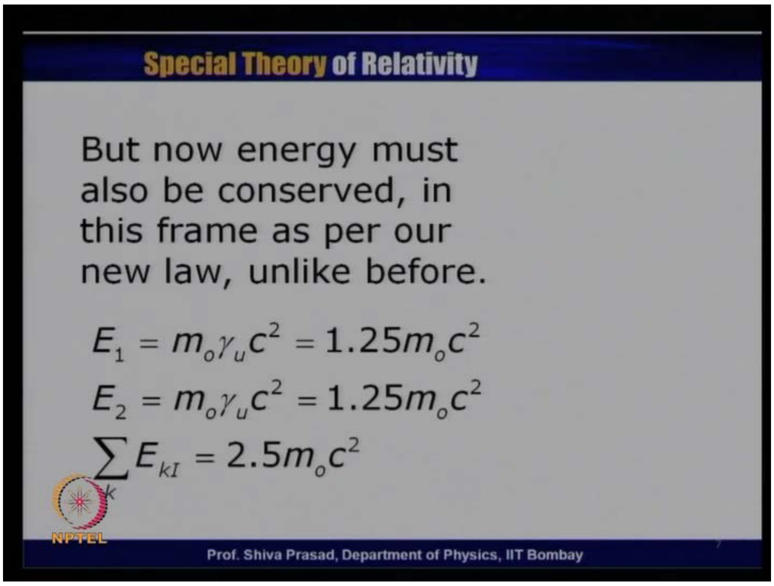

However, there is a different situation now. If you remember, when we worked out this problem earlier or when we work out a similar problem in a classical mechanics – traditional classical mechanics, we said that, this is an inelastic collision; and therefore, we never used to conserve the energy at that time. We at that time said, it is only momentum, which is conserved; energy is lost in some non-mechanical processes. In fact, we always meant that, overall energies never sort of disappearing; it is always conserved. But, what we mean at that particular time is of mechanical energy that, if you are taking potential energy and kinetic energy, those types of energies, the mechanical part of the energy; they are not conserved, because the energy is lost in some non-mechanical form like heat or whatever it is. So, we were never conserving the energy at that particular instant of time.

But, now we realised that, with the new definition of energy and momentum, the energy must also be conserved in this process, because remember, the conservation of momentum, universality of conservation of momentum depends critically on conservation of energy. So, unless energy is also conserved, momentum will not be found to be conserved in other frame. So, in this particular frame also, I must conserve energy, but with a new definition of energy and not the classical definition of energy, which involved only the kinetic energies. So, we conserve the energy. Therefore, I must write the energy of the first particle and energy of the second the particle. If I write the energy of the first particle, the expression is E is equal to m c square, which is the well-known expression.

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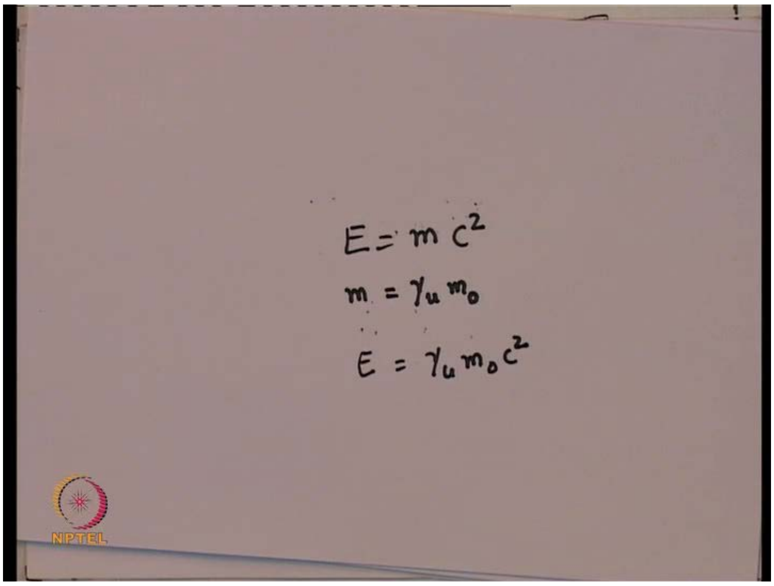

And, this m we have said, is gamma u times m naught, where m naught is the rest mass of the particle. So, if I have to write the energy of the particle, which is total relativistic energy of the particle, then what I have to use for E is gamma u m naught c square. So, this is what I have written here. For the first particle, E 1 is equal to m naught gamma u c square. Gamma u we just now discussed is 1.25. So, I put it 1.25; I get this equal to the energy of the first particle as 1.25 m naught c square. The second particle also moves with the same speed.

Therefore, the gamma value is again 1.25 though its speed is in negative direction. But, the energy is a scalar quantity. Therefore, I will not put a negative sign. It is the same energy, which is 1.25 m naught c square. Therefore, the total initial energy E – summation of E k I must be equal to this energy plus this energy; which means 2.5 m naught c square. So, what it says that, the initial energy of the system of the particle of the two particles was 2.5 m naught c square. And, I now want that, after the collision, again this energy should remain same. It means the combined particle, which has got stuck should also have the same energy of 2.5 m naught c square if this particular particle has to obey the conservation of energy, which we expected to be obeyed the according to the new rules of energy and momentum conservation. But, if I take the earlier expression, where I have assumed that, the two masses just gets stuck.

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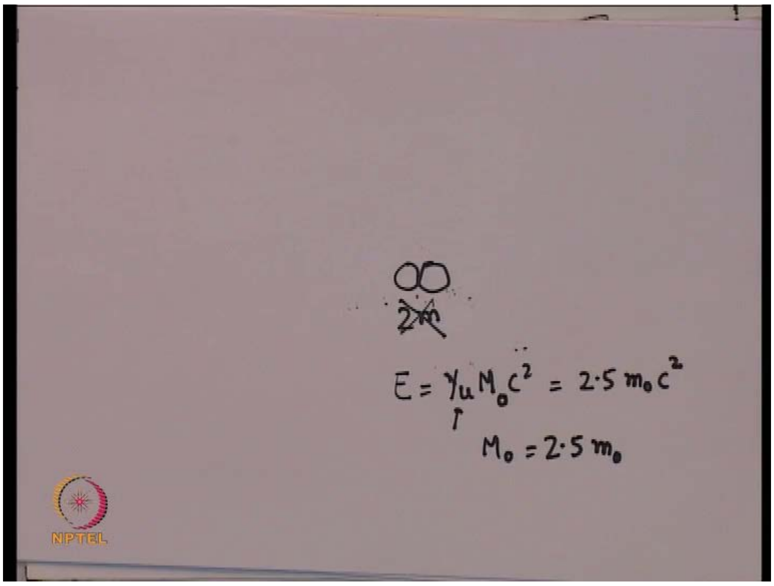

And, remember in that particular picture, I had drawn two masses and I have written 2 m at that particular point. Then this 2 m obviously cannot have energy of 2.5 m naught c square, because the particle at rest. But, essentially, it means that, this is not correct. When the two particles have got stuck to each other, of course, there is u, is 0. So, when I write the total energy E, gamma u is 1, because this is 1, because the particle is at rest. Therefore, this energy must result with a different m naught. It means the rest mass of the particle has changed. And, this gamma u m naught c square should be equal to 2.5 m naught c square, which I have written earlier. What it means that, after the particle have got stuck though they have come to rest, but their rest mass energy would have gone up, because this gamma u being 1, this means m naught is equal to 2.5 times m naught. So, the two particles once they have got stuck, their mass is no longer 2m, but it has become 2.5 m naught; their rest mass has gone up.

Why rest mass has gone up? How you would explain on the basis of relativity? You will say that, that energy, which was lost; it was lost into some non-mechanical form of energy. But, as far as E is equal to m c square is concerned, that does not look into that particular aspect, because overall energy must be conserved. And of course, that energy is no longer of the kinetic form. Therefore, u is still 0; gamma u still equal to 1. But, that infest; that particular energy shows itself in the form of an increased rest mass, because if the masses have become probably warmer, because of the collision, because of the loss of energy, this eventually result in its increase of rest mass. So, what it shows that, the two particles after they have got stuck to each other, their rest mass has gone up. That must be true if momentum and energy – both have to be conserved in this particular frame of reference.

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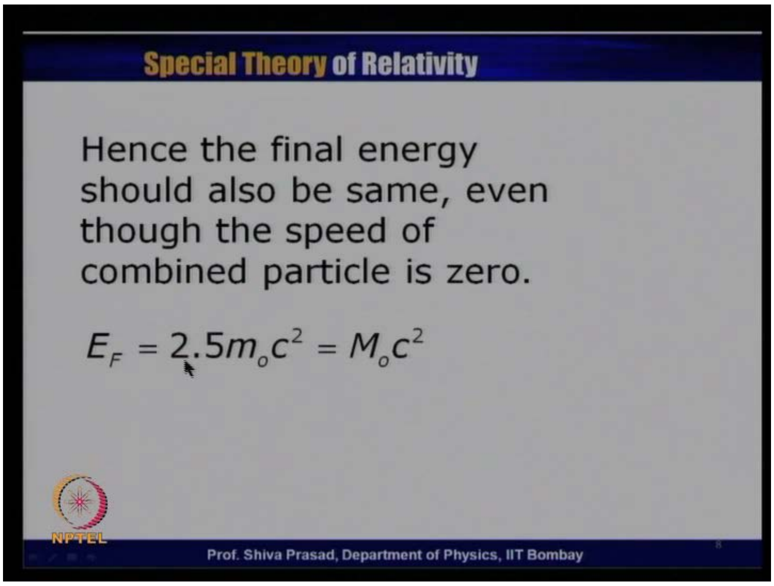

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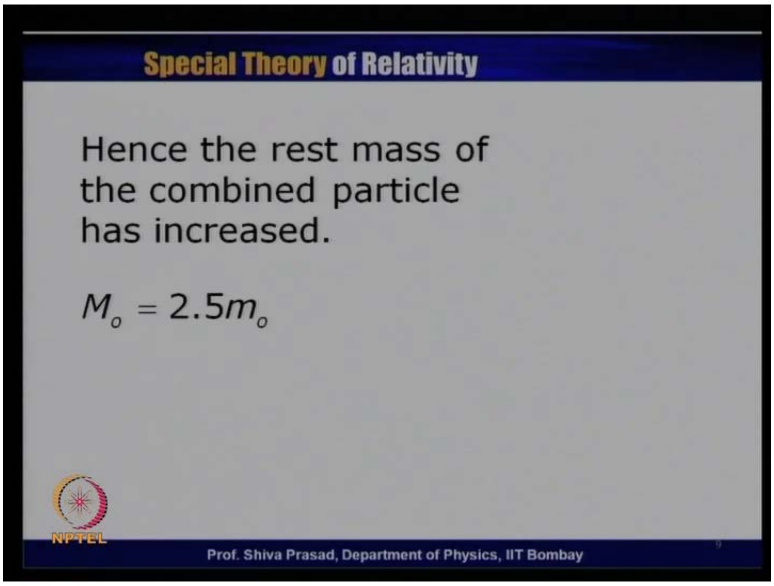

So, this is what I have written here. Hence, the final energy should also be the same even though the speed of combined particle is 0. So, gamma u is 1. So, 2.5 m naught c square must be equal to new M naught c square, because gamma u is equal to 1; which obviously means that, M naught is equal to 2.5 m naught; the rest mass of the particle has gone up.

This is what I have written here, is the rest mass of the combined particle has increased. And, M naught – the new rest mass of the particle is 2.5 times m naught. See remember that is what we have said that, if the two particles gain certain energy, then this rest mass goes up. If they lose certain amount of energy; if they tend to become cooler, then again their rest mass will go down. That is what is the new way of looking into the energy. Say as we said that, there is no difference between the energy and mass in relativity; mass and energy are related by fundamental constant c square.

If system has more energy, it has more mass; if system has a lower energy, it has a lower mass. Mass and energy are always related. In fact, as we have said earlier that, you can express energy in terms of masses or mass in terms of energy. So, many times, when we say, the mass of electron; many times we will tell within the unit of MeV or mass of the proton in terms of the MeV or GeV; while mega electron volt, giga electron volt – these are the units of energy and not of mass. But, then still we can express them, because the two are related with a fundamental constant c square.

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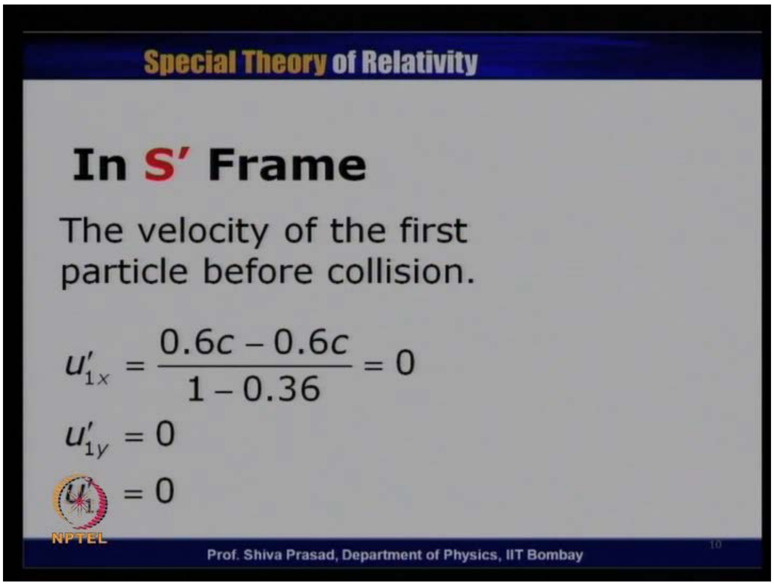

Now, let us look at the S prime frame of reference, because that is the way we have started. From S frame, we concluded that, the rest mass of the particular particle has gone up after the two particles have combined with each other. Now, remember this rest mass is a four scalar. So, this rest mass is not going to change even I go to S prime frame of reference. So, I must use the same rest mass of the particle even after I change the frame of reference from S to S prime. But, now, let us recalculate the momenta in S prime frame of reference. When I try to recalculate momenta, of course, the values of momenta individually will be different. But, eventually, momentum must be conserved and energy must be conserved, because I expect the conservation principles to be universal. So, let us go to S prime frame of reference.

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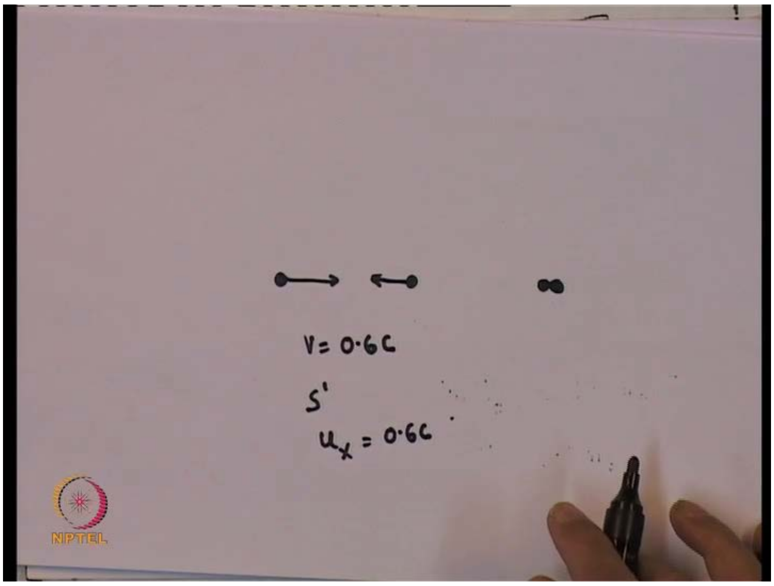

So, let us look at the velocity of the first particle just before the collision. Remember we have two particles in S frame: one going in this way; another going this way. Now, I go to a frame of reference of this particular particle. It means I go to a frame in which v is 0.6 c. Now, looking at this particular frame of reference, I go to this particular frame of reference with v is equal to 0.6 c. Calculate the speed of this particular particle, which obviously should be 0. Calculate the speed of this particular particle. Find out the total initial momentum. Then again, I look at that particular particle, which has been obtained after the collision. Find out the speed of that particular particle. Calculate the momentum of that particular particle. And, try to see whether I get the conservation of momentum and energy. That is what I want to do.

So, I go to this particular frame of reference S prime, which has v is equal to 0.6 c. First, I look at this particle itself for which u will be 0.6 c. I apply velocity transformation. So, this is what I have applied here. So, u 1 x prime is equal to u x minus v. So, u x is 0.6 c; v is 0.6 c; 1 minus u x into v divided by c square. Both u x and v are 0.6 c; c square gets divided. So, I get 1 minus 0.36; c square gets cancelled out. And, because numerator is 0; so u 1 x prime is 0, which is expected, because once you go to the frame of that particular particle, the speed of that particular particle must be 0. If you are in a frame, you do not notice its own speed with respect to anything. According to you, the speed of the frame is 0. So, obviously, u 1 x prime is 0.

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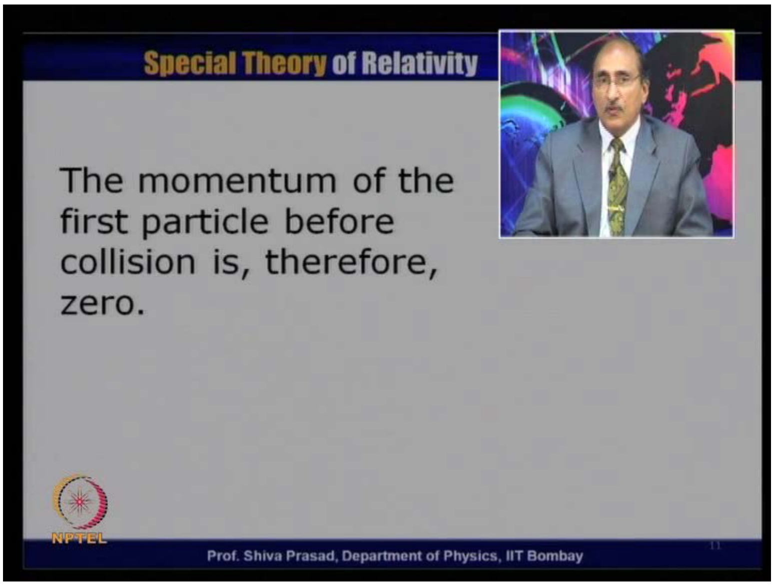

The momentum, then of course, is 0, because we have just now discussed, if the speed of the particle is 0, the momentum is 0 even with the new definition of momentum. So, there is no problem over there. So, momentum of the first particle is indeed 0. But, let us look at the momentum of the second particle. That will not be 0. So, let us find out the speed of the second particle.

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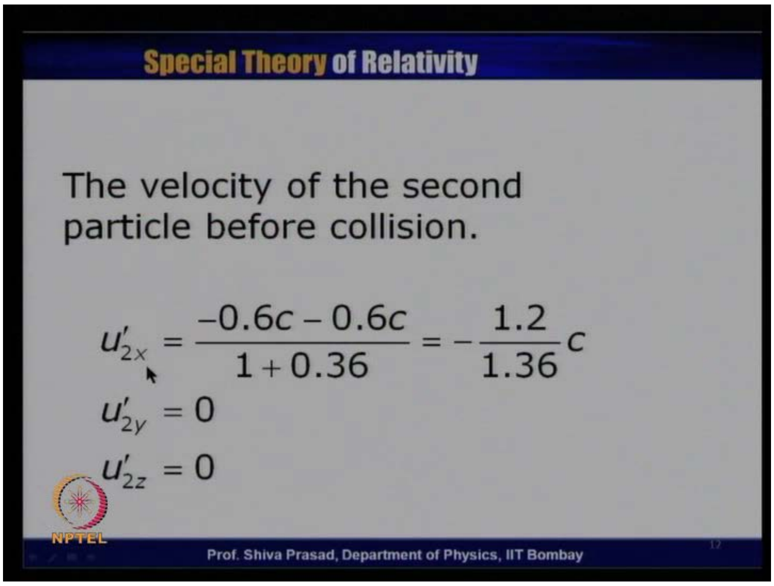

We use exactly the same expression. I write u prime 2 x. Now, for the second particle, the particle was moving to the left with the speed of minus... with a speed of 0.6 c. So, I write u x as minus 0.6 c; minus v, which is same – 0.6 c, because I am going to same frame of reference of the first particle divided by 1 minus u into v x, because of negative sign. This will become plus. And, this expression will remain as 0.36, because both u x and v are 0.6 c each; c square will cancel. So, you get the speed as minus 1.2 divided by 1.36 c. This calculation we have done earlier also when we had done this particular example earlier. Only thing, at that time, we had used the old definition of momentum; now, I will be using the new definition of momentum. So, using this particular u, I must calculate now gamma u, because the new definition of momentum not only involves the speed and mass, but also involves gamma u. So, let us calculate gamma u. This is being little longer, little big; you will find out that expression little bit more complicated. But, eventually, the result is simple.

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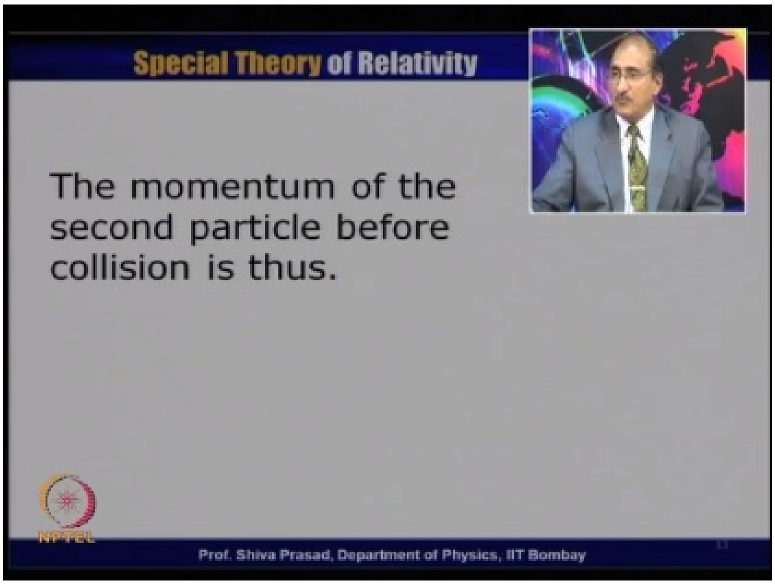

So, that is what I have written. The momentum of the second particle before collision is thus.

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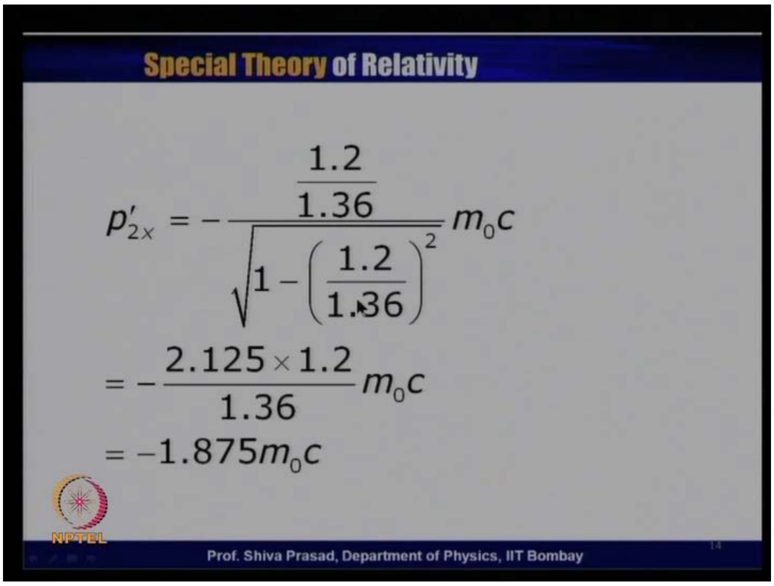

This is the gamma u value 1... So, not the numerator here; but 1 divided by under root 1 minus u square by c square. The speed was 1.2 divided by 1.36 c. So, c square and c square cancels. So, in numerator you are remained with just 1 upon under root 1 minus 1.2 divided by 1.36 square.

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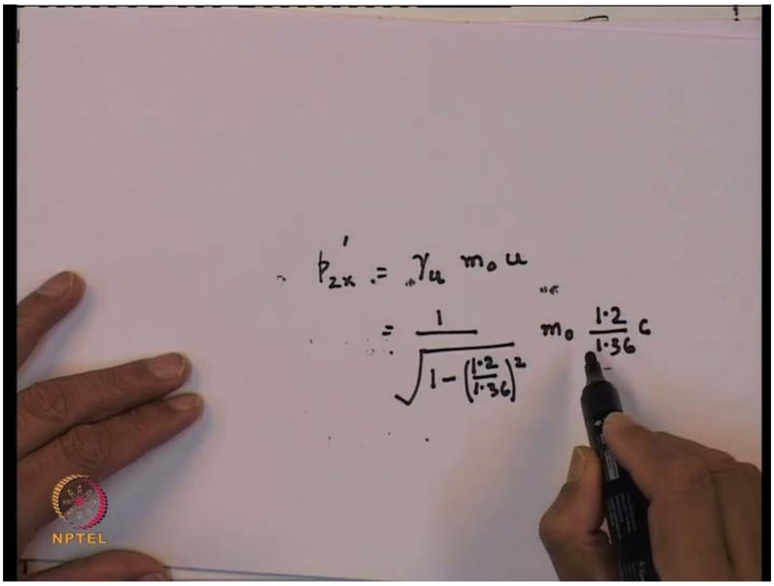

Let me just rewrite here just to make it clear. So, you have p 2 x prime, must be equal to gamma u m naught u. Now, this gamma u I can write as 1 upon under root 1 minus u square by c square. This was 1.2 divided by 1.36 c divided by – of course, you squared – divided by c square will cancel. What will remain is this square. This is m naught. And, u will become 1.2 divided by 1.36 c. So, what I have done in this transparency, I have brought this particular thing up into the numerator to express this in again the units of m naught c. So, this is what I have written here; p prime 2 x is equal to minus 1.2 divided by 1.36 divided by under root 1 minus 1.2 divided by 1.36 whole square m naught c.

If you calculate this 1 upon under root this particular gamma u value; that turns out to be equal to 2.125. This multiplied by 1.2 divided by 1.36. Eventually, you can calculate; this will give me a value of minus 1 875 m naught c. You remember this is minus, because the speed of the second particle was negative. If you just look, this speed is negative, because if you are sitting on the first particle, the second particle moves in minus x direction. So, we now realised that, once we go to the S prime frame of reference, then the momentum of the first particle is 0, but not of the second particle. And therefore, the initial momentum is not 0.

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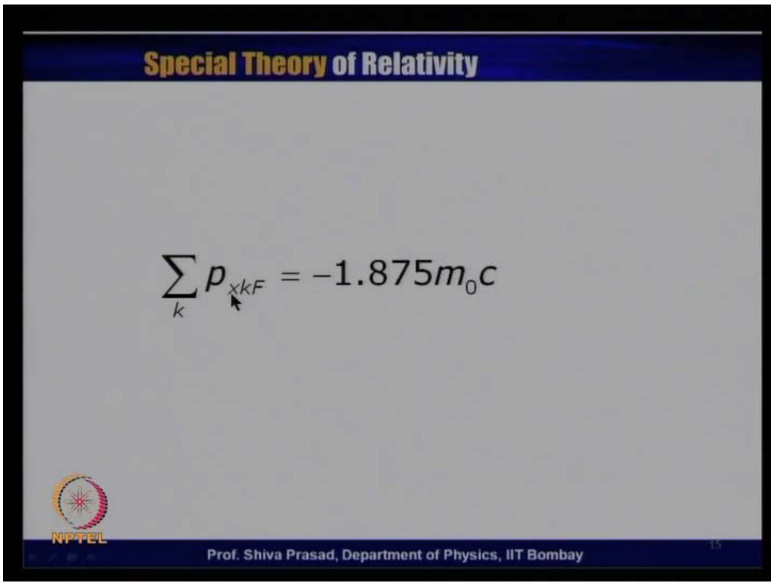

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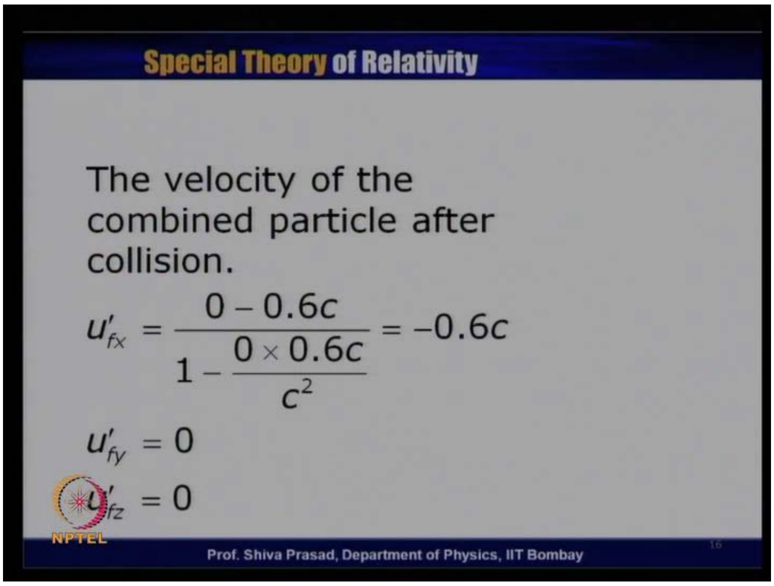

The initial momentum, which is the sum of the momentum of the two particles is equal to minus 1.875 m naught c. So, what we find? That the total value of momentum has changed once the frame has changed; which is obvious; which happens even in the classical mechanics that, once you change the frame of reference, you do find that, the speeds will change; and therefore, momentum will change. So, in S prime, this is the net momentum. Now, if whatever I am saying is correct, then with the new definitions of momentum, this should also be the momentum of the final particle, which is a combined particle. But, for that combined particle, I must use now a different rest mass, which is 2.5 m naught, and not 2 m naught.

So, let us look at the velocity of the combined particle after the collision. In S frame, which has the original frame in which we described the problem, this particular particle after the collision was at rest. So, it means u x was 0. And, we are going to a frame of reference with v is equal to 0.6 c. Therefore, this will be 0 minus 0.6 c divided by 1 minus u x v divided by c square; 0 multiplied by 0.6 c divided by c square. This term becomes 0. So, denominator becomes 1; and this is just equal to minus 0.6 c. Again, this particular result I had obtained even earlier when we had discussed this problem for the first time. This is the final speed. That is why I have written f here.

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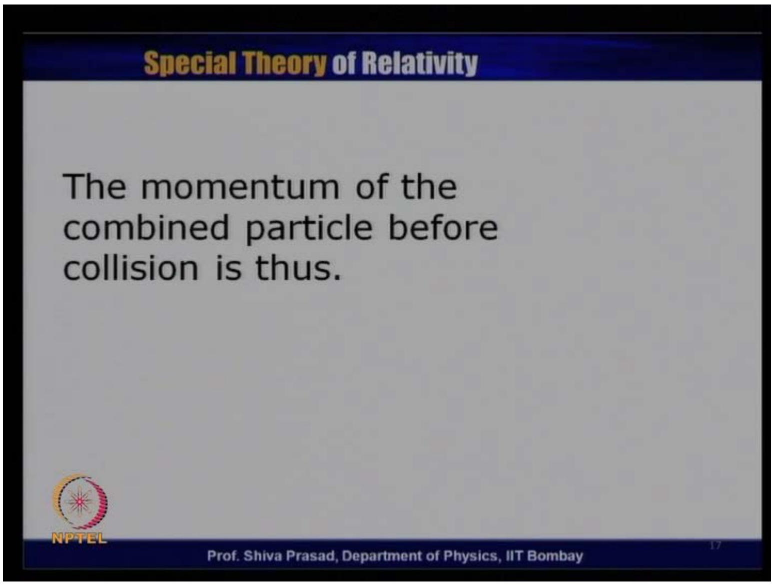

Now, I have to calculate the momentum of the combined particle. When I write the momentum of the combined particle, then in that particular case, I have to use this new speed and calculate its gamma; then multiply it by m naught; then multiply it by u.

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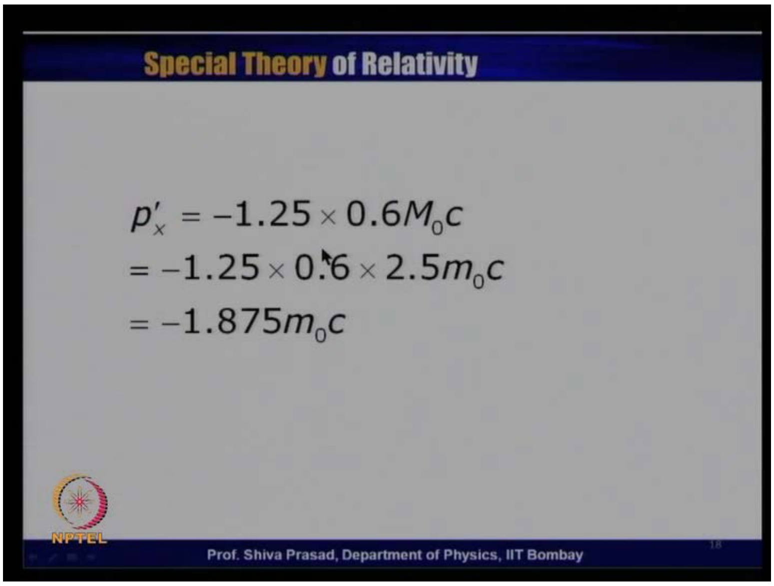

Now, this is 0.6 c. And, 0.6 c we have just now discussed, gives me a very clean value of gamma u, which is 1.25. So, gamma u is 1.25. The speed of the particle we have just now calculated is minus 0.6 c. This is what we have just now calculated here. It is minus 0.6 c. So, this is minus 0.6 c and m naught; where, this M naught is now the new M naught, because this M naught is scalar once I change; it is a four scalar. Once I change from one frame to another frame, this M naught is not going to change. So, for this M naught, I must use 2.5 m naught. Once I use this as 2.5 m naught and substitute these values and multiply, I indeed get minus 1.875 m naught c.

See you realise the trick here is to put M naught is equal to 2.5 m naught, which essentially comes according to the relativity from the conservation of energy; and as we have discussed that, conservation of energy is must in order to see conservation of momentum. So, we just now saw that, now that problem is solved in S prime frame of reference, also the momentum is conserved. Let us look at the energy whether in S prime frame of reference, even the energy is concerned. It should be for everything that we are saying is correct. So, now let us check whether energy is also conserved in this particular frame.

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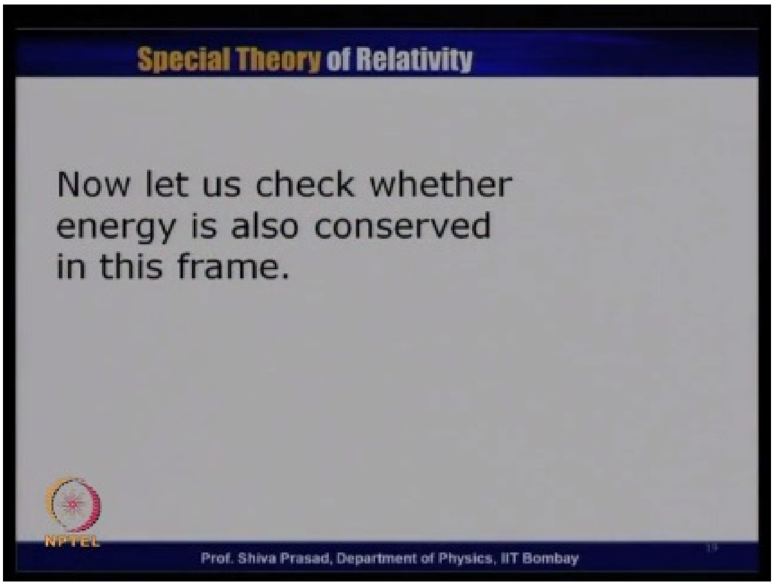

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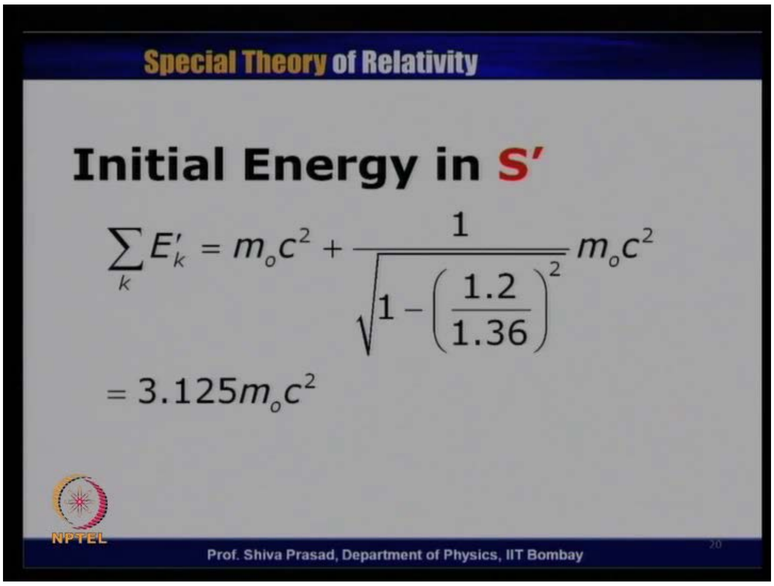

You go to S prime frame of reference and calculate the energy of the particle. How do we calculate the energy of the particle? Let us take the energy of the first particle. When we take the energy of the first particle, this particular particle was at rest. So, we have just now written E is equal to gamma u m naught c square. Now, once I go to the frame of reference of the first particle, then of course, gamma u is 1. We call the speed of the particle itself is 0. Therefore, this E is just equal to m naught c square. So, the energy of the first particle is equal to E is equal to just m naught c square. For the second particle, the speed is not 0, but it is minus 1.6 divided by 1.36 c. So, using that particular value, I must calculate gamma, which I have actually done earlier. So, that gamma u multiplied by that m naught, which is the rest mass of the second particle multiplied by c square; that would give me the energy of the second particle.

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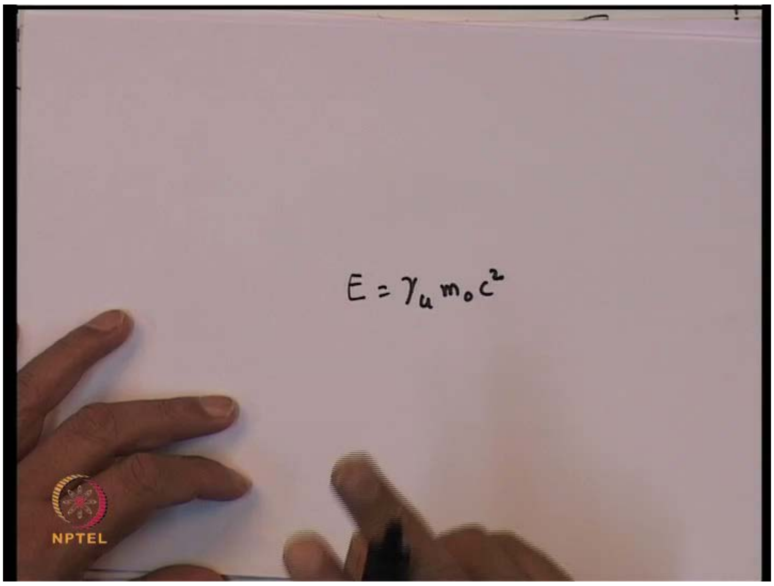

So, this is what I have written here in the transparency. First particle is just clean m naught c square, because this particle is at rest in S prime frame of reference. It is the second particle, which is moving in S prime frame of reference; and it is moving with its speed of 1.2 divided by 1.36 c; c square, c square cancels out. So, gamma is this much, which we have just now calculated. In fact, we have not calculated, but we have just now told that, this particular gamma u turns out to be 2.125 of course, multiplied by m naught c square. So, the total initial energy becomes 3.125 m naught c square. So, this is the total initial energy of the system.

Let us look at the final energy. As far as final energy is concerned, you have only one single particle now with a rest mass of capital M naught. And, that particular mass is not at rest in S prime frame of reference, but moves with a speed of minus 0.6 c, which we have just now seen. So, when I calculate gamma u for this particular particle, I should use 1 divided by under root 1 minus... This is speed of the particle, which is 0.6 c. And, c square of course cancels out.

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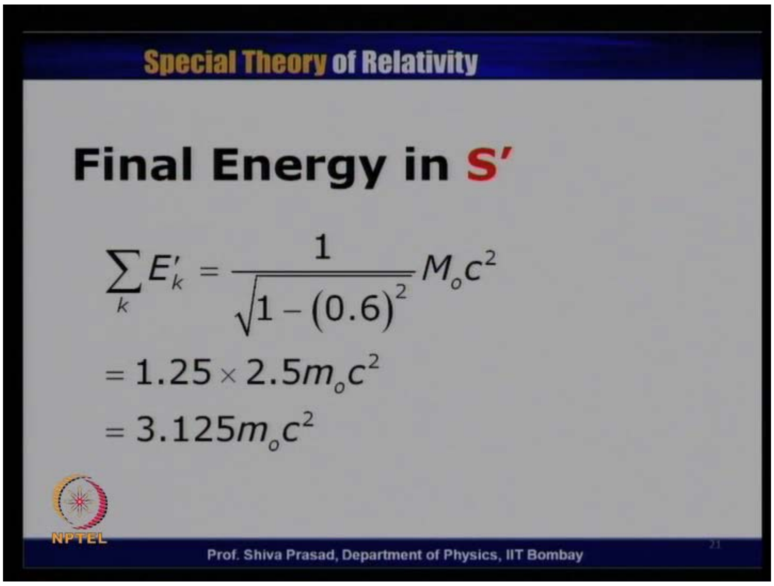

So, I get this gamma u, which I know is 1.25. So, for this gamma u, I will put 1.25. This m naught we have just now seen is the four scalar, must be equal to 2.5 m naught – small m naught times c square. You indeed get that, the total energy of the system is 3.125 m naught c square. So, we do find that, energy is also conserved. So, we worked out the same problem and tried to relook at the energy and momentum of conservation. And, we do find that, energy and momentum – both are conserved in S as well as the S prime frame of reference.

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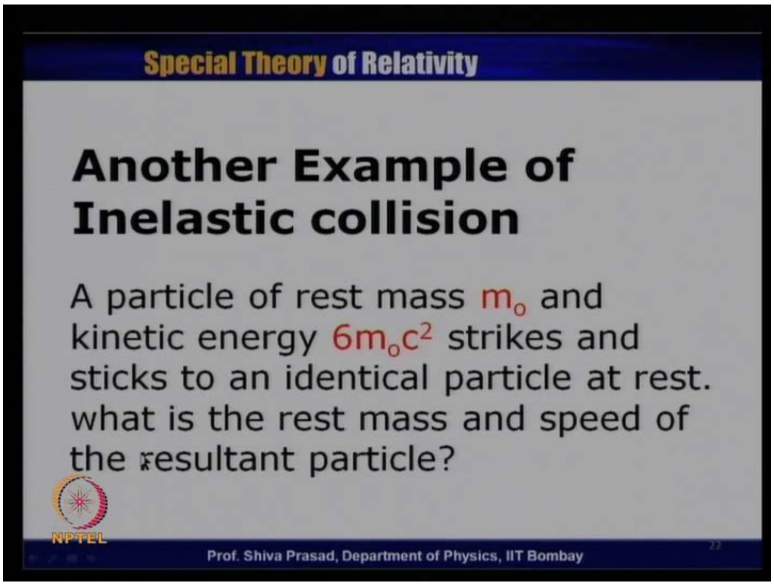

Let us look at another simple example, which is somewhat similar example. But, still I have chosen to describe this particular example of another inelastic collision, because this is the type of problem in which we will use the energy momentum conservation, which I had not used in the first problem. The first problem was more to illustrate that, the new definition of energy and momentum that we have now obtained, will lead to universality of energy and momentum conservation. Here I want to give an example, which is more like a problem in which we have been asked to calculate certain specific things. And, for which, I may require translating (( )) energy and momentum, knowing energy and momentum, calculate its speed, calculate momentum and (( )) – all those things. So, let me describe this particular example. So, there is a particular particle of rest mass m naught and it has a kinetic energy of 6 m naught c square. What has been given? Remember it is kinetic energy.

So, you should be careful when we are talking of kinetic energy and when we are talking of the total energy. So, of course, it has a very high energy, very high kinetic energy, because this is 6 times its rest mass energy. So, this particular particle strikes and sticks to an identical particle at rest. So, now, we have been given a problem in a frame in which one particle is at rest and another particle, which is coming and hitting it. So, one particle is at rest, another particle comes and hits it. So, this is the way we have described this particular problem.

Now, we say identical particle; it means its rest mass must also be available, because then only, this particle is identical particle. These two particles come and collide; one particle remains stationary; another particle comes and hits here. Let me first complete the problem. What is the rest mass and the speed of the resultant particle? What has been asked in this particular problem is, what is the rest mass and the speed of the resultant particle? These two particles strike and get stuck to each other just like in the problem that we have described earlier. Energy momentum conservation; not just momentum conservation, both momentum and energy conservation.

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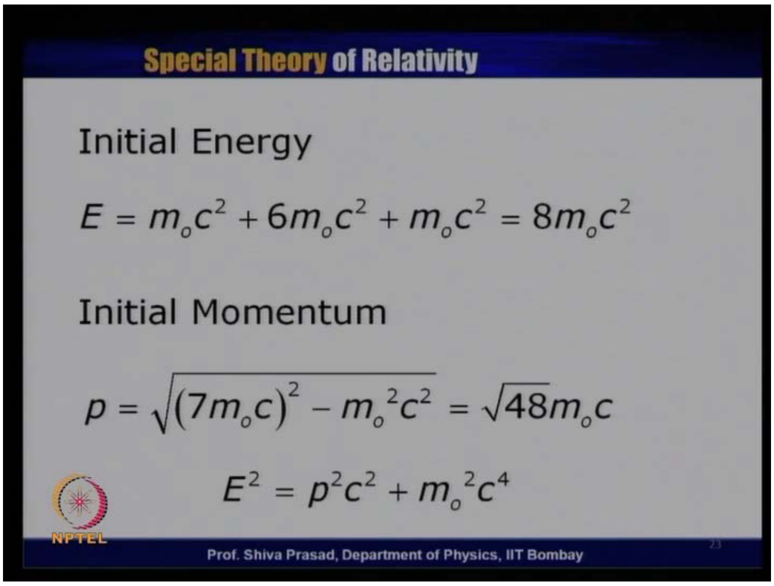

Let us write initial energy. When we write the initial energy, there was one particular particle, which was only at rest. So, that has energy of m naught c square. There is another particle, which had a kinetic energy of 6 m naught c square; obviously, that particle would have also had a rest mass energy of m naught c square. Therefore, if I take the total energy of that particular particle; that will be 6 m naught c square plus m naught c square, will be equal to 7 m naught c square. So, this will be for the particle, which is moving. This was its kinetic energy; this was its rest mass energy.

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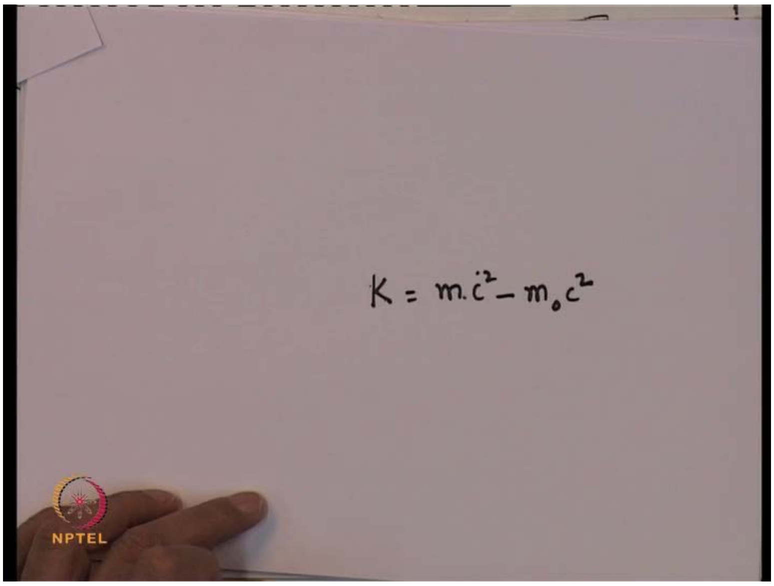

Remember the expression that we had written earlier that, K is equal to m c square... K is equal to m c square minus m naught c square. So, the total energy is sum of kinetic energy and rest mass energy. So, this is what I have written here. This is the total energy of the particle, which is moving. This is the total energy of the particle, which is stationary. And, this adds up to 8 m naught c square. So, total initial energy of the system is 8 m naught c square. Now, I have to calculate the momentum. Of course, momentum of the particle, which is at rest is 0; there is no problem, because that particular particle is at rest. So, u is 0. Therefore, momentum has to be 0. I had to calculate the momentum of the second particle. But, what has been given, is only the kinetic energy of the particle. Therefore, I must use an appropriate expression to find out from this energy, the momentum value.

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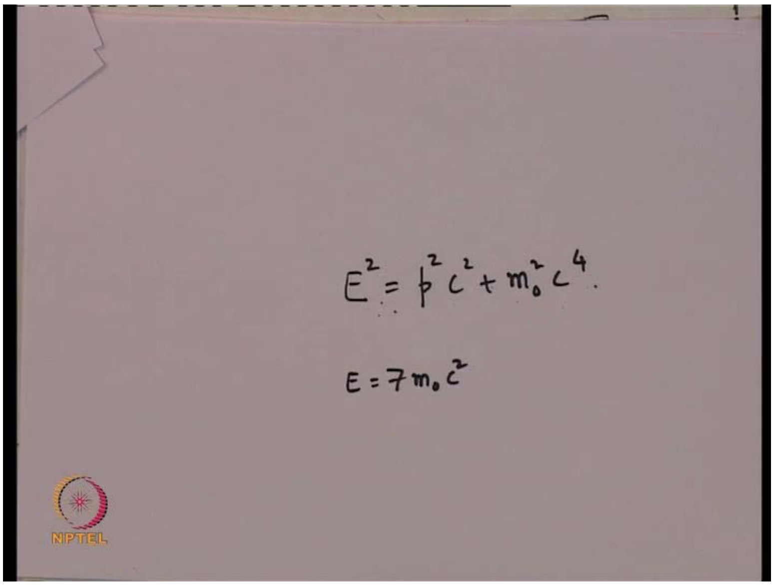

And, for that, we are using the expression, which is well-known, which we have described earlier. E square is equal to p square c square plus m naught square c to the power 4. Now, for the particle, which is moving, this E was 7 m naught c square as we have just now discussed. This is m naught square c to the power 4. Substituting this particular expression, I can find out what may be the value of momentum. So, this is an expression, which helps me for a single particle to convert energy into momentum or vice-versa. This is what I have written here. So, p square c square I have written as E square minus m naught square c to the power 4. This slightly simplify this equation taken the under root and divided by c square.

So, I will be getting this as under root E square, which is 7 m naught c square – 7 m naught c, not c square; 7 m naught c, because c square I have already divided; 7 m naught c minus m naught square c to the power c square, because the c square has already been divided. So, this becomes 49 minus 1, which is 48. So, I will get the momentum of this particular particle is under root 48 m naught c. So, the momentum of the first particle is 0; the momentum of the second particle is under root 48 m naught c.

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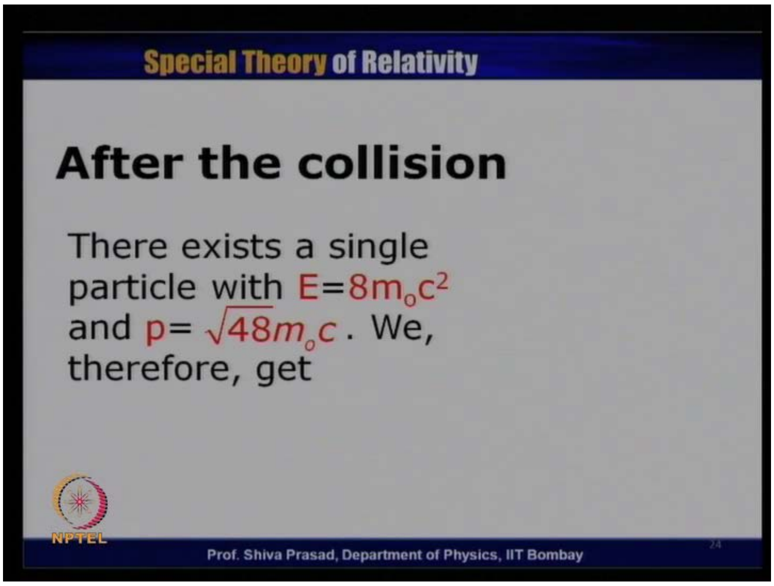

Hence, after the collision, we have just now discussed that, the two particles collide. And, that has been given in the problem. And, they get stuck to each other. If they get stuck to each other, what remains is only one single particle; and that single particle because of conservation of momentum must have the same momentum as the initial momentum of two particles; which is under root 48 m naught c. And, it must also have energy, which is equal to 8 m naught c square. So, all you know that, the new particle, which has been formed as a resultant of the combination of these two particles, will have now energy equal to 8 m naught c square and must have a momentum of under root 48 m naught c. Using again the same expression, I can find out what will be the value of M naught – capital M naught – the new mass – new rest mass of the particle, which we have obtained as a result the two particles getting stuck.

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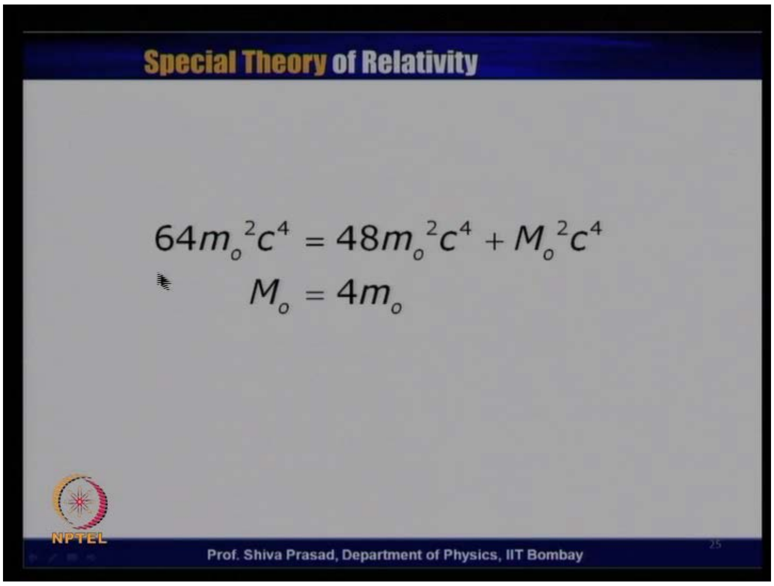

So, I use the same expression E square is equal to p square c square plus M naught square c to the power 4. E we just now said is 8 m naught c square. So, I squared it. So, this becomes 64 m naught square c to the power 4. So, this is the E square term – plus p square c square; p was under root 48 m naught c; I square it. So, I get 48 m naught c square; m naught square c square; and this was p square c square. So, there is another c square, which we will make it m naught square c to the power 4. Remember here this is the old m naught, because these momenta and energy have been written in terms of the mass of the original particles – rest mass of the original particle.

But, now, you have got a new particle as a combination of these two particles, which may have a different rest mass, which I am writing is capital M naught. So, E square is equal to p square c square plus capital M naught square c to the power 4, which must be the mass of the new particle. I just substitute it here; I just solve it; 64 minus 48. And, you will get M naught square. And, you take under root of that; you will get M naught is equal to 4 m naught. So, in this particular case, you will find that, the rest mass of the new particle that you obtained is 4 times the rest mass of the original particles with which this particular particle was formed. So, it has become... The rest mass is now 4 times that rest mass.

Next question is to find out the speed. Once we have obtained the mass, we have to find out rest mass and we know its momentum and energy; I can always find out its speed.

That is simple. Let us see how they are... I have given two different methods; but one can choose whatever is convenient.

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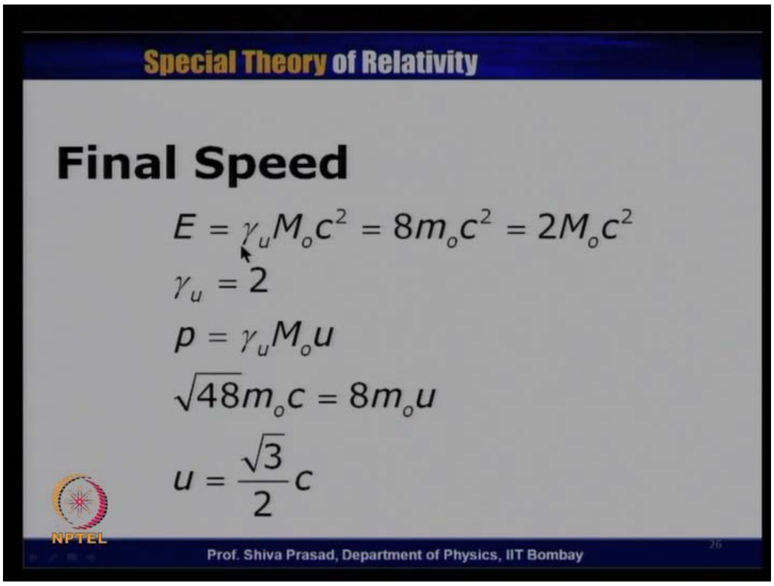

So, as far as a new particle is concerned, we have to find out the final speed. We have said E is equal to gamma u M naught c square. That is what we have written. This particular particle obviously is not at rest. That is why we have to find out its speed. This particular thing must be equal to gamma m naught c square, which must be equal to... because 4 times m naught was its capital M naught.

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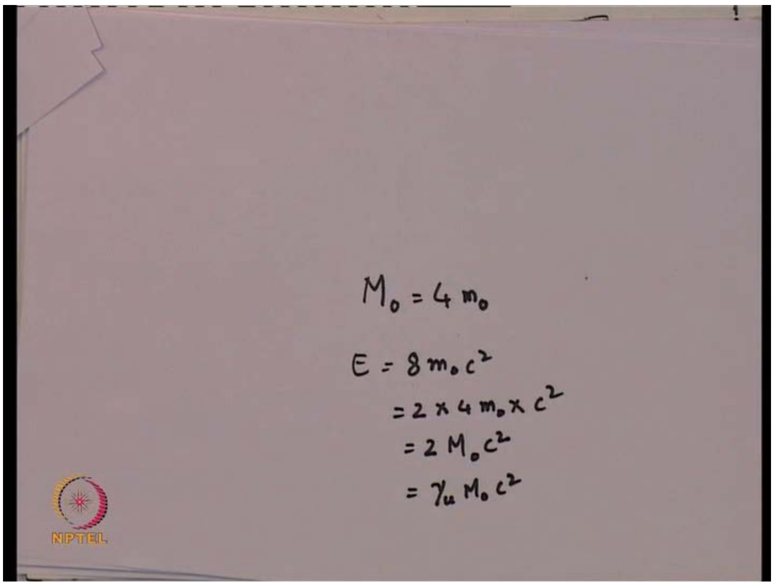

What we have written is that, M naught was equal to 4 small m naught. So, its energy E is equal to 8 m naught c square. This I can write this as 2 times 4 m naught into c square. This 4 m naught is capital M naught. So, I can write this as 2 capital M naught c square. And, this being equal to gamma u M naught c square. It means gamma u is equal to 2. That is what it will tell that, gamma u is equal to 2. So, this is what I have written here in this transparency that, E is equal to gamma u M naught c square. This 8 m naught can be written as 2 times capital M naught. So, this is 2 M naught – capital M naught c square giving me gamma u is equal to 2.

Now, either I can use the momentum expression here, which I know is gamma u times M naught times u to calculate this u, which I have done here. I know the momentum of the particle – combined particles – under root 48 m naught c. And, this is equal to gamma u M naught is equal to 8 times m naught, because this capital m naught is of course, small m naught. I can calculate u. This turns out to be equal to under root 3 divided by 2 c. This is other way that, you have already known gamma u. So, you use that expression.

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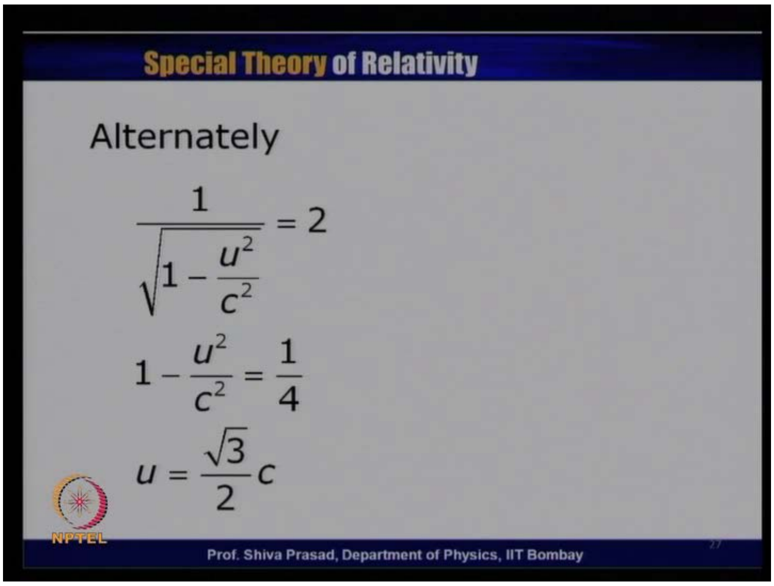

So, this is alternate way of doing it. Look at gamma u value directly. Gamma u we have just now discussed. It is 1 upon under root 1 minus u square by c square. And, we have seen that, this is equal to 2. So, I take inverse of this particular thing and take square of this thing. So, I get 1 minus u square divided by c square, which is equal to 1 upon 4. And, if you just solve, you get the same result, u is equal to under root 3 by 2 c. So, the new particle – the combined particle would now be travelling with the speed of under root 3 by 2 c as far as this particular problem is concerned. So, what we have seen in this particular problem that, how one can convert from energy to momentum or from speed. Now, we just get a practice of using these particular expressions the way we would like to use them to convert from energy to momentum or vice-versa or to convert into speed.

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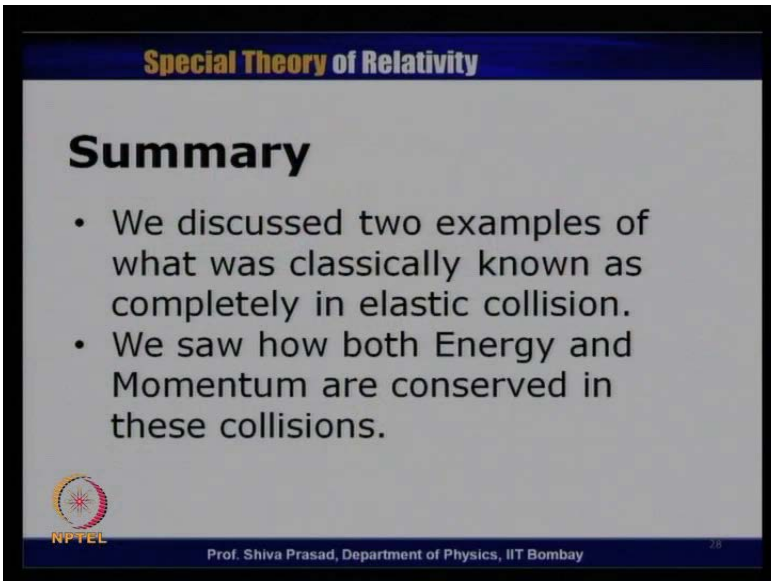

So, eventually, at the end of this particular lecture, I would just like to summarise that, in this particular lecture, we have just given two examples of what we classically call as completely inelastic collision. And, we saw that, both these examples – how energy and momentum – both have to be conserved together.

Thank you.